Maths class test 1

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Question 1

Suppose that U={1,2,3,4,5,6,7}is the universal set, A={x∈N|2< x <7}, B={3,7}, and C={3,4,5,6}.

Question 1 a

List the elements of the set B∪(A∩C)

A = 3, 4, 5, 6

Answer

(A∩C) = 3, 4, 5, 6 B∪(A∩C) = {3, 4, 5, 6, 7}

Question 1 b

What is the characteristic vector of (AΔB)∩C?

Answer

AΔB = {4, 5, 6, 7}

(AΔB)∩C = {4, 5, 6}

Question 2

Which of the following statements are true?

- {a, b, c}⊆{b, c, a}−{b}

So {b, c, a}−{b} is just {c, a}. Thus the question is is the set containing {a, b, c} a subset of the set containing {c, a}? Well no, because the first set contains {b} and the second set doesn't.

- {a}⊆{a, b} The set containing the element of "a" is indeed a subset of {a, b}.
- {b, c}⊆{c, b} Yes, b, c is a subset of c, b. The first set contains all the elements in the second set, and order doesn't matter. Thus it is a subset.
- {a, b, c}Δ{b}={a, c} {a, b, c}Δ{b} is all elements that are in set on the left and set on the right but not in both. Since the element b is in both it is removed. {a, c}. Thus the question becomes {a, c} = {a, c} Which is true for obvious reasons.

Question 3

Which of the following statements are true

- x∈{x} Is the singular element x an element of the set containing the element x? Yes.
- {x}⊆{x} Is the set containing the element x a subset of the set containing the element x? They are the same, so yes it is a subset.
- {x}∈{x} Is the set containing the element x an element of the set containing the element x? Nope. If it was, it would be like this {x}∈{{x}}.
- {x}∈{{x}} Yes, this is correct. See above.
- ∅⊆{x} Yes, the empty set is always a subset of *any* set.
- ∅∈{x} No, the empty set is not an element of the set containing *only* x.

Question 4

Which of the following are true for every negative integer x and every negative integer y?

So both integers are negative and they are integers (whole numbers).

- x−y is a positive integer This isn't *always* true. Let's try an example. -2 - -1 is -1, which is still negative.
- x×y is a natural number. Let's try an example. -2 * -2 is -4. Natural numbers are positive whole numbers, so this isn't true either. This is wrong! Any negative times a negative is a positive.
- x+y is a negative integer -6 + -3 = -9. If x and y are negative integers then this is true.
- x/y is a rational. This is true because -6 / -8 is true.
- y/x is a rational This is still true.

Question 5

How many prime numbers are odd?

Answer

All of them apart from the number 2.

Question 6

Let A={1,3,5} and B={x, y, z}. Define f:A→B by specifying that

> (1) = y, f(3) = z, f(5) = y Determine whether f is injective, surjective or bijective (remember, it can have more than one property).

Answer

Okay so injectivty is where not all answers have an output. This is not injective because all items in A have at least 1 output in B. Surjectivty is where B can have more than 1 A. This is surjective because y can be reached from f(5) and f(1). This isn't bijective because bijectity is where it is both injective and surjective.

Question 7

Let f:R→R be the function defined byf(x) = 2*x+1. Determine whether f is injective,surjective or bijective.

Answer

To check for injectivity you simply just do this 2*x1+1 = 2*x2+1 2*x1 = 2*x2 x1 = x2 Therefore it is injective

To check for surjectivity you just have to think about it (alot) So the function isn't surjective because 1 output can't be reached from 2 different inputs. If you have x = 2 so 2*2 = 4 + 1 = 5, now try to find a way to get 5 again. The closet thing we could do is -2 but -2 * 2 = -4 + 1 = -3, which isn't 5. Try again with 1. 2 * 1 = 2 + 1 = 3. 2 * -1 = -2 + 1 = -1. It's impossible to get the same output from 2 different inputs, so this isn't surjective.

Because it's not surjective, it's also not bijective.

Question 8

Let X={1,2,3},Y={a, b, c, d, e}and Z={x, y, z}. Define functions f:X→Y and g:Y→Zby specifying that:

> f(1) = b, f(2) = a, f(3) = c and g(a) = x, g(b) =y, g(c) = y, g(d) = z, g(e) = z. Determine g∘f. What is the range of g∘f?

Answer

so if you have pen and paper it's best to do this as a diagraph, but we'll try without. So GoF is defined as

> g(f(1)) = y g(f(2)) = x g(f(3)) = y So we take the output from f(1) etc and then given the output b we put this into g.

The range of GoF is all the possible outputs which is {y, x, y} but because set theory doesn't allow repition it's {x, y}.

Question 9

Let X={1,2},Y={a, b, c} and Z={x, y, z}. Let f:X→Y and g:Y→Z be such that:

> g(a) =x, g(b) =y, g(c) =x And g∘f(1) =x, g∘f(2) =x.

Determine F.

Answers

So GoF(1) = x. So we simply return it. like so So when 1 is input into F, it transform it into X. So therefore f(1) = a So when 2 is input into F, it transform into X. So therefore f(2) = c.

Question 10

How many people do you need to have in a room in order to guarantee that at least two of them have a birthday on the same day of the week?

Answer

So a person can be born on 1 of 7 days (monday tuesday wednesday etc...) and once you have 8 people in a room, two of them have to be born on the same day. So the answer is 8.

Question 11

How many integers from 100 through 999 must you pick in order to be sure that at least two of them have a digit in common? (For example,256 and 530 have the common digit 5.)

Answer

So this is a simple counting problem. There are 10 integers that it could be and 3 spaces

> X X X Any one of those X's could be one of 10 integers And you want at least 2 of them to have the same digit in common. So 10 digits and they can be put over any of the numbers Well, if you pick 2 random numbers there isn't a very high chance of getting 2 digits in common.

So in the worst case scenario you'll have

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111
222
333
444
555
666
777
888
999
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For a total of 9 numbers. But adding a tenth number like so

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989
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Would make it have a digit in common with 888. So therefore 10 is the worst case scenario to get 2 digits in common.