Maths Tutorial 1

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1 Question 1

Given an example of 2 natural numbers x and y such that x - y is not a natural number.

$$1 - 5$$

Reasoning

Natural numbers are numbers that are greater than 0 and are whole numbers (or sometimes natural numbers can include 0). $1 - 5$ results in negative 4, which is not a natural number.

2 Question 2

In mathematics we say that a datatype is closed under an operation if applying this operations to elements of the datatype produces a datatype element. For example, natural numbers are closed under addition (the sum of any two natural numbers is a natural number). This example shows that the natural numbers are not "closed under subtraction".

Give examples of integers x and y such that x/y is not an integer. Such an example shows that the integers are not "closed under division".

$$5/2$$

Reasoning

An integer is a whole positive number. To find x and y such that x/y is not an integer, we need to find a case whereby dividing x by y results in a floating point number. 5 divided by 2 is 2.5, which is not an integer.

3 Question 3

Consider an operation which takes numbers x and y and returns $x^2 - y$. Which of the following number systems are closed under this operation?

- The natural numbers
- The positive integers
- The integers
- The rationals

So it will be easier to find which of these systems are not closed under operation because we will only have to find a single test case where it is not true.

3.1 Natural numbers

Assume that x is 1. $1^2 = 1$. Assume that y is 15. $1 - 15 = -14$ This does not result in a natural number, therefore this is not closed under the natural numbers.

3.2 Positive Integers

Assume that x is 1. $1^2 = 1$. Assume that y is 15. $1 - 15 = -14$ This does not result in a positive integer, therefore this is not closed under the positive integers.

3.3 The Integers

The integers consist of all negative whole numbers, the number 0, and the positive whole numbers. This sum will remain closed under the integers because if you take away from a positive integer, it will always be an integer. Squaring a number does not make it change its datatype.

3.4 The rationals

Rational numbers are defined as $\frac{a}{b}$ where a and b can be any integer. Since any integer is closed under this operation, all rational numbers will also be closed under this operation.

4 Question 3

Prove that every integer n with $1 \leq n \leq 6, n^2 - n + 11$ is a prime number.
We have a set of numbers $1, 2, 3, 4, 5, 6$ to test. We will simply test all of these numbers.

$$1^2 - 1 + 11 = 11$$
$$2^2 - 2 + 11 = 13$$
$$3^2 - 3 + 11 = 17$$
$$4^2 - 4 + 11 = 23$$
$$5^2 - 5 + 11 = 31$$
$$6^2 - 6 + 11 = 41$$

All these numbers result in a prime number. Thus this has proven to be true. QED.

5 Question 5

Write down a list of all prime numbers that are even.
2 is the only prime number that is even.

6 Question 6

Prove that there exists integers m and n such that $m > 1, n > 1$ and $\frac{1}{m} + \frac{1}{n}$ is an integer.
$n = 2, m = 2$

Reasoning
Two halves added together make 1.

7 Question 7

Prove that there exists distinct ntegers m and n such that $m > 1, n > 1$ and $\frac{1}{m} + \frac{1}{n}$ is an integer.
$n = 2, m = -2$

Reasoning Obvious -2 is not 2, but $\frac{1}{2} + \frac{1}{-2} = \frac{1}{2} - \frac{1}{2} = 0$ and 0 is natural number.

8 Question 8

Try to prove that if any integers m and n are even then so is m - n. The definition of an even number is $2k$ therefore we can make m - n into $2k - 2m$ which can be simplfied to $2(k - m)$ whereby whatever the result of k - m is, it will always be even as it is multiplied by 2.

9 Question 9

Try to prove that the sum of any even integer and any odd integer is odd. Let m be an artbiary but fixed even integer and n be an arbitary but fixed odd integer. We want to prove that $m + n$ is odd. We can rewrite this using the definitions of odd and even. $2k + 2l + 1$ and this can be simplified to $2(k + l) + 1$ and by the definition of odd, this is odd.