Berechnung der Masse:

allg. $M(r) = \int_0^r 4\pi r'^2 \rho(r')\, dr'$

hier: $r = \alpha \xi \quad , \quad \rho(r) = \rho_c w^n$

$$\frac{dr}{d\xi} = \alpha \quad \Rightarrow dr = \alpha\, d\xi$$

$$\hookrightarrow M(r) = 4\pi \rho_c \alpha^3 \int_0^{\xi} \xi'^2 w^n \, d\xi'$$

dimensionslos: $M = \int_0^{\xi} \xi'^2 w^n \, d\xi'$

betrachte zunächst Lane-Emden-Gln.:

$$\frac{1}{\xi^2} \frac{d}{d\xi}\left(\xi^2 \frac{dw}{d\xi}\right) = -w^n \qquad | \cdot \xi^2$$

$$\frac{d}{d\xi}\left(\xi^2 \frac{dw}{d\xi}\right) = -\xi^2 w^n \qquad | \int_0^{\xi} d\xi$$

$$\int_0^{\xi} \frac{d}{d\xi'}\left(\xi'^2 \frac{dw}{d\xi'}\right) = -\int_0^{\xi} \xi'^2 w^n \, d\xi'$$

$$\left[\xi'^2 \frac{dw}{d\xi'}\right]_0^{\xi} = \xi^2 \frac{dw}{d\xi}\bigg|_{\xi} = -\int_0^{\xi} \xi'^2 w^n \, d\xi'$$

$$\Rightarrow M = -\xi^2 \frac{dw}{d\xi}\bigg|_{\xi}$$