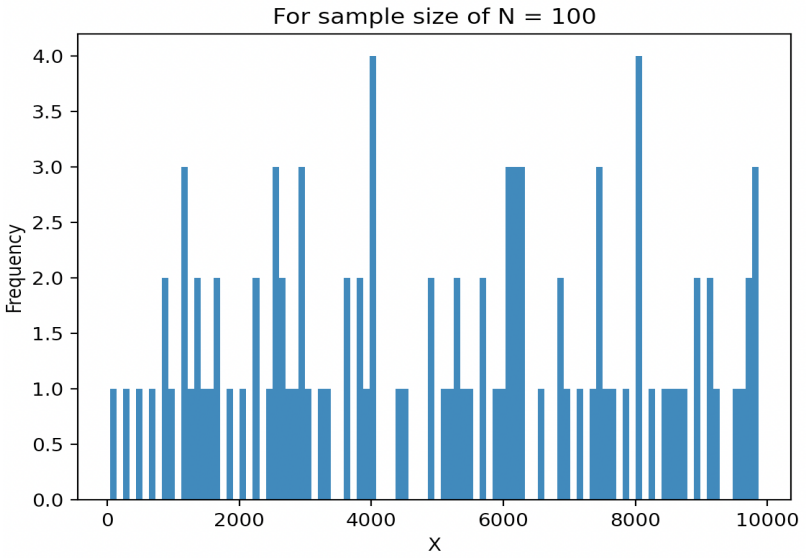
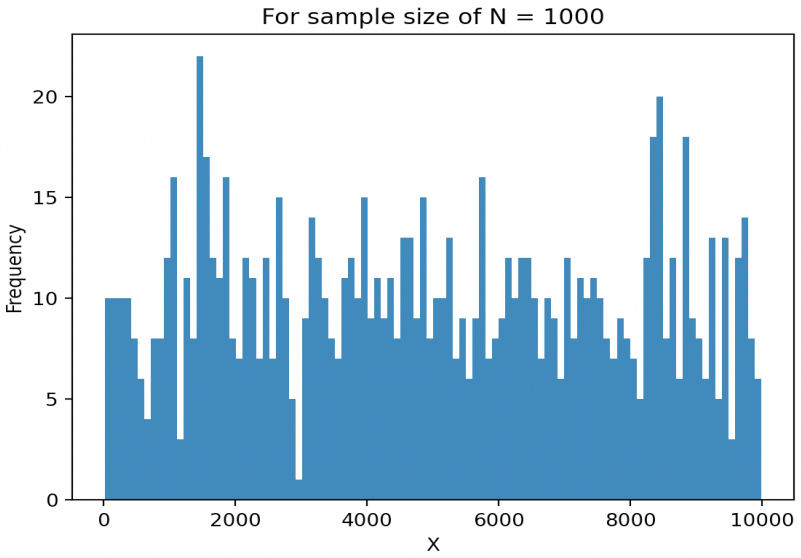
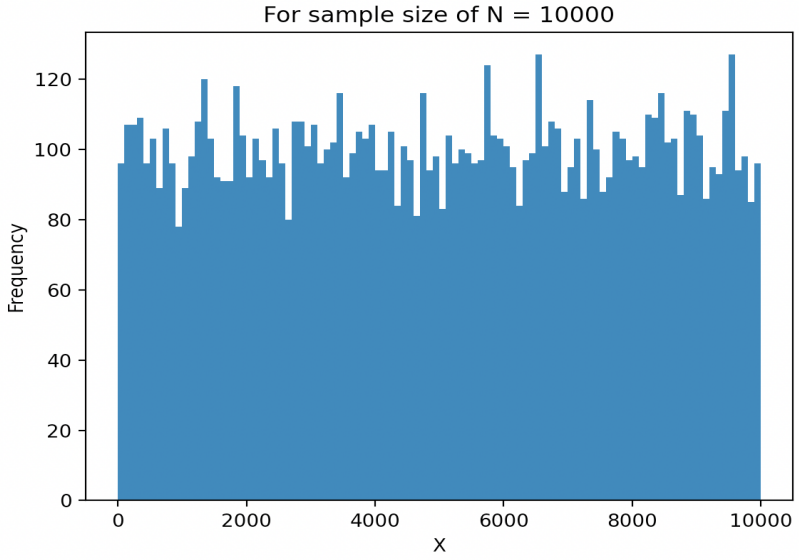
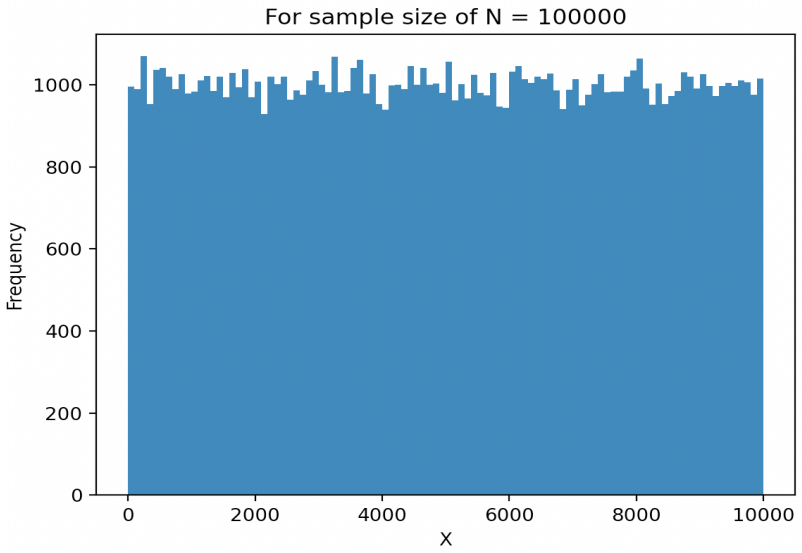
MA 323 (2020) Monte Carlo Simulation: LAB 03
Jay Vikas Sabale
180123019

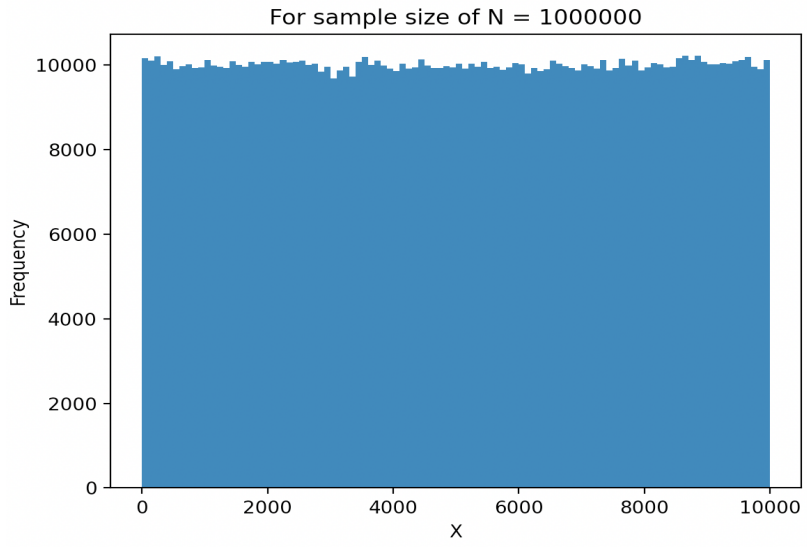
Problem I:











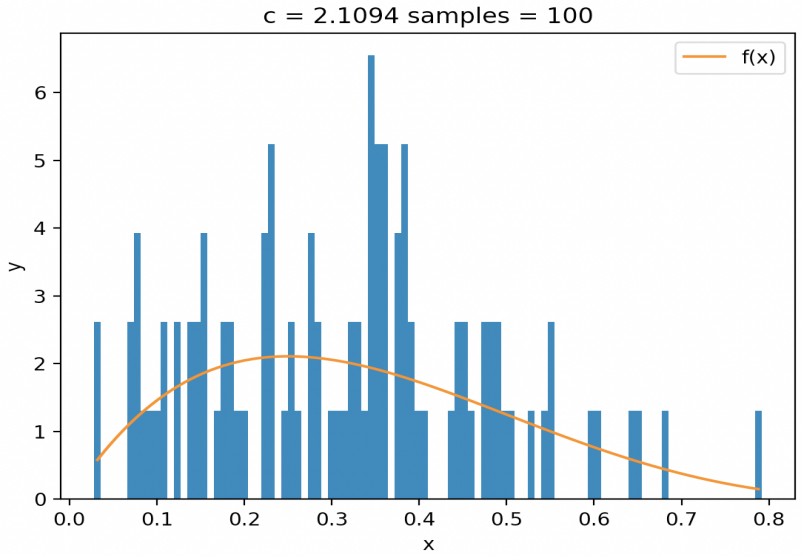
Observations:

1. As the sample size increases, the frequency of the discrete random values generated becomes more and more uniform.
2. The above histogram(s) with 100 bins justify the statement-(1).

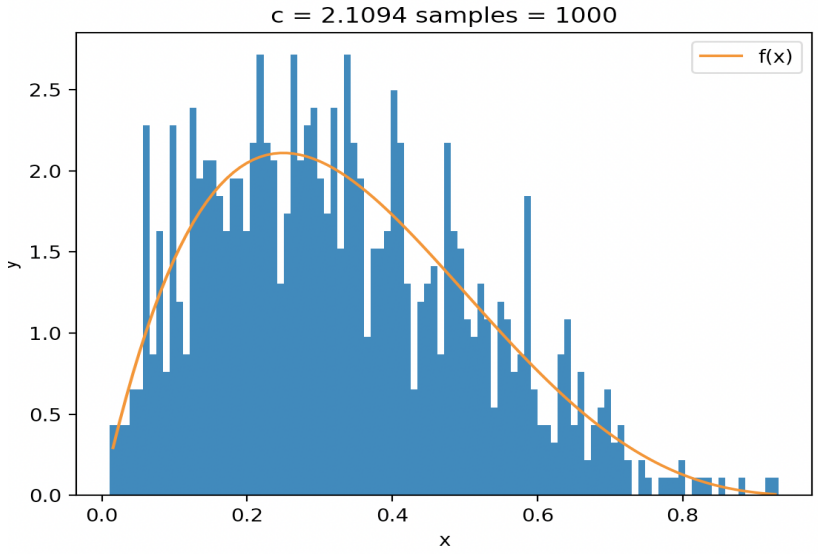
Problem II:

a) Since, $g(x) = 1, for\ U[0.00, 1.00]$, $f(x) <= c.g(x) => f(x) <= c$.
Using elementary calculus, tha maxima for $f(x)$ occurs at $x = 2.1094$.

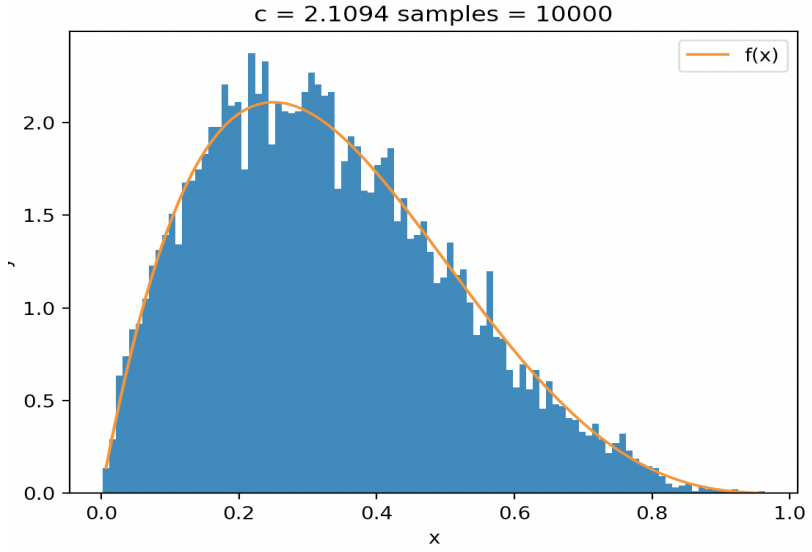
b) For sample size 100:



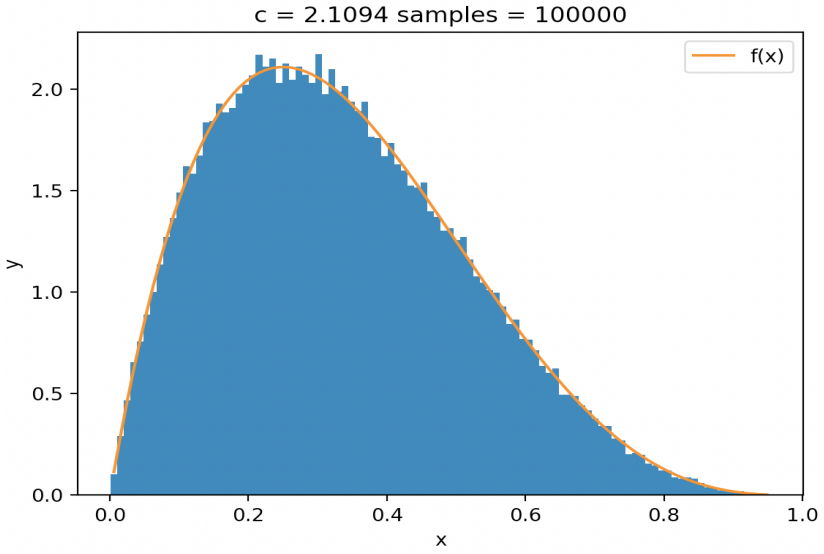
For sample size 1,000:



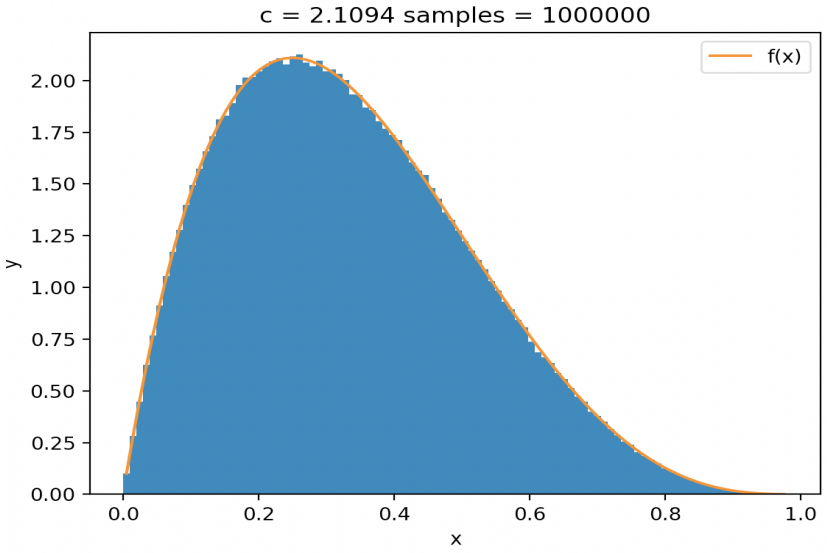
For sample size 10,000:



For sample size 1,00,000:



For sample size 10,00,000:



Clearly, from the above graphs, we can observe that for larger values of sample, f(x) and the frequency of random numbers generated converge!

c)

Sample Size	10	100	1,000	10,000	1,00,000	10,00,000
$\lvert c - avgIterations \rvert$	0.1094	0.1294	0.0136	0.0022	0.0070	0.0022

From the above table, we can observe that, as the sample size of random values generated increases, the computed value $c = 2.1094$, converges to the average iterations required to generate each random number!

d) For $c = 2.50 \; and \; c = 3.00$, I observed that in this case also, the average iterations required to generate random numbers converged to c for higher values of sample size!

For $c = 2.50$,

Sample Size	10	100	1,000	10,000	1,00,000	10,00,000
$\lvert c - avgIterations \rvert$	0.7000	0.4100	0.0110	0.0042	0.0023	0.0003

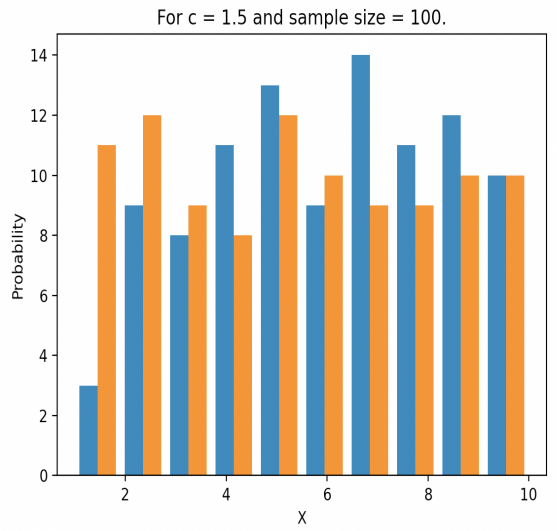
For $c = 3.00$,

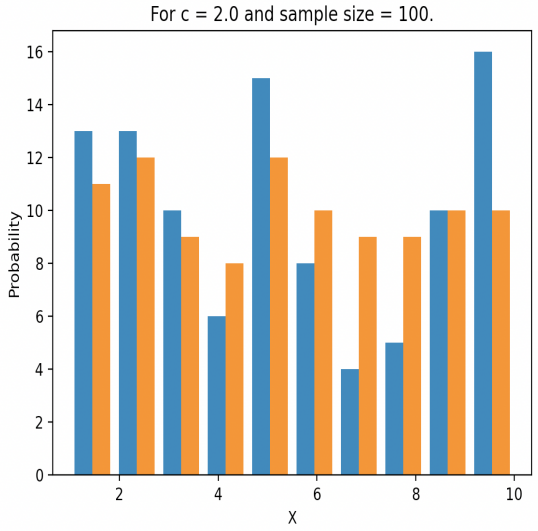
Sample Size	10	100	1,000	10,000	1,00,000	10,00,000
$\lvert c - avgIterations \rvert$	0.2000	0.0600	0.0060	0.0429	0.0026	0.0027

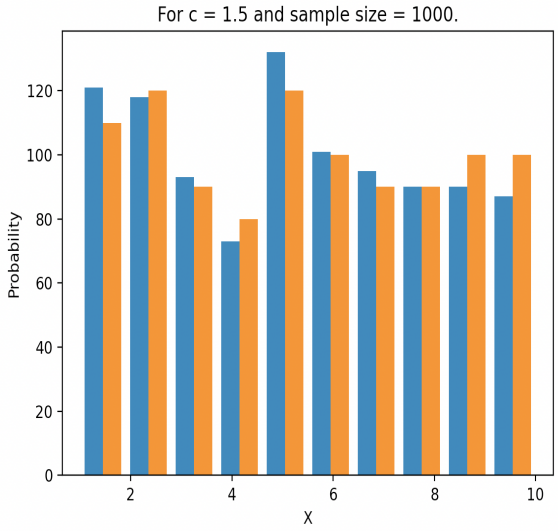
Problem III:

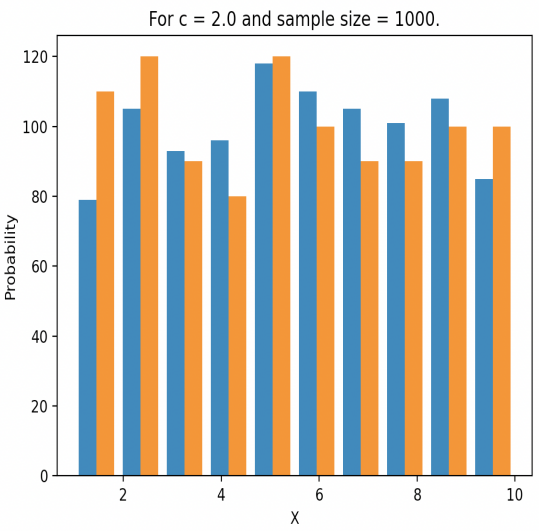
Legend:

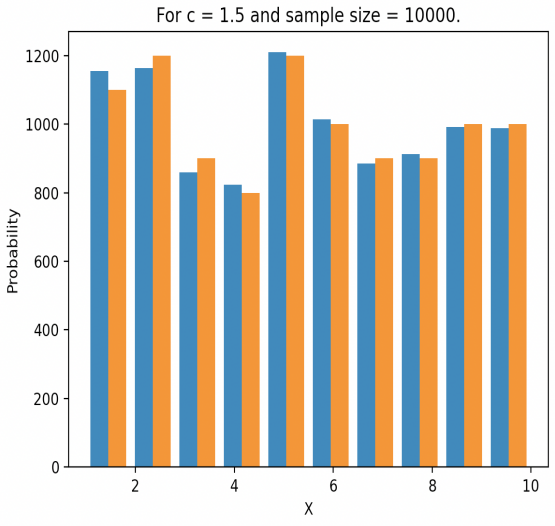
1. Orange = Actual Probability Distribution
2. Blue = Sample Probability Distribution

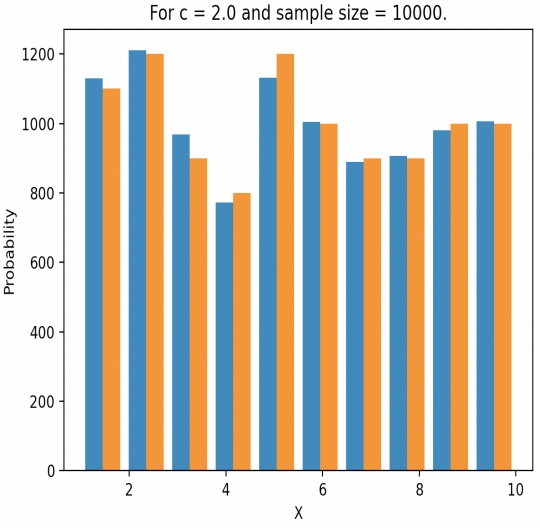


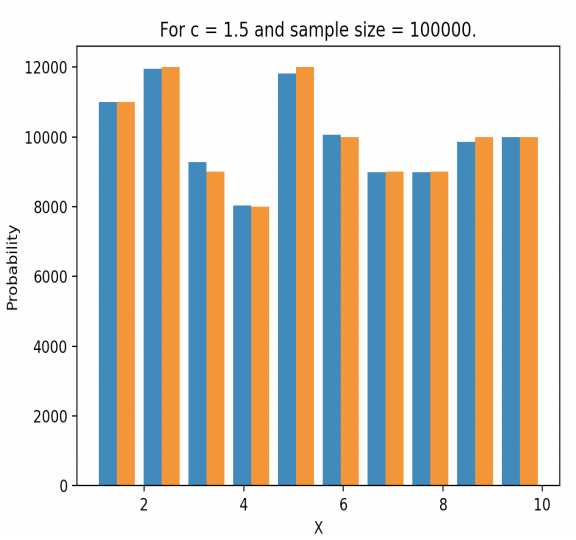


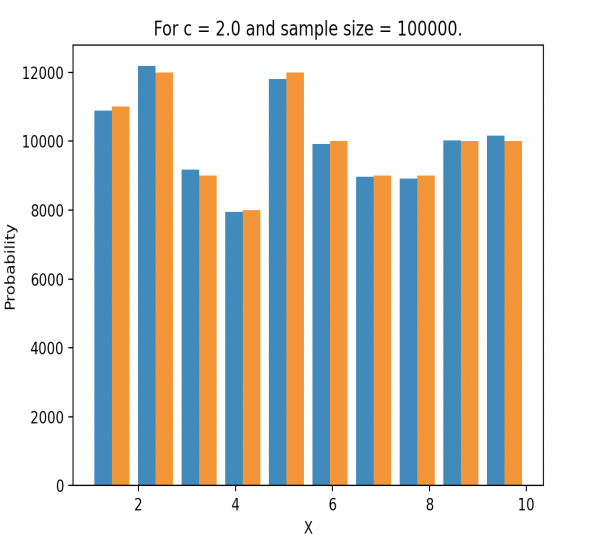


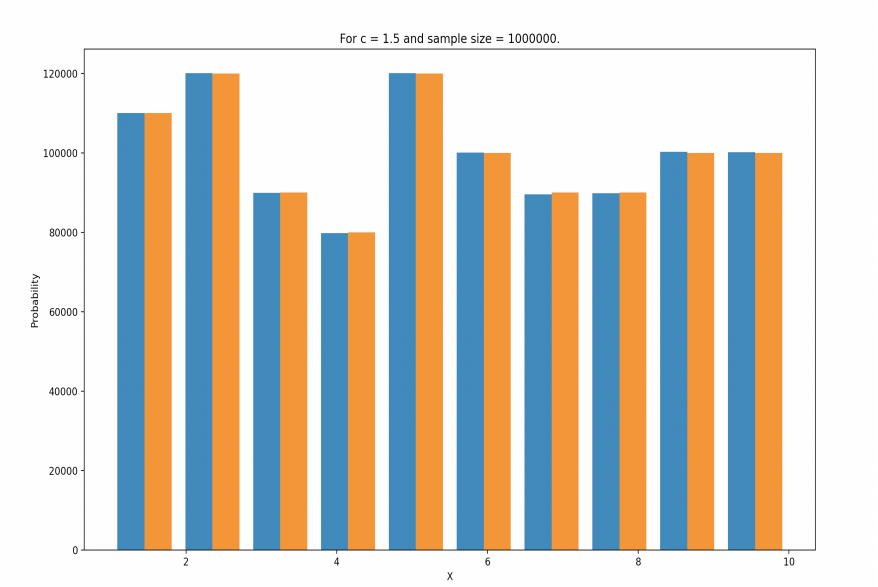


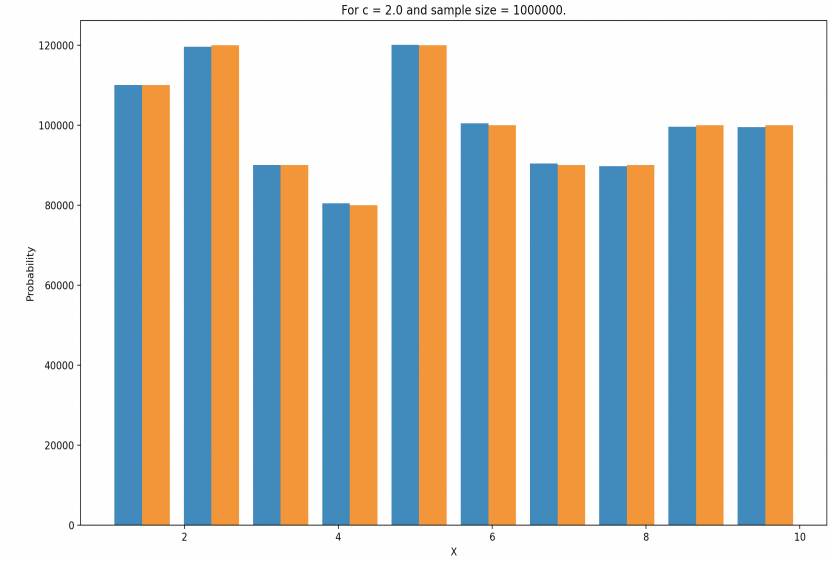





For c = 1.5 and sample size = 100000.
Probability
X
0
2000
4000
6000
8000
10000
12000
2
4
6
8
10


For c = 2.0 and sample size = 100000.
Probability
X
0
2000
4000
6000
8000
10000
12000
2
4
6
8
10


For c = 1.5 and sample size = 1000000.
Probability
X
0
20000
40000
60000
80000
100000
120000
2
4
6
8
10


For c = 2.0 and sample size = 1000000.
Probability
X
0
20000
40000
60000
80000
100000
120000
2
4
6
8
10

Observations:

1. As the sample size increases, the sample probability distribution converges with the required probability distribution!
2. The average number of iterations required to generate #sample_size random numbers, always converges to the chosen value of c, for a sufficiently large sample size!