Coupled Oscilator for 1D System with N Masses

For this task, a system of N Masses, each with displacement x_i, mass of m and spring coeffitients $k_{ij} = \kappa$ for $j = i + 1$ and 0 for others, $k_{ii} = k_{jj} = k$ will be inspected. The equation becomes:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2E}} \begin{pmatrix} \sqrt{m}\dot{x}_1(t) \\ \sqrt{m}\dot{x}_2(t) \\ \vdots \\ \sqrt{m}\dot{x}_N(t) \\ ix_1(t) \\ ix_2(t) \\ \vdots \\ ix_N(t) \\ i(x_1(t) - x_2(t)) \\ i(x_2(t) - x_3(t)) \\ \vdots \\ i(x_{N-1}(t) - x_N(t)) \end{pmatrix}$$

Now, in order to find matrix $\mathbf{B}$, we have:

$$\sqrt{\mathbf{M}}\mathbf{B}|1,1\rangle = \sqrt{k}|1\rangle$$

$$\sqrt{\mathbf{M}}\mathbf{B}|2,2\rangle = \sqrt{k}|2\rangle$$

$$\vdots$$

$$\sqrt{\mathbf{M}}\mathbf{B}|N,N\rangle = \sqrt{k}|N\rangle$$

$$\sqrt{\mathbf{M}}\mathbf{B}|1,2\rangle = \sqrt{\kappa}(|1\rangle - |2\rangle)$$

$$\sqrt{\mathbf{M}}\mathbf{B}|2,3\rangle = \sqrt{\kappa}(|2\rangle - |3\rangle)$$

$$\vdots$$

$$\sqrt{\mathbf{M}}\mathbf{B}|N-1,N\rangle = \sqrt{\kappa}(|N-1\rangle - |N\rangle)$$

Since $\mathbf{M} = \sqrt{m}\mathbf{I}$, we have:

$$\mathbf{B} = \begin{pmatrix} \sqrt{\frac{k}{m}} & 0 & 0 & \dots & 0 & \sqrt{\frac{\kappa}{m}} & 0 & \dots & 0 \\ 0 & \sqrt{\frac{k}{m}} & 0 & \dots & 0 & -\sqrt{\frac{\kappa}{m}} & \sqrt{\frac{\kappa}{m}} & \dots & 0 \\ 0 & 0 & \sqrt{\frac{k}{m}} & \dots & 0 & 0 & -\sqrt{\frac{\kappa}{m}} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sqrt{\frac{k}{m}} & 0 & 0 & \dots & -\sqrt{\frac{\kappa}{m}} \end{pmatrix}$$

Our hamiltonian will be:

$$\mathbf{H} = \begin{pmatrix} \mathbf{0} & \mathbf{B} \\ \mathbf{B}^\dagger & \mathbf{0} \end{pmatrix}$$

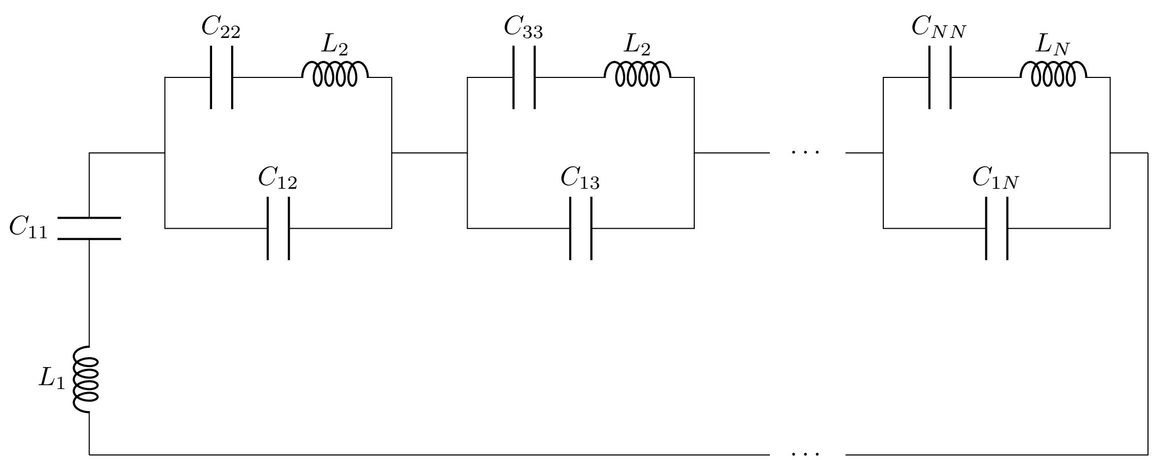
To find the Pauli string coeffitients, we use the Frobenius norm. The coefficient for i-th Pauli string will be achieved by

$$\alpha_i = \frac{\sqrt{Tr\left[H(\sigma_3^{(i)}\sigma_2^{(i)}\sigma_1^{(i)}\sigma_0^{(i)})^\dagger\right]}}{\sqrt{Tr\left[(\sigma_3^{(i)}\sigma_2^{(i)}\sigma_1^{(i)}\sigma_0^{(i)})(\sigma_3^{(i)}\sigma_2^{(i)}\sigma_1^{(i)}\sigma_0^{(i)})^\dagger\right]}}$$

$$\sigma^{(i)} \in \{I, X, Y, Z\}$$

Hamiltonian of a System of LC Circuits

Consider an LC circuit with the following components:



Let's write the equations of this system based on the charge on each branch. We have

$$L_1\ddot{q}_1 = \frac{q_2 - q_1}{C_{12}} + \frac{q_3 - q_1}{C_{13}} + \cdots + \frac{q_N - q_1}{C_{1N}} - \frac{q_1}{C_{11}}$$

$$L_2\ddot{q}_2 = \frac{q_1 - q_2}{C_{12}} - \frac{q_2}{C_{22}}$$

$$L_3\ddot{q}_3 = \frac{q_1 - q_3}{C_{13}} - \frac{q_3}{C_{33}}$$

$$\vdots$$

$$L_N \ddot{q_N} = \frac{q_1 - q_N}{C_{1N}} - \frac{q_N}{C_{NN}}$$

In order to solve this problem, we need to define:

$$m_j = L_j, \quad \kappa_{jk} = \frac{1}{C_{jk}}$$

So, the matrix B need to have:

$$\sqrt{\mathbf{M}}\mathbf{B}|1,1\rangle = \frac{1}{\sqrt{C_{11}}}|1\rangle$$

$$\sqrt{\mathbf{M}}\mathbf{B}|2,2\rangle = \frac{1}{\sqrt{C_{22}}}|2\rangle$$

$$\vdots$$

$$\sqrt{\mathbf{M}}\mathbf{B}|N,N\rangle = \frac{1}{\sqrt{C_{NN}}}|N\rangle$$

$$\sqrt{\mathbf{M}}\mathbf{B}|1,2\rangle = \frac{1}{\sqrt{C_{12}}}(|1\rangle - |2\rangle)$$

$$\sqrt{\mathbf{M}}\mathbf{B}|1,3\rangle = \frac{1}{\sqrt{C_{13}}}(|1\rangle - |3\rangle)$$

$$\vdots$$

$$\sqrt{\mathbf{M}}\mathbf{B}|1,N\rangle = \frac{1}{\sqrt{C_{1N}}}(|1\rangle - |N\rangle)$$

So, the matrix B is:

$$\mathbf{B} = \begin{pmatrix} \frac{1}{\sqrt{L_1 C_{11}}} & 0 & 0 & \dots & 0 & \frac{1}{\sqrt{L_1 C_{12}}} & \frac{1}{\sqrt{L_1 C_{13}}} & \dots & \frac{1}{\sqrt{L_1 C_{1N}}} \\ 0 & \frac{1}{\sqrt{L_2 C_{22}}} & 0 & \dots & 0 & -\frac{1}{\sqrt{L_2 C_{12}}} & 0 & \dots & 0 \\ 0 & 0 & \frac{1}{\sqrt{L_3 C_{33}}} & \dots & 0 & 0 & -\frac{1}{\sqrt{L_3 C_{13}}} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \frac{1}{\sqrt{L_N C_{NN}}} & 0 & 0 & \dots & -\frac{1}{\sqrt{L_N C_{1N}}} \end{pmatrix}$$

and the rest can be easily implemented as well.