Maths tutorial 2

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1 Question 1

Prove for all integers n, if n is even then n^2 is even
Answer By definition of even, $n = 2k$ and $n^2 = (2k)^2$

$$(2k)^2 = (2k)(2k) = 4k^2$$

4k is divisble by 2, thus if n is even then n^2 is even.

2 Answer 2

Prove for any natural number n that $n^2 + n + 1$ is always odd
Answer We have 2 cases, one where n is even and one where n is odd.

2.1 N is even

By definition of even, the equation now holds as $2k^2 + 2l + 1$ We can factor out the 2 $2(k^2 + l) + 1$ No matter what k and l are, $2(k^2 + l)$ will always be even as by definition of even and adding $+1$ will always make it odd as by definition of odd.

2.2 N is odd

By definition of odd, the equation now holds as $(2k + 1)^2 + 2k + 1 + 1$

$$(2k + 1)(2k + 1) = 4k^2 + 4k + 1$$

$$4k^2 + 4k + 1 + 2k + 1 + 1$$

$$4k^2 + 4k + 2k + 2 + 1$$

$$4k^2 + 6k + 2 + 1$$

$$2(2k^2 + 3k + 1) + 1$$

By the definition of odd, the above is odd.

3 Question 3

Prove by contradiction that there is no greatest even integer.

3.1 Answer

Assume there is a greatest even integer. By definition of even, this will be 2k where k is a value that is greater than the sum of every single value below it. Observe what happens when we add 1 to 2k.

$$2k + 1$$

2k, which was previously the greatest integer, is no longer the greatest integer as it contains $+ 1$ more than it did previously, which is a contradiction as it is the greatest possible integer. QED.

4 Question 4

Prove by contradiction that if n is an integer and n^3 is odd then n is odd