

# LABORPRAKTIKUM

KEVIN KLEIN

Some Active Motion and First-passage Time

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Kevin Klein: *Laborpraktikum*, Some Active Motion and First-passage Time, © August 2018

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## LISTINGS

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## ACRONYMS

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SRW	Simple isotropic random walk
MSD	Mean square displacement
BRW	Biased random walk
PRW	Persistent random walk
CRW	Correlated random walk

## Part I

### PREPARATION

Not sure if an introduction is needed for the Laborpraktikum.



## INTRODUCTION

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Write some kind of intro here.





## ACTIVE MOTION

*Active motion* describes the process of converting energy resources into directed motion. Living beings using active motion are e.g. humans, animals and microorganisms such as cells or bacteria.

While it is everyday experience that animals and humans are able to move directed, it is a non trivial fact for microorganisms. Especially considering cells and bacteria which are often surrounded by fluids and therefore experience thermal *diffusion*. The diffusion alone would lead to so called *Brownian motion*, a random motion named after its famous discoverer Brown and his studies on the motion of pollen particles [5]. However, in many experiments non-diffusive motion patterns have been observed for microorganisms. Figure 2.1 shows two examples of cells and bacteria for which directed and persistent motion has been recorded.

*The motion of a human floating in a sea is only subject to the current, however, by using energy and muscle power the human can swim and therefore move directed.*

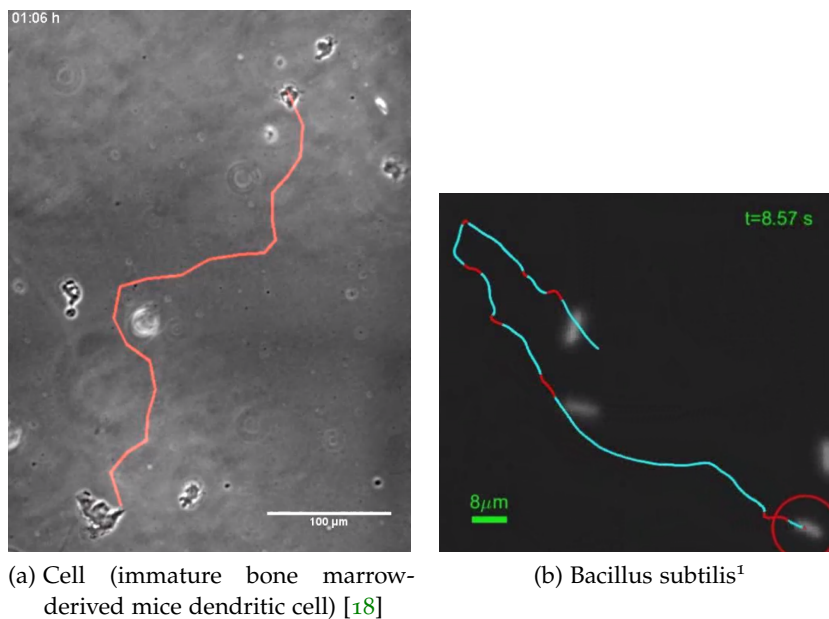


Figure 2.1: Directed and piecewise persistent migration paths for a cell and bacterium.

In the following, reasons for active motion are given, a selection of research in different fields is presented and the search problem aspect is explained.

<sup>1</sup> J. Najafi *et al.*, under review (2018)

## 2.1 MOTIVATION AND EXAMPLES

*A mouse might need  
to hide or run from a  
fox while a human  
might be looking for  
lost keys.*

The reasons for active motion are manifold and depend among others on the species and environmental conditions. They include:

- *Survival*: evading predators, escaping hazardous locations, finding shelter.
- *Foraging*: finding food or nutrients, hunting prey.
- *Reproduction*: finding mates.
- *Search*: searching for objects or locations of interest.
- *Migration*: finding and exploring new habitats and environments.
- *Biological tasks*: e. g. morphogenesis, wound healing, immune response, etc.

From this list one can extract that most of the time a specific search problem is the cause of active motion. Furthermore, it is reasonable to assume that, depending on the motivation, also the movement patterns of active motion vary. To stick with the example of the mouse and the human: the movement pattern of a mouse running for its life will most certainly look different than that of a human looking for its lost keys. Therefore, there is a lot of research with the goal of quantifying and qualifying the many different kinds of active motion. Considering the tasks and functions of cells and bacteria, understanding their migrational properties is especially relevant in the fields of biology and medicine.

To give a few examples and an impression of the research that has been done, a small selection is presented below.

**HUMANS** For humans almost any activity in everyday life is connected to active motion (e. g. doing groceries, sports, work, ...). However, on a macroscopic scale ancient migration and colonization can be analyzed as active motion. In this sense, the colonization of America and the Neanderthal replacement in Europe has been studied [8].

**ANIMALS** Here, the term animals includes all kinds of terrestrial animals, birds, fishes and insects.

There is quite a lot of literature on active motion of animals such as the *Encyclopedia of Animal Behavior* [3] and the book *Animal Behavior* [4] which cover many interesting aspects such as *search, navigation, migration, dispersal, foraging, self-defense, mating*, and many more in great generality.

More specific research concentrates on e. g. the movement of fish and crustaceans [24], the 'zigzagging' and 'casting' in the

flight of moths [14], the foraging behavior of squirrels [28], planktivorous fish [21], and foraging/moving animals in general [15, 25]. The search for prey has been studied for e.g. toads [17], different ant-eating jumping spiders (*Salitricidae*) [13], buried bivalves [10], and predator search in general [1].

**MICROORGANISMS** Directed and persistent motion has also been observed for microorganisms as it was stated before and shown in Figure 2.1. Considering the diverse tasks and functions of different microorganisms, this is not surprising as diffusive Brownian motion is highly inefficient in most (search) applications.

In the past 30 years there has been a focus on understanding migrational processes of cells and bacteria, the underlying mechanics and their properties. These include basic as well as specific studies of *in vitro* and *in vivo* experiments, the comparison of random and directed motion, migration in complex environments, theoretical and mathematical models, and many properties of microscopic migration in general [7, 9, 11, 12, 16, 18–20, 22, 23, 27, 29].

**ARTIFICIAL PARTICLES** Nowadays researchers have even developed artificial active particles on the micro- and nanometer scale [2], which could turn out to be very useful in many different fields.

The focus in this work mainly lies on the qualitative and quantitative properties of abstract search problems and the results can then be used to better understand the motion of biological organisms for example. Therefore, now that active motion has been introduced and explained, we want to look at it from the search problem point of view.

## 2.2 SEARCH PROBLEMS

As it was mentioned before, most of the time a search problem is the cause of active motion. Naturally, for search problems the time required to find a target is a limiting factor and will be called the *search time*. Considering the search for food or nutrients, the search time is crucial in survival or death by starvation. Therefore, minimizing the search time often turns out to be essential.

Looking at the amount of different species and organisms, which again have different abilities and conditions to actively move and search for targets, it is normal that over many years of evolution many different search strategies have evolved. However, an interesting question is whether there is an optimal strategy under given circumstances and how does such a strategy look like.

Even though it might be too much to ask for the one and only optimal strategy, it is still possible to compare and tweak different strategies.

One approach is to identify and study strategies that have been adopted by different organisms and living beings. In the sense of the *survival of the fittest* one could assume that natural selection has yielded highly efficient strategies. And indeed there is evidence....  
**TODO: find evidence and cite .**

A slightly different approach is to define and look at abstract search problems and theoretical models that can be compared to real-life examples.

Even for the first approach it is interesting to model the search strategies that have been observed in order to quantify and better describe the strategy. As the search for a target starts with (active) motion, ways of modelling the same are needed. Therefore, they will be introduced in [Chapter 3](#).

## MODELING STOCHASTIC ACTIVE MOTION

In [Chapter 2](#) the importance and relevance of understanding migration, motion and search problems in many different fields has been outlined. In order to understand and analyze such processes mathematical modelling is a commonly used tool. A broad field of possible modelling approaches is based on the so called *Random Walk* and different extensions on it.

The random walk is a stochastic process that describes successive random steps on a mathematical space such as a hypothetical particle that walks on the integers  $\mathbb{Z}$ . The most basic version of the random walk is the *Simple isotropic random walk (SRW)*. It is unbiased (isotropic), meaning that the walker has no preference for one specific direction, and uncorrelated in direction, meaning that the history of previous steps' directions has no influence on the step direction at a given time.

More complex random walk versions build on the [SRW](#) and extend it.

### 3.1 THE SIMPLE ISOTROPIC RANDOM WALK

Consider a one-dimensional lattice which is split into discrete sites as it is shown in [Figure 3.1](#). On this lattice, in each discrete timestep, a hypothetical particle (the *walker*) is able to jump from its current site to the neighbouring sites, each with equal probabilities  $p = 1/2$ . Therefore, the state of the walker can be described by the discrete time  $n \in \mathbb{N}$  and position  $m \in \mathbb{Z}$ . Having the walker start at the origin ( $m = 0$ ), after one time step, it will either be at site  $m = -1$  or  $m = 1$ , each with probability  $p = 1/2$ . After another time step, accordingly, the walker can be at sites  $m = -2$  or  $m = 2$ , each with probability  $p = 1/4$ , or at the origin  $m = 0$  with probability  $p = 1/2$ . In this manner one can continue in order to find the probabilities for being at each site at a given time.

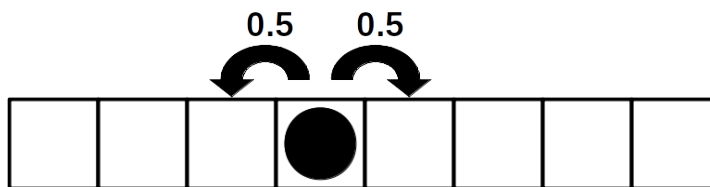


Figure 3.1: [SRW](#) on a one-dimensional lattice. Black arrows indicate the possible sites in the next step with according probabilities.

*REZA: Should I define the random walk in a more mathematical approach?*

Some important and useful quantities, however, are the mean position  $\mathbb{E}[M_n]$  and the *Mean square displacement (MSD)*  $\mathbb{E}[M_n^2]$ , which are defined as

$$\begin{aligned}\mathbb{E}[M_n] &= \sum_{m=-\infty}^{\infty} mp(m, n), \\ \mathbb{E}[M_n^2] &= \sum_{m=-\infty}^{\infty} m^2 p(m, n).\end{aligned}\tag{3.1}$$

Here,  $p(m, n)$  denotes the *probability mass function*.

As the single steps of the walk are independent from each other, these quantities are easily derived. One obtains

$$\mathbb{E}[M_n] = 0,$$

which illustrates the isotropy or absence of a bias, and

$$\mathbb{E}[M_n^2] = n,$$

the typical property of *diffusion* (MSD linear in time). Indeed, the SRW is used to model diffusive motion [6] and therefore cannot be used for active, directed motion. However, it is still a good starting point for extended models.

### 3.2 THE BIASED RANDOM WALK

The SRW is a special case of the BRW for  $p = 1/2$ .

By introducing a hopping probability  $p \neq 1/2$  in the SRW one obtains the *Biased random walk (BRW)*. The situation for the one-dimensional case is depicted in Figure 3.2.

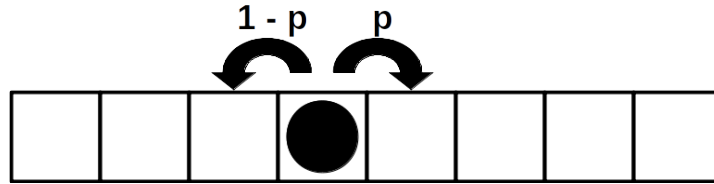


Figure 3.2: BRW on a one-dimensional lattice. Black arrows indicate the possible sites in the next step with according probabilities.

Again one can easily derive the mean position and the MSD. One obtains

$$\mathbb{E}[M_n] = n(2p - 1),$$

which is nonzero (except for  $p = 1/2$ ) as a result of the introduced asymmetrical hopping rates. This drift to one side illustrates the walker's preference for one direction. For the MSD one gets

$$\mathbb{E}[M_n^2] = 4np(1 - p) + n^2(4p^2 - 4p + 1),$$

and thus  $\mathbb{E}[M_n^2] \propto n^2$ , which is typical for *ballistic* motion. However, since the mean position drifts, in this case it is more meaningful to take a look at the dispersal about the mean position, which is defined as

$$\sigma_n^2 = \sum_{m=-\infty}^{\infty} (m - \mathbb{E}[M_n])^2 p(m, n). \quad (3.2)$$

This quantity is easily obtainable as well and one derives

$$\sigma_n^2 = 4np(1-p).$$

This shows that the dispersal about the mean position is only linear in time and therefore the walker diffuses about its mean position. In other words: in its own rest frame the walker performs diffusive motion.

Because of its drift component the [BRW](#) is a possible model to describe directed motion, however, it is only applicable under certain circumstances and given certain requirements, which will be explained later on. For now, one more random walk model will be introduced.

### 3.3 THE PERSISTENT RANDOM WALK

So far the introduced random walk models have been uncorrelated and steps were independent from each other. In the *Persistent random walk* ([PRW](#)) (or also *Correlated random walk* ([CRW](#))) this is not the case. Instead, at a given time the probabilities for the different directions of the next step are dependent on the direction of the very previous step. In other words: it matters from which direction the walker came from in the previous step, the walker has some kind of short memory.

The [PRW](#) model defines a *persistency* parameter  $p \in [0, 1]$  which gives the probability to keep going in the same direction. Therefore, in the one-dimensional case there are two possible ways of how a walker has reached its current site, leading to two possible scenarios of how it will continue its walk which are shown in [Figure 3.3](#).

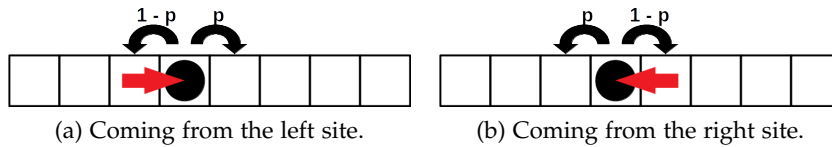


Figure 3.3: The two possible scenarios which can happen during the one-dimensional [PRW](#). Red arrows indicate the direction of movement in the previous step, black arrows indicate the possible sites in the next step with according probabilities.

Consequently, walks with  $p < 0.5$  tend to reverse the direction of movement leading to *anti-persistent motion*, while walks with  $p > 0.5$

*A [PRW](#) with persistency parameter  $p = 1/2$  is equivalent to the [SRW](#).*

reinforce the direction of movement leading to *persistent motion*. For the trivial cases of  $p = 0$  the walker only jumps back and forth and for  $p = 1$  it keeps going in one direction without ever reversing, which is a ballistic flight.

For the mean position and the MSD one obtains [26]

$$\begin{aligned}\mathbb{E}[M_n] &= 0, \\ \mathbb{E}[M_n^2] &= \frac{1+p}{1-p}n + \frac{2p}{(1-p)^2}(p^n - 1).\end{aligned}$$

The result for the mean position is not surprising as there is no bias or preferred direction in the PRW. However, the result for the MSD is more interesting. Here one needs to differentiate between short and long term behavior.

For short times one derives

$$\mathbb{E}[M_n^2] \propto n^\alpha,$$

where  $\alpha = 1 + \ln(1+p) / \ln 2$  [26]. Considering only the non-trivial case of  $p \in (0, 1)$  gives  $\alpha \in (1, 2)$  and therefore the short term behavior is *superdiffusive*.

For long times ( $n \rightarrow \infty$ ) the second part of the right-hand side becomes a constant and the first part defines the behavior. This means that the MSD is linear in time and the motion becomes diffusive.

Therefore, the PRW shows a transition from superdiffusive to diffusive motion, which makes it a possible model to describe active, persistent motion. And indeed, later on, it will be the model of choice in order to study different aspects of motion. For this purpose, the model is extended to two dimensions and a model in continuous space is introduced.

### 3.3.1 Two-dimensional lattice

On a two-dimensional lattice the persistency parameter  $p$  gives the probability to keep going in the same direction. However, instead of only one probability for the reversal of movement, there are two additional probabilities for turning to the right or left in respect to the direction of movement. To distinct between them, they will be denoted by  $p_l$  and  $p_r$ , respectively. The probability of reversing the direction of motion is then computed by  $p_b = 1 - p - p_l - p_r$ . This means that for a walker at any given time there are four possible ways of how it has reached its current site and therefore there are four possible scenarios of how it will continue its walk which are shown in Figure 3.4.

Here, the analytical derivation of the mean position and the MSD is much more complex than in the one-dimensional case and, since those quantities are not of much importance now, will be skipped. However, there are different other meaningful quantities that can be derived under certain simplifications, which will be explained below.

REZA: I might want to use a smaller grid in Figure 3.4 to save some space



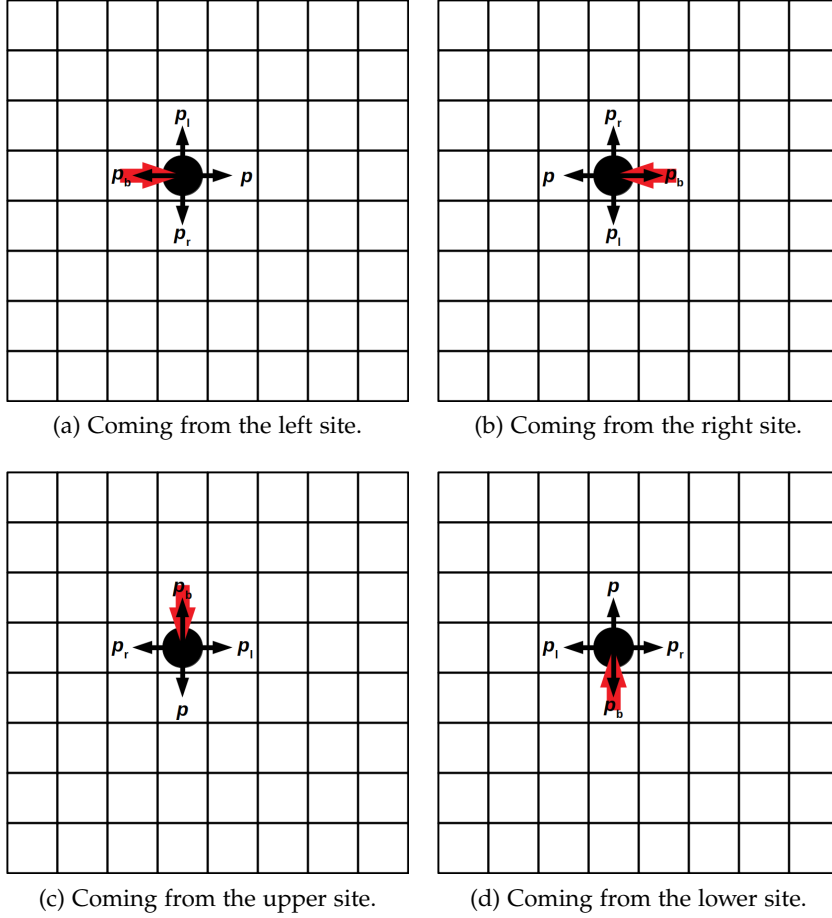


Figure 3.4: The four possible scenarios which can happen during the PRW on a two-dimensional lattice. Red arrows indicate the direction of movement in the previous step, black arrows indicate the possible sites in the next step with according probability.

### 3.3.2 Two-dimensional continuous space

For the two-dimensional continuous PRW, instead of hopping probabilities, one needs a continuous turning angle distribution in order to determine the direction of movement. Additionally, the step-length can be chosen from a step-length distribution. Nevertheless, one can still define a persistency parameter.

Considering the turning angle distribution  $R(\phi)$ , one can define the mean cosine  $c$  and mean sine  $s$  of the turning angle as

$$\begin{aligned}
 c &= \mathbb{E}[\cos(\phi)] = \int_{-\pi}^{\pi} \cos(\phi) R(\phi) d\phi, \\
 s &= \mathbb{E}[\sin(\phi)] = \int_{-\pi}^{\pi} \sin(\phi) R(\phi) d\phi.
 \end{aligned} \tag{3.3}$$

From these quantities one can extract information about the PRW. The mean sine measures the relative probability of clockwise and anti-clockwise turns. For most applications, however, the turning angle distributions are symmetric and, hence, the mean turning angle  $\phi_{\text{mean}}$  as well as the mean sine  $s$  are zero. In this case, the quantity  $c$  is a measure of the correlation or persistency and therefore, the mean cosine  $c$  as defined in Equation 3.3 will be called persistency parameter  $p$  hereinafter. Note that  $p \in [-1, 1]$  can be negative and depending on its value the motion is either anti-persistent, diffusive or persistent. An example distribution for each regime is shown in Figure 3.5.

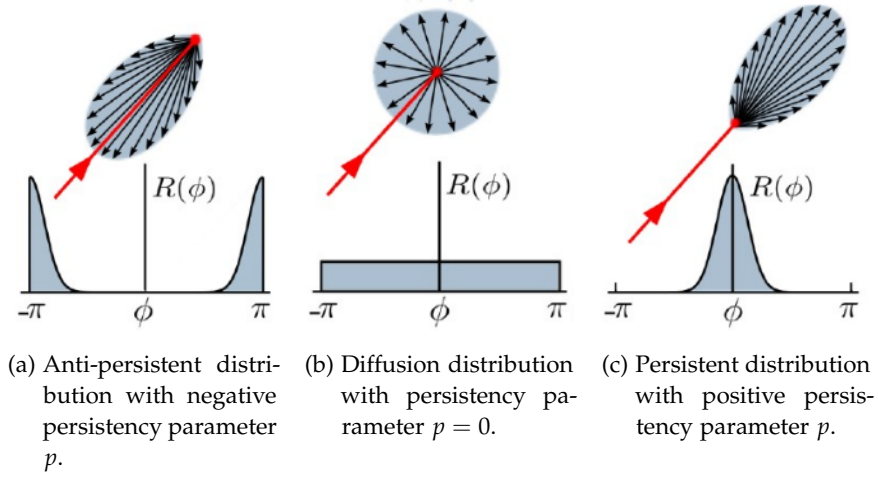


Figure 3.5: Exemplary turning angle distributions and directions of movement in the context of the two-dimensional continuous PRW. Red arrows indicate the direction of movement in the previous step, black arrows indicate possible directions of movement in the next step, with length being proportional to the probability. [26]

### 3.4 A TWO DIMENSIONAL LATTICE MODEL

## Part II

### PART 2



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