MA 323 (2020) Monte Carlo Simulation: LAB 05
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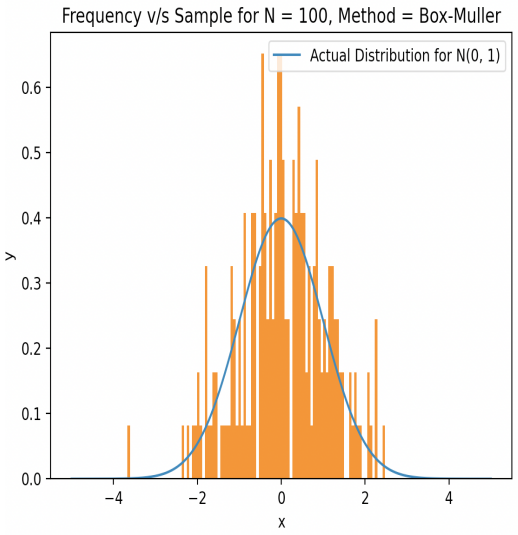
Problem I

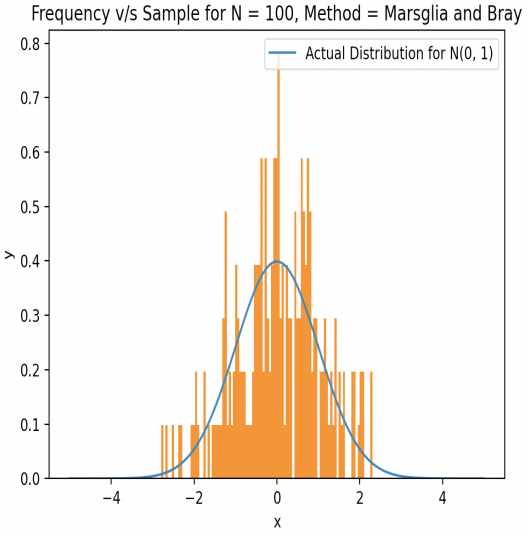
a.

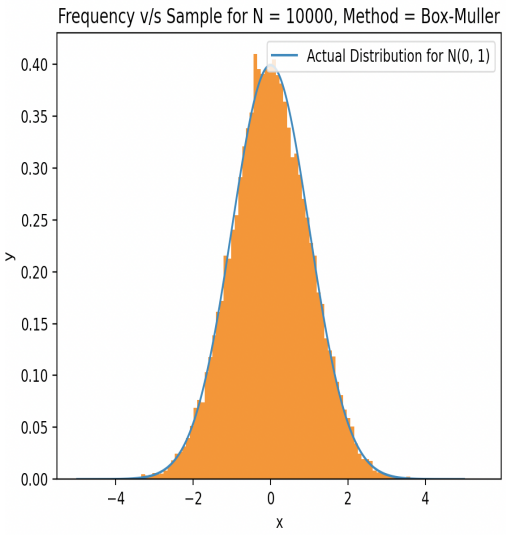
Sample Size (N)	Sample Mean			
	Box-Muller		Marsglia and Bray	
	Z1	Z2	Z1	Z2
100	0.0625	0.0214	0.0433	0.0144
10,000	-0.00633	0.0065	0.0010	0.0042

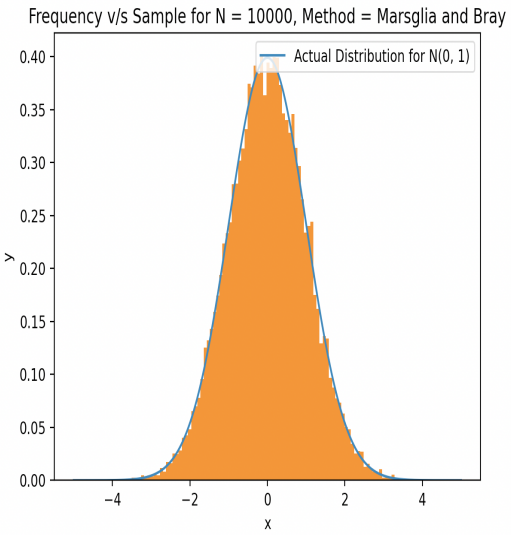
Sample Size (N)	Sample Variance			
	Box-Muller		Marsglia and Bray	
	Z1	Z2	Z1	Z2
100	1.3712	0.8290	1.0416	1.0980
10,000	1.0232	0.9979	1.0054	1.0155

b.

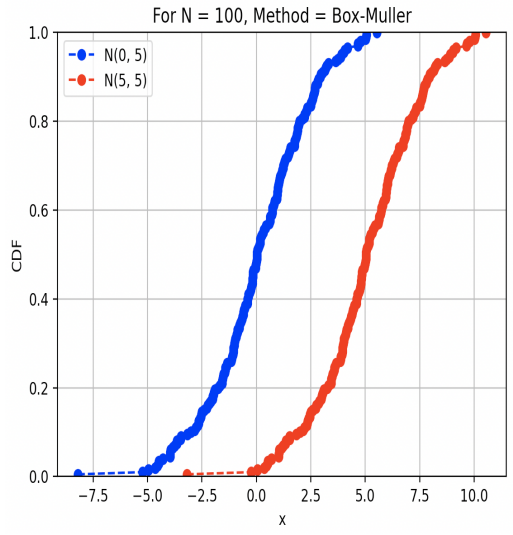


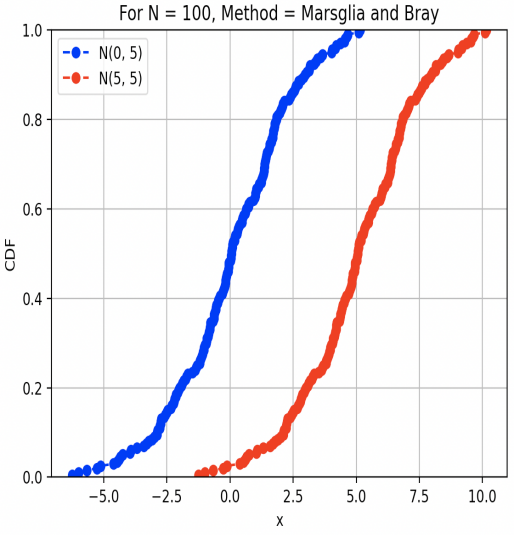


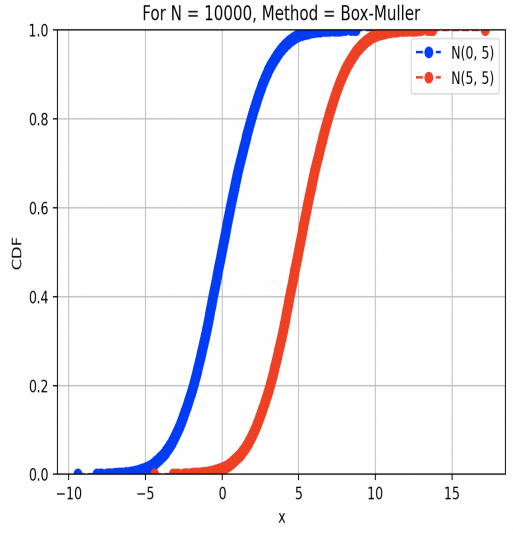


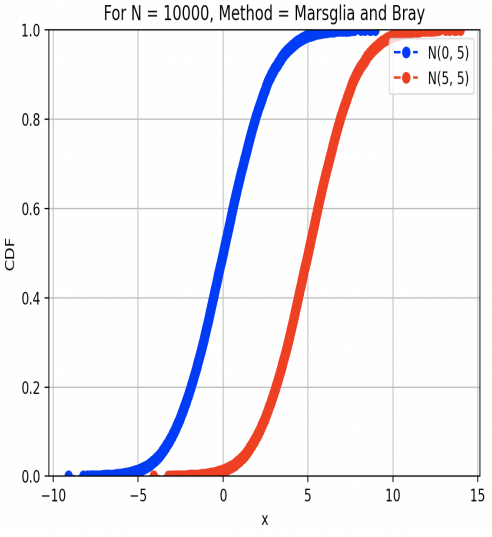


C.









Observations:

1. The red part appears to be just a translated version of the blue part. The major reason for this is:
    a. The variance / standard deviation for both $N(0, 5)$ and $N(5, 5)$ is 5. This justifies that the shape of both graphs has to be similar.
    b. The translation can be justified using the fact that the mean of the blue part and the red part is $0 \; and \; 5$ respectively, which would mean that the values in both the parts should differ by 5 on an average!
    c. The observation (2) can be confirmed by seeing that the centers of the blue part and the red part are at 0 and 5 respectively!

Problem II

Sample Size (N)	Time Taken (in ns)		Faster Method
	Box-Muller	Marsglia and Bray	
100	6,96,000	8,95,000	**Box-Muller**
10,000	6,14,02,000	8,33,78,000	**Box-Muller**

Observations:

1. From the above table, we can observe that for large as well as small values of N, $Box - Muller \; beats \; Marsglia \; and \; Bray$.
2. Theoretically, it should not have been the case. From the lecture notes, the reason for the same was pointed out to be time taken for computing $sine \; and \; cosine \; values \; in \; Box - Muller$!
3. In my opinion, if we assume that $sine \; and \; cosine \; functions \; work \; in \; O(1)$, then $Box - Muller \; should \; beat \; Marsglia \; and \; Bray$, since the latter works on $Acceptance - Rejection \; Principle$, which would imply that it would take more than $O(N)$ operations for generating N numbers!
4. Python is an interpreted, object-oriented, high-level programming language. Since $Marsglia \; and \; Bray$ is an implementation heavy algorithm, it and the language of choice being Python here, it takes a longer amount of run-time than $Box - Muller$!

Problem III

$Rejection \; Ratio \; (R.R) \; = \; (Number \; of \; rejections) \; / \; (Number \; of \; iterations)$

Sample Size (N)	$abs(1 - \pi/4 - R.R)$
100	0.0400
10,000	0.0012

Observations:

1. For large values of N the $Rejection \; Ratio$ converges to $1 - \pi/4$.

2. Tha observation (1) can be verified using the above table.