Maths Tutorial 5

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1 Question 1

Let A and B be sets. Prove the following statements Note: I'm skipping proof questions

2 Question 3

List all distinct functions from the set A={1,2} to the set B={a, b}. How many such distinct functions exist?

2.1 Answer

So this is just a mapping. A function cannot have more than 2 answers for 1 input, so this is just a simple mapping of A > B. The question asks for every distinct function of this mapping.
f(1) = a, f(2) = a
f(1) = b, f(2) = b
f(1) = a, f(2) = b
f(1) = b, f(2) = a

3 Question 4

Consider f: $R- > R$ given by f(x) = x^2. What are the domain, codomain and range of this function?

3.1 Answer

The domain is all the inputs into a function, the codomain is all the possible answers and the range is all the actual answers.

So the domain is the set of all real numbers, the codomain is the set of all real numbers and the range is the set of all non negative numbers (square numbers)

4 Question 5

Which of the following functions are injective? Which are surjective? Injective is where b cant have many a. Not all outputs in B can be reached by A. Surjective is where every single output has some a. 2 different inputs can lead to the same output.

4.1 $f : Z- > Z - given - by - f(x) = x^2 + 1$

Z is the class of all integers.

4.2 Answer

$$x_1^2 + 1 = x_2^2 + 1$$

$$x_1^2 = x_2^2$$

$$\pm x_2 = \pm x_2$$

Therefore this is not injective. TO see if it is surjective, we need to find a counter example. Something in the range of integers which this function cannot reach. We'll slowly work our way up. f(x) = 1, is possible if x = 0. f(x) = 2 is possible where x = 1. f(x) = 3 is not possible. Therefore this is not surjective.

4.3 $g : N \rightarrow N - given - by - g(x) = 2^x$

N is natural numbers (0 to infinity).

4.4 Answer

$$2^x = 2^y$$

Assume x is x1 and y is y1. I cannot write x1 as powers. If

$$2^x = 2^y$$

then it is assumed that

$$x = y$$

therefore this is injective For surjectivity we again need to find an example of natural numbers that cannot be reached. So g(x) will alwasy result in a natural number, we need to find a natural number that isn't a power of 2. So given g(x) = 1, 2^1 is 1. What about g(x) = 3? 2^3 is not equal to 3, therefore it is surjective.

4.5 $h : R \rightarrow R - given - byh(x) = 5x - 1$

R is the set of all real numbers (any number that is countable).

4.6 Answer

$$5x_1 - 1 = 5x_2 - 1$$

$$5x_1 = 5x_2$$

$$x_1 = x_2$$

therefore it is injective We can assume that no matter what the input is, h(x) will always result in a real number. To prove it is not a real number, it will have to be something uncountable like Infinity. Since this maths operation uses countable numbers and only has countable numbers in the sum, it will always result in a countable real number; thus it is surjective.

5 Question 6

Let f be the function from {a, b, c} to {1,2,3} such that f(a) = 2, f(b) = 3, and f(c) = 1. Is f a bijection, and if it is, what is its inverse?

5.1 Answer

It's best to draw graphs with these, so once you draw the graph you'll see it's bijective. Once we know it's bijective we can find the inverse by just switching things around. The inverse, formally, is

$$f(2) = a, f(3) = b, f(1) = c$$

6 Question 7

Consider the function f:R->R given by f(x) = 3x and the function g:R->R given by g(x) = x + 9. Calculate $g_0 f, f_0 g, f_0 f and g_0 g$.