

Maths tutorial 6

You

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1 Question 1

Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is f a bijection, and if it is, what is its inverse?

1.1 Answer

Yes, f is a bijection because every input has 1 output and every output can be reached from every input. The inverse is $f(2) = a$, $f(3) = b$. $f(1) = c$.

2 Question 2

Consider the function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 3x$ and the function $g: \mathbb{R} \rightarrow \mathbb{R}$ given by $g(x) = x + 9$. Calculate $g \circ f$, $f \circ g$, $f \circ f$ and $g \circ g$.

2.1 Answer

$$\begin{aligned}g(f(x)) &= g(3x) = 3x + 9 \\f(g(x)) &= f(x+9) = 3x + 27 \\f \circ f &= f(3x) = 9x \\g \circ g &= g(x+9) = x+18\end{aligned}$$

3 Question 3

Use the pigeonhole principle to give solutions to the following problems:

3.1 1

How many times must a single die be rolled to guarantee that some number is obtained at least twice?

3.2 Answer

7 because once a dice is rolled 6 times all possible numbers are reached, the seventh time guarantees it'll roll on a number twice.

3.3 2

How many times must two dice be rolled to guarantee that the same total score is obtained at least twice?

3.4 Answer

12 times. It's important to visualise what you're thinking about. 2 dice can be thrown, each can land on any number between 1 - 6, resulting in 12 possible different scores.

3.5 3

How many times must two dice be rolled to guarantee that the same total score is obtained at least three times?

3.6 Answer

Answer 23

4 Question 4

A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

4.1 1

How many socks must he take out to be sure that he has at least two socks of the same colour?

4.2 Answer

3

4.3 2

How many socks must he take out to be sure that he has at least two black socks?

4.4 Answer

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