

First-passage Times for persistent Random Walks

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Active motion

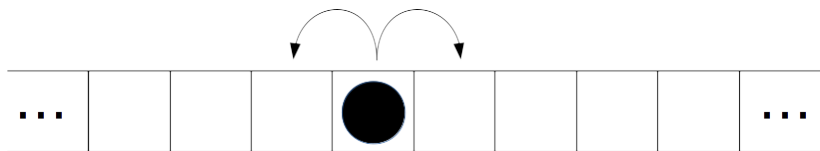
Active particles consume energy and turn it into persistent motion.

Migrating cell

Bacillus subtilis

Simplest approach to model migration

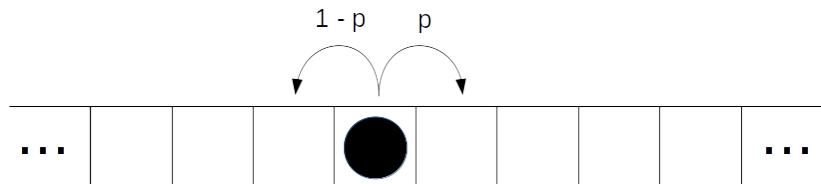
- How can we model the movement of microorganisms?
- A possible approach: simple 1D random walk (*RW*) on a lattice



- Equal hopping probabilities lead to *diffusion*.
- For diffusion the first two moments are:
 - $\mathbb{E}[X_n] = 0$
 - $\mathbb{E}[X_n^2] = n$

Random walk with bias

- Asymmetrical hopping rates lead to a bias/drift.



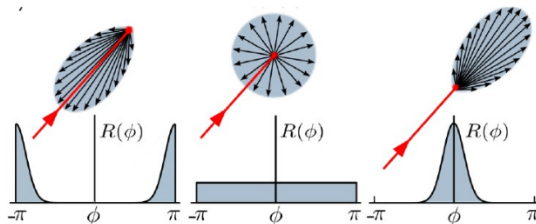
- Therefore the first two moments are:
 - $\mathbb{E}[X_n] = n(2p - 1)$
 - $\mathbb{E}[X_n^2] = n^2$
- But here the dispersal about the mean location is more meaningful. $\sigma_n \propto n$

Persistent random walks

- The direction of movement in the previous step influences the hopping probabilities:

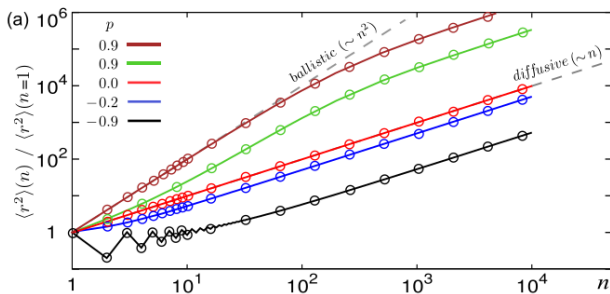


- In two-dimensional continuous walks: turning angle distribution:

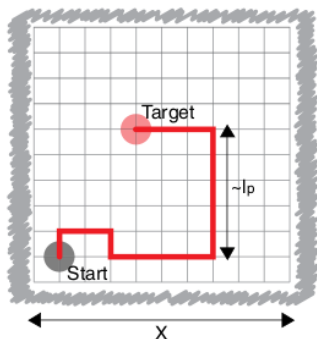


Some properties

- $\mathbb{E}[X_n] = 0$
- For the MSD one obtains: $\mathbb{E}[X_n^2] = \frac{1+p}{1-p}n + \frac{2p}{(1-p)^2}(p^n - 1)$
- Therefore longterm: diffusion
- Shortterm: $\mathbb{E}[X_n^2] \propto n^\alpha$ with $\alpha = 1 + \ln(1+p)/\ln 2$



A 2D lattice model



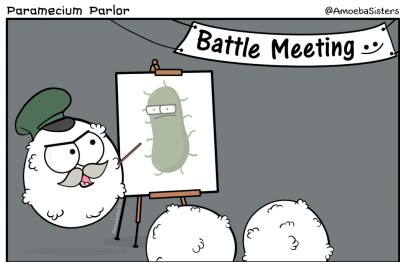
- 2 dimensional lattice
- Periodic boundary conditions
- System size of $X \gg 1$
- Persistent direction with p_1
- Other directions with $p_2 = \frac{1-p_1}{3}$

Tejedor et al., Phys. Rev. Lett. 108, 088103, 2012

- Therefore $P(l) = (1 - p_1) p_1^{l-1}$
- Persistence length $l_p = \sum_{l=1}^{\infty} l P(l) = \frac{1}{1-p_1}$

Motivation

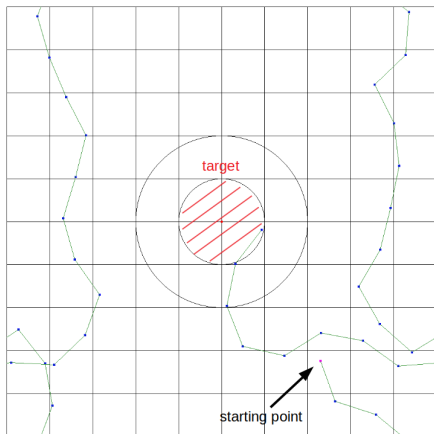
- Microorganisms need to migrate in order to adapt to environmental changes:
 - Finding nutrients
 - Finding target organisms
 - Escaping hazardous locations
 - Relocation for various other biological tasks
- An important measure is the time needed to find places of interest.



White blood cells: waging war with pathogens while you're watching cat videos.

First-passage time

- How to define the efficiency of a search?
- Time or number of steps it takes to find a hidden target
- Mathematically described by the mean first-passage time (MFPT).



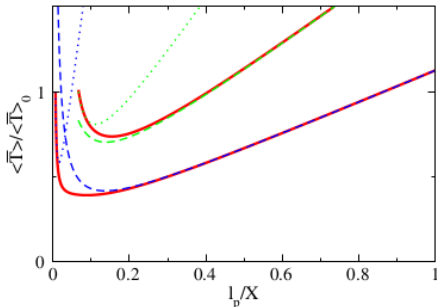
Sources of complexity

- Different types of searches:
 - Diffusion
 - Lévy-Walk
 - Intermittent (“Run and Tumble”)
 - Single-state search strategy
- Parameters:
 - Persistency
 - System size
 - Step length / velocity
 - Size of target / detection range
 - Information in the environment
 - Crowded environment
 - Interactions between targets and/or searchers

→ Simple model with constant persistency

MFPT of a persistent walker on a 2D lattice

- Reminder: $l_p = \frac{1}{1-p}$
- By using backward equations and fourier transform an exact expression for the MFPT τ is obtained.

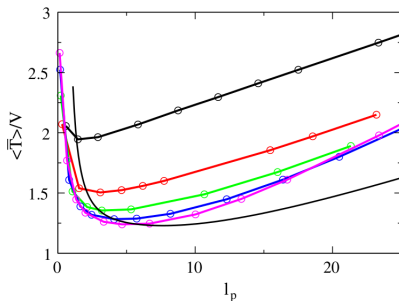


- Upper set of curves:
 $X = 10$
- Lower set of curves:
 $X = 100$
- Red line: exact results
- Dotted/dashed lines: limits

- Search time can be minimized by selecting optimal persistency p .

Comparison to Lévy walk

- Tejedor et al. compare their results to Lévy walk strategies.
- One target and system size $X = 50$
- For the Lévy walk:
 $P(l) \propto \frac{1}{l^{1+\mu}}$, with
 $\mu \in \{1.2, 1.4, 1.6, 1.8, 2\}$ from top to bottom
- The persistent random walk with constant state outperforms the Lévy walk strategy here if only one target is available.



Tejedor et al., Phys. Rev. Lett. 108, 088103, 2012

Summary I

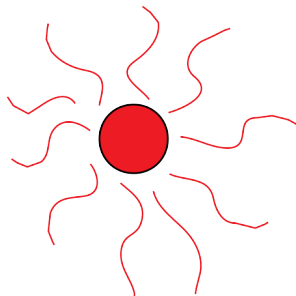
- Microorganisms do need to efficiently migrate.
- This migration can be modeled by different types of random walks.
- The mean first-passage time is an important measure for the efficiency of a search.
- Even using a simple search strategy the MFPT can be minimized.

Variable activity

- In biological systems microorganisms show activity.
- This activity can be expressed by the persistency p .
- But why do organisms show activity?

Hidden vs. visible targets

- There might be information about the target location available in the environment.
- This information can be sensed by the searcher and therefore it can adapt its activity accordingly.
- One can distinguish between deterministic and stochastic navigation.

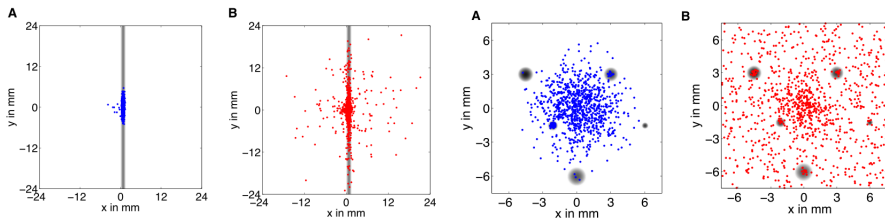


Deterministic navigation

- Local gradient is used to determine the right direction to the target.
- E.g.: Chemotaxis, gravitaxis, thermotaxis, magnetotaxis, ...
- The tactical response to any information profile is of interest.

E. coli - Precision vs. motility

- Chemotaxis signal transmission pathway is modified.
- Population A has exponentially distributed lengths of runs.
- Population B has power-law distributed lengths of runs.

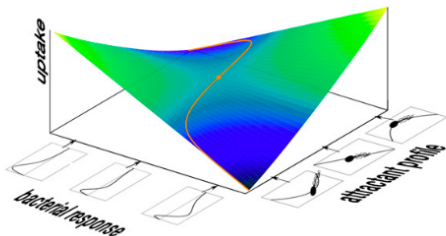


- Different behavior/responses advantageous in different scenarios.

F. Matthäus et al., Biophysical Journal 97, p. 946, 2009

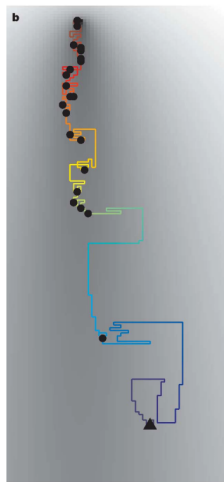
E. coli chemoattractant profiles and Robot-Infotaxis

- Rapidly fluctuating, diverse/complex environments: chemotaxis or shutting down signaling pathway preferable?
- Chemotactic response: bias in fraction of time spent in run/tumble.



- “Maximin” strategy always outperforms motile but nonchemotactic bacteria.

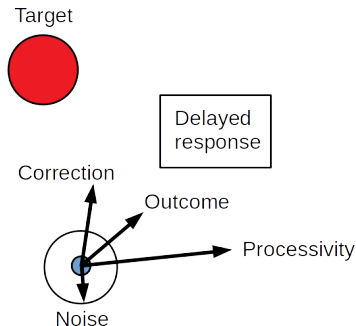
Infotaxis



M. Vergassola et al., nature 445, p. 406,

Stochastic navigation

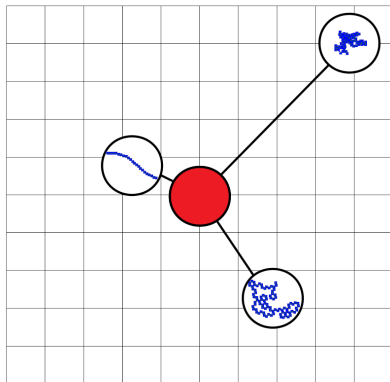
- Shouldn't deterministic navigation lead straight to the target?
- There are several confounding factors:
 - Weak gradients or chaotic fields
 - Sparse information / no gradient
 - External (and internal) noise
 - Wrong navigation caused by ligand-receptor binding



- Under such conditions it is reasonable to give up on deterministically choosing a direction.
- Therefore, directions will be chosen randomly and only the activity changes.

Implementation of chemokinesis

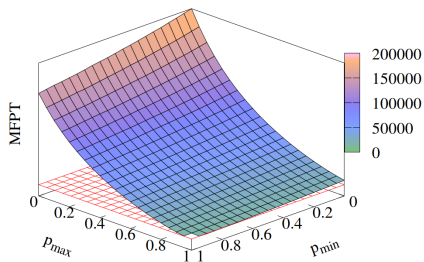
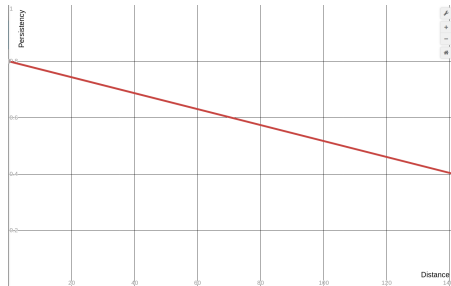
- For now: 2D lattice model
- We assume simple dependencies between distance and persistency.
- After each step the persistency is adapted according to the new distance to the target.
- Therefore, the movement patterns vary from diffusive to ballistic motion.



Results - Linear profile

- Linear distance-persistency relation:

$$p(d) = p_{max} - (p_{max} - p_{min}) \cdot \frac{\sqrt{2}}{X} \cdot d$$

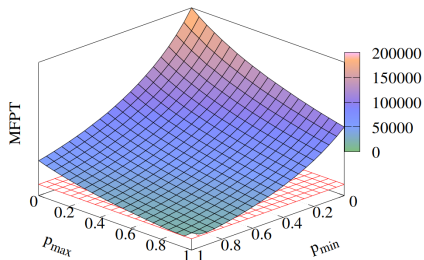
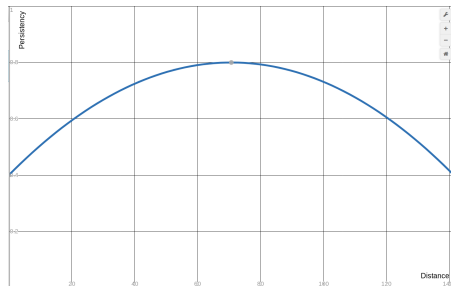


- Parameter sets close to (1,1) outperform constant search strategy

Results - Parabolic profile

- Parabolic distance-persistency relation:

$$p(d) = p_{max} - (p_{max} - p_{min}) \cdot \left(\frac{2\sqrt{2}}{X}\right)^2 \cdot \left(d - \frac{X}{2\sqrt{2}}\right)^2$$



- Parameter sets close to (1,1) outperform constant search strategy

Outlook

- The preliminary results (as well as other not shown results) suggest to consider the regime of high persistence search in more detail.
- Considering the persistence length / higher resolution of p .
- Also other mathematical relations between persistency and target-distance need to be explored.
- Identify properties of efficient profiles.
- Consider a continuous chemokinesis model.

Summary

- Microorganisms need to efficiently search for targets or locations of interest.
- There are different search strategies with their own specific advantages, e.g.:
 - Run and tumble
 - Single-state (constant persistency)
 - Chemotaxis (gradient)
 - Chemokinesis (concentration)
- Search strategies with concentration/distance dependent persistency can be a useful strategy in order to minimize search times.