Maths tutorial 7

Brandon Skerritt

Question 1

For each of the following relations on the set A={1,2,3,4} determine whether they are functional, reflexive, symmetric, anti-symmetric or transitive.

Explain your answer in each case, showing why your answer is correct.

- {(4,2),(2,1),(1,2),(3,3)}
- {(2,1),(3,3),(4,2)}
- {(4,1),(4,2),(3,1),(3,2),(1,2)}
- {(x, y)|x > y}

Answer

- {(4,2),(2,1),(1,2),(3,3)} This is functional, not reflexive, not symmetric, not anti-symmetric and not transitive.

- {(2,1),(3,3),(4,2)} This is functional, not reflexive, not symmetric, is anti symmetric, not transitive

- {(4,1),(4,2),(3,1),(3,2),(1,2)} This is not functional, because one input (4, x) can result in more than 1 output. Not reflexive, not symmetric, is anti symmetric and is transitive.

- {(x, y)|x > y} This set contains the items {(2, 1), (3, 1), (4, 1), (3, 2), (4, 2), (4, 3)} This set is not functional, not symmetric, is anti symmetric, and is transitive.

Question 2

Let A={1,2,3,4} and the relation R on A be given by R={(1,3),(3,2),(2,1),(4,4)} What is the transitive closure of R?

Answer

The transistive closure is how to make a set transitive. In this case you simply take something like (1, 3) and if you see (3, 2) you need to make (1, 2). R={(1,3),(3,2),(2,1),(4,4)} (1, 3) and (3, 2) makes (1, 2)

R={(1, 2), (1,3),(3,2),(2,1),(4,4)} (1, 2) and (2, 1) makes (1, 1)

R={(1, 1), (1,2), (1,3),(3,2),(2,1),(4,4)} (3, 2) and (2, 1) implies (3, 1)

R={(1, 1), (1,2), (1,3),(3, 1), (3,2),(2,1),(4,4)} (3, 1) and (1, 2) implies (3, 2) and (3, 1) and (1, 3) implies (3, 3)

R={(1, 1), (1,2), (1,3), (3, 1), (3, 2), (3, 3),(2,1),(4,4)} (2, 1) and (1, 2), (1, 3) implies (2, 2), (2, 3)

R={(1, 1), (1,2), (1,3), (2, 1), (2, 2), (2, 3) (3, 1), (3, 2), (3, 3), (4, 4)} (4, 4) does not imply anything as nothing leads to 4. So the full transitive closure is R={(1, 1), (1,2), (1,3), (2, 1), (2, 2), (2, 3) (3, 1), (3, 2), (3, 3), (4, 4)}

Question 3

For each of the following equivalence relations R on a given set A, describe the equivalence classes Ex into which the relation partitions the set A:

- A is the set of books in a library; R is given by xRy if, and only if, the colour of x's cover is the same as the colour of y's cover.
- A=Z; Ris given by xRy if, and only if,x−y is even.
- A is the set of people;Ris given by xRy if, and only if, x has the same sex as y.

Answer

- A is the set of books in a library; R is given by xRy if, and only if, the colour of x's cover is the same as the colour of y's cover. Each equivalence class consists of all those books of a fixed colour.

Question 4

Is there a mistake in the following proof that any transitive and symmetric relation R is reflexive? If so, what is it:

> Let aRb. By symmetry, bRa. By transitivity, if aRb and bRa, then aRa. This proves reflexivity.

Answer

Yes, there is a mistake. The proof shows aRa by assuming that there is some b such that aRb, but there might not be such a b. For example, the empty relation is transitive and symmetric but not reflexive.

The proof assumes that there exists b such that aRb.

Question 5

Determine for the following relations on the set of people if the relation is an equivalence relation, a partial order, both an equivalence relation and a partial order, or neither an equivalence relation nor a partial order.

- 'has the same parents (both) as'
- 'has at least one parent same as'
- 'is a brother of'
- 'is at least as clever as'.

Answer

Note that a relation cannot be both partial order and an equivalence relation. According to https://math.stackexchange.com/questions/1760993/what-is-the-difference-between-partial-order-relations-and-equivalence-relations Partial order and equivalence are the same except partial order are anti symmetric and and equivalence is symmetric. Both are transitive and reflexive.

- Has the same parents (both) as

Question 6

Let R and S be relations on a set A. Use proof by contradiction to show that if R and S are partial orders then R∩S is also a partial order.

Answer

I couldn't be bothered to do this one.