MA 323 (2020) Monte Carlo Simulation: LAB 02
Jay Vikas Sabale
180123019

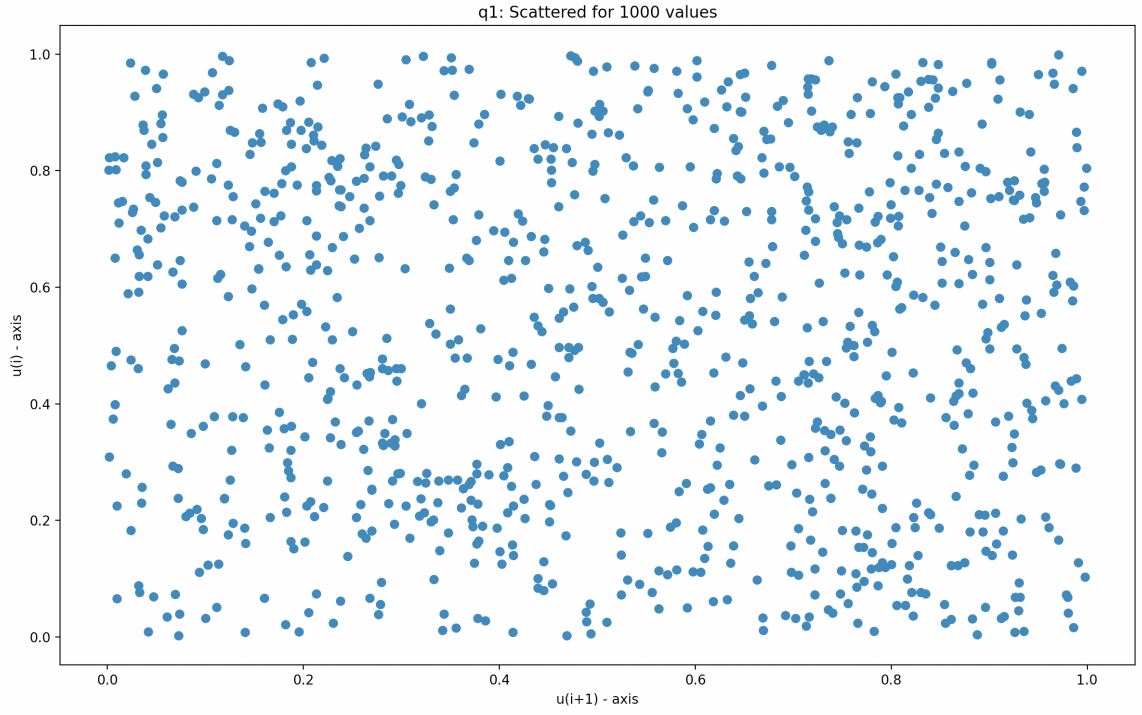
Problem I:

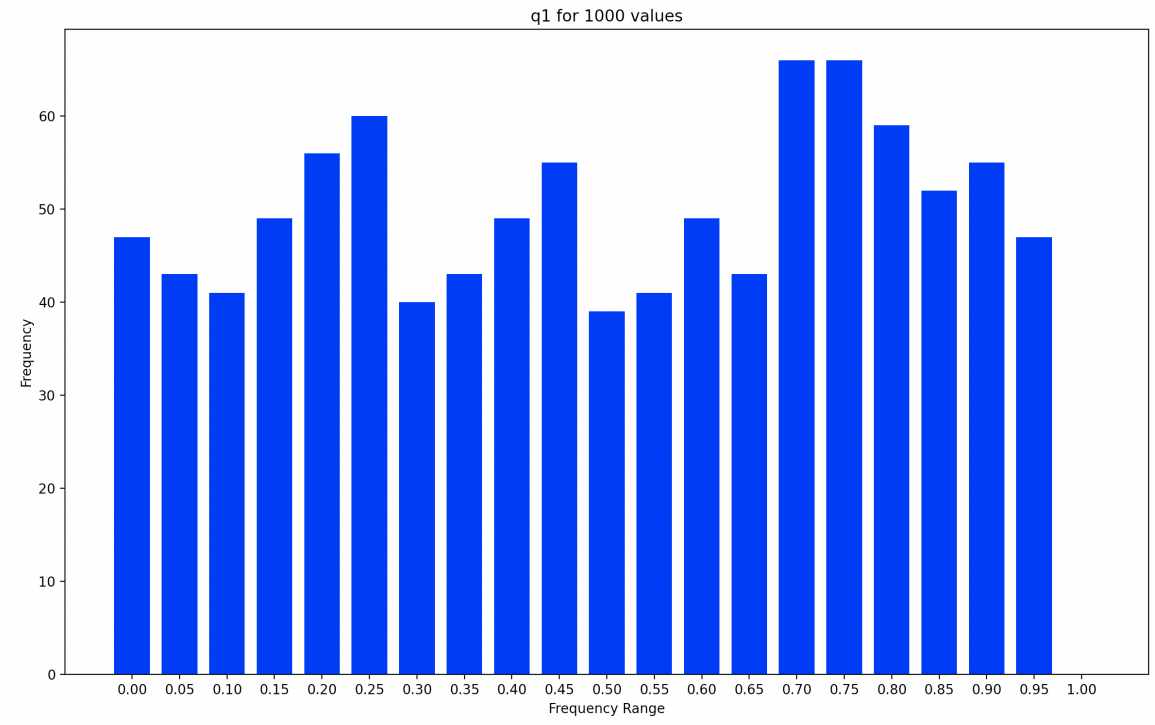
a) First 17 Values of U_i are:

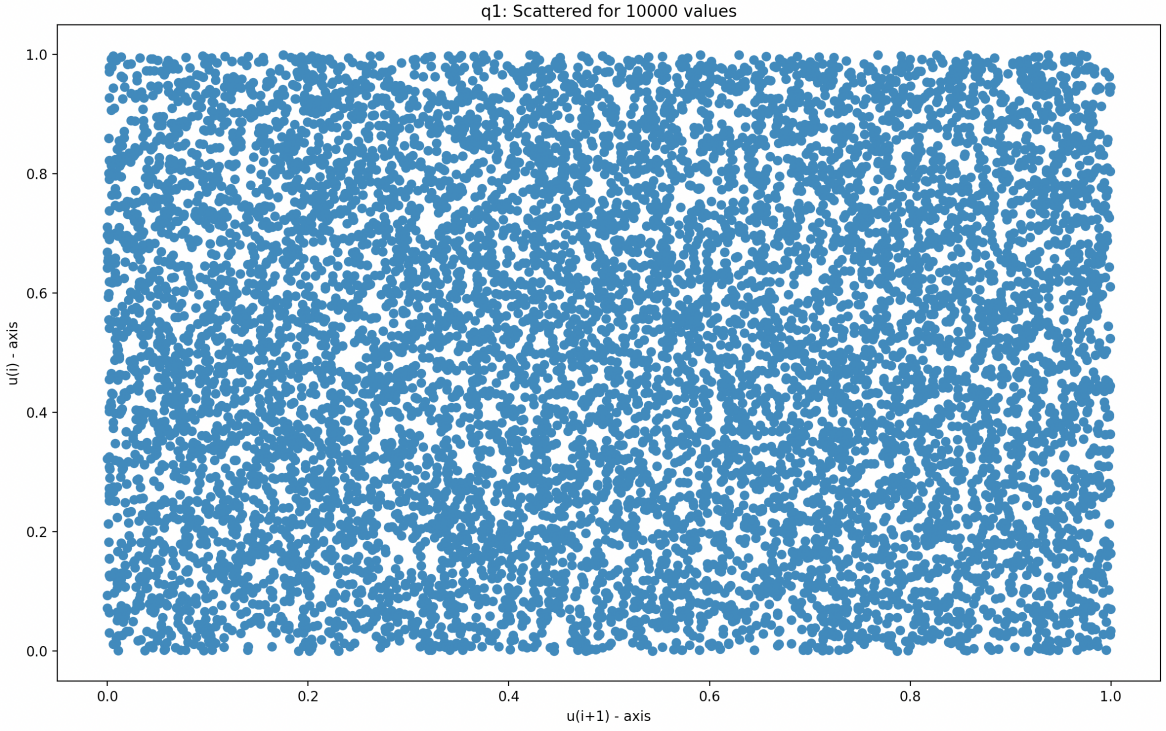
U(0)	0.00146484375
U(1)	0.80078125
U(2)	0.16064453125
U(3)	0.4326171875
U(4)	0.68701171875
U(5)	0.337890625
U(6)	0.26806640625
U(7)	0.4541015625
U(8)	0.09130859375
U(9)	0.21875
U(10)	0.84423828125
U(11)	0.5693359375
U(12)	0.71435546875
U(13)	0.943359375
U(14)	0.38916015625
U(15)	0.2783203125
U(16)	0.05615234375

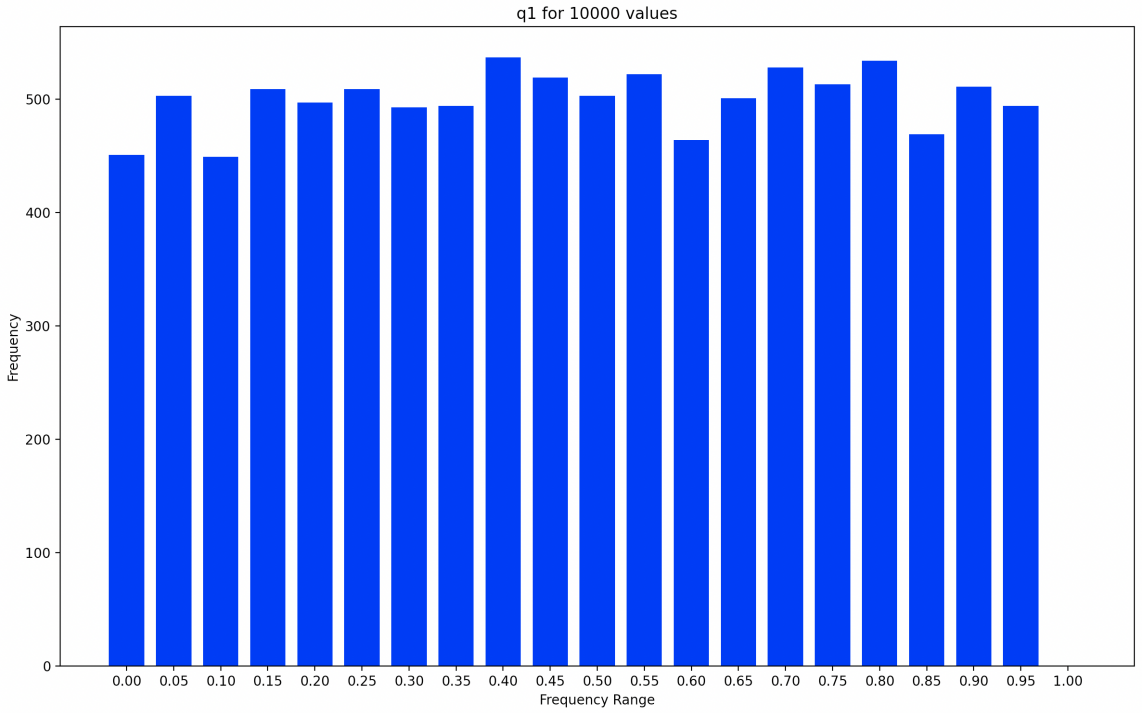
b) *Implicit Implementation*

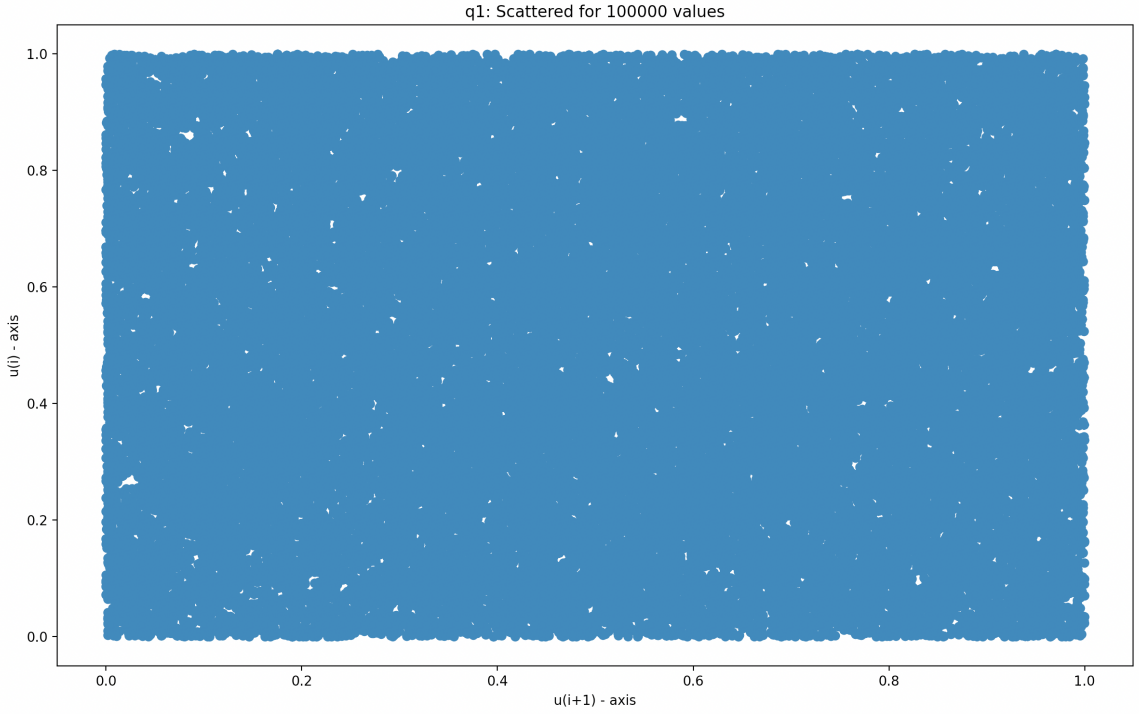
c)

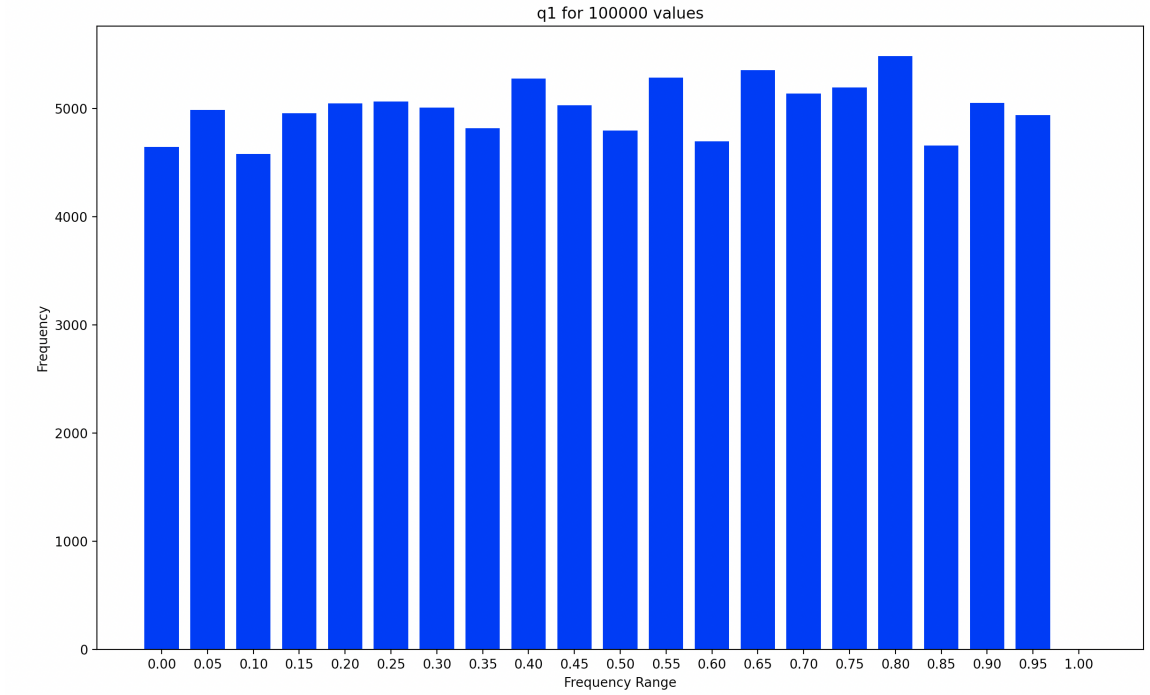



q1 for 1000 values
Frequency
Frequency Range
0
10
20
30
40
50
60
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00


q1: Scattered for 10000 values
u(i) - axis
u(i+1) - axis
0.0
0.2
0.4
0.6
0.8
1.0


q1 for 10000 values
Frequency
0
100
200
300
400
500
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Frequency Range


q1: Scattered for 100000 values
u(i) - axis
0.0
0.2
0.4
0.6
0.8
1.0
u(i+1) - axis

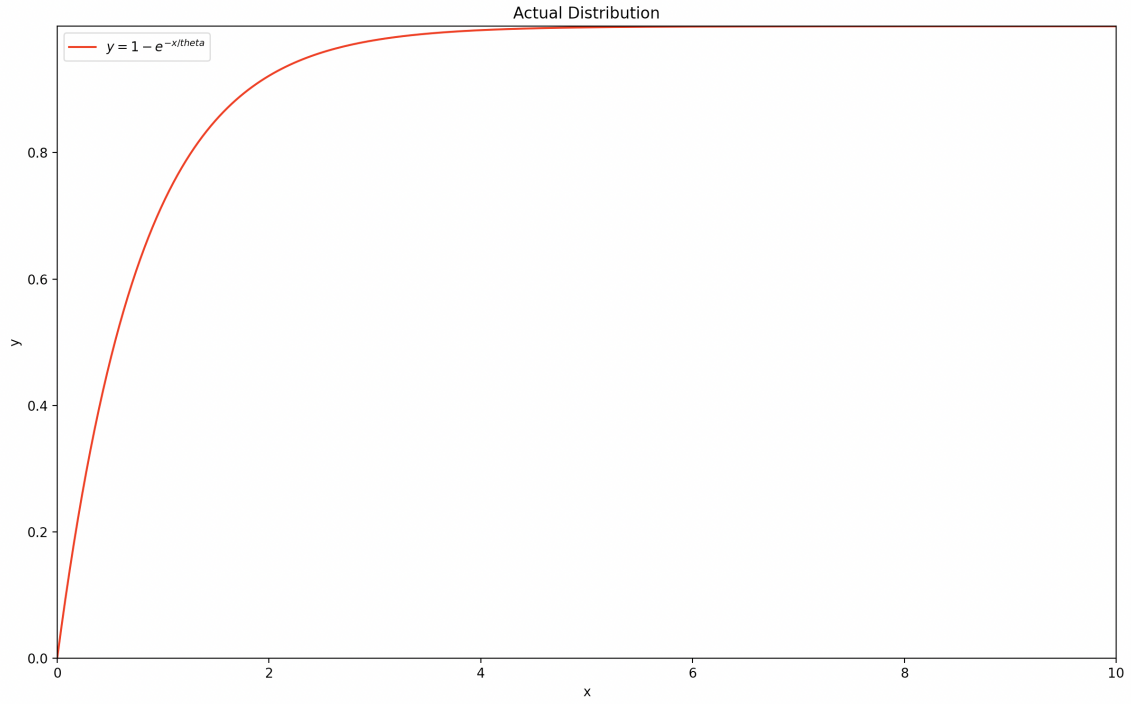


Observations:

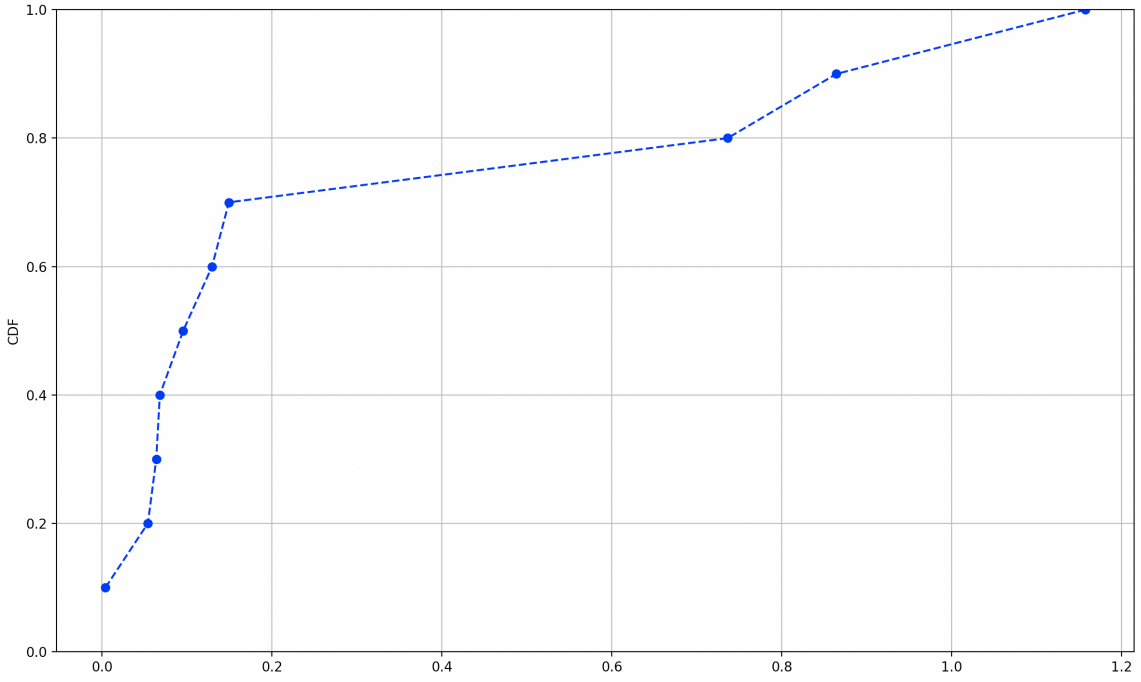
1. The plot between U_i *and* U_{i+1} suggests that the values generated are uniformly distributed between 0 and 1 since the shape of the graph is rectangular.
2. The bar graph showing frequency vs range of $U_i's$ generated supports our (1) Observation since the frequency of all ranges is almost similar. Also as the size of the set of numbers being generated keeps on increasing this observation becomes more and more prominent.

Problem II:

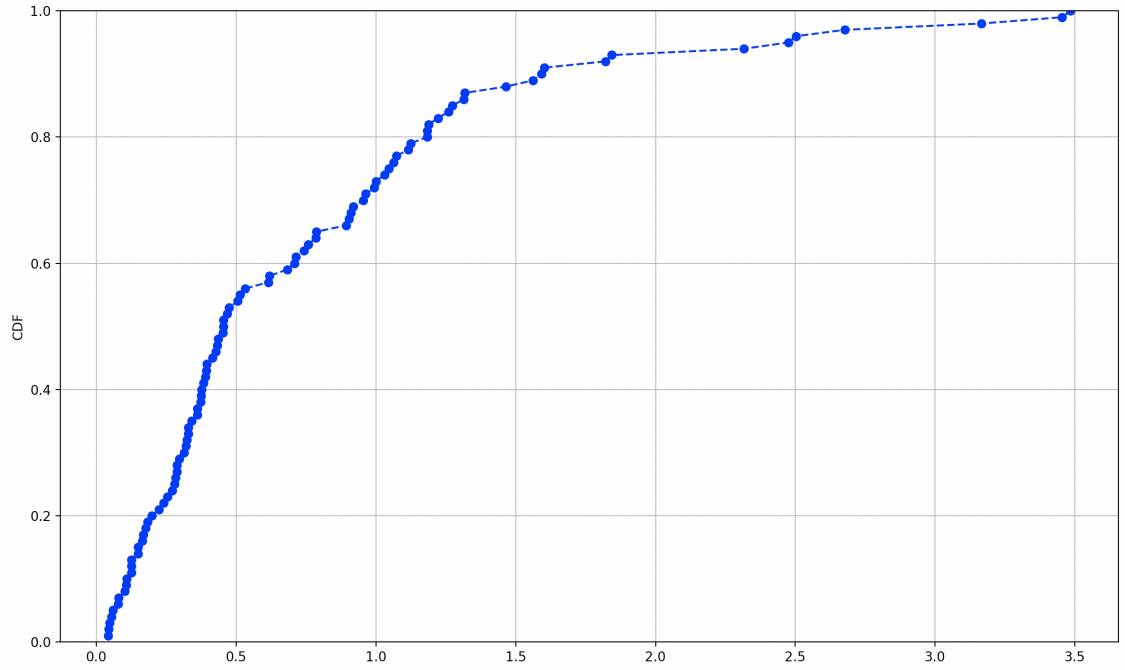
Actual Distribution



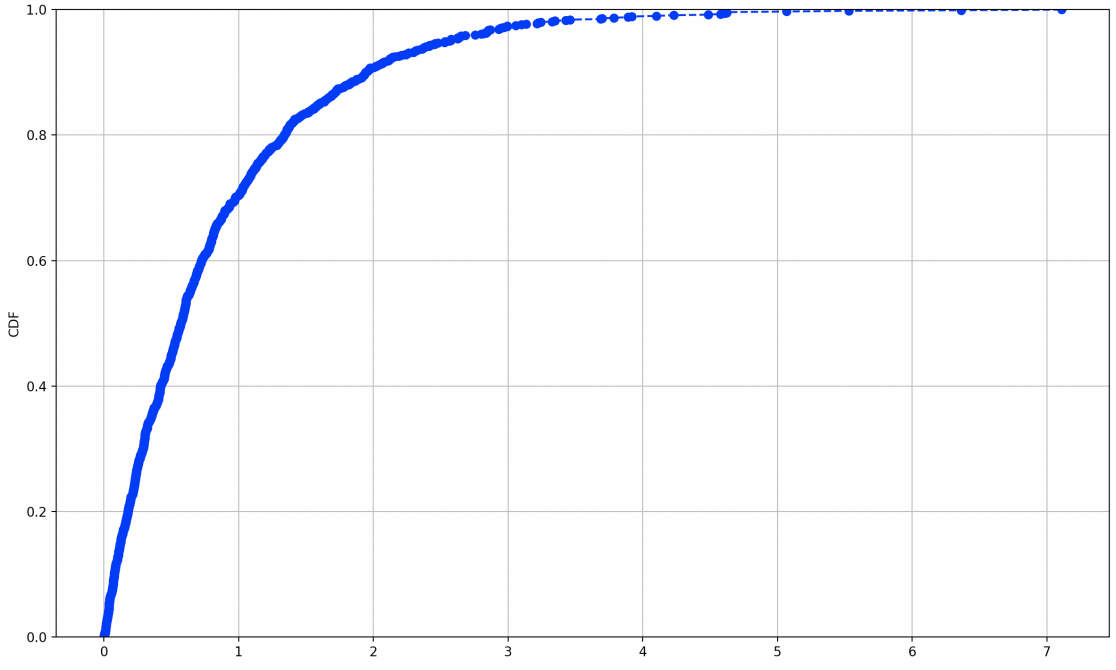
CDF for 10 Values:



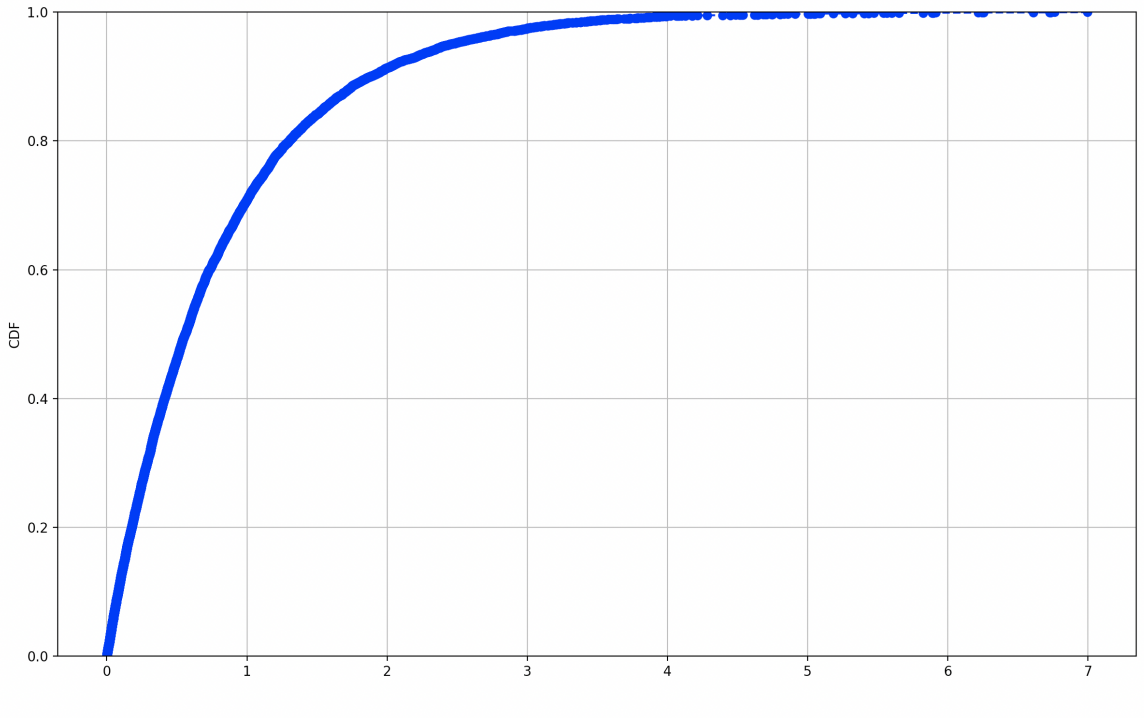
CDF for 100 Values:



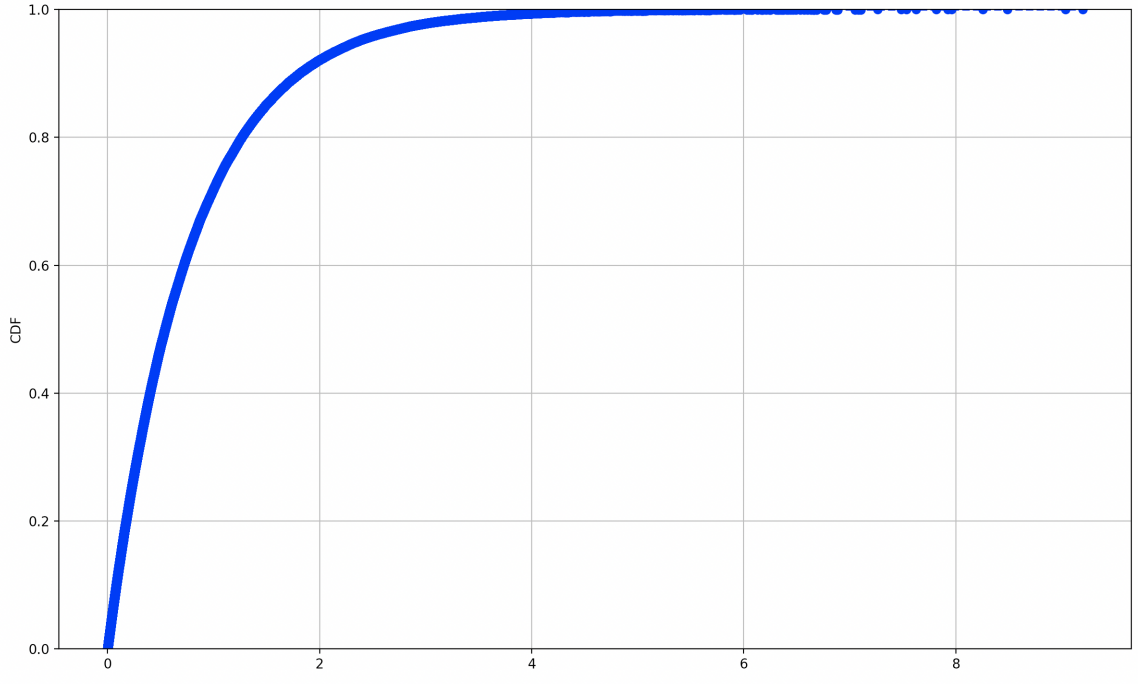
CDF for 1000 Values:



CDF for 10000 Values:



CDF for 100000 Values:



Actual Mean: **0.7853981633974483**
Actual Variance: **0.6168502750680849**

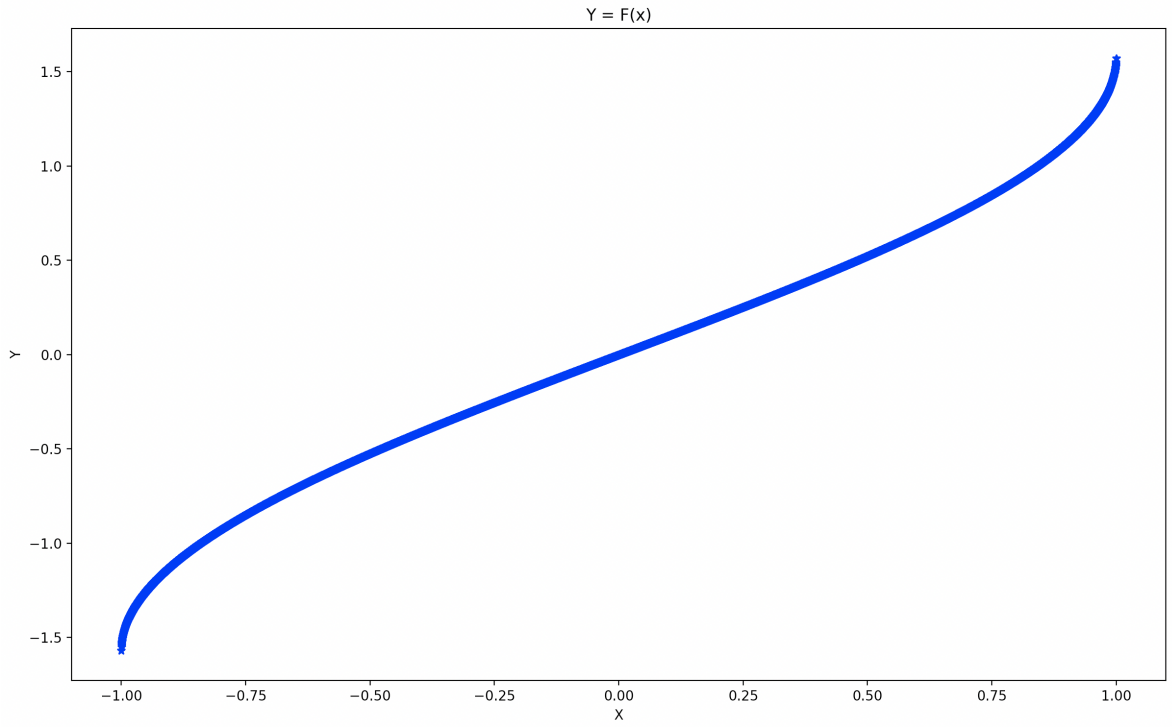
Size of Input	Sample Mean	abs(Actual Mean - Sample Mean)	Sample Variance	abs(Actual Variance - Sample Variance)
10	0.33207635528563506	0.45332180811	0.17600279636410646	0.4408474787
100	0.7598086613050881	0.02558950209	0.5478735916657148	0.0689766834
1,000	0.832494739109147	0.04709657571	0.7573424271881648	0.14049215212
10,000	0.80984048857847	0.02444232518	0.6685470412040024	0.05169676613
1,00,000	0.784452507888851	0.0009456555	0.6198169735971717	0.00296669852

Observations:

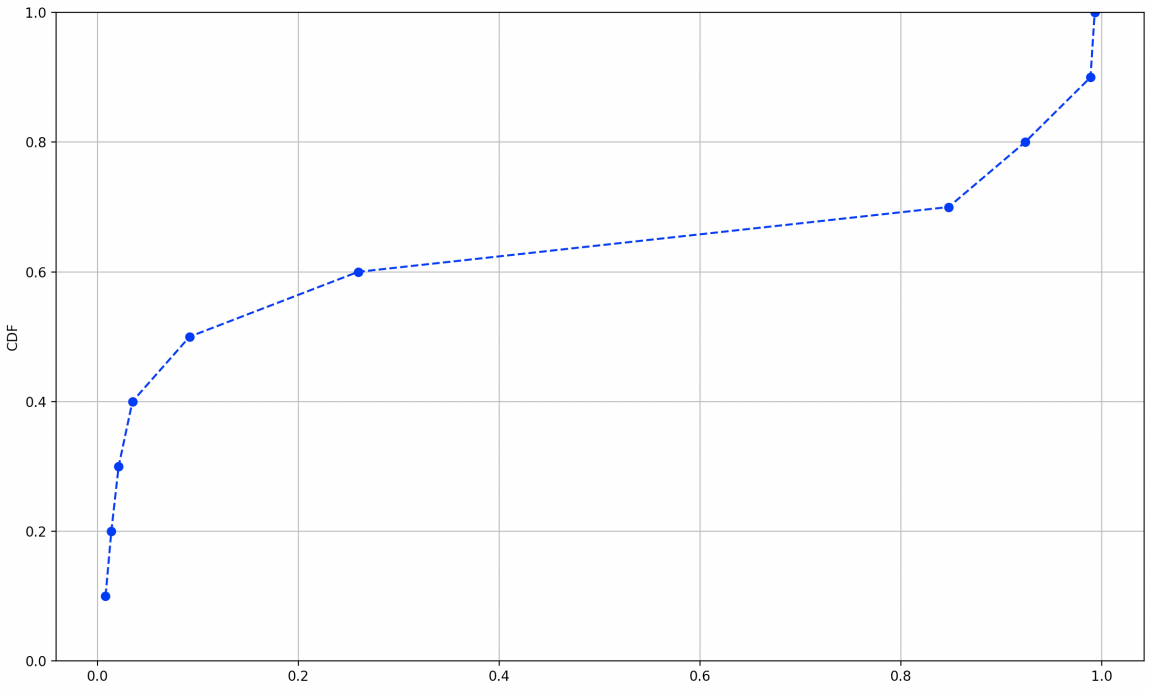
1. Our experiment verifies the **Law of Large Numbers**.
    - The law of large numbers, in probability and statistics, states that as a sample size grows, its mean gets closer to the average of the whole population.
2. We can observe that as the size of our input/sample size grows the absolute difference between the actual mean/variance and sample mean / variance keeps getting smaller and smaller. At an input size of 1,00,000, both of the above values seem to converge!
3. The graph of **actual distribution** and **cumulative distribution frequency** keeps getting more and more similar as the size of input increases.

Problem III:

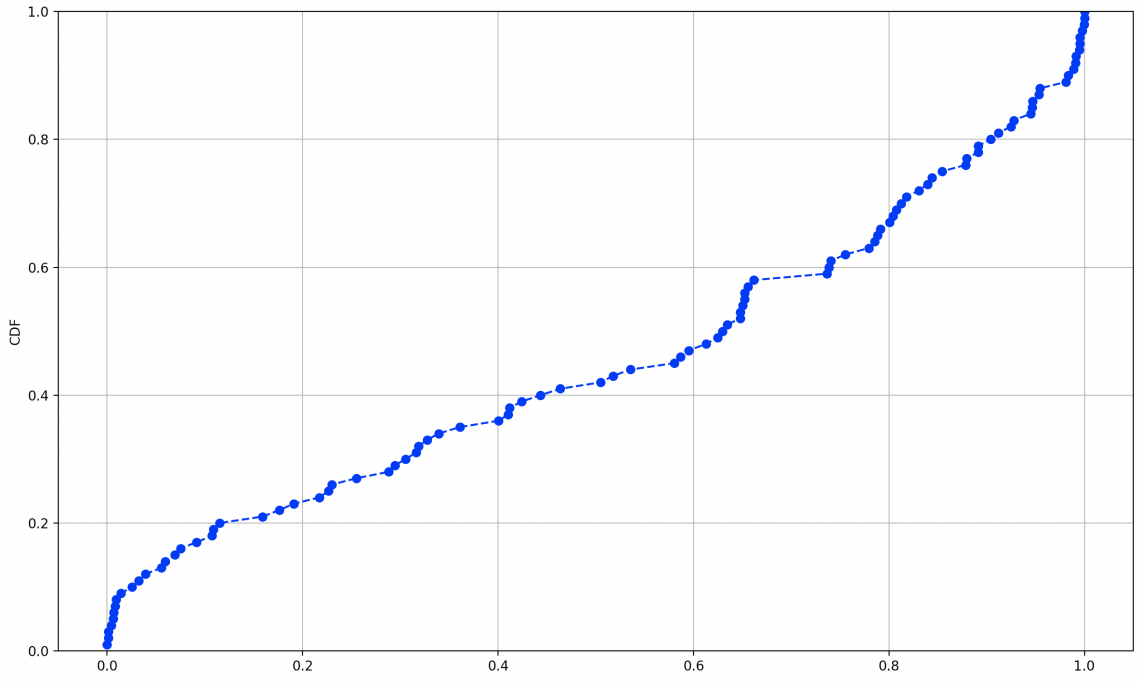
Actual Distribution



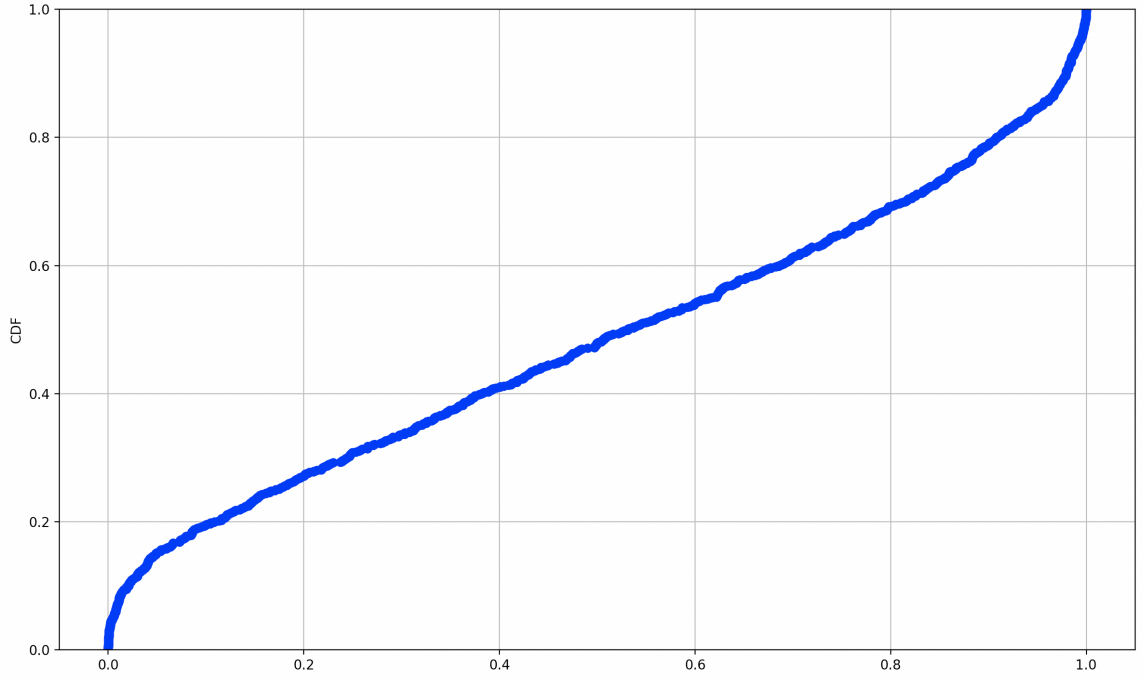
CDF for 10 Values:



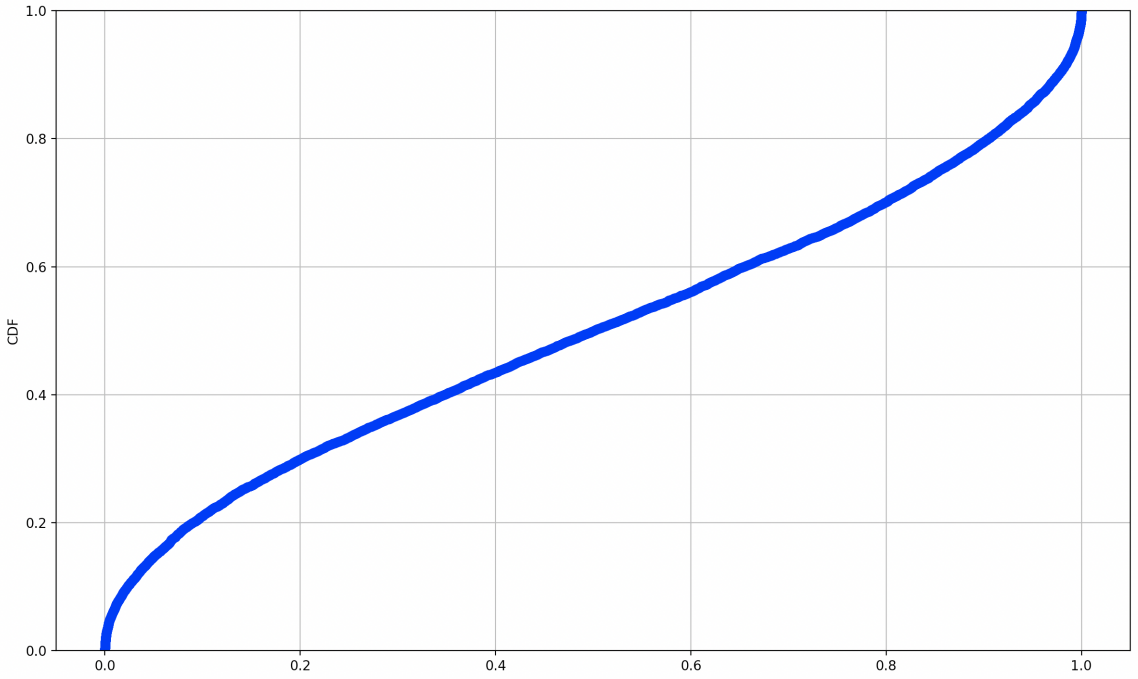
CDF for 100 Values:



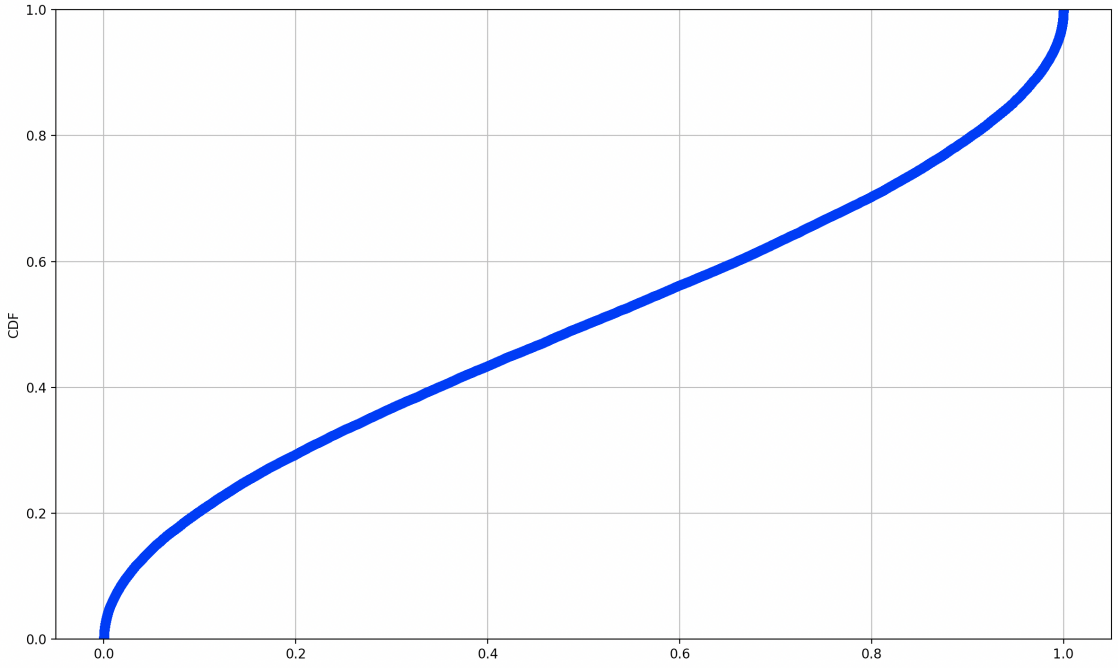
CDF for 1000 Values:



CDF for 10000 Values:



CDF for 100000 Values:



Size of Input	Sample Mean	Sample Variance
10	0.41833255854890466	0.20713322004573728
100	0.551888265760434	0.11964450861461831
1,000	0.5180800694400134	0.12435837300571814
10,000	0.5000832199304189	0.12609428349059829
1,00,000	0.5014229705045473	0.12495057006350475

Observations:

1. The graph of **actual distribution** and **cumulative distribution frequency** keeps getting more and more similar as the size of input increases.