First-passage Times for persistent Random Walks

Kevin Klein

Naturwissenschaftlich-Technische Fakultät II
- Physik und Mechatronik -
Universität des Saarlandes

Forschungsseminar zur Masterarbeit, 2018

Contents

Active motion

Active particles consume energy and turn it into persistent motion.

Migrating cell

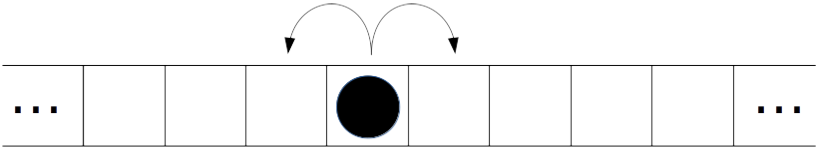
Bacillus subtilis

P. Maiuri, ..., F. Lautenschläger, ..., Cell 161, p. 374, 2015

J. Najafi et al., under review (2018)

Simplest approach to model migration

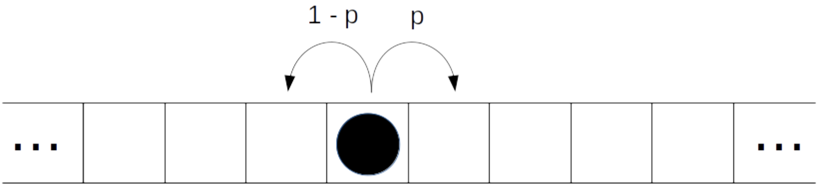
- How can we model the movement of microorganisms?
- A possible approach: simple 1D random walk (*RW*) on a lattice



- Equal hopping probabilities lead to *diffusion*.
- For diffusion the first two moments are:
  - $\mathbb{E}[X_n] = 0$
  - $\mathbb{E}[X_n^2] = n$

Random walk with bias

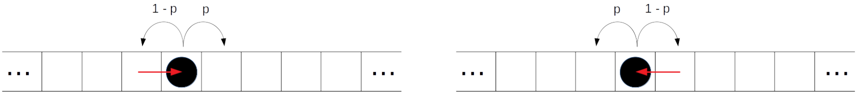
- Asymmetrical hopping rates lead to a bias/drift.



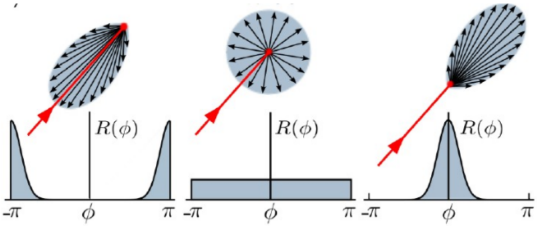
- Therefore the first two moments are:
  - $\mathbb{E}\left[X_n\right] = n\left(2p - 1\right)$
  - $\mathbb{E}\left[X_n^2\right] = n^2$
- But here the dispersal about the mean location is more meaningful. $\sigma_n \propto n$

Persistent random walks

- The direction of movement in the previous step influences the hopping probabilites:



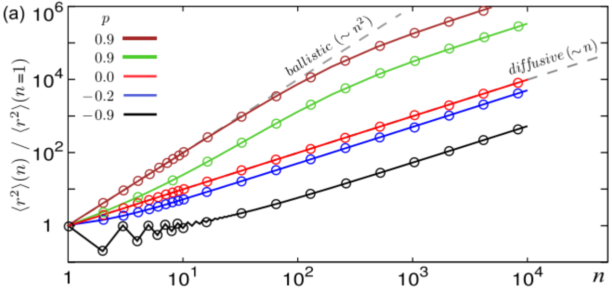
- In two-dimensional continuous walks: turning angle distribution:



M. Reza Shaebani et al., Phys. Rev. E 90, 030701, 2014

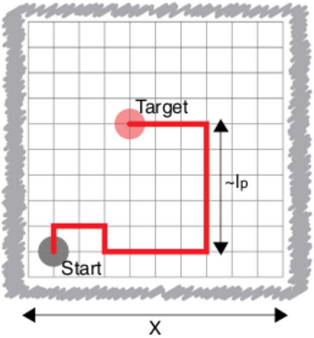
Some properties

- $\mathbb{E}\left[X_n\right] = 0$
- For the MSD one obtains: $\mathbb{E}\left[X_n^2\right] = \frac{1+p}{1-p}n + \frac{2p}{(1-p)^2}\left(p^n - 1\right)$
- Therefore longterm: diffusion
- Shortterm: $\mathbb{E}\left[X_n^2\right] \propto n^\alpha$ with $\alpha = 1 + \ln\left(1+p\right)/\ln 2$



M. Reza Shaebani et al., Phys. Rev. E 90, 030701, 2014

A 2D lattice model



- 2 dimensional lattice
- Periodic boundary conditions
- System size of $X \gg 1$
- Persistent direction with p_1
- Other directions with $p_2 = \frac{1-p_1}{3}$

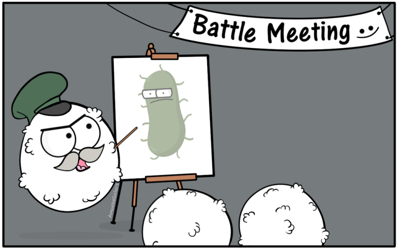
Tejedor et al., Phys. Rev. Lett. 108, 088103, 2012

- Therefore $P\left(l\right) = \left(1 - p_1\right) p_1^{l-1}$
- Persistence length $l_p = \sum_{l=1}^{\infty} lP\left(l\right) = \frac{1}{1-p_1}$

Motivation

- Microorganisms need to migrate in order to adapt to environmental changes:
  - Finding nutrients
  - Finding target organisms
  - Escaping hazardous locations
  - Relocation for various other biological tasks
- An important measure is the time needed to find places of interest.

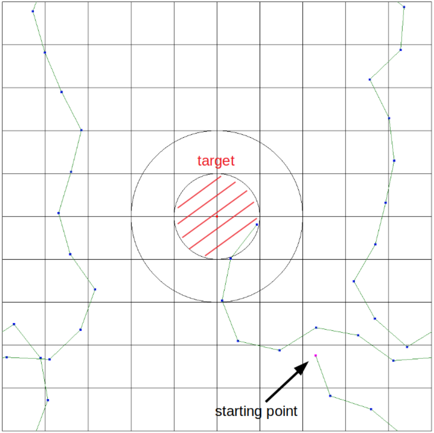
Paramecium Parlor @AmoebaSisters



White blood cells: waging war with pathogens while you're watching cat videos.

First-passage time

- How to define the efficiency of a search?
- Time or number of steps it takes to find a hidden target
- Mathematically described by the mean first-passage time (MFPT).



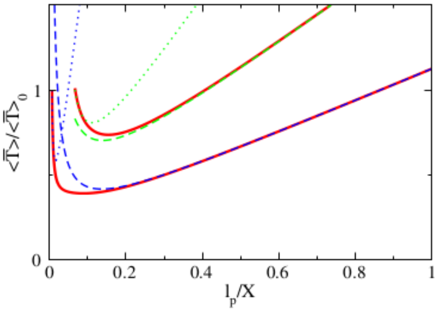
Sources of complexity

- Different types of searches:
  - Diffusion
  - Lévy-Walk
  - Intermittent (“Run and Tumble”)
  - Single-state search strategy
- Parameters:
  - Persistency
  - System size
  - Step length / velocity
  - Size of target / detection range
  - Information in the environment
  - Crowded environment
  - Interactions between targets and/or searchers

$\longrightarrow$ Simple model with constant persistency

MFPT of a persistent walker on a 2D lattice

- Reminder: $l_p = \frac{1}{1-p}$
- By using backward equations and fourier transform an exact expression for the MFPT τ is obtained.



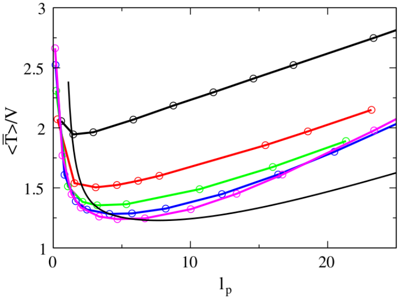
- Upper set of curves: $X = 10$
- Lower set of curves: $X = 100$
- Red line: exact results
- Dotted/dashed lines: limits

- Search time can be minimized by selecting optimal persistency p.

Tejedor et al., Phys. Rev. Lett. 108, 088103, 2012

Comparison to Lévy walk

- Tejedor et al. compare their results to Lévy walk strategies.
- One target and system size $X = 50$
- For the Lévy walk: $P(l) \propto \frac{1}{l^{1+\mu}}$, with $\mu \in \{1.2, 1.4, 1.6, 1.8, 2\}$ from top to bottom



- The persistent random walk with constant state outperforms the Lévy walk strategy here if only one target is available.

Tejedor et al., Phys. Rev. Lett. 108, 088103, 2012

Summary I

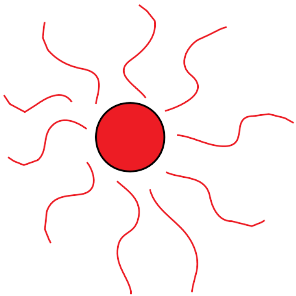
- Microorganisms do need to efficiently migrate.
- This migration can be modeled by different types of random walks.
- The mean first-passage time is an important measure for the efficiency of a search.
- Even using a simple search strategy the MFPT can be minimized.

Variable activity

- In biological systems microorganisms show activity.
- This activity can be expressed by the persistency p.
- But why do organisms show activity?

Hidden vs. visible targets

- There might be information about the target location available in the environment.
- This information can be sensed by the searcher and therefore it can adapt its activity accordingly.
- One can distinguish between deterministic and stochastic navigation.

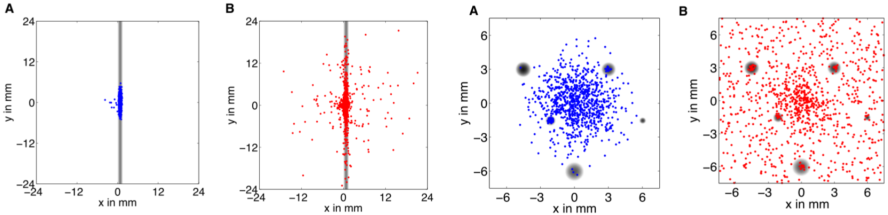


Deterministic navigation

- Local gradient is used to determine the right direction to the target.
- E.g.: Chemotaxis, gravitaxis, thermotaxis, magnetotaxis, ...
- The tactical response to any information profile is of interest.

E. coli - Precision vs. motility

- Chemotaxis signal transmission pathway is modified.
- Population A has exponentially distributed lengths of runs.
- Population B has power-law distributed lengths of runs.

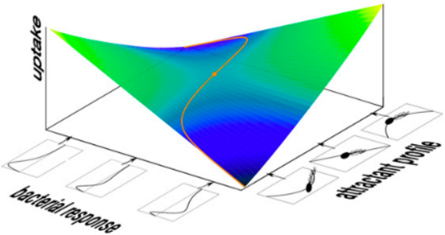


- Different behavior/responses advantageous in different scenarios.

F. Matthäus et al., Biophysical Journal 97, p. 946, 2009

E. coli chemoattractant profiles and Robot-Infotaxis

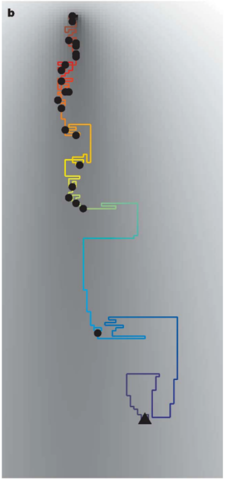
- Rapidly fluctuating, diverse/complex environments: chemotaxis or shutting down signaling pathway preferable?
- Chemotactic response: bias in fraction of time spent in run/tumble.



- “Maximin” strategy always outperforms motile but nonchemotactic bacteria.

A. Celani, M. Vergassola, PNAS 107, p. 1391, 2010

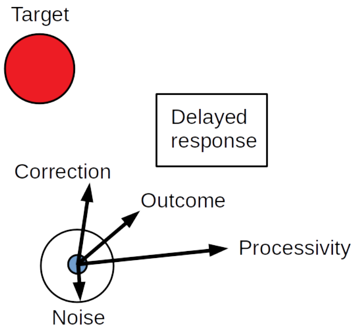
Infotaxis



M. Vergassola et al., nature 445, p. 406, 2007

Stochastic navigation

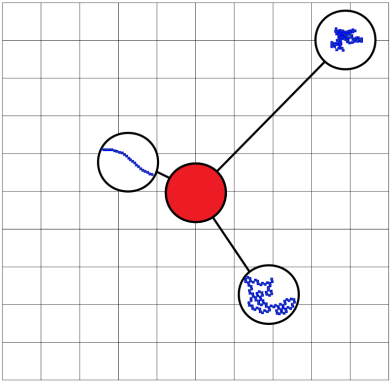
- Shouldn't deterministic navigation lead straight to the target?
- There are several confounding factors:
  - Weak gradients or chaotic fields
  - Sparse information / no gradient
  - External (and internal) noise
  - Wrong navigation caused by ligand-receptor binding



- Under such conditions it is reasonable to give up on deterministically chosing a direction.
- Therefore, directions will be chosen randomly and only the activity changes.

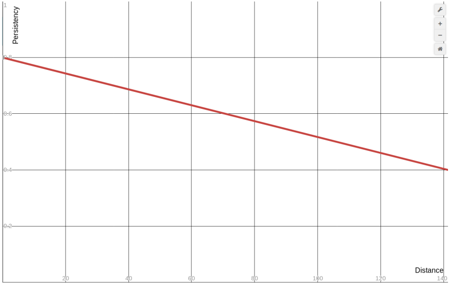
Implementation of chemokinesis

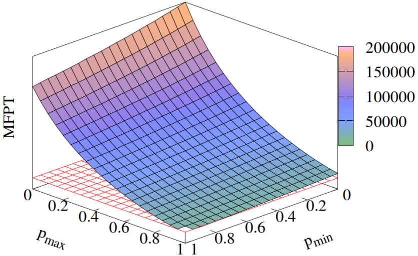
- For now: 2D lattice model
- We assume simple dependencies between distance and persistency.
- After each step the persistency is adapted according to the new distance to the target.
- Therefore, the movement patterns vary from diffusive to ballistic motion.



Results - Linear profile

- Linear distance-persistency relation:
  $p\,(d) = p_{max} - (p_{max} - p_{min}) \cdot \frac{\sqrt{2}}{X} \cdot d$



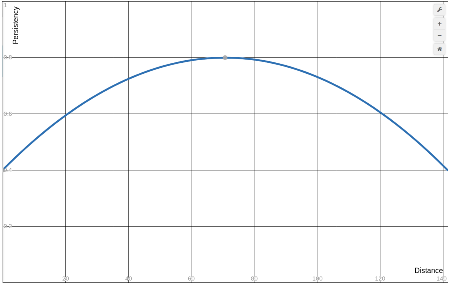


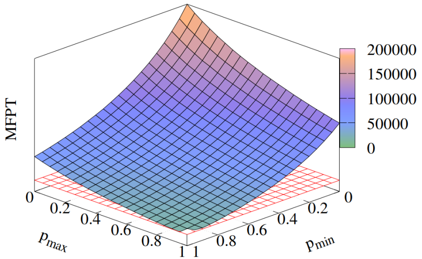
- Parameter sets close to (1,1) outperform constant search strategy

Results - Parabolic profile

- Parabolic distance-persistency relation:

$$p(d) = p_{max} - (p_{max} - p_{min}) \cdot \left(\frac{2\sqrt{2}}{X}\right)^2 \cdot \left(d - \frac{X}{2\sqrt{2}}\right)^2$$





- Parameter sets close to (1,1) outperform constant search strategy

Outlook

- The preliminary results (as well as other not shown results) suggest to consider the regime of high persistence search in more detail.
- Considering the persistence length / higher resolution of p.
- Also other mathematical relations between persistency and target-distance need to be explored.
- Identify properties of efficient profiles.
- Consider a continuous chemokinesis model.

Summary

- Microorganisms need to efficiently search for targets or locations of interest.
- There are different search strategies with their own specific advantages, e.g.:
  - Run and tumble
  - Single-state (constant persistency)
  - Chemotaxis (gradient)
  - Chemokinesis (concentration)
- Search strategies with concentration/distance dependent persistency can be a useful strategy in order to minimize search times.