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# UNINSURED IDIOSYNCRATIC RISK AND AGGREGATE SAVING\*

S. RAO AIYAGARI

We present a qualitative and quantitative analysis of the standard growth model modified to include precautionary saving motives and liquidity constraints. We address the impact on the aggregate saving rate, the importance of asset trading to individuals, and the relative inequality of wealth and income distributions.

## I. INTRODUCTION

This paper has two main goals. The first is to provide an exposition of models whose aggregate behavior is the result of market interaction among a large number of agents subject to idiosyncratic shocks. This class of models involves a considerable amount of individual dynamics, uncertainty, and asset trading which is the main mechanism (in the models) by which individuals attempt to smooth consumption. However, aggregate variables are unchanging. This contrasts with representative agent models in which individual dynamics and uncertainty coincide with aggregate dynamics and uncertainty. The exposition is motivated by two facts: (i) the behavior of individual consumptions, wealths, and portfolios is strongly at variance with the complete markets model implicit in the representative agent framework; and (ii) recently several authors have found versions of such models useful for analyzing a variety of issues including asset pricing, monetary policy, business cycles, and taxation.<sup>1</sup>

The exposition is built around the standard growth model of Brock and Mirman [1972] modified to include a role for uninsured idiosyncratic risk and liquidity/borrowing constraints. This is done by having a large number of agents who receive idiosyncratic labor endowment shocks that are uninsured, as in the models of Bewley [1986, undated]. As a result of this market incompleteness in combination with the possibility of being borrowing constrained in future periods, agents accumulate excess capital in order to smooth consumption in the face of uncertain individual labor incomes. The

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1. See, for example, Aiyagari [1994], Diaz-Gimenez and Prescott [1992], Imrohroglu [1989, 1992], Aiyagari and Gertler [1991], and Huggett [1990].

interest rate, aggregate capital, and the wealth distribution are all jointly determined in the presence of precautionary motives and borrowing constraints.

The second main goal of this paper is to use such a model to study the quantitative importance of individual risk for aggregate saving. This study is motivated by the debate concerning the sources of aggregate capital accumulation, in particular, the suggestion that precautionary saving may be a quantitatively important component of aggregate saving.<sup>2</sup> For example, Modigliani [1988, p. 39] argues that the pure bequest motive is likely important only for people in the highest income and wealth brackets and that "some portion of bequests, especially in lower income brackets, is not due to a pure bequest motive but rather to a precautionary motive reflecting uncertainty about the length of life, although it is not possible at present to pinpoint the size of this component." Several other authors have suggested that the precautionary motive may contribute importantly to wealth accumulation. For example, Zeldes [1989a, p. 289] has conjectured that "a significant fraction of the capital accumulation that occurs in the United States may be due to precautionary savings." Skinner [1988] and Caballero [1990] contain similar suggestions.

The following are some of the key features of our exercise: (i) endogenous heterogeneity, (ii) aggregation, (iii) infinite horizons, (iv) borrowing constraint, and (v) general equilibrium, i.e., endogenously determined interest rate. In a steady state consumers face a constant interest rate because the shocks are purely idiosyncratic and, hence, there is no aggregate uncertainty. For a given interest rate optimal individual saving behavior leads to a distribution of agents with different levels of assets reflecting different histories of labor endowment shocks. Aggregation implies some level of per capita assets. In a steady-state equilibrium the per capita amount of capital must equal the per capita asset holdings of consumers, and the interest rate must equal the net marginal product of capital (as determined by a standard neoclassical production function). These features in combination explain why the interest rate is necessarily less than the time preference rate and, hence, the aggregate capital stock and the saving rate are necessarily greater than under certainty (equivalently, complete markets). In particular, the convexity of marginal utility (which is the traditional criterion for generating precautionary saving) becomes unneces-

2. See Kotlikoff and Summers [1981], Kotlikoff [1988], and Modigliani [1988].

sary for this result once features (i)–(v), especially (iii) and (iv), are taken into account.

The quantitative results of this paper suggest that the contribution of uninsured idiosyncratic risk to aggregate saving is quite modest, at least for moderate and empirically plausible values of risk aversion, variability, and persistence in earnings.<sup>3</sup> The aggregate saving rate is higher by no more than three percentage points. However, for sufficiently high variability and persistence in earnings, the aggregate saving rate could be higher by as much as seven to fourteen percentage points.

Some additional implications of our analysis are as follows. In contrast to representative agent models (see Cochrane [1989]), it turns out that access to asset markets is quite important in enabling consumers to smooth out earnings fluctuations. In one example, by optimally accumulating and decumulating assets, an individual can cut consumption variability by about half and enjoy a welfare gain of about 14 percent of per capita consumption, or about 8 percent of per capita GNP, compared with a situation in which he had no access to asset markets.<sup>4</sup>

The model is also consistent, at least qualitatively, with certain features of income and wealth distributions. The distributions are positively skewed (median < mean), the wealth distribution is much more dispersed than the income distribution, and inequality as measured by the Gini coefficient is significantly higher for wealth than for income.

The rest of this paper is organized as follows. In Section II we review the relevant empirical and theoretical-quantitative literature which suggests that the precautionary motive and liquidity constraints may be important for a variety of phenomena. In

3. We use a trend stationary specification of the individual earnings process. Since the shocks are purely idiosyncratic (by assumption), per capita earnings will be growing deterministically. Therefore, the empirically reasonable requirement that the cross-section distribution of earnings, wealth, income, consumption, etc. (normalized by the respective per capita values) be stationary requires that the individual earnings process be trend stationary.

4. The above calculation requires a complete model since good data on consumption at the individual level are not available. Otherwise, one could use the consumption data to get an idea of consumption variability at the individual level and combine it with some specification of the utility function to obtain an estimate for the welfare gain as in Lucas [1985]. Since data on earnings are available, this can be used together with a complete model to estimate how much individual consumption varies in a stochastic steady-state equilibrium. The present model seems much more appropriate for addressing this type of question than a representative agent model because in a representative agent model agents face only aggregate risk and this seems quite unrealistic. The representative agent model may be a useful abstraction for other questions but not for this one.

Section III we offer an exposition of models with uninsured idiosyncratic risk and liquidity constraints. In Section IV we describe the specification and parameterization and the computational procedure. Section V contains the results, and Section VI concludes with some suggestions for further work.

## II. PRECAUTIONARY MOTIVE AND LIQUIDITY CONSTRAINTS

There is a considerable literature that emphasizes precautionary savings and liquidity/borrowing constraints for understanding household consumption/saving behavior as well as a variety of aggregate phenomena.<sup>5</sup> The behavior of individual consumptions, wealths, and portfolios are at considerable variance with the predictions of complete markets models.<sup>6</sup>

Casual empiricism as well as formal evidence indicates that individual consumptions are much more variable than aggregate consumption [Barsky, Mankiw, and Zeldes 1986; Deaton 1991]. Further, individual consumptions are not very highly correlated either with each other or with aggregate consumption as would be the case with complete frictionless Arrow-Debreu markets. This suggests that heterogeneity due to incomplete markets may be important. Heterogeneity is clearly necessary for studying the importance of borrowing constraints.

Further and more detailed evidence for the importance of precautionary saving is described by Carroll [1991] and summarized below. Individual wealth holdings appear to be highly volatile with large fractions of households moving from one wealth decile to another over a few years.<sup>7</sup> It would be hard to explain such mobility across the wealth distribution over a fairly short period of time (suggesting that age and life-cycle-related factors are not the reasons) in the absence of temporary idiosyncratic shocks. Avery, Elliehausen, and Canner [1984] present evidence to the effect that

5. A partial list of such work includes Aiyagari [1994], Diaz-Gimenez and Prescott [1992], Aiyagari and Gertler [1991], Carroll [1991], Deaton [1991], Caballero [1990], Huggett [1990], Zeldes [1989a, 1989b], Imrohoroglu [1989, 1992], Kimball and Mankiw [1989], and Scheinkman and Weiss [1986].

6. See Aiyagari [1993b, pp. 22–24] for a lengthier discussion of the predictions of models with complete frictionless markets and their empirical shortcomings.

7. According to Avery and Kennickell [1989], 60 percent of households were in a different wealth decile in 1985 than in 1982. Approximately 30 percent moved up, and 30 percent moved down. Only people in the topmost and the bottommost deciles were more likely to stay put than move to another decile.

the ratio of median wealth to median income is higher for individuals in occupations with greater income uncertainty, e.g., farmers and self-employed businessmen.

The evidence on portfolios indicates considerable diversity in portfolio compositions for households with different wealth levels. Mankiw and Zeldes [1991] present evidence that only about 25 percent of U. S. households own any stocks in spite of the fact that the expected return on stocks has been so much higher than the risk-free rate. According to evidence presented by Avery, Elliehausen, and Kennickell [1988], the ownership of stocks is highly concentrated at the top end of the wealth distribution whereas the ownership of liquid assets is concentrated in the lower portion of the wealth distribution.<sup>8</sup> The portfolios of households with low wealth contain a disproportionately large share of low return risk-free assets and a disproportionately small share of high return risky assets. The portfolios of high wealth households exhibit the opposite characteristic.<sup>9</sup> Such wide disparities in portfolio compositions would be hard to explain under complete frictionless markets assuming that individuals have roughly constant and equal relative risk aversion coefficients. Last, it would be hard to reconcile the vast amount of trading in asset markets and the pattern of transaction velocities across assets with a complete frictionless markets story.

The above facts constitute quite strong a priori evidence in favor of the importance of uninsured idiosyncratic risk.

In the next section we provide an intuitive overview of the workings of general equilibrium dynamic economies with heterogeneous agents, uninsured idiosyncratic shocks, and borrowing constraints.

8. For example, the top 1 percent of wealth holders own about 60 percent of all equity, but only about 10 percent of all liquid assets. In contrast, the bottom 90 percent of households own about 53 percent of all liquid assets and only about 9 percent of all equity. Greenwood [1983] presents similar evidence to the effect that the top 5 percent of wealth holders own about 85 percent of all corporate stock and about 60 percent of all debt instruments [Table 4, p. 35; Figure 2, p. 34].

9. Kessler and Wolff [1991] calculate that the lowest wealth quintile's portfolio contains over 80 percent of liquid assets (currency, demand deposits, and time deposits), only about 9 percent of financial securities and corporate stock, and only about 3 percent of other real estate (i.e., not including housing) and unincorporated business. In contrast, the highest wealth quintile's portfolio contains only about 15 percent of liquid assets, about 22 percent of financial securities and corporate stock, and over 42 percent of other real estate and unincorporated business [Table 6, p. 263]. Similar evidence is presented in Mankiw and Zeldes [1991].

### III. ECONOMIES WITH HETEROGENEOUS AGENTS, UNINSURED IDIOSYNCRATIC SHOCKS, AND BORROWING CONSTRAINTS: AN EXPOSITION

The chief ingredient in this class of models is the “income fluctuation problem.”<sup>10</sup> In this problem an individual facing uncertain earnings and a constant return on assets makes consumption and asset accumulation/decumulation decisions optimally in order to maximize the expected value of the discounted sum of one-period utilities of consumption. The individual may be permitted to borrow (hold negative assets) up to some limit. Under some conditions this is a well-defined problem and gives rise to unique decision rules and a unique long-run distribution of asset holdings, and, hence, unique long-run average asset holdings.

The solution of this problem can be turned into a stochastic steady state of a general equilibrium dynamic capital accumulation model in the following way. Imagine that there is a continuum of individuals (of size unity) subject to idiosyncratic earnings uncertainty and among whom asset holdings are distributed according to the long-run distribution mentioned above. By construction, then, the cross-section distribution of assets will be constant over time even though individual asset holdings vary stochastically over time. Further, the long-run average asset holdings for an individual will equal the constant per capita assets of the population.

We now introduce a neoclassical aggregate production function into this economy in which per capita output depends on per capita capital (the only outside asset) and per capita labor supply. Idiosyncratic earnings uncertainty is generated by assuming that individual labor supplies are randomly inelastic and independent across agents. Due to the idiosyncratic nature of the labor supply shocks, the expected value of labor supply for an individual equals the per capita labor supply. Therefore, per capita labor supply is constant and may be normalized to unity.<sup>11</sup> Individual earnings are then given by the wage (which equals the marginal product of labor) times individual labor supply. Last, the return on assets faced by individuals must equal the net marginal product of capital, and the per capita amount of capital must equal per capita asset holdings.

In the absence of earnings uncertainty (equivalently, with full

10. See Schechtman and Escudero [1977]. The ensuing exposition is based on results from that paper and from Clarida [1987, 1990] and Bewley [1986, undated]. See also Laitner [1992].

11. An equivalent description is to imagine that individual labor supplies are inelastic at unity but that individual productivities are idiosyncratically random and that average productivity is normalized to unity.



insurance markets) all agents are alike and face no uncertainty. The model collapses to the representative agent Brock-Mirman [1972] model of capital accumulation whose steady state is characterized by an interest rate equal to the rate of time preference and per capita capital given by the modified golden rule. However, with idiosyncratic earnings uncertainty and no insurance markets, the combination of the precautionary motive and limited borrowing leads to an interest rate lower than the rate of time preference and, therefore, to a per capita capital higher than the modified golden rule capital.<sup>12</sup> Aggregate saving and the saving rate are higher.

To see these points more clearly, we start by describing the income fluctuation problem and some properties of its solution.<sup>13</sup>

### *The Individual's Problem*

For simplicity, we assume that labor endowment shocks (equivalently, earnings) are i.i.d. over time.<sup>14</sup> We also permit some borrowing.<sup>15</sup> Let  $c_t$ ,  $a_t$ , and  $l_t$  denote period  $t$  consumption, assets, and the labor endowment. Let  $U(c)$  be the period utility function,  $\beta$  be the utility discount factor with  $\lambda = (1 - \beta)/\beta > 0$  being the time preference rate,  $r$  be the return on assets, and  $w$  be the wage. The individual's problem is to maximize

$$(1a) \quad E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\}$$

subject to

$$(1b) \quad c_t + a_{t+1} = wl_t + (1 + r)a_t; \\ c_t \geq 0, a_t \geq -b, \text{ almost surely (a.s.),}$$

where  $b$  (if positive) is the limit on borrowing and  $l_t$  is assumed to be i.i.d. with bounded support given by  $[l_{\min}, l_{\max}]$ , with  $l_{\min} > 0$ .

Some discussion of the borrowing constraint seems appropriate here. Clearly, if  $r < 0$ , some limit on borrowing is required. Otherwise the problem is not well posed, and a maximum does not exist. The present value of earnings is infinite (a.s.) and nothing prevents the individual from running a Ponzi scheme. If  $r > 0$ , then a less restrictive alternative to imposing a borrowing limit is

12. Throughout this paper we abstract from technical progress and aggregate growth. It is straightforward to incorporate this and is indicated later.

13. The technical details can be found in the appendix to Aiyagari [1993a].

14. In our quantitative analysis we permit the labor endowment shock to be serially correlated so that anticipation effects will be present.

15. The purpose of permitting borrowing is to emphasize the point that it only serves to reduce aggregate saving and the saving rate. That is, the impact on the saving rate would be even less if borrowing were to be allowed.



to impose present value budget balance (a.s.). This is equivalent to requiring  $\lim a_t/(1+r)^t \geq 0$  (a.s.). In turn, this limit condition together with the nonnegativity of consumption is equivalent to the period-by-period borrowing constraint  $a_t \geq -wl_{\min}/r$  (a.s.).<sup>16</sup> Thus, a borrowing constraint is necessarily implied by nonnegative consumption. Further, if  $b$  exceeds  $wl_{\min}/r$ , then the borrowing limit  $b$  will never be binding, and  $b$  may be replaced by the smaller amount  $wl_{\min}/r$ . Therefore, without loss of generality we may specify the limit on borrowing as

$$(2a) \quad a_t \geq -\phi,$$

$$(2b) \quad \phi \equiv \min\{b, wl_{\min}/r\}, \text{ for } r > 0; \quad \phi \equiv b, \text{ for } r \leq 0.$$

If the borrowing limit  $b$  is tighter than  $wl_{\min}/r$  (e.g., if  $b$  is zero so that no borrowing is permitted), then the borrowing limit  $b$  may be regarded as ad hoc in the sense that it is not a consequence of present value budget balance and nonnegativity of consumption. Note that  $\phi$  is to be regarded as a function of  $b$ ,  $w$ , and  $r$ . Equation (2) will be referred to as a “fixed” borrowing limit. Figure IIa shows the typical shape of the borrowing limit as a function of  $r$  (the curve marked  $\phi$ ).

We now define  $\hat{a}_t$  and  $z_t$  as follows:

$$(3a) \quad \hat{a}_t = a_t + \phi,$$

$$(3b) \quad z_t = wl_t + (1+r)\hat{a}_t - r\phi,$$

where  $z_t$  may be thought of as the total resources of the agent at date  $t$ . Using (2) and (3), we can rewrite (1b) as follows:

$$(4a) \quad c_t + \hat{a}_{t+1} = z_t, \quad c_t \geq 0, \quad \hat{a}_t \geq 0,$$

$$(4b) \quad z_{t+1} = wl_{t+1} + (1+r)\hat{a}_{t+1} - r\phi.$$

Let  $V(z_t, b, w, r)$  be the optimal value function for the agent with total resources  $z_t$ . This function is the unique solution to the following Bellman's equation:

$$(5) \quad V(z_t, b, w, r) \equiv \max \left\{ U(z_t - \hat{a}_{t+1}) + \beta \int V(z_{t+1}, b, w, r) dF(l_{t+1}) \right\},$$

where the maximization on the right side is over  $\hat{a}_{t+1}$  subject to (4).

The optimal asset demand rule for an agent is obtained by solving the maximization on the right side of (5). This yields the

16. See Proposition 1 in Aiyagari [1993a]. Since  $r > 0$ , it is easy to see that the borrowing constraint implies the limit condition. To see the reverse, note that  $wl_{\min}/r$  is the maximum amount that the consumer can repay for sure without violating nonnegativity of consumption.

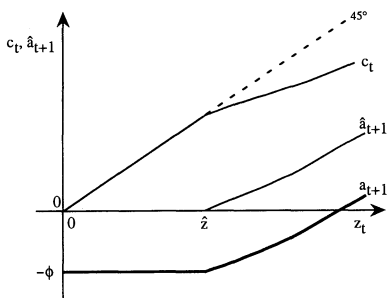


FIGURE 1a  
Consumption and Assets as Functions  
of Total Resources

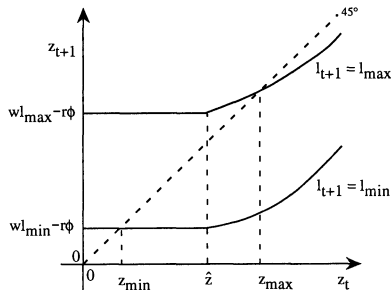


FIGURE 1b  
Evolution of Total Resources

following single-valued and continuous asset demand function:

$$(6) \quad \hat{a}_{t+1} = A(z_t, b, w, r).$$

Substituting (6) into (4b), we obtain the transition law for total resources  $z_t$ :

$$(7) \quad z_{t+1} = w l_{t+1} + (1 + r)A(z_t, b, w, r) - r\phi.$$

In Figures 1a and 1b we show some typical shapes for the functions on the right sides of (6) and (7), under the assumption that the interest rate  $r$  is less than the rate of time preference  $\lambda$ . Clearly, the agent would like to borrow but is limited by the borrowing limit. As total resources get smaller and smaller, the individual borrows more and more in order to maintain current consumption, and his debt approaches the borrowing limit. At some point when total resources are too low, it would be optimal to borrow up to the limit and consume all of the total resources. Thus, there exists a positive value  $\hat{z} > z_{\min} \equiv w l_{\min} - r\phi \geq 0$ , such that whenever  $z_t \leq \hat{z}$ , it is optimal to consume all the total resources (i.e., set  $c_t = z_t$ ) and set  $\hat{a}_{t+1}$  to its lowest permissible value, which is zero (see Proposition 3 in Aiyagari [1993a]). That is, it is optimal to exhaust the borrowing limit. For  $z_t \geq \hat{z}$ , both  $c_t$  and  $\hat{a}_{t+1}$  are strictly increasing in  $z_t$ ; i.e.,  $A(\cdot)$  is strictly increasing with a slope less than unity. In this situation the borrowing limit is not currently binding.<sup>17</sup>

17. The excess sensitivity of consumption to a transitory earnings innovation is apparent here. In the liquidity-constrained region consumption responds one- to-one even to transitory earnings shocks. Note that in this i.i.d. case, given current consumption, other currently known variables will not improve the forecast of future consumption. (This will not be true when the earnings shocks are serially correlated.) Thus, tests such as Hall's [1978] do not necessarily throw light on whether liquidity constraints are or are not important.

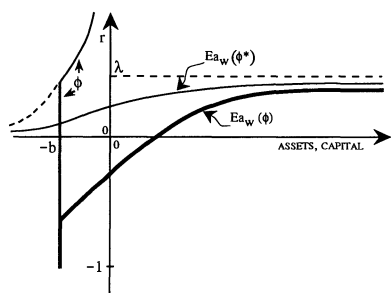


FIGURE IIa

Interest Rate versus Per Capita Assets

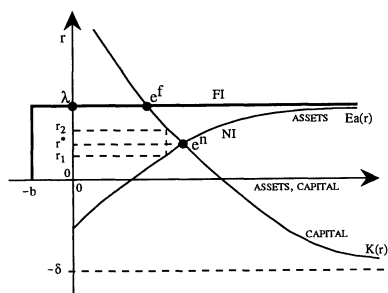


FIGURE IIb

Steady-State Determination

Under some additional assumptions, the support of the Markov process defined by (7) is bounded; specifically there is a  $z^*$  such that for all  $z_t \geq z^*$ ,  $z_{t+1} \leq z_t$  with probability one (see Figure Ib).<sup>18</sup> These conditions also guarantee that there exists a unique, stable stationary distribution for  $\{z_t\}$  which behaves continuously with respect to the parameters  $b$ ,  $w$ , and  $r$  (see Aiyagari [1993a], Proposition 5). Let  $Ea_w$  (the subscript reflecting the fact that for now  $w$  is being treated as fixed) denote long-run average assets. Using (3a) and (6), this is given by

$$(8) \quad Ea_w = E[A(z, b, w, r)] - \phi,$$

where  $E\{\cdot\}$  denotes expectation with respect to the stationary distribution of  $z$ .

### Endogenous Heterogeneity and Aggregation

The distribution of  $\{z_t\}$  and the value of  $Ea_w$  reflect the endogenous heterogeneity and the aggregation features mentioned in the introduction.  $Ea_w$  represents the aggregate assets of the population consistent with the distribution of assets across the population implied by individual optimal saving behavior. In Figure IIa we show a typical shape of the graph of  $Ea_w$  versus  $r$  (the curve marked  $Ea_w(\phi)$ ).

The most important feature of this graph is that  $Ea_w$  tends to infinity as  $r$  approaches the rate of time preference  $\lambda$  from below.<sup>19</sup>

18. The key condition here is that the relative risk aversion coefficient should be bounded [Aiyagari 1993a, Proposition 4]. This condition is violated by, for example, negative exponential utility, in which case there exist values of  $r$  below  $\lambda$  and a probability distribution for  $\{l_t\}$  with bounded support such that the consumer's assets will wander off to infinity a.s. (see Schechtman and Escudero [1977], pp. 159–61).

19. See Bewley [undated, Figure 1, p. 4] and Clarida [1990, Proposition 2.4, p. 548].  $Ea_w$  is a continuous function of  $r$  (and also of  $b$  and  $w$ ) but need not be monotone in  $r$ .

This reflects the infinite horizon of consumers. If  $r$  equals or exceeds the rate of time preference, then the individual will accumulate an infinitely large amount of assets, and  $Ea_w$  may be thought to be infinity. Intuitively, if  $r$  exceeds  $\lambda$ , then the individual wants to postpone consumption to the future and be a lender. The consumption profile will be upward sloping, and the agent will accumulate an infinitely large amount of assets to finance an infinitely large amount of consumption in the distant future.<sup>20</sup> This conclusion carries over to the borderline case of  $r$  equal to  $\lambda$ . In this case the consumer attempts to maintain a smooth marginal utility of consumption profile. At the margin it is costless for the consumer to acquire an additional unit of the asset. However, since there is a positive probability of getting a sufficiently long string of bad draws of labor shocks, maintaining a smooth marginal utility of consumption profile is only possible if the consumer has an arbitrarily large amount of assets to buffer the shocks.<sup>21</sup>

Next, note that for values of  $r < \lambda$ ,  $Ea_w$  is always higher under uncertainty than if earnings were certain, at least so long as  $r$  is not too much below  $\lambda$ ; i.e., assets are not too costly to hold. This result is independent of whether  $U'$  is convex or concave and arises due to the borrowing constraint and the infinite horizon. In a single consumer problem with a two-period horizon, there is no heterogeneity, and the borrowing constraint may be ignored by making suitable assumptions about the time profile of earnings. However, with an infinite horizon, repeated shocks, and  $r < \lambda$ , neither the heterogeneity nor the borrowing constraint can be ignored.

If earnings were certain (or, equivalently, markets complete),  $Ea_w$  would equal  $(-\phi)$  for all  $r < \lambda$ . That is, per capita assets under certainty are at their lowest permissible level since all agents are alike and everyone is constrained. However, in a steady state under

20. The first-order necessary condition (the Euler equation) for individual optimization is  $U'(c_t) \geq \beta(1+r)\bar{E}_t[U'(c_{t+1})]$ , with equality if  $\hat{a}_{t+1} > 0$ . Therefore, on average marginal utility is decreasing at least geometrically over time and will converge to zero (a.s.). More formally,  $\beta'(1+r)^t U'(c_t)$  is a nonnegative supermartingale and converges a.s. to a finite random variable. Since  $\beta(1+r) > 1$ , it follows that  $U'$  must converge to zero a.s. and, hence,  $c_t$  must converge to  $\infty$  a.s. It follows that  $z_t$  and  $\hat{a}_t$  also converge to infinity a.s. The conclusion holds also for the borderline case of  $\beta(1+r) = 1$ . See Chamberlain and Wilson [1984] for the details of the arguments.

21. It is easy to see that there cannot be a fixed point in Figure 1b corresponding to  $l_{t+1} = l_{\max}$  when  $\beta(1+r)$  is unity. If there is such a finite fixed point (denoted  $z_{\max}$ ), then we have (combining the envelope condition with the first-order condition for problem (5) and using the strict concavity of the value function):  $V'(z_{\max}) \geq E_t[V'(y_{t+1} + (1+r)A(z_{\max}))] > V'(y_{\max} + (1+r)A(z_{\max})) = V'(z_{\max})$  which is a contradiction. Consequently, the support of the distribution of  $\{z_t\}$  is unbounded. In fact,  $z_t$ ,  $c_t$ ,  $a_t$ , and  $\hat{a}_t$  all go to infinity a.s. Thus, there is a crucial difference between the solutions to the individual problem with and without uncertainty when  $\beta(1+r)$  equals unity.

incomplete markets, there is a distribution of agents with different total resources reflecting different histories of labor endowment shocks. Those with low total resources will continue to be liquidity constrained, whereas those with high total resources will accumulate assets beyond the constrained level regardless of the convexity of marginal utility simply because their current total resources are quite high relative to average future total resources.<sup>22</sup> Aggregation then implies that per capita assets must necessarily exceed their level under certainty.<sup>23</sup>

### General Equilibrium

The crucial features that explain how uninsured idiosyncratic shocks and borrowing constraints lead to higher aggregate saving are that  $Ea_w$  is finite only if  $r$  is less than  $\lambda$  and that it tends to infinity as  $r$  approaches  $\lambda$  from below. To see this, let  $f(k, 1)$  denote per capita output as a function of per capita capital ( $k$ ) and per capita labor (which equals unity), and let  $\delta$  be the depreciation rate of capital. Now consider the curve labeled  $K(r)$  in Figure IIb. This is a graph of  $k$  versus  $r$  defined by the marginal condition arising from producer profit maximization,  $r = f_1(k, 1) - \delta$ , which must hold in a steady state of this economy. Under standard assumptions, this curve is downward sloping, tends to  $\infty$  as  $r$  tends to  $(-\delta)$ , and tends to zero as  $r$  tends to  $\infty$ . In addition, we can express the wage  $w$  as a function of  $r$  since  $w$  equals  $f_2(k, 1)$ . Denote this by  $\omega(r)$ , which is a decreasing function under standard assumptions, tends to zero as  $r$  tends to  $\infty$ , and tends to  $\infty$  as  $r$  tends to  $(-\delta)$ . For a given  $r$ , let  $Ea$  denote the value of  $Ea_w$  when  $w$  equals  $\omega(r)$ , and let  $NI$  (for no

22. It should be noted that for  $r \in (0, \lambda)$  such that  $wl_{\min}/r < b$ , the borrowing constraint under incomplete markets is more stringent than under complete markets since under complete markets the borrowing limit would be  $\min\{b, w/r\}$  since  $E[1] = 1$ . This, in itself, makes per capita assets potentially higher under incomplete markets.

23. As can be seen in Figure IIa, if  $r$  is too low, then even under uncertainty everyone continues to be liquidity constrained, and per capita assets are the same as in the certainty case. This will happen for values of  $r$  which satisfy  $U'(wl_{\max} - r\phi) \geq \beta(1 + r)E_t[U'(wl_{t+1} - r\phi)]$ . A related question is what happens to  $Ea_w$  if uncertainty is increased. Sibley [1975] and Miller [1976] show that if  $U'$  is convex, then a mean-preserving spread in the distribution of  $\{l_t\}$  will lower the consumption function, or equivalently, raise the asset demand function (6). (In their papers  $r > 0$  and the borrowing constraint is of the form  $a_t \geq -wl_{\min}/r$ ). That is, the consumer will consume less and save more for each level of total resources  $z_t$ . (This may be thought of as the infinite horizon analogue of the standard two-period analyses of precautionary saving.) However, whether this will increase per capita assets or decrease them is hard to say since a shift in the asset demand function changes the stationary distribution of total resources and, thereby, per capita assets in a complicated way. The quantitative results reported in Section V always indicated that an increase in the variability of  $\{l_t\}$  shifted the  $Ea_w$  curve to the right.

insurance) be the graph of  $Ea(r)$  versus  $r$ .<sup>24</sup> A steady state of this economy is then characterized by the condition  $K(r) = Ea(r)$ . This is shown in Figure IIb by the intersection point (labeled  $e^n$ ) of these two curves.<sup>25</sup> Intuitively, one may think of  $K(r)$  as the capital desired by firms at the interest rate  $r$ , and  $Ea(r)$  as the capital supplied by households at the interest rate  $r$ .

Now consider what the steady state of the economy would be if there were no uncertainty, or, equivalently, there were full insurance markets. Then the economy consists of a representative agent who receives the constant earnings  $w$  in each period. If  $r$  is less than  $\lambda$ , the agent would always be up against his borrowing limit, and his asset holdings would be  $(-\phi)$ . If  $r$  equals  $\lambda$ , his asset holdings equal his initial holdings whatever they may be. If  $r$  exceeds  $\lambda$ , then he would accumulate an infinitely large amount of assets. Therefore, the right-angled line labeled  $FI$  (for full insurance) consisting of the horizontal segment at the height  $\lambda$  and the vertical segment corresponding to the binding borrowing constraint represents the individual's desired asset holdings as a function of  $r$ . The steady state of this full insurance economy is at the point  $e^f$ .

It is clear from the preceding argument that the aggregate capital stock is higher and the interest rate is lower in the economy with uninsured idiosyncratic shocks and borrowing constraints as compared with the standard economy.<sup>26</sup> The saving rate, which is given by  $\delta k/f(k,1)$  is also higher.<sup>27</sup>

24. This curve also has the same general shape as the  $Ea_w$  curve in Figure IIa in which  $w$  was treated as a separate parameter, instead of as a function of  $r$ . It is also continuous and tends to  $\infty$  as  $r$  tends to  $\lambda$  from below. It need not be monotonic. To understand this, suppose that  $b$  is zero,  $r$  is fixed, and the utility function is isoelastic. Then it is easy to see that an increase in  $w$  leads to a proportional increase in  $\hat{a}$ ,  $z$ ,  $c$ , and  $a$ . That is, the  $Ea_w$  curve shifts to the right with an increase in  $w$ . However, since  $\omega(r)$  is decreasing in  $r$ , it is quite possible that the  $NI$  curve is nonmonotonic in  $r$ .

25. Note that since the  $NI$  curve need not be monotonic there is no guarantee that the steady state is unique.

26. Deterministic growth in the aggregates can easily be accommodated in the same way as it can be in the standard growth model assuming that the utility function is isoelastic (see Section 5.4, pp. 105–07, in Stokey and Lucas with Prescott [1989]). Assume that due to labor-augmenting technical progress, effective per capita labor supply, and, hence, the wage, grow as  $(1+g)^t$ . Assume that the borrowing limit also grows as  $(1+g)^t$ , and let  $\mu$  be the elasticity of marginal utility. Then the steady-state interest rate with complete markets will equal  $[(1+\lambda) \times (1+g)^\mu - 1]$ , whereas with incomplete markets the interest rate would be less than  $[(1+\lambda)(1+g)^\mu - 1]$ . The distributions of earnings, total resources, assets, consumption, and income after normalizing by their respective per capita values would be stationary.

27. This follows since  $k/f(k,1)$  is an increasing function of  $k$ . It is strictly increasing if the capital share is less than unity. Note that since there is no growth in this economy net saving is zero in the steady state.

The above analysis also explains why general equilibrium considerations in combination with the shape of the  $Ea(r)$  curve can play an important role in limiting the impact of idiosyncratic risk and liquidity constraints on aggregate saving. Since  $Ea$  approaches infinity as  $r$  approaches  $\lambda$  from below, average household assets are extremely sensitive to slight variations in the interest rate when it is close to (but below) the rate of time preference. In a partial equilibrium analysis, by choosing an interest rate close enough to the rate of time preference, one can generate arbitrarily large precautionary saving in excess of the certainty case. However, in general equilibrium  $r$  is determined endogenously and how much  $r$  is reduced relative to the certainty case and, thereby, how much aggregate saving is increased is then a quantitative issue.<sup>28</sup>

We shall briefly describe the effects of varying the borrowing limit  $b$ . This is related to two other features of the  $Ea_w$  curve in Figure IIa. When  $r$  equals negative unity (so that the gross return on assets is zero), assets will always equal  $(-b)$ . When  $r$  equals zero,  $b$  is not an argument of the asset demand function  $A(\cdot)$ . Hence,  $Ea_w$  decreases one-to-one with increases in  $b$  when  $r$  is zero. These two features suggest that the  $Ea_w$  curve shifts to the left when  $b$  increases. Therefore, permitting a higher borrowing limit serves to lower aggregate capital and raise the interest rate toward the time preference rate (see Figure IIb). The intuition behind this conclusion is that when borrowing is permitted individuals need not rely solely on holdings of capital to buffer earnings variation. Borrowing can also be used to buffer these shocks and, hence, leads to smaller holdings of capital.<sup>29</sup>

It follows from the previous remarks that if uninsured idiosyncratic risk and no borrowing ( $b = 0$ ) lead to a small increase in aggregate capital relative to the certainty case, then permitting some borrowing ( $b > 0$ ) will lead to an even smaller increase in aggregate capital.

We briefly describe the effects of setting  $a_t \geq -\phi^* \equiv -w_{\min}/r$

28. Clearly, the elasticity of the  $K(r)$  curve plays an important role in determining whether the impact of uninsured idiosyncratic shocks and borrowing constraints is mainly on the interest rate or on aggregate saving. However, it should be kept in mind that the  $K(r)$  and  $\omega(r)$  functions are related through the production function. Therefore, the  $K(r)$  and  $Ea(r)$  functions are not independent of each other.

29. Note that the effects of an increase in  $b$  occur only in the range of interest rates for which  $w_{\min}/r > b$ . If  $w_{\min}/\lambda < b$ , then there is a range of values of  $r$  below and near  $\lambda$  for which marginal changes in  $b$  do not affect  $Ea_w$  or the  $Ea(r)$  curves. Therefore, if the steady state is in this range, then marginal changes in  $b$  have no effect on the steady state.



and restricting  $r$  to be positive. As noted earlier, this is the appropriate form of the borrowing constraint implied by present value budget balance and nonnegativity of consumption when  $r > 0$ , and will be referred to as the “present value” borrowing constraint. If  $l_{\min}$  is zero, this is equivalent to the case of no borrowing ( $b = 0$ ). However, if  $l_{\min}$  is positive, then  $Ea_w$  tends to  $(-\infty)$  as  $r$  tends to zero (see Figure IIa, the curve marked  $Ea_w(\phi^*)$ ). This can be seen by substituting  $wl_{\min}/r$  for  $\phi$  in equations (3)–(8). The first term on the right side of (8) remains finite, whereas the second term tends to  $(-\infty)$  as  $r$  tends to zero. Intuitively, as  $r$  becomes smaller, the borrowing limit becomes larger, permitting the individual to carry large amounts of debt. That is, the present value of minimum earnings is tending to infinity, enabling the individual to service large amounts of debt. The main difference between this case and the case of a fixed borrowing limit is that under the present value borrowing constraint there always exists a steady state with a positive interest rate. With a fixed borrowing limit there may be no steady state with a positive interest rate though there does exist a steady state with a negative interest rate.<sup>30</sup>

### *Some Alternative Interpretations*

The model of individual optimization in (1) can be turned into a pure exchange model with government debt in order to analyze the effects of changing the level of government debt. Let the government have outstanding a constant per capita amount of debt denoted  $d$ , the interest on which is financed by an equal (across agents) lump sum tax  $\tau$  ( $= rd$ ). Then the consumer’s budget constraint (1b) is altered to  $c_t + a_{t+1} = wl_t - rd + (1 + r)a_t$ . The steady-state equilibrium condition is  $Ea_w = d$ , where  $Ea_w$  denotes per capita asset holdings in the steady state.

In this model, whether debt neutrality holds or not depends crucially on how the borrowing constraint is specified. With a fixed borrowing limit as in (2), debt neutrality will not hold. However,

30. This can be seen by referring to Figure IIb. With a fixed borrowing limit  $b > 0$ , the steady-state interest rate will be negative if  $Ea$  is positive when  $r$  equals zero and the curve  $K(r)$  is such that it intersects the  $Ea$  curve at a negative  $r$ . (This occurs in one of our numerical examples.) With the present value borrowing constraint, there is always a positive interest rate at which  $Ea$  is zero, ensuring that there is always a steady state with a positive interest rate (see Figure IIa). Again, note that the  $Ea_w$  curves corresponding to a fixed  $b$  and corresponding to the present value borrowing constraint may coincide for a range of values of  $r$  near and below  $\lambda$  if it happens that  $wl_{\min}/\lambda < b$ . This is because, for values of  $r$  in this range the borrowing limit  $b$  is not binding (see equation (2)).

with a present value borrowing constraint, i.e., when the borrowing limit equals the present value of minimum earnings *adjusted for tax obligations*, so that we have  $a_t \geq -(wl_{\min} - rd)/r$ , then debt neutrality does hold. The equilibrium interest rate, the distribution of asset holdings net of government debt, and consumption are invariant to the level of  $d$ . This can be seen by using the transformation  $a_t^* = a_t - d$  in the above equations. Thus, the validity of debt neutrality in this framework with incomplete markets is entirely dependent on whether the borrowing limit takes account of changing tax obligations.

The model of individual optimization in (1) can also be turned into an "optimum quantity of money" model as in Bewley [1983] by interpreting  $a_t$  as  $(m_t - 1)/p$ ,  $b$  as  $1/p$ , and  $r$  as the interest paid on money, where  $m_t$  is an agent's nominal money holding at the beginning of period  $t$ ,  $p$  is the price level, and the per capita nominal money supply is constant at unity. Note that  $r/p$  is the real value of per capita lump sum taxes (equal across agents) levied to finance interest payments on money. The borrowing constraint is equivalent to nonnegativity of money holdings. The steady state equilibrium condition is that per capita asset holdings must be zero; equivalently, per capita nominal money holdings equal unity. The problem can be posed as finding an equilibrium  $p$  for a given  $r$  (as Bewley posed it) or as finding an equilibrium  $r$  for a given  $p$  (as is done here). The latter way of posing the problem makes it easier to see why it is not possible to have monetary equilibria with  $r$  being arbitrarily close to the rate of time preference [Bewley 1983].

In Figure IIa let  $r\#$  be the interest rate at which average assets are zero, corresponding to the  $Ea_w$  curve with the present value borrowing limit given by  $wl_{\min}/r$ . This will be a monetary equilibrium with a price level  $p\# = r\#/(wl_{\min})$ ; equivalently a borrowing limit  $b\# = 1/p\#$ . One cannot support an interest rate higher than  $r\#$  by lowering the price level (raising real balances, or, equivalently, raising the borrowing limit) since the portion of the curve  $Ea_w$  at and above  $r\#$  is unaffected. This is because when  $p < p\#$  and  $r > r\#$ , we have  $wl_{\min}/r < wl_{\min}/r\# = 1/p\# < 1/p$ . Therefore, the constraint  $a_t \geq -1/p$  will never bind. Raising the price level above  $p\#$  (equivalently, lowering the borrowing limit) only serves to increase  $Ea_w$  in a neighborhood of  $r\#$  and, hence, lowers the interest rate.

In the next section we describe model specification, parameterization, and the computation procedure for a version of the capital accumulation model with serially correlated labor endowment

shocks. In the section after that we describe the results on the contribution of precautionary saving to aggregate saving and some other results.

#### IV. MODEL SPECIFICATION, PARAMETERIZATION, AND COMPUTATION

The model period is taken to be one year, and the utility discount factor  $\beta$  is chosen to be 0.96. The production function  $f(\cdot)$  is assumed to be Cobb-Douglas with the capital share parameter (denoted  $\alpha$ ) taken to be 0.36. The depreciation rate of capital ( $\delta$ ) is set at 0.08. The period utility function is of the constant relative risk aversion (CRRA) type, i.e.,  $U(c) = [c^{1-\mu} - 1]/(1 - \mu)$ , where  $\mu$  is the relative risk aversion coefficient. Results are reported for three different values of  $\mu \in \{1, 3, 5\}$ . The above technology and preference specifications and parameter values are chosen to be consistent with aggregate features of the postwar U. S. economy and are commonly employed in aggregative models of growth and business cycles.<sup>31</sup>

For the labor endowment shocks we use a Markov chain specification with seven states to match the following first-order autoregressive representation for the logarithm of the labor endowment shock (equivalently, earnings):

$$(9) \quad \log(l_t) = \rho \log(l_{t-1}) + \sigma(1 - \rho^2)^{1/2} \epsilon_t, \epsilon_t \sim \text{Normal}(0, 1)$$

$$(10) \quad \sigma \in \{0.2, 0.4\}, \rho \in \{0, 0.3, 0.6, 0.9\}.$$

The coefficient of variation equals  $\sigma$  and the serial correlation coefficient equals  $\rho$ . We then follow the procedure described in Deaton [1991, p. 1232] and Tauchen [1986] to approximate the above autoregression by a seven-state Markov chain.<sup>32</sup> Table I reports the  $\sigma$  and  $\rho$  values implied by the Markov chain and shows that the approximation is quite good.

The values of  $\sigma$  and  $\rho$  were based on the following studies. Kydland [1984] reports that the standard deviation of annual hours worked from PSID data is about 15 percent. Abowd and Card [1987, 1989] use data from the PSID and NLS and calculate that the standard deviations of percentage changes in real earnings and annual hours are about 40 percent and 35 percent, respectively. The implied value for the coefficient of variation (c.v.) in earnings depends on the serial correlation in earnings. If earnings are i.i.d.,

31. See, for example, Prescott [1986].

32. See Aiyagari [1993a, p. 21, note 28] for the details.

TABLE I  
MARKOV CHAIN APPROXIMATION TO THE LABOR ENDOWMENT SHOCK:  
MARKOV CHAIN  $\sigma$ /MARKOV CHAIN  $\rho$

$\sigma \backslash \rho$	0	0.3	0.6	0.9
0.2	0.21/0	0.21/0.3	0.21/0.59	0.24/0.9
0.4	0.43/0	0.43/0.28	0.44/0.58	0.49/0.89

this yields a figure of 28 percent for the c.v. of earnings. Positive correlation would lead to a larger figure.<sup>33</sup> The covariances reported in Abowd and Card [1987, Table 3, p. 61; 1989, Tables IV, V, VI, pp. 418–22] suggest a first-order serial correlation coefficient of about 0.3. This would give a figure of 34 percent for the c.v. of earnings. Heaton and D. Lucas [1992] also use PSID data to estimate several versions of equation (9). Their estimates (see their Tables A.2–A.5) indicate a range of 0.23 to 0.53 for  $\rho$  and a range of 0.27 to 0.4 for the c.v. of earnings. These studies suggest that a c.v. of 20–40 percent in earnings at an annual rate may be reasonable.

Note that we have made no allowance for the possibility that the reported earnings variabilities contain significant measurement error. As the discussion in the papers by Abowd and Card suggests, this is a serious possibility, and the relevant degree of idiosyncratic earnings variability may be somewhat lower. However, this is balanced by the possibilities that the data do not include uninsured losses and taste shocks.

Last, the borrowing limit  $b$  is set to zero; i.e., borrowing is prohibited. As explained in the previous section, permitting some borrowing would lead to even smaller effects on the aggregate saving rate.

### Computation

We approximate the asset demand as a function of total resources (for each of seven possible current labor endowment shocks) by a continuous, piece-wise linear function over an interval. The corresponding partition of the interval is chosen to be finer at the lower end of the interval and coarser at the upper end of the interval.<sup>34</sup>

33. Let  $y$  be the log of earnings,  $\sigma_y$  be the standard deviation (s.d.) of  $y$ , and  $\sigma_g$  be the s.d. of  $(y_t - y_{t-1})$ . Suppose that  $y$  follows the first-order process:  $y_t = \text{trend}_t + \rho y_{t-1} + \epsilon_t$ , where  $\epsilon$  is i.i.d. It is straightforward to calculate that  $\sigma_y/\sigma_g = [2(1 - \rho)]^{-1/2}$ .

34. The reason is that for low levels of total resources assets will be zero since the borrowing constraint will bind. At some critical level of total resources, assets

The algorithm for approximating the steady state uses simulated series and the bisection method. We start with some value of  $r$  (say,  $r_1$ ) close to but less than the rate of time preference (see Figure IIb). We then compute the asset demand function as described above. We then simulate the Markov chain for the labor endowment shock using a random number generator and obtain a series of 10,000 draws.<sup>35</sup> These are used with the asset demand function to obtain a simulated series of assets. The sample mean of this is taken to be  $Ea$ . We then calculate  $r_2$  such that  $K(r_2)$  equals  $Ea$ . If  $r_2$  exceeds the rate of time preference, it is replaced by the rate of time preference. Now note that by construction  $r_1$  and  $r_2$  are on opposite sides of the steady-state interest rate  $r^*$ . Without loss of generality we may suppose that  $r_1 < r^* < r_2$  (by relabeling, if necessary). We then define  $r_3 = (r_1 + r_2)/2$  (this is the bisection part) and calculate  $Ea$  corresponding to  $r_3$ . If  $Ea$  exceeds  $K(r_3)$ , then  $r_2$  is replaced by  $r_3$ , and we use bisection again. If  $Ea$  is less than  $K(r_3)$ , then  $r_1$  is replaced by  $r_3$ , and we use bisection again. Typically, this yields an excellent approximation to the steady state within ten iterations.<sup>36</sup>

Once the steady state is approximated, we use the solution to calculate the following objects of interest. We calculate the mean, median, standard deviation, coefficient of variation, skewness, and serial correlation coefficient for labor income, asset (capital) holdings, net income, gross income, gross saving, and consumption.<sup>37</sup> These descriptive statistics are based on the simulated series obtained in the manner described before. We also calculate measures of inequality for each of these variables. We use the simulated series for each variable to construct its distribution, and then we compute the Lorenz curves and calculate the associated Gini coefficients.

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will become positive. This introduces a high degree of nonlinearity into the asset demand function. Consequently, it is important to have a finer partition at the lower end of the interval to obtain a good approximation. It turned out that throughout the upper half of the interval the asset demand function was very nearly linear so that a coarse partition was adequate to obtain a good approximation in this region. See Aiyagari [1993a] for the details and Figure 3 in Aiyagari [1993a] for an example of asset demand functions corresponding to  $l_{\min}$  and  $l_{\max}$  for a particular set of parameter values.

35. We repeated all the calculations using 20,000 draws and found that the changes in the results were very minor.

36. Figure 4 in Aiyagari [1993a] shows the graphs of  $Ea(r)$  and  $K(r)$  for a particular case.

37. The skewness measure is (mean-median)/standard deviation. This is one-third of Pearson's second coefficient of skewness. For a log-normal distribution with standard deviation  $\sigma$ , the skewness measure is approximately  $\sigma/2$ . Net income is defined as  $wl_t + ra_t$ . Gross income is net income plus depreciation, which is  $\delta a_t$ . Gross saving is gross income minus consumption.

TABLE II

A. Net return to capital in %/aggregate saving rate in % ( $\sigma = 0.2$ )

$\rho \backslash \mu$	1	3	5
0	4.1666/23.67	4.1456/23.71	4.0858/23.83
0.3	4.1365/23.73	4.0432/23.91	3.9054/24.19
0.6	4.0912/23.82	3.8767/24.25	3.5857/24.86
0.9	3.9305/24.14	3.2903/25.51	2.5260/27.36

B. Net return to capital in %/aggregate saving rate in % ( $\sigma = 0.4$ )

$\rho \backslash \mu$	1	3	5
0	4.0649/23.87	3.7816/24.44	3.4177/25.22
0.3	3.9554/24.09	3.4188/25.22	2.8032/26.66
0.6	3.7567/24.50	2.7835/26.71	1.8070/29.37
0.9	3.3054/25.47	1.2894/31.00	-0.3456/37.63

## V. RESULTS

*Aggregate Saving*

In Tables IIA and IIB we present the net return to capital in percent (before the solidus) and the saving rates in percent (following the solidus) for  $\sigma$  (coefficient of variation of earnings) equal to 0.2 and 0.4 and various values of  $\rho$  (serial correlation in earnings) and  $\mu$  (the relative risk aversion coefficient).<sup>38</sup> For comparison it is easy to calculate that the full insurance net return to capital is 4.17 percent, and the saving rate is 23.67 percent.<sup>39</sup>

The main point to note is that the differences between the saving rates with and without insurance are quite small for moderate and empirically plausible values of  $\sigma$ ,  $\rho$ , and  $\mu$ . However, for high values of  $\sigma$ ,  $\rho$ , and  $\mu$ , the presence of idiosyncratic risk can raise the saving rate quite significantly by up to seven percentage points. The extreme case when  $\sigma$  equals 0.4,  $\rho$  equals 0.9, and  $\mu$  equals 5 leads to a considerable increase in the saving rate of almost fourteen percentage points.

These results may be related to the concepts of relative prudence (*RP*) and equivalent precautionary premium (*EPP*)

38. The  $\sigma$  and  $\rho$  values reported in Table II are the ones used in computing the Markov chain approximation to the labor endowment shock—not the values of  $\sigma$  and  $\rho$  implied by the Markov chain approximation. These are described in Table I. Note that for high values of  $\sigma$ , the Markov-chain-based value of  $\sigma$  is higher and hence indicates even greater earnings variability than is indicated in Table II.

39. Recall that the production function is Cobb-Douglas with  $\alpha$  being the capital share parameter. The saving rate equals  $\delta k/f(k,1)$ , which may be written as  $\delta[kf_1/f]/f_1$  which equals  $\delta\alpha/(r + \delta)$ . With full insurance,  $r$  equals  $\lambda$ .

developed by Kimball [1990]. For the CRRA preferences used here  $RP$  equals  $(\mu + 1)$ , and  $EPP$  equals  $RP(\sigma_c)^2/2$ , where  $\sigma_c$  is the coefficient of variation of consumption. For a given value of  $r$ , increases in earnings variability ( $\sigma$ ) or persistence ( $\rho$ ) shift the  $Ea$  curve in Figure IIb to the right and also increase  $\sigma_c$  and, hence, the  $EPP$ . As can be seen from Table II, the equilibrium interest rate falls, and the saving rate goes up. An increase in  $\mu$  also shifts the  $Ea$  curve to the right and directly increases  $RP$  and the  $EPP$ . Again, from Table II, the equilibrium interest rate falls, and the saving rate goes up.<sup>40</sup>

Some studies (e.g., Caballero [1990] and Deaton [1991, Section 2.1]) use earnings processes that are difference stationary instead of trend stationary. This may be approximated by making  $\rho$  approach unity and simultaneously letting  $\sigma$  approach infinity in such a way as to keep  $\sigma[2(1 - \rho)]^{1/2}$  fixed and positive (see note 33). Table II suggests that this would depress the return to capital and increase the saving rate enormously.

### Variabilities

Aiyagari [1993a, Tables 3A and 3B] contains results for the variabilities (measured by the coefficient of variation) of consumption, income (net and gross), gross saving, and assets which are summarized here. Consumption varies about 50–70 percent as much as income. Saving and assets are much more volatile than income. Saving varies about three times as much as income, and assets vary about twice as much as income. Risk aversion tends to reduce the variabilities of all these variables. Variability in earnings ( $\sigma$ ) has a relatively smaller effect on the variability of consumption and relatively larger effects on the variabilities of other variables. Consumption variability rises with persistence in earnings and falls with risk aversion. Variability of consumption relative to income behaves similarly.

### Importance of Asset Trading

An approximate expression for the welfare loss from consumption variability, measured as the percentage of average consumption the consumer is willing to give up, is given by  $\mu\sigma_c^2/2$ , where  $\mu$  is

40. As noted in the introduction and explained in Section III, infinite horizons and binding borrowing constraints play important roles in our results. Further, the  $Ea$  curve represents the result of aggregating the saving behavior of a large number of consumers. Kimball's [1990] concepts and results are developed for a single consumer with a two-period horizon and ignoring borrowing constraints.



the relative risk aversion coefficient and  $\sigma_c$  is the coefficient of variation of consumption. In contrast to representative agent models (see Cochrane [1989]), the results here imply that consumers are able to accomplish a significant amount of consumption smoothing by accumulating and decumulating assets and, hence, enjoy significant welfare benefits from participating in asset markets. To see this, consider how variable consumption would be if an individual could not trade in asset markets. Suppose that a consumer held a fixed quantity of assets equal to the per capita amount and consumed his earnings plus the return on the assets. In this case  $\sigma_c$  would be given by  $\sigma/[1 + \alpha r / \{(1 - \alpha)(r + \delta)\}]$ .<sup>41</sup> If, as an example, we take  $\mu = 3$ ,  $\sigma = 0.4$ , and  $\rho = 0.6$ , then  $\sigma_c$  with a fixed amount of assets equals 0.35. Actual consumption variability from Aiyagari [1993a, Table 3B] is 0.17. Thus, by optimally accumulating and depleting assets, consumption variability is cut in half, yielding a welfare benefit of about 14 percent of per capita consumption, or about 8 percent of per capita GNP.

### *Cross-Section Distributions and Inequality Measures*

Since long-run distributions for an individual coincide with cross-section distributions for the population, results for variabilities of individual consumption, income, and assets have immediate implications for cross-section distributions. These results are qualitatively consistent with casual empiricism and more careful empirical observation. There is much less dispersion across households in consumption compared with income and much greater dispersion in wealth compared with income (see Aiyagari [1993a], Tables 3A and 3B and Figures 5A and 5B for an example). In all cases, the fraction of liquidity-constrained households is close to zero.<sup>42</sup> Skewness coefficients reveal another aspect of inequality. All of the cross-section distributions are positively skewed (median < mean).<sup>43</sup> However, the degree of skewness is somewhat less than in the data. For example, median net income, gross

41. This may be derived as follows. With fixed assets equal to  $k$  individual consumption  $c_t = rk + w l_t$ . Hence,  $\sigma_c = \sigma/[1 + rk/w]$ . Now note that  $k/w = (k f_1/f)/\{f_1(w/f)\} = \alpha/[(r + \delta)(1 - \alpha)]$ , since the production function is Cobb-Douglas.

42. Of course, it is the possibility of being constrained (and its utility cost to the household) that affects behavior and leads to this result.

43. Note that the labor shock distribution is positively skewed since it is an approximation of a log-normal distribution. However, we found that even when the labor shock distribution is symmetric the mechanics of the model naturally generate positive skewness (median < mean) in the wealth distribution, even though the income and consumption distributions were roughly symmetric.

income, and capital are all over 90 percent of their respective mean values. In contrast, U. S. median household income is about 80 percent of U. S. mean household income.<sup>44</sup> Lorenz curves and Gini coefficients (see Figure 6 in Aiyagari [1993a] for an example) show that the model does generate empirically plausible relative degrees of inequality. Consumption exhibits the least inequality, followed by net income, gross income, and then capital, and saving exhibits the greatest inequality. However, the model cannot generate the observed degrees of inequality. For example, when  $\mu$  is 5,  $\rho$  is 0.6, and  $\sigma$  is 0.2, the Gini coefficients for net income and wealth are 0.12 and 0.32, respectively [Figure 6, Aiyagari 1993a]. In U. S. data, however, the Gini coefficient for income is about 0.4 and that for net wealth is about 0.8.<sup>45</sup>

## VI. CONCLUDING REMARKS

In this paper a version of the Brock-Mirman [1972] growth model with a large number of agents subject to uninsured idiosyncratic shocks was described and its qualitative and quantitative implications for the contribution of precautionary saving to aggregate saving, importance of asset trading, and income and wealth distributions were analyzed. This class of models may also be useful in resolving various asset return puzzles. Mehra and Prescott [1985, p. 145] suggested that these puzzles cannot be “accounted for by models that abstract from transactions costs,

44. This figure is for 1985 from the *Statistical Abstract of the United States* 1988, [U. S. Bureau of the Census, 1987] using numbers for median and mean household incomes from Tables 693 and 694, p. 424. The corresponding ratio for male persons in 1985 is 0.79 and for female persons in 1985 is 0.71 (both from Table 710, p. 432, *ibid*). It should be possible to lower the ratio of median to mean income in the model by adjusting  $\sigma$ , the coefficient of variation of earnings. Since earnings are approximately log-normal, median/mean  $\approx \exp(-\sigma^2/2)$ . This ratio equals 0.98 when  $\sigma$  is 0.2, and it equals 0.92 when  $\sigma$  is 0.4.

45. See, for example, Greenwood [1983, Table 9, p. 41] and Kessler and Wolff [1991, Table 4, p. 260]. According to Greenwood [Table 4, p. 35, Table 7, p. 40], in 1973, the top 5 percent of wealth holders held close to 60 percent of net wealth, and the top 5 percent of income earners earned about 23 percent of total income. Kessler and Wolff [1991, Table 3, p. 259] present a very similar number for net wealth in 1983. There are several reasons for the discrepancy between the model and the data. First, the way income and wealth distribution data are put together does not match well with the model. Since the model is one of infinitely lived agents, a household in the model should probably be thought of as consisting of members of all the different generations of a family. Implicitly, the model assumes that the different generations of a family are linked by bequests and, therefore, focuses only on total family income, wealth, and consumption where the family is more broadly defined than in the data. Second, the model focuses on only one source of inequality (that due to different histories of labor endowment shocks) and abstracts from other sources of inequality like differences in the endowments of human capital (broadly interpreted).

liquidity constraints and other frictions absent in the Arrow-Debreu set-up." However, this requires that the models be generalized to include aggregate dynamics and uncertainty.<sup>46</sup> This is a very hard problem computationally since the distribution of assets across households can no longer be taken to be constant. Instead, the cross-section distribution is part of the state vector that evolves stochastically over time in response to aggregate shocks. This is an issue that remains to be explored.

This class of models can also differ from the infinite-lived agent complete markets model on some important policy issues. For instance, with complete markets, dynamic optimal factor taxation leads to the result that the capital income tax should be zero in the long run (see Chamley [1986]). However, Aiyagari [1994] shows that with idiosyncratic shocks and incomplete markets the capital income tax is strictly positive even in the long run. Therefore, the large welfare gains of reducing the capital income tax to zero calculated by Lucas [1990] in a complete markets model may well turn out to be welfare losses in an incomplete markets model.

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46. See Aiyagari and Gertler [1991] for an early attempt based on differential transaction costs but without aggregate uncertainty. Because of the absence of aggregate uncertainty, that model could only analyze the risk-free rate and the transaction premium but not the risk premium.

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