A Splitting Algorithm for Minimization under Stochastic Linear Constraints

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Many applications in machine learning, statistics or signal processing require the solution of the following optimization problem :

$$\min_{(x,z)\in\mathsf{X}\times\mathsf{Z}} F(x) + G(z), \quad \text{s.t} \quad \mathsf{A}x + \mathsf{B}z = \mathsf{c}$$

where X, Z are Euclidean spaces, F, G are convex functions, A, B are matrices and c is a vector. In order to solve this problem, primal-dual methods typically generate a sequence of primal estimates $(x_n, z_n)_{n\in\mathbb{N}}$ and a sequence of dual estimates $(\lambda_n)_{n\in\mathbb{N}}$ jointly converging to a saddle point of the Lagrangian function.

We consider the case where *all* the quantities used to define the minimization problem are likely to be unavailable : $F, G, \mathsf{A}, \mathsf{B}, \mathsf{c}$ are define as expectations. These expectations are unknown but revealed across time through i.i.d realizations of a random variable. Among the instances of this problem are the Markowitz portfolio optimization and large scale minimization problems.

We provide a new stochastic primal dual algorithm and establish its a.s convergence. It generalizes the stochastic proximal gradient algorithm.