

Maths tutorial 3

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1 Question 1

Use proof by contradiction to show that if p is rational and r is irrational then $p + r$ is irrational. Hint: Use the fact that the difference of two rational numbers is rational.

1.1 Answer

Suppose not. Suppose that if p is rational and r is irrational, $p + r$ is rational. Since the difference of two rational numbers is rational, $(p+r) - p = r$, which is irrational. Thus a contradiction. QED.

1.2 Answer written out

Okay so the difference of 2 rational numbers is rational. And we need to do this. So suppose we have 2 rational numbers, $x - y = \text{rational}$. We just substitute in our values, so we have p which is rational, $p + x = \text{rational}$ and then we know $p+r$ is rational because its the contradiction we're trying to make, so $p - p+r = \text{rational}$

2 Question 2

Prove by contradiction that the square root of any positive irrational number is irrational.

2.1 Proof

Assume that the square root of any positive irrational number is rational. Then $\sqrt{p} = \frac{a}{b}$ for some integers, p, a, b . Then it holds that squaring p would result in $p = \frac{a^2}{b^2}$. This would make p rational, thus a contradiction. Because we assumed that the square root of any positive irrational number is rational, that means that p must be irrational but here it is rational.

3 Question 4

Determine which statements in III.3.a–III.3.d are true and which are false. Prove those that are true and disprove those that are false

3.1 $6 - 7\sqrt{2}$ is irrational.

3.2 Answer - WRONG

Suppose that it is rational. So $6 - 7\sqrt{2} = \frac{m}{a}$ Then it holds that $6a - 7a\sqrt{2} = m$ where m and a are rational. We can rewrite this as

$$6a - 7a\sqrt{2} = m$$

$$-7a\sqrt{2} = m - 6a$$

$$\sqrt{2} = \frac{m - 6a}{7a}$$

$$\sqrt{2} = \frac{1}{7n}(6n - m)$$

3.3 $3\sqrt{2}-7$ is rational

This statement is false.

3.4 Answer

Suppose $3\sqrt{2}-7$ is rational. Then it holds that $3\sqrt{2}-7 = \frac{m}{n}$.

$$3n\sqrt{2} - 7n = m$$

$$3n\sqrt{2} = m + 7n$$

$$\sqrt{2} = \frac{m + 7n}{3n}$$