Coupled Oscilator for 1D System with 2 Masses

Assume a system of 2 Masses, each with displacement x_i, mass of m and spring coeffitients $k_{12} = \kappa$, $k_{11} = k_{22} = k$. We have:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2E}} \begin{pmatrix} \sqrt{m}\dot{x}_1(t) \\ \sqrt{m}\dot{x}_2(t) \\ ix_1(t) \\ ix_2(t) \\ i(x_1(t) - x_2(t)) \end{pmatrix}$$

Now, in order to find matrix $\mathbf{B}$, we have:

$$\sqrt{\mathbf{M}}\mathbf{B}|1,1\rangle = \sqrt{k}|1\rangle$$

$$\sqrt{\mathbf{M}}\mathbf{B}|2,2\rangle = \sqrt{k}|2\rangle$$

$$\sqrt{\mathbf{M}}\mathbf{B}|1,2\rangle = \sqrt{\kappa}(|1\rangle - |2\rangle)$$

Since $\mathbf{M} = \sqrt{m}\mathbf{I}$, we have:

$$\mathbf{B} = \begin{pmatrix} \sqrt{\frac{k}{m}} & 0 & \sqrt{\frac{\kappa}{m}} \\ 0 & \sqrt{\frac{k}{m}} & -\sqrt{\frac{\kappa}{m}} \end{pmatrix}$$

Hence, Our hamiltonian will be:

$$\mathbf{H} = \begin{pmatrix} 0 & 0 & \sqrt{\frac{k}{m}} & 0 & \sqrt{\frac{\kappa}{m}} \\ 0 & 0 & 0 & \sqrt{\frac{k}{m}} & -\sqrt{\frac{\kappa}{m}} \\ \sqrt{\frac{k}{m}} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{k}{m}} & 0 & 0 & 0 \\ \sqrt{\frac{\kappa}{m}} & -\sqrt{\frac{\kappa}{m}} & 0 & 0 & 0 \end{pmatrix}$$

In order to make this to be implemented using 3 qubits, we must extend the hamiltonian to have the form:

$$\mathbf{H} = \begin{pmatrix} 0 & 0 & \sqrt{\frac{k}{m}} & 0 & \sqrt{\frac{\kappa}{m}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\frac{k}{m}} & -\sqrt{\frac{\kappa}{m}} & 0 & 0 & 0 \\ \sqrt{\frac{k}{m}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{k}{m}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\frac{\kappa}{m}} & -\sqrt{\frac{\kappa}{m}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Therefore, we have this decomposition for the pauli strings:

$$H = 0.5\sqrt{\frac{\kappa}{m}}XII + 0.5\sqrt{\frac{k}{m}}IXI + 0.5\sqrt{\frac{\kappa}{m}}XZI + 0.5\sqrt{\frac{k}{m}}ZXI$$

$$+0.5\sqrt{\frac{\kappa}{m}}XIZ - 0.25\sqrt{\frac{\kappa}{m}}XIX - 0.25\sqrt{\frac{\kappa}{m}}YIY$$

$$+0.5\sqrt{\frac{\kappa}{m}}XZZ - 0.25\sqrt{\frac{\kappa}{m}}XZX - 0.25\sqrt{\frac{\kappa}{m}}YZY$$

For the simulations, we will start with these parameters:

$$\kappa = 0.5, \quad k = 4, \quad m = 1$$

$$x_1(0) = 1, \quad x_2(0) = 0, \quad \dot{x}_1(0) = 0, \quad \dot{x}_2 = 0$$

So, the normalized initial state will be

$$|\psi(0)\rangle = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

This is equivalent to creating the bell state $|\psi^+\rangle$ on qubits 0 and 1 and leave the qubit 2 alone. The simulation files are provided.