Design of a noise barrier wall using fractal protections

HARESS El Mehdi

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1. Presentation of the problem

1.1. Noise pollution

In France, about 12% of the population is exposed to noise levels exceeding the threshold of 65 dB(A) during the day and is therefore subject to a high disturbance [1]. Road noise is the main contributortthis noise population. For this reason, circumstantial measures have been taken to limit this noise. The most effective and practical measure is the construction of noise barriers.

Noises spread very easily and the distance does not decrease their power enough to eliminate their nuisance. That is why noise barriers are not built over large distances away from the source of the noise, but the protection is achieved close to the source, or in its immediate vicinity.

Thus, in this report, our goal is to build a noise barrier on a road and test its performance. For this we will need to model it in a more specific way.

1.2. Noise barrier

It is not an easy task to build a noise barrier. The difficulty lies mainly in the choice of materials and in the choice of the shape of the wall. The choice of the material determines the dissipativity of the wall but the shape plays another role.

Indeed, one can imagine that a complex shape could somehow "enclose" the sound waves coming from the source. We speak more precisely of "localization".

The idea is to increase the surface of the wall (the surface of contact with the wave) without changing its volume. To achieve this, fractal protections are used again.

Indeed, we know that to locate a wave of frequency λ, we can choose a fractal of characteristic size $\lambda/2$.

1.3. Fractals

A fractal figure is a mathematical object, such as a curve or a surface, whose structure is invariant by change of scale. In this report, we do not deal with the problem of optimizing the shape of the wall, i.e. choosing an optimal fractal for the wall we want to build, so we will set a fractal and work with it throughout the report.

This is the "broccoli" fractal. Mathematically it is first of all a question of defining a transformation g on a segment $[a, b]$ such that $g(a) = g(b)$. Then we define a sequence of function $f0 = cst$ on $[a, b]$ and construct f_n+1 by applying g in each segment where f_n is differentiable.

The basic transformation of the "broccoli" fractal is as follows

FIGURE 1 - "Broccoli" fractal pattern

In this report, we consider up to 2nd generation of the fractal for our applications. Expressing the fractal using mathematical functions facilitates its coding in Python or Matlab for example. However, one can also code the fractal by writing a code that returns a list of all the points that make up the fractal (an ordered list, for example, by browsing the fractal from left to right).

FIGURE 2 -Second generation of the Brocoli fractal

Although this fracral is very nice, it is not completely suitable for the our applications.

We have one problem. In our case, we do not want to change the volume of our wall. That's why we don't choose exactly the following pattern but rather a pattern we will can « inverted Brocoli ».

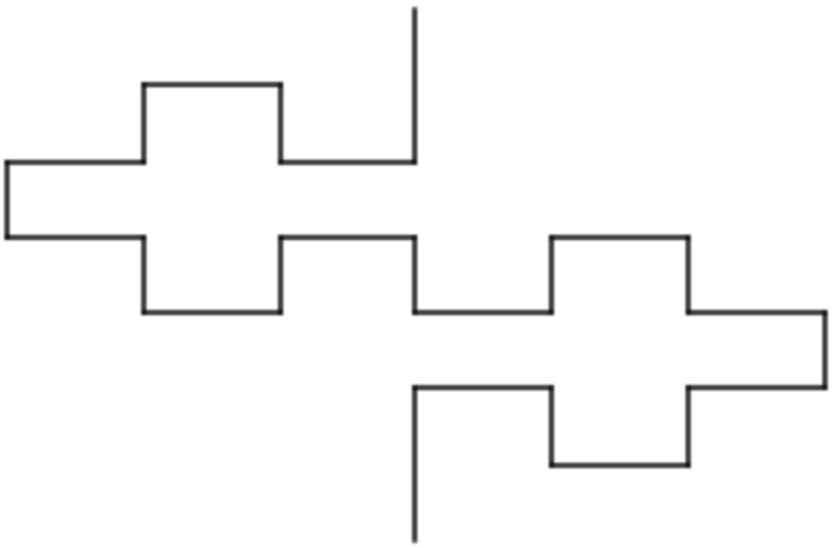

FIGURE 3 - New Pattern

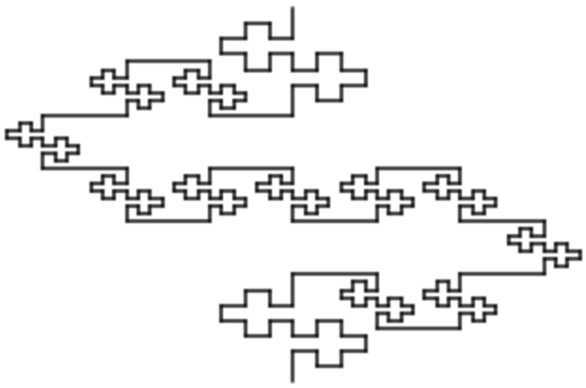

FIGURE 4 -Second generation of the New Fractal

1.4. Definition of the problem

Finally, we can define from before correctly the problem we will deal with. First, we will treat the problem in 2D. Our domain Ω will be a rectangle. The left side ΓD will represent the road and therefore the source of the noise. The right side Γ will represent the wall and therefore our fractal. It's like taking a picture from a point of view at the height of the road and neglecting the reflections of the wave on the ground.

Obviously, there is interest in building the wall along a limited length of the road because that is where the noise is significant. This is modeled by the bottom and top sides of the ΓN rectangle, considering that there is no exchange with what is outside the rectangle.

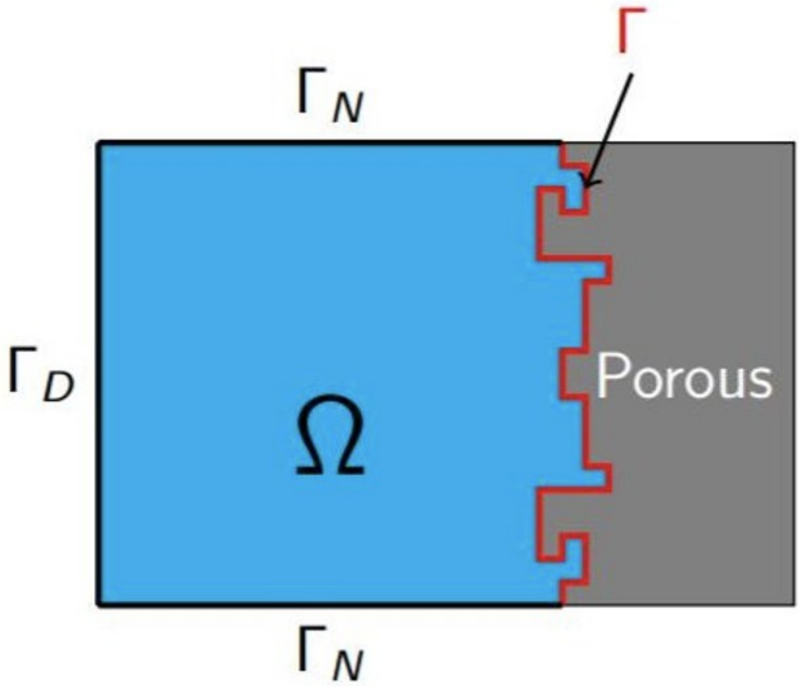

FIGURE 5 - Problem Modeling

1.5. Specification of some parameters

We progress towards the definition of a mathematical problem that can be solved. Physically, we want our wall to be protected against noises that have a frequency in a given range. It is the intersection between the frequency range that are 'annoying' and those emitted by cars. The noise emitted by a vehicle is mainly due to the motor, the rolling on the road and the airborne emissions.

People are sensitive to low frequencies, we can choose the following frequency interval: [100Hz,1KHz]. Indeed as can be seen in figure 6, this interval is 6dB wide, we consider that this is sufficient for our application (note that the power is divided by 4).

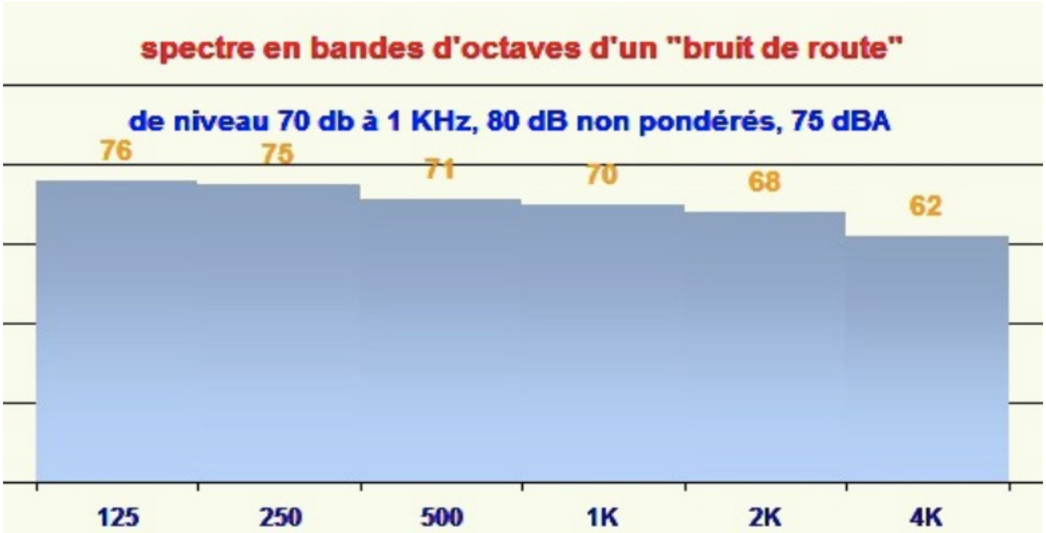

FIGURE 6 - Road noise spectrum [2].

Next, we have to choose how to model the noise source. So we make the hypothesis that the source noise does not depend on the position in the wall. Indeed, vehicle noise is harmful when there are many vehicles on the road (so the noise does not depend on the position) or when several vehicles pass at a very high speed, we can assume that in stationary state, the noise is the same everywhere on the road (assuming that all vehicles emit the same noise).

Finally, we assume that our noise depends on the frequence. Low frequency noise emitted by cars tends to be louder than high frequency noise.

2. Wall performance

2.1. First of all, which material to choose?

Three types of materials are studied: ISOREL, ITFH and B5. These are porous, hydrogen and anisotropic. As mentioned above, these materials will determine the absorptivity of the wall. The characteristics we are interested in are: resistivity, porosity and tortuosity. From these three values, we will be able to calculate a complex coefficient a that writes the contact of the wave with our wall. The real part of this coefficient reflects the reflection on the wall and the imaginary part reflects the absorption of the wall. What will interest us will be the relationship between these two values: On average, does our wall have more tendency to absorb or to reflect?

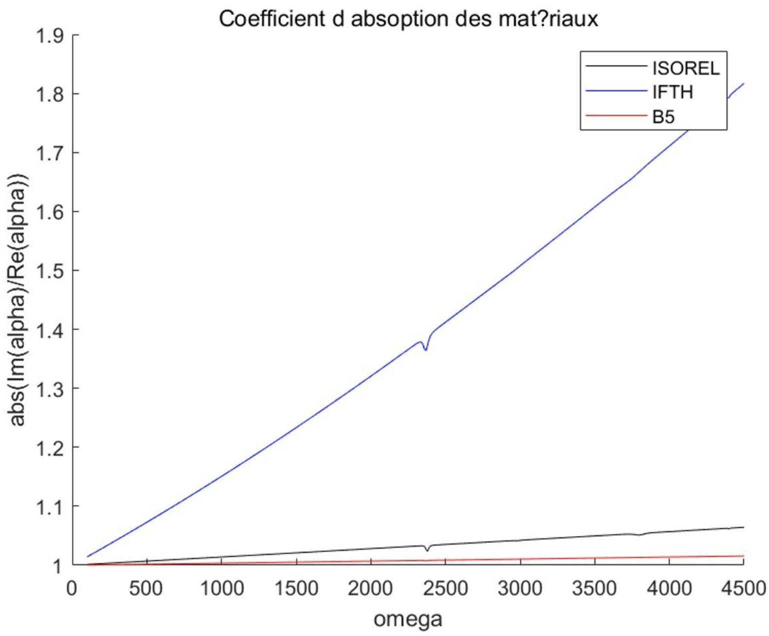

FIGURE 7 - Absorption coefficient a

We notice that in the range of frequencies that interest us, the ITFH is more efficient. Our wall will therefore be built in ITFH.

2.2. Location of eigenmodes

It is believed that choosing a complex geometry for the wall allows better localization of the wave. In this section, we will understand why. The *u-wave* propagating from the source to the wall/fractal is the solution of a 'differential equation'. This solution can be decomposed as a linear combination of eigenmodes. This is why we are interested in those that are localized. The existence of several localized eigenmodes allows us to hope that our solution will be localized too. Indeed, it is enough that there is only one localized eigenmode that dominates in the expression to have a localized solution.

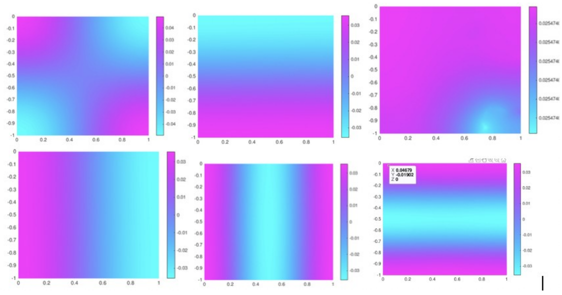

FIGURE 8 - Some eigenmodes for a flat wall

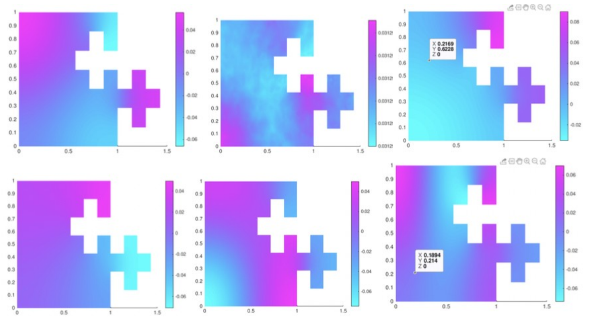

FIGURE 9 - Some eigenmodes for the first generation

In these figures, we see that the more fractal it is, the more localized the wave is.

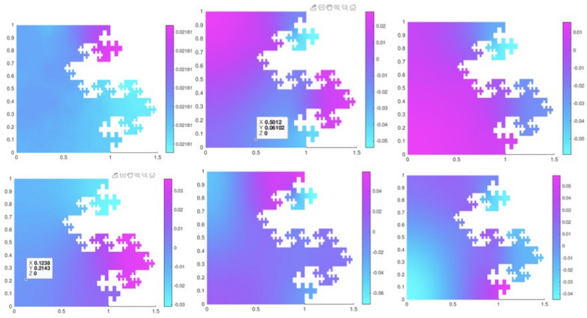

FIGURE 10 - Some eigenmodes for the second generation

2.3. Surfaces of Existence and Dissipation Energy

In order to better describe the phenomenon of localization, we introduce the notion of existence surface. The existence surface represents the effective surface area occupied by the eigenmode. So naturally, the smaller the area, the more localized the eigenmode will be.

For example, here are two figures (11 and 12)wthe surface is represented of existence as a function of the n-th proper mode, for the plane wall and the second generation.

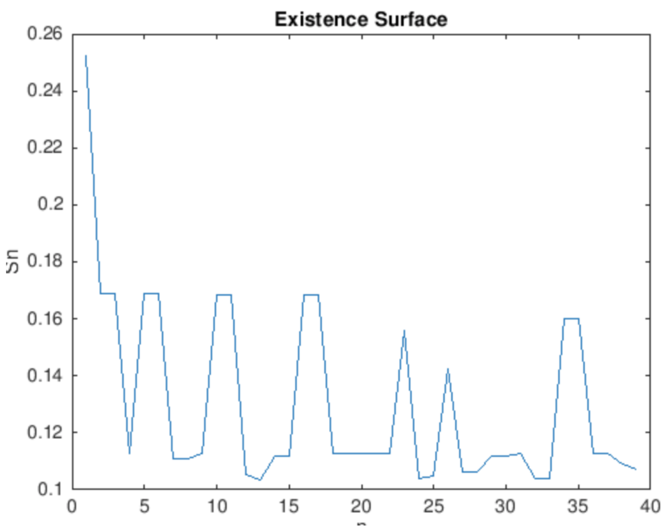

FIGURE 11 - Existence surface for the flat wall

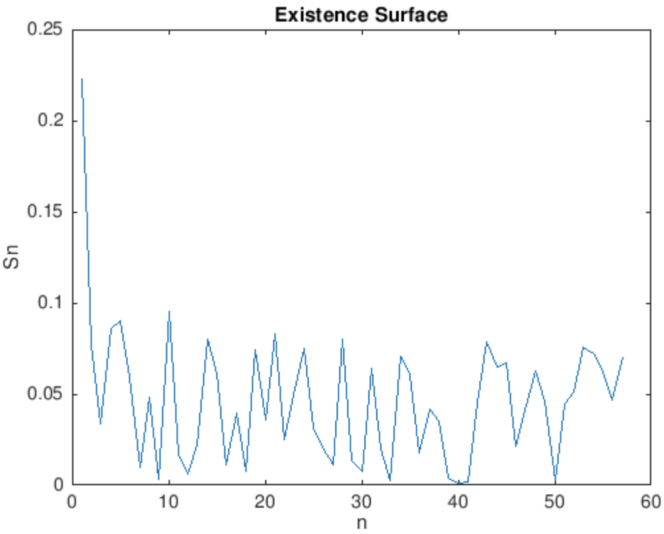

FIGURE 12 - Existence area for the second generation

Then, the dissipation energy w is introduced, which is equal to the total energy of the square module of the eigenmode on the wall.

This energy describes how much the wall "absorbs" the waveThe bigger it is, the more the wall does its job.

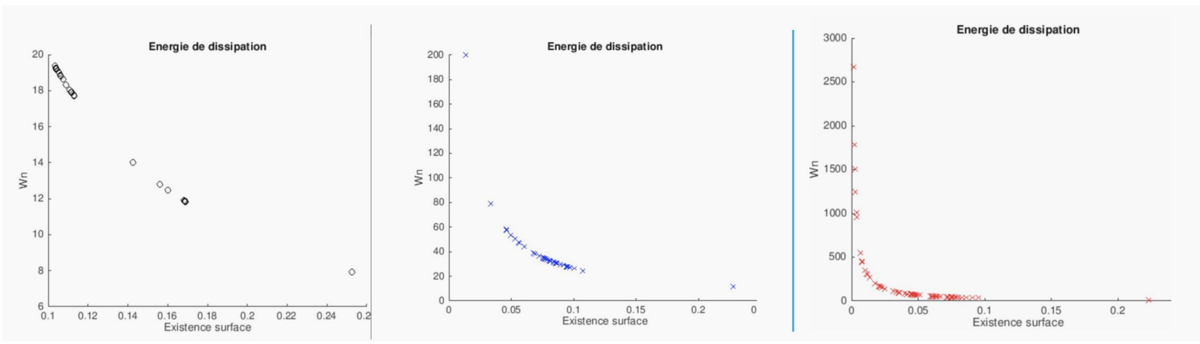

FIGURE 13 - Dissipation energy as a function of the existence surface for the plane wall, the first and second fractal respectively.

These figures explain the importance of localized modes: avery high dissipation energy!

2.4. Energy of the wave

In this part, we no longer deal with the modes themselves, but we are interested in the solution of our problem. One way to compare the performance of the different generations is to calculate the energy of our solution with respect to the frequencies. Indeed, this allows us to see that in the range of frequencies that we set, the energy of the wave tends to fall for high generations.

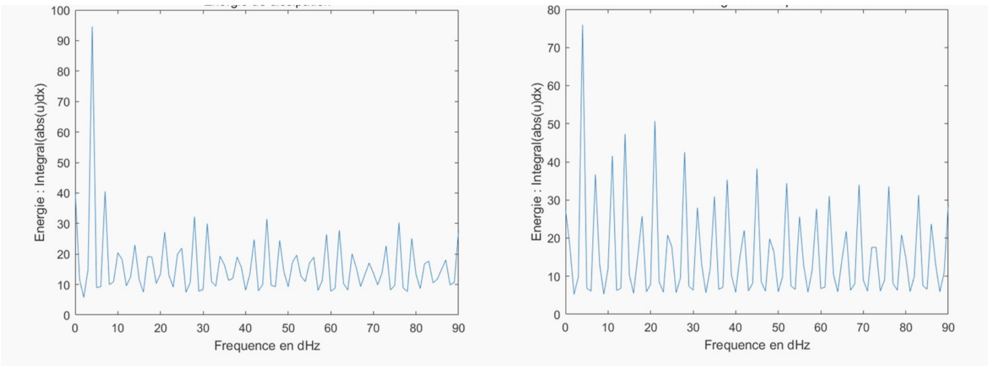

FIGURE 14 - Wave energy for the flat wall and the first generation

The energy is therefore tending to decrease but we notice something of abnormal. For low frequencies (< *20dHz),* the energy of the flat wall is smaller than that of the first generation. This may be due, for example, to the fact that the chosen shape is not optimal. However, for the second generation, it is clearly noticeable that the energy is decreasing.

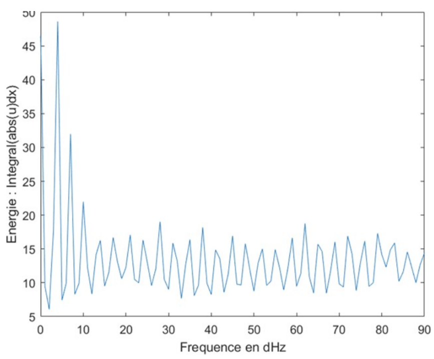

FIGURE 15 - Wave energy for the second generation

3. Mathematical models

3.1. Equations

The problem we are solving is the following: (also called Helm- Holtz equation)

$$\begin{cases} \Delta u + \omega^2 u = f & x \in \Omega \\ u = g(\omega) & on\ \Gamma_D \\ \frac{\partial u}{\partial n} = 0 & on\ \Gamma_N \\ \frac{\partial u}{\partial n} + \alpha(\omega) u = 0 & on\ \Gamma,\ Re(\alpha) < 0\ and\ Im(\alpha) > 0 \end{cases}$$

ω is a frequency selected from our range of frequencies set at the beginning. g and f are terms that represent the source. Either we represent the source by its value at the edge, or by the way it propagates.

In our case, we choose to put f =0 and set $g(\omega)$=cste/w. a is the coefficient

of absorption mentioned above for ITFH material. Since $Im(\alpha) > 0$, there will always be exchanges at wall level and therefore the model is quite dissipative.

3.2. Theoretical solution

One might think that this is a classic problem with Dirichelt or Neumann conditions on the edges, but this is not quite the case. Indeed, classical theory does not allow to solve equations on domains Ω such that the edge ∂Ω is not regular (Lipschitz). In particular, defining a vector that is orthogonal to the surface of a domain that is not regular is not an easy task.

Fortunately, when ∂Ω is a fractal, the orthogonal derivative exists in the weak sense (as a linear form on the image of the operator of the trace on the fractal). We can thus define a precise theoretical framework to solve these equations in the weak sense (variational formulation) where we have Green's formula and the discretization of the Laplacian operator.

4. Numerical methods

4.1. Solving the Helmholtz Equation

To solve the Helmholtz equation, a finite discretization is realized. We define a rectangle R which contains our domain Ω and a matrix D which indicates whether the element $(i, j), i, j\ N$ *of* our mesh belongs to Ω or not.

First, we explain how to compute the Laplacian matrix in the case Ω coincides with the inside of R by explaining how to approximate the edge points (i.e. $i = 1, i = N, j = 1, j = N$).

Then, in case Ω does not coincide with the inside, we use the same discretization on ω than before, but with the following subtlety: if we are on a point (i, j) that is in Ω but has one of its neighbors outside of Ω, we use the above mentioned edge conditions to approximate those neighbors.

Thus, we can define the Laplacian matrix and solve our problem numerically (by writing an algorithm that calculates this matrix for example). The convergence schemes will not be different from the classical case. For more information, see [3] where the authors give examples of exact solutions and approximate solutions of the same problem.

4.2. Resolution using the PDE toolbox

If you use Matlab, you have the PDE toolbox which is a very powerful tool for solving partial differential equations on complex geometries.

In the PDE toolbox, you can define the geometry you want (to define the fractal, simply enter all the points that make up the fractal). Then, we can put the conditions we want (Dirichlet, Neumann, general Neumann) on the different sides of our domain. Then, we can specify the different coefficients of the equation in question. Finally, we can define our mesh, the initial conditions and solve our problem.
equation.

5. Conclusion

— Road noise is a problem that needs to be solved.
— Fractal protections are widely used to build noise barriers.
— Fractals are very complex and irregular structures.
— Their geometry allows to locate the sound waves and their material increases their absorption.
— Walls built with fractals perform better.
— There is a mathematical framework that allows the definition of the wave propagation equation on a fractal edge.
— Numerical tools exist to solve differential equations on complex geometries, especially fractals.

6. References

1. Noise Information Center - Road Traffic
2. http ://www.acouphile.fr/bruit-trafic.html

3. Thi Phuong Kieu Nguyen. Shape optimization for the Helmholtz equa-tion with the Robin boundary condition. Ecole Centrale De Paris Laboratory of Applied Mathematics and Systems. 2011