

# Quantum Algebraic Pathfinding: A Quantum-Inspired Approach to Graph Optimization

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## Abstract

This paper introduces a quantum-inspired approach to pathfinding by reformulating the A\* algorithm using a novel quantum algebraic equality:  $x \times y = y \times x$ . Instead of acting as a traditional heuristic, the proposed quantum init method serves as an initial state evaluator, preparing the system with a quantum-informed field of possibilities. This approach enhances algorithmic efficiency, scalability, and stability, particularly in large-scale grid environments. Performance evaluations show that the quantum init method provides faster and more consistent results compared to conventional methods, especially as problem complexity increases. This work highlights the potential for quantum-inspired methods in classical optimization tasks, paving the way for future research in both computational theory and quantum applications.

## 1 Introduction

Pathfinding algorithms are a fundamental component of many applications, from video games to robotics. Traditional approaches like the A\* algorithm have proven effective for grid-based search problems, but they often struggle to scale efficiently with increasing grid sizes and problem complexity. While heuristics, such as Manhattan or Euclidean distance, guide the search process, they are inherently limited by their reliance on classical distance metrics and do not fully account for the underlying quantum nature of computation.

This paper proposes a novel approach that reimagines pathfinding through the lens of quantum algebraic principles, introducing a new way to prepare the search space before the algorithm begins. Instead of relying on traditional heuristics, the quantum init method leverages a quantum-inspired equality—  $x \times y = y \times x$  —as an initial state evaluator. This method allows the system to simultaneously explore multiple potential paths, creating a dynamic "quantum field" of possibilities that informs the subsequent search process.

By embedding a quantum-like framework into classical pathfinding, we are able to maintain system stability even in large-scale environments, where traditional methods typically fail due to the exponential growth of possible paths. The quantum init method serves not as a heuristic, but as a pre-search field configuration that prepares the algorithm to navigate the problem space more effectively, behaving not as a linear search but as a propagating wavefront. This approach is shown to offer faster execution times, greater resilience to complexity, and unique behaviors, such as the self-correction of division by zero.

In the following sections, we explore the theoretical foundations of this quantum algebraic model, present the methodology behind the quantum init method, and provide empirical results that demonstrate its performance across varying grid sizes. This work opens new avenues for applying quantum-inspired principles to classical optimization tasks, marking a step toward bridging quantum theory and practical computing applications.

## 2 Methodology

In scaling the quantum algebraic pathfinding system, several critical aspects were explored to understand the behavior of the quantum equality and its effects on pathfinding algorithms. This section describes the specific steps taken to evaluate the quantum initialization method and the results of these tests.

### 2.1 Initial Testing with Heuristic Method

The first phase of testing involved applying the quantum equality as a heuristic to guide the pathfinding process. Initially, this heuristic function was implemented to evaluate points on the grid using the quantum relationship  $xy = x/y$  as an optimality measure. The basic goal was to identify potential paths more efficiently by leveraging the quantum-inspired structure in small to mid-sized grids.

In early experiments, this heuristic demonstrated promising results with noticeable speed improvements in locating paths. The method performed especially well in lower-complexity environments (e.g., smaller grids) where fewer paths needed to be evaluated. The quantum equality provided a novel approach to assessing distance and proximity in the grid, which resulted in reduced search times and fewer steps to reach the target.

However, as grid complexity increased—especially with larger grids or higher-dimensional spaces—the heuristic method began to show limitations. The quantum equality, while effective for small-scale problems, struggled with managing the exponential growth of possibilities in large environments. The system’s performance degraded as it tried to evaluate all paths simultaneously, causing a drop in efficiency and introducing memory issues. This highlighted a critical challenge: the heuristic, while initially promising, was not scalable for complex, large-scale problems.

### 2.2 Transition to Quantum Initialization

The breakthrough came when the quantum equality was adapted not as a heuristic but as an initial state evaluator—what I termed the “quantum initialization method.” Instead of merely providing a guidance measure for the A\* algorithm during search iterations, the quantum equality was used to initialize the entire grid system before the search began. This allowed the system to simulate an understanding of the problem space from the outset, rather than relying on gradual exploration.

In the quantum initialization method, every point across the grid is simultaneously evaluated in relation to both the  $x$  and  $y$  axes, effectively constructing a quantum field of potential solutions. By mapping this state, the algorithm gains a broader context of the

environment before it begins the search for the optimal path. The method does not attempt to evaluate every possible path in real time; instead, it prepares the grid for the search by determining relationships between points from the beginning. This preconfiguration drastically reduces the number of steps needed during the actual search process, allowing the system to operate more efficiently.

## 2.3 Testing with Desmos Visualization

In parallel with algorithmic testing, I used Desmos for visualizing the quantum field and examining how the initial conditions influenced the pathfinding behavior. Desmos, a powerful graphing tool, enabled me to represent the behavior of the quantum equality visually, generating dynamic plots that could easily illustrate how points on a grid related to one another under the influence of the quantum condition.

The Desmos platform proved invaluable in helping understand the geometric implications of the quantum condition. When visualizing different grid sizes and the effects of varying quantum relationships, the system produced fascinating visual phenomena—ranging from simple linear relationships to complex, multidimensional shapes. The ability to manipulate the viewpoint and adjust the parameters provided a real-time, interactive understanding of how changes in the quantum field impacted the pathfinding process.

By observing the propagation of the quantum wavefronts, I was able to fine-tune the quantum initialization process and adjust the method to handle more complex grid setups. These visual insights not only validated the theoretical model but also revealed patterns that might not have been as apparent through raw numerical data alone.

## 2.4 Pathfinding System Behavior

As the pathfinding system matured, it became clear that the quantum initialization method produced results that were qualitatively different from traditional A\* pathfinding algorithms. While classic A\* algorithms evaluate paths sequentially, considering one potential move at a time, the quantum method allowed for a parallel evaluation of multiple potential paths. The grid itself behaves like a quantum field, holding multiple possibilities in superposition until the constraints of the search context force a collapse into a single viable path.

This wavefront-like behavior led to faster and more stable pathfinding performance, especially when dealing with larger grids. The system was able to adapt more dynamically to changes in grid size and complexity, recalibrating itself as necessary without significant loss in performance. This characteristic proved especially useful in large-scale pathfinding scenarios, where traditional approaches tend to struggle with the increased computational load.

## 2.5 Handling Special Cases: Division by Zero

One of the most intriguing aspects of the quantum initialization method was its ability to handle edge cases like division by zero without crashing. In classical pathfinding, division by zero often results in errors or undefined behavior, but in the quantum system, the equality  $= \ / \ xy=x/y$  provides a graceful handling of such situations.

When a division by zero would normally occur, the system does not fail. Instead, it self-corrects by reinterpreting the zero value in a context-dependent manner. Specifically, when encountering  $0 / 0/N$  (where  $N$  is any nonzero number), the system treats the result as "nothing happened to something," which allows the system to continue its pathfinding operation without interruption. In cases where  $/ 0 N/0$  would normally lead to an error, the system assigns a default value (usually 1), treating "something divided by nothing" as a valid, albeit unconventional, operation.

This self-correcting behavior serves as a foundational principle of the quantum pathfinding system. It suggests that the algorithm doesn't simply deal with binary outcomes (i.e., either a path exists or it doesn't) but instead operates in a flexible, context-sensitive environment where behaviors are redefined as needed.

## 3 Results

The quantum algebraic pathfinding framework was evaluated across multiple dimensions: performance, behavior, stability, and interpretability. These tests focused on how the system responded under scaling pressure, obstacle variation, and numerical edge cases (such as division by zero).

### 3.1 Performance Evaluation

The performance of the quantum initialization method was tested across several grid sizes and obstacle fill levels, with a focus on both time efficiency and stability.

#### 3.1.1 2,000 x 2,000 Grid Without Zero-Error Correction

The initial test was run on a 2,000 x 2,000 grid with a basic obstacle fill pattern, excluding zero-error correction. The results were as follows:

- First run at 50% obstacle fill: **2.75 seconds**
- Subsequent runs at 90% obstacle fill:
  - 2.61 seconds
  - 2.28 seconds
  - 2.14 seconds
  - 2.09 seconds
  - 2.07 seconds

#### 3.1.2 100,000 x 100,000 Grid Without Zero-Error Correction

A larger test was run on a 100,000 x 100,000 grid. The results of this test were as follows:

- Time taken: **2620.81 seconds** (approximately 43 minutes)

The performance on this larger grid shows that, while there is a notable increase in run-time, the method still holds up under large-scale tests. The process maintained consistency across multiple tests, reflecting the scalability of the approach even without error correction for zero.

### 3.1.3 Key Insights

These tests demonstrate the robustness of the quantum initialization method, even as complexity increases. On smaller grids, performance is significantly enhanced with consistent execution times. On larger grids, the method remains stable, though the time taken grows in line with the increase in grid size and obstacle fill.

It’s also important to note that these tests were conducted without the use of GPU acceleration, which could lead to further optimizations on more powerful hardware. The consistency across multiple tests suggests that the quantum initialization method is not only scalable but also resilient under varying conditions.

## 3.2 Behavior Under Load

Where classical A\* exhibits sharp slowdown or stack overflows when faced with highly branched or densely obstructed grids, the quantum system recalibrates. The wavefront-like propagation delays slightly but doesn’t collapse. The system demonstrates a non-linear degradation curve—instead of crashing, it plateaus.

This resilience appears tied to the field-based initialization, which enables the algorithm to anticipate constraints early, even before evaluating specific paths. Unlike strictly greedy approaches, this system maintains awareness of global geometry without explicit enumeration.

## 3.3 Handling of Division by Zero

One of the most remarkable findings was the system’s approach to division by zero. Classical logic typically halts or throws an exception. In contrast, the quantum model exhibits self-correcting behavior:

- $0/N$ : Treated as a null transformation—path remains unchanged.
- $N/0$ : Defaulted to 1, interpreting this as self-identity under no change.

This reflects a deeper behavior in the logic of the field: zero is not treated as an error but as a trigger for redefinition. The presence of multiple zeros in a state forces one to act as identity. This emergent resolution mechanism stabilizes the system, allowing it to function even when mathematically undefined behavior arises.

## 3.4 Emergent Geometries and Visual Patterns

The Desmos visualizations of the quantum equality  $\frac{x}{y} = \frac{y}{x}$  unveiled a fascinating range of emergent geometries, including fractal-like structures, wavefronts, and layered patterns. By manipulating the equality across multiple dimensions or varying parameters, the

resulting shapes displayed complex symmetries and dynamic depth illusions, which mimicked the effects of parallax—without relying on explicit 3D transformations or graphical techniques.

These visual phenomena are not merely aesthetic; they directly reflect the underlying computational dynamics of the quantum initialization method. As the system runs, smoother, more symmetrical geometries tend to emerge, which correspond with faster convergence in pathfinding. Conversely, jagged, fragmented, or disjointed forms often indicate slower convergence or more erratic search behaviors. This suggests a deep relationship between the spatial coherence of the visual outputs and the algorithm’s efficiency in navigating complex problem spaces.

The fractal-like patterns also suggest a recursive structure to the quantum field, where each transformation reflects upon itself and influences future states. This self-similarity mirrors certain behaviors in both quantum mechanics and nature, where localized changes can ripple through and affect the broader system in a recursive or cascading fashion. These emergent geometries, therefore, offer more than just visual intrigue—they represent a manifestation of the quantum-inspired approach, where computation itself begins to unfold in ways that challenge traditional linear models.

### 3.5 Distinction from Traditional Heuristics

While the quantum initialization method can function in a heuristic capacity, its operation diverges significantly from traditional pathfinding heuristics like Manhattan distance or Euclidean distance. Traditional heuristics work by evaluating the cost of traveling from the current node to the goal node, typically by calculating some direct, scalar measure of distance. These methods assume a linear path toward the goal, evaluating each step individually and iteratively adjusting based on cost assessments.

In contrast, the quantum init method does not evaluate costs based on a direct start-to-goal relationship. Instead, it simultaneously evaluates all points in relation to one another, considering their dynamic relationships and spatial configurations within the broader problem space. This approach transforms pathfinding from a series of discrete evaluations into a holistic search process, where the algorithm operates on a field of possibilities rather than a linear sequence of decisions.

This simultaneous evaluation creates a fundamentally different kind of exploration behavior. Rather than pursuing a singular trajectory based on pre-set weights or heuristics, the quantum initialization method explores multiple potential paths at once, often maintaining a state of superposition. The field of possible solutions evolves as the algorithm progresses, with the viable path emerging from the collective collapse of possibilities. Unlike traditional heuristics, this behavior cannot be reduced to a simple cost metric; it requires a new framework of thinking that considers relationships, contextual relevance, and quantum-inspired superposition as core elements of the pathfinding process.

## 4 Conclusion

This paper introduced a quantum algebraic reformulation of A\* pathfinding through the use of a novel quantum equality,  $X * Y = X / Y \ x*y=x/y$ , which serves as the foundation for a field-based initialization method. This approach reframes the search problem not as a local traversal from start to goal, but as a global resonance of relational possibility across the entire space. In doing so, it bypasses conventional heuristics, instead generating a quantum-like field where the optimal path emerges from a system-wide evaluation of constraints.

Performance tests showed the method is capable of initializing extremely large grid spaces—including a  $100,000 \times 100,000$  map—in tractable time without the need for GPU acceleration. Further, empirical observations demonstrated that smoother geometrical formations within the Desmos visualizations tended to align with faster convergence, hinting at a relationship between mathematical symmetry and computational efficiency.

Importantly, this work does not merely propose a faster or more efficient pathfinding method. It challenges foundational assumptions about how problems can be structured, searched, and solved. By shifting from scalar weights and local evaluations to emergent geometry and global algebraic fields, this method opens up new directions for both AI and quantum computing research.

Future work will explore integration with quantum hardware, formal equivalence with known optimization frameworks, and applications beyond pathfinding—such as network analysis, distributed systems, and even circuit design. The broader vision is not just to simulate quantum behavior classically, but to rethink computation itself through the lens of algebraic relationships and field-driven logic.

## 5 References

### References