Reformulation of the KKT conditions

- $\nabla f = \sum_{i \in E} \mu_i \nabla h_i(x) + \sum_{j \in I} \lambda_j \nabla g_j(x)$
- $h_i(x) = 0$
- $g_j(x) \geq 0$
- $\lambda_j \geq 0$
- $g_j(x)\, \lambda_j = 0$

- $\lambda_i = 0$ $\quad y_i(w^T x + b) \geq 1$
- $\lambda_i = C$ $\quad y_i(w^T x + b) \leq 1$
- $0 < \lambda_i < C$ $\quad y_i(w^T x + b) = 1$

From a complex problem to a simpler one

- $max_\lambda \sum_{i=1}^{l} \lambda_i - \frac{1}{2} \sum_{i,j=1}^{l} y_i \, y_j \lambda_i \lambda_j x_i, x_j$
- $\sum_{i=1}^{l} \lambda_i \, y_i = 0$
- $0 \leq \lambda_i \leq C$

If we fix all λ_i except λ_1 and λ_2

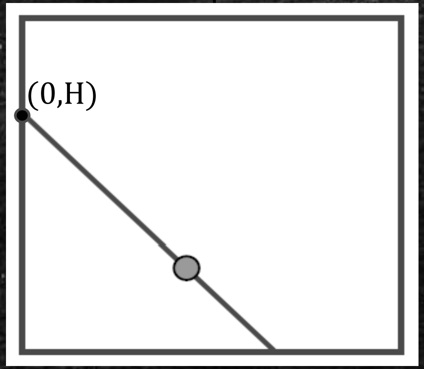
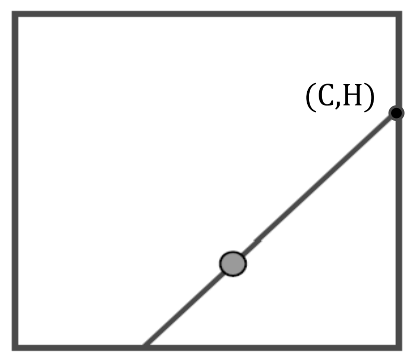
$$\sum_{i=1}^{l} \lambda_i \, y_i = 0$$

$$\lambda_1 y_1 + \lambda_2 y_2 = -\sum_{i=3}^{l} \lambda_i \, y_i$$

$$\lambda_1 y_1 + \lambda_2 y_2 = \xi$$

$y_1 = y_2$ $\lambda_1 + \lambda_2 = \xi$	$y_1 \neq y_2$ $\lambda_1 - \lambda_2 = \xi$

Finding the bounds

$y_i = y_j$ $\lambda_i + \lambda_j = \xi$	$y_i \neq y_j$ $\lambda_i - \lambda_j = \xi$
$\lambda_j = C$ $\lambda_i = 0$  $\lambda_i = C$ $\lambda_j = 0$	$\lambda_j = C$ $\lambda_i = 0$  $\lambda_i = C$ $\lambda_j = 0$
$L_j = \max(0, \lambda_i + \lambda_j - C)$ $H_j = min(C, \lambda_i + \lambda_j)$	$L_j = \max(0, \lambda_j - \lambda_i)$ $H_j = min(C, C + \lambda_j - \lambda_i)$

Finding the optimized λ_j

- $min_{\lambda} \frac{1}{2} \sum_{i,j=1}^{l} y_i \, y_j \lambda_i \lambda_j x_i, x_j - \sum_{i=1}^{l} \lambda_i$
- $\sum_{i=1}^{l} \lambda_i \, y_i \; = 0$

$$\lambda_j^{new} = \lambda_j - \frac{y_j(E_i - E_j)}{\eta}$$