# WISD EDUCATION

 $\begin{array}{c} {\rm Leads\ Scoring} \\ {\rm using\ logistic\ regression} \\ {\rm FSDM} \end{array}$ 

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# Introduction

In many fields and sectors we face many problems which are solved and interpreted based upon numerical values using statistical analysis .In our case the problem is trying to augment the leads conversion rate, but how?. Take the example of a company named WISD education sells online courses. On any given day, many professionals who are interested in the courses land on their website and browse for courses. The company markets its courses on several websites and search engines like Google. Once these people land on the website, they might browse the courses or fill up a form for the course or watch some videos. When these people fill up a form providing their email address or phone number, they are classified to be a lead. Moreover, the company also gets leads through past referrals. Once these leads are acquired, employees from the sales team start making calls, writing emails, etc. Through this process, some of the leads get converted while most do not. The typical lead conversion rate at WISD education is around 30%. To make this process more efficient, the company wishes to identify the most potential leads, also known as 'Hot Leads' If they successfully identify this set of leads, the lead conversion rate should go up as the sales team will now be focusing more on communicating with the potential leads rather than making calls to everyone.

In another form we will create a logistic regression model who is able to predict the probability of a conversion to specific individual professional based on different parameters .

Thus, to do this task we need a dataset at the first place then, a complete description for our model and its parameters , also some inferences and diagnostics to see if we could make our model more and more efficient .

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# Chapter 1

# DATA PRE-PROCESSING

### 1.1 DATASET PRESENTATION:

Creating a logistic regression for leads scoring (aka conversion scoring) needs a good dataset that represent the real data as a company could have thus, we will use a dataset consists of various attributes such as Lead Source, Lead Origin, Total Visits, Last Activity, etc. which may or may not be useful in ultimately deciding whether a lead will be converted or not. The target variable, in this case, is the column 'Converted' which tells whether a past lead was converted or not wherein 1 means it was converted and 0 means it wasn't converted. So: the goal here is to Build a logistic regression model in R language to assign a lead score between 0 and 100 to each of the leads which can be used by the company to target potential leads. A higher score would mean that the lead is hot, i.e. is most likely to convert whereas a lower score would mean that the lead is cold and will mostly not get converted.

So without further to do lets dive in.

#### Data loading:

First we can load our dataset from a csv file using this command .

1 > leads\_data = read.csv(leads.csv")



Figure 1.1: Leads dataset

we can check whether a dataset contain a duplicated values like so.

```
1 > sum(duplicated(leads_data)) == 0 # True => No duplicate values
2 [1] TRUE
```

#### Data Inspection:

By dimension we mean the number of rows and columns represented in the dataset .

```
1 > dim(leads_data)
2 [1] 9240 37
```

we could see a lot of information about our dataset just by using the summary () method  $\ .$ 

```
> summary(leads_data)
                      Lead.Number
                                       Lead.Origin
                                                          Lead.Source
  Prospect.ID
             Do.Not.Email
                           :579533
  Length:9240
                     Min.
                                      Length:9240
                                                          Length: 9240
             Length:9240
  Class : character 1st Qu.:596485
                                       Class : character
                                                          Class :
     character Class : character
  Mode : character
                    Median :615479
                                       Mode :character
                                                          Mode
     character Mode : character
                     Mean :617188
6
                      3rd Qu.:637387
8
                     Max. :660737
9
                     . . .
```

## 1.2 DATA CLEANING:

data cleaning consist of removing or correcting the unused values and columns . Now we will see different cleaning processes on different attributes or what we call it in data science Features . For example here we replace the unused value "Select" in the column Specialization with NA ( which is a none value ) .

After this we need to know the NA frequency in each column to make a decision if this column is contain a significant information or not .

```
> sum_na = sapply(leads_data, function(x) sum(is.na(x)))
2 > round(100*(sum_na/nrow(leads_data)), 2)
3 Prospect.ID
                                                      Lead. Number
4 0.00
                                                      0.00
5 Lead.Origin
                                                      Lead.Source
6 0.00
                                                      0.00
7 Do. Not. Email
                                                      Do. Not. Call
8 0.00
                                                      0.00
9 Converted
                                                      TotalVisits
10 0.00
                                                      1.48
```

```
11 Total.Time.Spent.on.Website
                                                        Page. Views. Per.
12 0.00
                                                        1.48
13 Last.Activity
                                                        Country
14 0.00
                                                        0.00
15 Specialization
                                                        How.did.you.hear.
       about.WISD
16 21.02
                                                        0.00
17 Search
                                                        Magazine
18 0.00
                                                        0.00
                                                        WISD..Forums
19 Newspaper.Article
20 0.00
                                                        0.00
21
22 Asymmetrique. Activity. Score
                                                        Asymmetrique.
       Profile.Score
23 45.65
                                                        45.65
```

For now we will drop the columns having more than 70% NA values because it mean that the column doesn't represent any information and that is possible by using this for loop:

```
1 > for( i in 0:ncol(leads_data)){
2 +    sum_col_na = sum(is.na(leads_data[i]))
3 +    na_percentage = round(100*(sum_col_na/nrow(leads_data)), 2)
4 +    print(na_percentage)
5 +    if( na_percentage > 70){
6 +        leads_data = subset(leads_data, select = -c(i)) #select(leads_data, i)
7 +    }
8 + }
```

Now we will take care of empty values in each column one by one.

```
1 > leads_data[leads_data == ""] = NA
```

#### Lead Quality:

Indicates the quality of lead based on the data and intuition the employee who has been assigned to the lead

```
1 > describe(leads_data$Lead.Quality)
2 leads_data$Lead.Quality
        n missing distinct
      4473
              4767
 lowest : High in Relevance Low in Relevance Might be
                                                                   Not
                    Worst
      Sure
 highest: High in Relevance Low in Relevance Might be
                                                                   Not
                    Worst
      Sure
             High in Relevance Low in Relevance
                                                            Might be
9 Value
             Not Sure
                                   Worst
10 Frequency
                            637
                                              583
                                                                1560
                 1092
                                     601
11 Proportion
                          0.142
                                            0.130
                                                               0.349
                0.244
                                   0.134
```

There is too much variation in these parameters so its not reliable to impute any value in it.

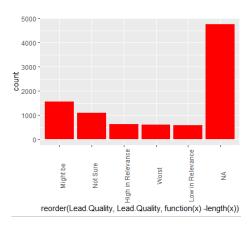


Figure 1.2: Leads Quality Bar plot

So as Lead quality is based on the intuition of employee, so if left blank we can impute 'Not Sure' in NA safely.

```
1 > leads_data$Lead.Quality = ifelse( is.na( leads_data$Lead.Quality
      ),"Not Sure",leads_data$Lead.Quality)
2 > ggplot(leads_data, aes(x=reorder(leadQuality, leadQuality,
      function(x)-length(x)))) +
3 + geom_bar(fill='red') + labs(x='leads Quality')
```

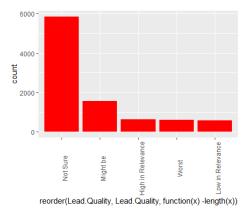
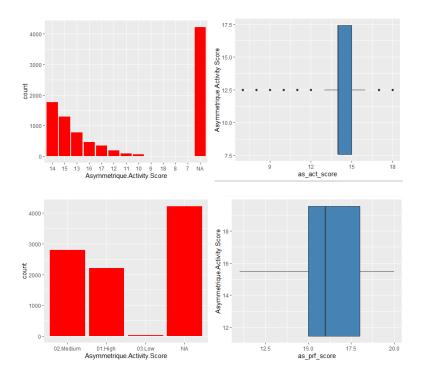


Figure 1.3: Leads Quality Bar plot

#### Leads Asymmetrique:

An index and score assigned to each customer based on their activity and their profile . and we can visualize their variations from this graphs :

```
1 > as_act_score = leads_data$Asymmetrique.Activity.Score
2 > ggplot(leads_data, aes(x=reorder(as_act_score, as_act_score, function(x)-length(x))))
3 + geom_bar(fill='red') + labs(x='Asymmetrique.Activity.Score')
4 > ggplot(leads_data, aes(x=as_act_score, y=as_act_score)) +
5 + geom_boxplot(fill='steelblue') + labs(x='Asymmetrique.Activity.Score') + coord_flip()
6 > as_prf_indx = leads_data$Asymmetrique.Profile.Index
7 > ggplot(leads_data, aes(x=reorder(as_prf_indx, as_prf_indx, function(x)-length(x)))) +
8 + geom_bar(fill='red') + labs(x='Asymmetrique.Activity.Score')
9 > as_prf_score = leads_data$Asymmetrique.Profile.Score
10 > ggplot(leads_data, aes(x=as_prf_score, y=as_prf_score)) +
11 + geom_boxplot(fill='steelblue') + labs(x='Asymmetrique.Activity.Score')
```



#### City:

This is the city related to the user profile.

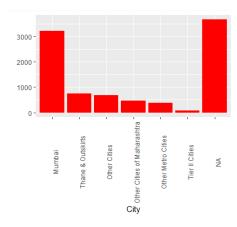


Figure 1.4: City BarPlot

Around 60% of the data is Mumbai so we can impute Mumbai in the missing values (NA or Select).

#### Specailization

This column specifies the specialization domain of the lead ( i.e the specialization of the user that represent this lead ).

```
8 > ggplot(leads_data, aes(x=reorder(Specialization, Specialization, function(x)-length(x)))) +
9 + geom_bar(fill='red') + labs(x='Specialization', y="") +
10 + theme(axis.text.x = element_text(angle = 90))
```

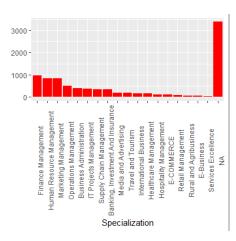


Figure 1.5: Specialization BarPlot

It maybe the case that lead has not entered any specialization if his/her option is not available on the list, may not have any specialization or is a student , Hence we can make a category "Others" for missing values.

#### Tags:

Tags are able to determine some extra information about the user .

```
> describe(leads_data$Tags)
 leads_data$Tags
           missing distinct
        n
      5887
              3353
6 lowest : Already a student
                                                 Closed by Horizzon
                                   Diploma holder (Not Eligible)
                      Graduation in progress
 highest: switched off
     University not recognized
                                                         Want to take
     admission but has financial problems Will revert after reading
     the email
                              wrong number given
```

Blanks in the tag column may be imputed by 'Will back after reading the email'.

```
1 > leads_data$Tags = ifelse(is.na(leads_data$Tags),"Will back after
    reading the email",leads_data$Tags)
```

#### Occupation:

Occupation are able to dusting the occupation of the user (could represent work, job, activity, etc..) .

```
> describe(leads_data$What.is.your.current.occupation)
2 leads_data$What.is.your.current.occupation
       n missing distinct
3
      6550
              2690
6 lowest : Businessman
                               Housewife
                                                    Other
          Student
                               Unemployed
7 highest: Housewife
                               Other
                                                    Student
          Unemployed
                              Working Professional
                   Businessman
8 Value
                                           Housewife
      Other
                      Student
                                           Unemployed
9 Frequency
                             8
                                                  10
         16
                             210
                                                5600
                           0.001
10 Proportion
                                                0.002
                           0.032
                                                0.855
      0.002
11
12 Value
             Working Professional
13 Frequency
14 Proportion
                           0.108
```

We can see from the proportion row that 86% entries are of Unemployed so we can impute "Unemployed" in it.

```
1 > leads_data$What.is.your.current.occupation = ifelse(is.na(leads_data$What.is.your.current.occupation),"Unemployed",leads_data$
What.is.your.current.occupation)
```

This is the most important cleaning parts the same work is done for the other columns such as exploring, replacing, and finally cleaning. So we continue the same work on all the dataset until Data will be cleaned to start with the analysis part.

### 1.3 DATA ANALYTIC:

#### Univariate Analysis:

#### Converted:

Converted is the target variable, Indicates whether a lead has been successfully converted (1) or not (0), So lets explore it first.

#### Lead Origin:

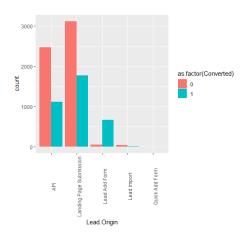


Figure 1.6: Leads dataset

The values API and Landing Page Submission have 30-35% conversion rate but count of lead originated from them are considerable. And the value Lead Add Form has more than 90% conversion rate but count of lead are not very high. In the other hand Lead Import are very less in count. To improve overall lead conversion rate, we need to focus more on improving lead conversion of API and Landing Page Submission origin and generate more leads from Lead Add Form. Lead Source:

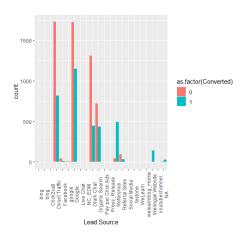


Figure 1.7: Leads dataset

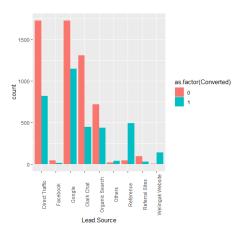
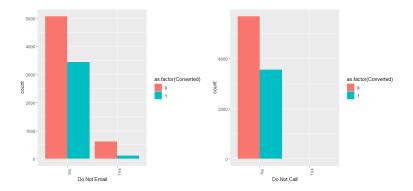


Figure 1.8: Leads dataset

From this plot we can make some inferences like so . Google and Direct traffic generates maximum number of leads. Conversion Rate of reference leads and leads through welingak website is high. To improve overall lead conversion rate, focus should be on improving lead conversion of olark chat, organic search, direct traffic, and google leads and generate more leads from reference and welingak website.

#### Do Not Email Do Not Call:

These two columns contain Yes or No.



#### **Total Visits:**

We can say that TotalVisits represent the engaging rate of the users , so a higher visits mean a higher possibility to convert .

```
1 > ggplot(leads_data, aes(x=TotalVisits, y=TotalVisits)) + geom_
    boxplot(fill='steelblue') + labs(x='TotalVisits') + coord_flip
    ()
```

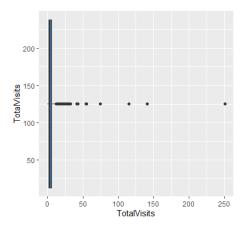


Figure 1.9: Total Visits barplot

As we can see there are a number of outliers in the data. We will cap the outliers to 95% value for analysis.

```
1 > fun <- function(x){
2 +    quantiles <- quantile( x, c(.05, .95 ), na.rm = T )
3 +    x[ x <= quantiles[0] ] <- quantiles[0]
4 +    x[ x >= quantiles[1] ] <- quantiles[1]
5 +    x
6 + }
7 > leads_data$TotalVisits <- squish(leads_data$TotalVisits, quantile (leads_data$TotalVisits, c(.05, .95), na.rm = T))</pre>
```

```
8 > ggplot(leads_data, aes(x=Converted, y=TotalVisits))+
9 + geom_boxplot(fill='steelblue') + labs(x='TotalVisits') + coord
_flip()
```

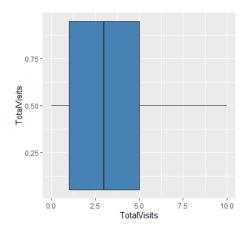


Figure 1.10: Total Visits

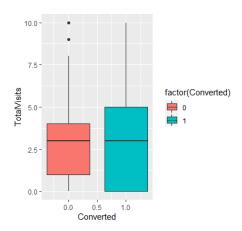


Figure 1.11: Total Visits converted

Here we see that Median for converted and not converted leads are the same . Which mean nothing conclusive can be said on the basis of Total Visits. **Total time spent on website:** 

post(leads\_data, aes(x = Converted, y = Total.Time.Spent.on.
Website, fill=factor(Converted))) + geom\_boxplot()

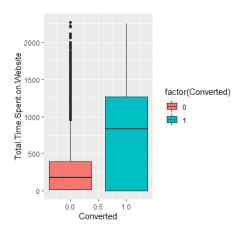


Figure 1.12: Total Visits converted

We remark that Leads spending more time on the weblise are more likely to be converted. and Website should be made more engaging to make leads spend more time.

we apply the same processing on all the dataset and we get this conclusion from the result of each study:

- Nothing can be said specifically for lead conversion from Page Views Per Visit
- 2. Most of the lead have their Email opened as their last activity.
- 3. Conversion rate for leads with last activity as SMS Sent is almost 60
- 4. Most values are 'India' so no such inference can be drawn in country
- 5. Focus should be more on the Specialization with high conversion rate.
- 6. Working Professionals going for the course have high chances of joining it.
- 7. Unemployed leads are the most in numbers but has around 30-35% conversion rate.
- 8. Most entries are 'Better Career Prospects'. No Inference can be drawn with this parameter.
- 9. Most entries are 'No' so no inference can be drawn with these parameters (search, Magazine, Newspaper Article, WISD Education Forums, Newspaper, Newspaper, Through Recommendations, Receive More Updates About Our Courses, Update me on Supply Chain Content, Get updates

on DM Content, I agree to pay the amount through cheque, A free copy of Mastering The Interview)

10. Most leads are from mumbai with around 30% conversion rate.

#### Recapitulate:

Based on the uni-variant analysis we have seen that many columns are not adding any information to the model, hence we can drop them for further analysis .

### 1.4 DATA PREPARATION:

In this part we will prepare our data for building the model . The first step is converting some binary variables (Yes/No) to 1/0

For categorical variables with multiple levels, create dummy features (one-hot encoded) using these liens of code .

```
1 > X = subset(leads_data, select = -c(Prospect.ID))
2 > X = X[!duplicated(X),]
3 > X$TotalVisits = c(scale(X$TotalVisits, center = TRUE, scale = TRUE))
4 > X$Total.Time.Spent.on.Website = c(scale(X$Total.Time.Spent.on. Website, center = TRUE, scale = TRUE))
5 > X$Page.Views.Per.Visit = c(scale(X$Page.Views.Per.Visit, center = TRUE, scale = TRUE))
```

Until this point we can say that our data is ready for training part using the binomial logistic regression model to be able to predict the conversion state and score the leads from 0 to 100.

# Chapter 2

# MODEL BUILDING

**F** Irst we will deep dive in describing this model, if you are not interested in the first section you can passe directly to model implementation section (number 2.2) and you will consider the method that creates the model as a black box, don't worry you will be fine and able to understand the other parts.

### 2.1 LOGISTIC REGRESSION

To understand the purpose and to make sure that all of us in the same road lets Take an example where we want to know if a student will succeed in the exam based on the hours of studying . The problem here is that we use a non-categorical variable to predict a binary one (  $0\ {\rm or}\ 1$  , succeed or fail ). For that we need to use a probability concept where we can take the working hours and calculate the probability to succeed in the exam from 0 to 1 . So we get the S-shaped curve ( see the figure ) . the shape of our probability where the y axis is the probability value goes from 0 to 1 and the x axis for the studying hours .

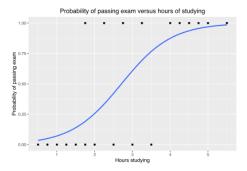


Figure 2.1: S-shape

If we want to read this graph we could say the students that work for 2 hours have 25% to succeed in the exams and the students with 5 hours working rate are 95% more likely to success in the exam ..etc.

This is one of the use cases where we want to find the correlation between Y axis the probability of success and the studying hours . Before , anything we need to present the model . so if you don't follow along in the mathematics part don't worry every will be getting more clear while reading . Lets deep dive in the mathematics . First we have the logistic function with this form :

$$Probability(Y = succeed | X = x, aspeci fic value of X) = \pi = \frac{e^{\beta_0 + \beta x}}{1 + e^{\beta_0 + \beta x}}$$
(2.1)

 $\pi$  is the probability to succeed in the exam ,  $\beta_0$  is the Y intercept ,  $\beta$  is the regression coefficient related to the variable X which can be categorical or continuous , but Y is always categorical more specifically binary in the case of binary logistic regression . But curved shapes such a S-shaped , often referred to as sigmoidal , is difficult to describe with a linear equation for two reasons. First, the extremes do not follow a linear trend. Second, the errors are neither normally distributed nor constant across the entire range of data . Logistic regression solves these problems by applying the logit transformation to the dependent variable. In essence, the logistic model predicts the logit of Y from X. As stated earlier, the logit is the natural logarithm (ln) of odds of Y, and odds are ratios of probabilities  $(\pi)$  of Y happening ( i.e., a student is succeeded ) . Based on all of that , the simple logistic model has the form :

$$logit(Y) = ln\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta x$$
 (2.2)

According to Equation (1.2), the relationship between logit (Y) and X is linear. Yet, according to Equation (1.1), the relationship between the probability of Y and X is nonlinear. For this reason, the natural log transformation of the odds in Equation (1.2) is necessary to make the relationship between a categorical outcome variable and its predictor(s) linear. Extending the logic of the simple logistic regression to multiple predictors (say X1 = working hours and X2 = gender), one can construct a complex logistic regression for Y (recommendation for remedial reading programs) as follows:

$$logit(Y) = ln\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$$Therefore,$$

$$\pi = Probability(Y = outcomeofinterest | X_1 = x_1, X_2 = x_2)$$

$$= \frac{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2}}{1 + e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2}}$$

where  $\pi$  is once again the probability of the success,  $\beta_0$  is the Y intercept,  $\beta_i$  are regression coefficients, and  $X_i$  are a set of predictors.  $\beta_0$  and  $\beta_i$  are estimated by the maximum likelihood (ML) method which is preferred over the weighted least squares approach. The ML method is designed to maximize the likelihood of reproducing the data given the parameter estimates ( we will deep dive on this method in the next chapter -chapter 2-).

Finally, we have the general logistic regression model as follow:

$$logit(Y) = ln\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \sum_{i=1}^n \beta_i x_i$$

$$Therefore,$$

$$\pi = Probability(Y = outcomeofinterest | X_1 = x_1, ..., X_n = x_n)$$

$$= \frac{e^{\beta_0 + \sum_{i=1}^n \beta_i x_i}}{1 + e^{\beta_0 + \sum_{i=1}^n \beta_i x_i}}$$

This is the logistic regression model a simple linear equation between probability of success and a set of independent variables ( studying hours, gender ). Now

, you'd say yes this is the model a bunch of dummy parameter but how can I use it? in other form, how can I train the model based on my dataset? Good question, the model is a bunch of parameters needs to be calculated or estimated, Exactly that what we call it estimation, we can't define values from our heads, or making a magic function or formula that works for all datasets, but instead we find an estimated value for each parameter that depend on the dataset that we are using. And that what we will see in the next chapter.

#### Parameters Estimation:

To estimate the parameters in the logistic regression model we can't minimize the residual sum of squares like was done in linear regression. Instead, we use a statistical technique called maximum likelihood. Before we dive into how the parameters of the model are estimated from data, we need to understand what logistic regression is calculating exactly .

This might be the most confusing part of logistic regression, so we will go over it slowly.

The equation for the standard logistic function is given by:

$$f(z) = \frac{e^z}{1 + e^z}$$

The graph for this equation can be visualized as (RMK: its similar to s-shaped figure ). In any proposed model, to predict the likelihood of an outcome, the variable



Figure 2.2: standard logistic equation

z needs to be a function of the input or feature variables  $X_1, X_2, ..., X_n$ . In logistic regression, z is often expressed as a linear function of the input variables as follows:

$$z = \beta_0 + \sum_{i=1}^{n} \beta_i x_i$$
  
So,  $z = logit(Y)$   
keepitinmind.

Suppose we have the data of n o bservations  $(y_1, y_2, ..., y_n)$  and  $(x_1, x_2, ..., x_n)$ . How do we estimate  $\beta_0$  and  $\beta_0$ ? Let's first take a slight detour and take a very simple example using maximum likelihood estimation:

#### Coin Toss:

The star of examples in statics world is coin toss. you don't hear about it .Okay here it is, suppose we have a coin, and we need to estimate the probability that it lands on the heads. Naturally, the first thing to do would be to toss it several times (say, n times) and note down the results (as binary outcomes: 1 for heads and 0 for tails).

Suppose we record the observations as  $(y_1, y_2, ..., y_n)$ . We might then be tempted to find the sample mean of the observations and use it as an estimate for the probability of landing heads. So, if we get 3 heads and 7 tails in 10 tosses, we might conclude that the probability of landing heads is 0.3. Let's see if we can use rigorous mathematics to confirm our intuition:

Let's first identify the probability distribution of the coin toss example. Since the outcome variable is binary 0 and 1, it follows a Bernoulli distribution . The probability mass function of a Bernoulli distribution is defined as:

$$p(x) = \begin{cases} p, y = 1\\ 1 - p, y = 0 \end{cases}$$

Often, the above function is simplified to a single line equation as follows:

$$p(y) = p^{y}(1-p)^{(1-y)}$$

We'll now define the likelihood function for our distribution. In general, the likelihood function is defined as follows for discrete random variables as follows:

$$L_n(Y_1, Y_2, ..., Y_n, \theta) = \mathbb{P}_{\theta}[Y_1 = y_1, Y_2 = y_2, ..., Y_n = y_n]$$

Furthermore, while  $Y_1, Y_2, \ldots, Y_n$  are independent,

$$\begin{split} L_n(Y_1,Y_2,...,Y_n,\theta) &= \mathbb{P}_{\theta}[Y_1=y_1]\mathbb{P}_{\theta}[Y_2=y_2]...\mathbb{P}_{\theta}[Y_n=y_n] \\ \text{we define the probability mass function, for i in 1...n }, &\mathbb{P}_{\theta}[Y_i=y_i]=p(y_i)_{\theta} \\ L_n(Y_1,Y_2,...,Y_n,\theta) &= p_{\theta}[y_1]\mathbb{P}_{\theta}[Y_2=y_2]...\mathbb{P}_{\theta}[Y_n=y_n] \end{split}$$

Using this we can compute the likelihood function of the coin toss problem: Parameter:  $\theta$ =p Probability Mass Function:

$$p_{\theta}(y) = p^{y}(y)(1-p)^{(1-y)}$$

Likelihood function:

$$L_n(Y_1, Y_2, ..., Y_n, \theta) = \prod_{i=1}^n p_{\theta}[y_i]$$

$$\implies L_n(Y_1, Y_2, ..., Y_n, \theta) = p^{\sum y_i} (1 - p)^{n - \sum y_i}$$

Now that we have the likelihood function, we can easily find the maximum likelihood estimator (MLE) for the parameter p. The MLE is defined as the value of  $\theta$  that maximizes the likelihood function:

$$\hat{\theta}^{MLE} = \underset{\theta \in \Theta}{argmax} \prod_{i=1}^{n} p_{\theta}[y_i]$$

arg max return the parameter that maximize the likelihood function.

Note that  $\Theta$  refers to the parameter space i.e., the range of values the unknown parameter  $\theta$  can take. For our case, since p indicates the probability that the coin lands as heads, p is bounded between 0 and 1. Hence,  $\theta = [0, 1]$ . We can use the

tools of calculus to maximise the likelihood function. However, it's often very tricky to take the derivatives. So, we use logarithmic differentiation by calculating the log-likelihood function and maximizing it instead of the likelihood function. Since  $\log x$  is an increasing function,

$$\hat{\theta}^{MLE} = \underset{\theta \in \Theta}{\operatorname{argmax}} \prod_{i=1}^{n} p_{\theta}[y_{i}] = \underset{\theta \in \Theta}{\operatorname{argmax}} \left[ \log \left( \prod_{i=1}^{n} p_{\theta}[y_{i}] \right) \right]$$

Now that we're equipped with the tools of maximum likelihood estimation, we can use them to find the MLE for the parameter p of Bernoulli distribution :

$$log(L_n(Y_1, Y_2, ..., Y_n, p)) = log(p^{\sum y_i} (1-p)^{n-\sum y_i})$$

$$= \sum_{i=1}^n y_i log(p) + \left(n - \sum_{i=1}^n y_i\right) log(1-p)$$

Maximum Likelihood Estimator:

$$\hat{\theta}^{MLE} = \hat{p}^{MLE} = \underset{\theta \in \Theta}{\operatorname{argmax}} \left[ \sum_{i=1}^{n} y_i log(p) + \left( n - \sum_{i=1}^{n} y_i \right) log(1-p) \right]$$

Calculation of the First derivative:

$$\frac{\partial}{\partial \theta} [log(L_n(Y_1, Y_2, ..., Y_n, \theta))] = \frac{\partial}{\partial p} \left[ \sum_{i=1}^n y_i log(p) + \left( n - \sum_{i=1}^n y_i \right) log(1-p) \right]$$
$$= \frac{\sum_{i=1}^n y_i}{p} - \frac{n - \sum_{i=1}^n y_i}{1-p}$$

Calculation of Critical Points in (0, 1):

$$\frac{\partial}{\partial \theta} [log(L_n(Y_1, Y_2, ..., Y_n, \theta)] = 0$$

$$\implies \frac{\sum_{i=1}^n y_i}{p} - \frac{n - \sum_{i=1}^n y_i}{1 - p} = 0 \implies \frac{\sum_{i=1}^n y_i}{p} = \frac{n - \sum_{i=1}^n y_i}{1 - p}$$

$$\implies p = \frac{1}{n} \sum_{i=1}^n y_i$$

Calculation of the Second derivative:

$$\frac{\partial^2}{\partial \theta^2}[log(L_n(Y_1, Y_2, ..., Y_n, \theta)] = \frac{\partial}{\partial \theta} \left[ \frac{\sum_{i=1}^n y_i}{p} - \frac{n - \sum_{i=1}^n y_i}{1 - p} \right] = -\frac{\sum_{i=1}^n y_i}{p^2} - \frac{n - \sum_{i=1}^n y_i}{(1 - p)^2}$$

Substituting the estimator we obtained earlier in the above expression, we obtain,

$$-\frac{n^2 \sum_{i=1}^n y_i}{(\sum_{i=1}^n y_i)^2} - \frac{n^2 (n - \sum_{i=1}^n y_i)}{(n - \sum_{i=1}^n y_i)^2} = -\left(\frac{n^2}{\sum_{i=1}^n y_i} - \frac{n^2}{(n - \sum_{i=1}^n y_i)}\right) < 0$$

Therefore,  $p = \frac{1}{n} \sum y_i$  is the maximiser of the log-likelihood. Therefore,

$$\hat{p}^{MLE} = \frac{1}{n} \sum_{i=1}^{n} y_i)$$

Yes, the MLE is the sample-mean estimator for the Bernoulli distribution. Isn't it amazing how something so natural as a simple intuition could be confirmed using rigorous mathematical formulation and computation! We can now use this method of MLE to find the regression coefficients of the logistic regression.

## Using MLE for the Logistic Regression Model

Let's first attempt to answer a simple question: what the probability distribution for our problem is? Is it the logistic function that we talked about earlier? Nope. It's again the Bernoulli distribution. The only difference from the previous case is that this time the parameter p (probability that Y=1) is the output of the logistic function. The data that we have is inputted into the logistic function, which gives the output:

$$p = P(Y = 1|X) = \frac{e^{\beta_0 + \beta_1 x_1}}{1 + e^{\beta_0 + \beta_1 x_1}}$$

We can make the above substitution in the probability mass function of a Bernoulli distribution to get:

Parameters:  $\theta = [\beta_0, \beta_1]$  probability mass function :

$$p(y) = p^{y} (1 - p)^{(1 - y)}$$

$$= \left(\frac{e^{\beta_0 + \beta x}}{1 + e^{\beta_0 + \beta x}}\right)^{y} \left(1 - \frac{e^{\beta_0 + \beta x}}{1 + e^{\beta_0 + \beta x}}\right)$$

$$= \frac{(e^{\beta_0 + \beta x})^{y}}{(1 + e^{\beta_0 + \beta x})^{y} (1 + e^{\beta_0 + \beta x})^{1} - y}$$

$$= \frac{e^{y(\beta_0 + \beta x)}}{1 + e^{\beta_0 + \beta x}}$$

Likelihood Function:

$$L_n(Y_1, Y_2, ..., Y_n, \theta) = \prod_{i=1}^n p_{\theta}[y_i]$$

$$\implies L_n(Y_1, Y_2, ..., Y_n, \beta_0, \beta_1) = \prod_{i=1}^n \frac{e^{y_i(\beta_0 + \beta x_i)}}{1 + e^{\beta_0 + \beta x_i}}$$

 ${\bf Log\text{-}likelihood\ Function:}$ 

$$log(L_n(Y_1, Y_2, ..., Y_n, \beta_0, \beta_1)) = log\left(\prod_{i=1}^n \frac{e^{y_i(\beta_0 + \beta x_i)}}{}\right)$$

$$= \sum_{i=1}^n log\left(\frac{1}{1 + e^{\beta_0 + \beta x_i}}\right)$$

$$= \sum_{i=1}^n e^{y_i(\beta_0 + \beta x_i)} - \sum_{i=1}^n 1 + e^{\beta_0 + \beta x_i}$$

$$= \sum_{i=1}^n y_i(\beta_0 + \beta x_i) - \sum_{i=1}^n 1 + e^{\beta_0 + \beta x_i}$$

Now that we're derived the log-likelihood function, we can use it to determine the MLE:

Maximum Likelihood Estimator:

$$\hat{\theta}^{MLE} = \underset{\beta_0, \beta_1 \in (-\infty, \infty)}{\operatorname{argmax}} \left[ \log(L_n(Y_1, Y_2, ..., Y_n, \beta_0, \beta_1)) \right]$$

Unlike the previous example, this time we have 2 parameters to optimise instead of just one. Thus, we'll have to employ tools in the domain of multi-variable calculus (gradients and partial derivatives) to solve our problem. We maximize the multi-dimensional log-likelihood function as follows: Computing the Gradient of the Log-likelihood:

$$\nabla log(L_n(Y_1, Y_2, ..., Y_n, \beta_0, \beta_1)) = \begin{pmatrix} \frac{\partial}{\partial \beta_0} log(L_n(Y_1, Y_2, ..., Y_n, \beta_0, \beta_1)) \\ \frac{\partial}{\partial \beta_1} log(L_n(Y_1, Y_2, ..., Y_n, \beta_0, \beta_1)) \end{pmatrix}$$

$$\implies \nabla log(L_n(Y_1, Y_2, ..., Y_n, \beta_0, \beta_1)) = \begin{pmatrix} \frac{\partial}{\partial \beta_0} \left(\sum_{i=1}^n y_i (\beta_0 + \beta x_i) - \sum_{i=1}^n 1 + e^{\beta_0 + \beta x_i}\right) \\ \frac{\partial}{\partial \beta_1} \left(\sum_{i=1}^n y_i (\beta_0 + \beta x_i) - \sum_{i=1}^n 1 + e^{\beta_0 + \beta x_i}\right) \end{pmatrix}$$

$$\implies \nabla log(L_n(Y_1, Y_2, ..., Y_n, \beta_0, \beta_1)) = \begin{pmatrix} \sum_{i=1}^n y_i - \frac{e^{y_i (\beta_0 + \beta x_i)}}{1 + e^{\beta_0 + \beta x_i}} \\ \sum_{i=1}^n x_i y_i - \sum_{i=1}^n \frac{x_i e^{\beta_0 + \beta x_i}}{1 + e^{\beta_0 + \beta x_i}} \end{pmatrix}$$

Setting the gradient equal to the zero vector, we obtain,

$$\nabla log(L_n(Y_1, Y_2, ..., Y_n, \beta_0, \beta_1)) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \implies \begin{pmatrix} \sum_{i=1}^n y_i - \frac{e^{y_i(\beta_0 + \beta_1 x_i)}}{1 + e^{\beta_0 + \beta_1 x_i}} \\ \sum_{i=1}^n x_i y_i - \sum_{i=1}^n \frac{x_i e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

On comparing the first element, we obtain:

$$\sum_{i=1}^{n} y_i - \frac{e^{y_i(\beta_0 + \beta_1 x_i)}}{1 + e^{\beta_0 + \beta_1 x_i}} = 0$$

On comparing the second element, we obtain:

$$\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} \frac{x_i e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}} = 0$$

Thus, we have obtained the maximum likelihood estimators for the parameters of the logistic regression in the form of a pair of equations. Note that, there is no closed-form solution for the estimators. The solutions to the above pair of equations can be computed using various mathematical algorithms e.g., the Newton Raphson algorithm.

## Simple Example

Finally, to make some more sense of all the math we did, let's plug in some real numbers. Suppose we have the following data where  $x_i$  is the studying hours for the student number i , and  $y_i$  indicates whether the student succeeded or not , To simplify the calculations and the analysis, we have considered the case for only 3 students: Let's first use R to perform the calculations. We will then compare the results obtained by R with those obtained by using our equations:

xi	yi
4	1
1	0
-1	1

Figure 2.3: students and studying hours

(Intercept) x 0.5427435 0.1206257

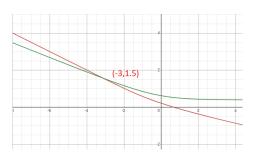
Now, we'll use the equations we derived:

$$\sum_{i=1}^{n} y_i = 2; \sum_{i=1}^{n} x_i y_i = 4$$

Thus, we have the following set of score equations:

$$2 - \frac{e^{(\beta_0 + \beta_1 4)}}{1 + e^{\beta_0 + \beta_1 4}} - \frac{e^{(\beta_0 + \beta_1 1)}}{1 + e^{\beta_0 + \beta_1 1}} - \frac{e^{(\beta_0 + \beta_1 3)}}{1 + e^{\beta_0 + \beta_1 3}} = 0$$
$$4 - \frac{4e^{(\beta_0 + \beta_1 4)}}{1 + e^{\beta_0 + \beta_1 4}} - \frac{e^{(\beta_0 + \beta_1 1)}}{1 + e^{\beta_0 + \beta_1 1}} + \frac{e^{(\beta_0 - \beta_1)}}{1 + e^{\beta_0 - \beta_1}} = 0$$

We can plot the above equations to solve them graphically: The intersection of the 2



graphs gives us the optimal value of the coefficients: (-3, 1.5). Voila! That's incredibly close. Thus, we have been able to successfully use the tools of calculus and statistics to decode the computation processes that determine the coefficients of logistic regression.

Thus, our model is defined as follow:

$$logit(Y) = ln\left(\frac{\pi}{1-\pi}\right) = -3 + 1.5x$$

Therefore, student that study 2 hours,

$$\pi = Probability(Y = outcome of interest | X = 2)$$

$$= \frac{e^{-3+1.5\times2}}{1+e^{-3+1.5\times2}}$$

Create the model on simple data gives us a high accuracy easily but with complex data we need more than just a method, Lets get back to leads scoring and trying to apply all these concepts and if does it works or not . And see how we can improve our model and do what the WISD education needs from us.

#### Inference

#### **R-Squared** $R^2$

the  $R^2$  statistics measures the amount of variance explained by the regression model. The value of  $R^2$  ranges in [0,1], with a larger value indicating more variance is explained by the model the values closer to 1 indicating better fit. For logistic regression, there have been many proposed pseudo- $R^2$  in this project we chose the methode of DL McFadden.

- $\bullet$  DL McFadden stated that a pseudo- $R^2$  higher than 0.2 represents an excellent fit.
- Additionally, McFadden's  $R^2$  can be negative.
- these pseudo- $R^2$  values may be wildly different from one another.

$$R^{2} = 1 - \frac{\ln(\hat{L_full})}{\ln(\hat{L_null})}$$

- ullet  $\hat{L_full}$  is the estimated likelihood of the full model
- $\hat{L_n ull}$  is the estimated likelihood of the null model (model with only intercept)

#### Score Test

The score test relies on the fact that the slope of the log-likelihood function is 0 when  $\beta = \hat{\beta}_j$  The idea is to evaluate the slope of the log-likelihood for the "reduced" model (does not include  $\beta_1$ ) and see if it is "significantly" steep. The score test is also called Rao test. The test statistic, S, follows a  $\chi_r^2$  distribution when  $H_0$  is true and n is large (r is the number of of variables set to zero, in this case = 1). The null hypothesis is rejected when  $S > \chi_r^2(\alpha)$ .

An advantage of the score test is that is only requires fitting the reduced model. This provides computational advantages in some complex situations (generally not an issue for logistic regression). Like the LRT, the score test doesn't depend upon the model parameterization

## 2.2 MODEL IMPLEMENTATION

Finally we are here in the most excited part in our project , now we will explore the magic of all this stuff , and see every thing in action.

So First lets checking the conversion rate to see if our data is balanced .

as you see we have almost 38% conversion Next we split the dataset to 2 parts one for training (70% of the dataset) and the others for test (30% of the dataset)

```
set.seed(22)
sample <- createDataPartition(X$Converted, p = .7, list = F)
leads_data_train = X[sample, ]
leads_data_test = X[-sample, ]</pre>
```

Lets do it !! yes finally!! training the logistic regression model is here using this lines of code

```
1 >logistic_reg <- glm(Converted~Do.Not.Email+Do.Not.Call+TotalVisits</pre>
      +Total.Time.Spent.on.Website+Page.Views.Per.Visit+Lead.
      OriginLanding.Page.Submission, family = "binomial", data =
      leads_data_train)
2 > summary(logistic_reg)
4 Call:
5 glm(formula = Converted ~ Do.Not.Email + Do.Not.Call + TotalVisits
      Total.Time.Spent.on.Website + Page.Views.Per.Visit + Lead.
      OriginLanding.Page.Submission,
      family = "binomial", data = leads_data_train)
9 Deviance Residuals:
           1Q Median
10
     Min
                                  30
                                           Max
11 -2.2173 -0.8558 -0.6146 0.9635
                                        2.4025
13 Coefficients:
                                       Estimate Std. Error z value Pr
      (>|z|)
                                                   0.05552 -0.620
15 (Intercept)
                                       -0.03442
      0.535
                                       -1.18578
                                                   0.14148
                                                           -8.381 <
16 Do.Not.Email
      2e-16 ***
                                       12.34464 190.43154
                                                            0.065
17 Do. Not. Call
      0.948
                                       0.06233
                                                            1.472
                                                   0.04233
18 TotalVisits
      0.141
19 Total.Time.Spent.on.Website
                                       0.94165
                                                   0.03408 27.632 <
     2e-16 **
20 Page.Views.Per.Visit
                                       -0.24534
                                                   0.04551
                                                            -5.391
      7.01e-08
Lead.OriginLanding.Page.Submission -0.55493
                                                   0.07115 -7.800
      6.21e-15 ***
23 Signif. codes: 0
                     ***
                              0.001
                                     **
                                             0.01
                                                           0.05
          0.1
                      1
25 (Dispersion parameter for binomial family taken to be 1)
      Null deviance: 7067.1 on 5260 degrees of freedom
_{\rm 28} Residual deviance: 5965.6 on 5254 degrees of freedom
29 AIC: 5979.6
```

```
30
31 Number of Fisher Scoring iterations: 11
```

Yes this is our model as simple as it is, lets make some diagnostics.

as you see the results show that 455 which is true positive and 1185 is false negative , it seems that something wrong here is our model working well ? .

Okay, lets check the accuracy of this model

```
1 > accuracy_Test <- sum(diag(table_mat))/ sum(table_mat)
2 > accuracy_Test
3 [1] 0.7275954
```

Oh! here we go again why? how? Don't worry I'm just kidding, unfortunately when we try to train the model using all columns it seems like some of them isn't a good predictor which make our model to generate wrong predictions, as you can see this model give 70% in the accuracy. but, what about R2? there is no such R2 value for logistic regression (for more details see the previous section in the estimation part). Instead, we can compute a metric known as McFadden's R2, which ranges from 0 to just under 1. Values close to 0 indicate that the model has no predictive power. In practice, values over 0.40 indicate that a model fits the data very well. We can compute McFadden's R2 for our model using the pR2 function from the pscl package:

```
1 > pscl::pR2(logistic_reg2)["McFadden"]
2 fitting null model for pseudo-r2
3 McFadden
4 0.159280
```

Here we are! our model doesn't fit the model. This is why we need to make a decision about which column is a good predictor and which is not. That's possible using the RFE algorithm (Recursive Feature Elimination), in the short form is a widely used algorithm for selecting features that are most relevant in predicting the target variable in a predictive model — either regression or classification.

```
17 [3] "Total.Time.Spent.on.Website"
  [4] "TagsClosed.by.Horizzon"
19 [5] "TagsRinging"
20 [6] "Last.Notable.ActivitySMS.Sent"
21 [7] "TagsWill.back.after.reading.the.email"
22 [8] "Last.Notable.ActivityModified"
23 [9] "Last.ActivitySMS.Sent"
24 [10] "Lead.QualityHigh.in.Relevance"
25 [11] "Lead.QualityNot.Sure"
26 [12] "TagsInterested.in.other.courses"
27 [13] "TagsBusy"
28
29 . . . .
31 [89] "Last.ActivityPage.Visited.on.Website"
32 [90] "SpecializationMarketing.Management'
33 [91] "CityOther.Cities"
34 [92] "SpecializationE.Business"
35 [93] "Last.Notable.ActivityPage.Visited.on.Website"
36 [94] "Last.ActivityOlark.Chat.Conversation"
```

We see that all columns are sorted in the way that the first are the best predictors and the last are the worst. Based on this we can make another model on almost the same data but with less columns where we let just which seems the best from the RFE output.

With the same process as showing previously , we can show the accuracy and the R2, like so.

```
1 > pscl::pR2(logistic_reg2)["McFadden"]
2 fitting null model for pseudo-r2
3 McFadden
4 0.6403674
5 > accuracy_Test <- sum(diag(table_mat))/ sum(table_mat)
6 > accuracy_Test
7 [1] 0.9214729
```

Wow! with just few simple steps we made it our model now has a 92% chance to make the right prediction, are you excited like me to see the real test? I create a simple function that take as input the lead vector ( the vector that represent leads information but more of that it needs to be encoded or converted to something our model could use) and next to it the real model parameter which holds our model and a Boolean parameter that say if the input lead is vector or not ( if not an additional pre-processing is needed ).

It's clear that the method returns the score from 0 to 100 by a simple multiplication of the model output. Next, lets use it.

Like we see here the function return the score 94 for the lead that already converted . Now lets try the opposite, here the conversion is 0 so the score must be low.

```
1 > i = 2
2 > lead_score = score_lead(leads_data_test[i,], logistic_reg2, TRUE)
3 > lead_id = leads_data[i,] Prospect.ID
4 > conv = leads_data_test[i,] Converted
5 > sprintf('The lead with id %s and with convertion equal to %s is scored by the model as %.2f percent', lead_id, conv,lead_score [1])
6 [1] "The lead with id 2a272436-5132-4136-86fa-dcc88c88f482 and with convertion equal to 0 is scored by the model as 12.64 percent "
```

Yes indeed, it works again, the score for the none converted lead is 12 isn't awesome how this is done . for me it's like the magic .

# Conclusion

Our model working very well, Now WISD Education could determine the hot leads and passe it to the sales team and without doubt the conversion rate will be more higher from the past because now the employees in this team will have a big chance to find the people which more or just few more interested in the WISD Education or its products with that the possibility of the conversion being higher for sure. Yes! we did it!! now every one happy we are in the top level of data science technologies ... Wait a minute there is more? Yes, there is more. For now based on specific data we can determine the score of a lead but what about the time and the competition which is something that couldn't represented in the tables .or even his emotions at the moment, take an example where some one is very interested in the books provided in WISD Education, but after a while he will see a new competitive brand with new offers , so when the someone from the team call him he will say no or am not interested . All these are very hard to be implemented in a static model like ours. So there is a much more complicated concept named Time Series Models which are simply another type of models that uses variations through time and adapt based on this changes or even predict the future. Here a bunch of questions appear in our heads, how we could do that? How it is possible to use time? How we train a model that uses the time to predict through the time? and more questions. Like the quote says "The future is made of the same stuff as the present". Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

# bibliothèques

dataset

 $https://www.kaggle.com/code/ashydv/lead-scoring-logistic-regression/data?select=Leads.csv\\ Inference$ 

https://www.joshuapkeller.com/page/introregression/logisticinference.html

Article

An Introduction to Logistic Regression Analysis and Reporting

September 2002 The Journal of Educational Research DOI:10.1080/00220670209598786