UNIVERSITÀ DEGLI STUDI DI MILANO
FACOLTÀ DI SCIENZE E TECNOLOGIE

Master degree in Physics

Development of an open-source calibration framework for superconducting qubits

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A.A. 2024/2025

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Notes on quantum computing

1.1 Qubits

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1.2.1 Density matrix

1.3 Quantum operations

A quantum operation is a mathematical transformation that describes how a quantum state changes as a consequence of a physical process. Formally, it is a map $\mathcal{E}$ that transforms a quantum state described by a density operator $\hat{\rho}$ into another state described by a new density operator $\hat{\rho}'$:

$$\mathcal{E}(\rho) = \rho'. \tag{1.1}$$

The simplest example of a quantum operation is the evolution of a quantum state $\hat{\rho}$ of a closed quantum system, under a unitary operator $\hat{U}$, which can be written as $\mathcal{E} \equiv \hat{U}\hat{\rho}\hat{U}^\dagger$.

Depolarizing chennel A depolarizing channel describes a process in which the current state of the n-qubit system ρ, is replaced by $\frac{\mathbb{I}}{2^n}$, with probability d. This process can be represented with a quantum map as follows:

$$\mathcal{E}_{dc}(\rho) = d\frac{\mathbb{I}}{2^n} + (1-d)\rho \tag{1.2}$$

1.4 Superconducting qubits

Chapter 2

Qibo

Chapter 3

Results

Tutti i risultati che sono presentati nel seguito sono stati ottenuti utilizzando il software di `Qibolab`per l'interazione con gli strumenti del laboratorio e `Qibocal`per il controllo delle operazioni sui qubit. L'hardware è un chip ... di QunatumWare. Durante il lavoro condotto per questo progetto di tesi entrambe le libereria, sia Qibocal che Qibolab undergo update and release, for this reason the first part of this work was realized using `Qibocal`v0.1 and `Qibolab`v0.1 while the second part of the work, dato che puntava anche allo sviluppo di routine ch epotessero essere utili per la calibrazione dei qubit è stato realizzato direttamente con `Qibocal`v0.2 e `Qibolab`v0.2.

3.1 RB fidelity optimization

3.1.1 Randomized Benchmarking

A strong limitation to the realization of quantum computing technologies is the loss of coherence that happens as a consequence of the application of many sequential quantum gates to to the quibts. A possible approach to characterize gate error is the quantum process tomography which allows the experimenter to establish the behaviour of a quantum gates; the main drawback of this approach is that process tomography can be very time consumig since its time complexity scales exponentially with the number of qubits involved [1] and the result is affected by state preparation and measurements (SPAM) errors.

To overcome these limitations, randomized benchmarking (RB) was introduced and is currently widely used to quantify the avarage error rate for a set of quantum gates.

The main idea is that the error obtained from the combined action of random unitary gates drawn from a uniform distribution with respect to the Haar measure [2] and applied in sequence to the qubit will avarage out to behave like a depolarizing channel [3]. This last consideration simplifies the characterization of noise because it removes dependence on specific error structures and allows fidelity to be extracted through a simple exponential decay.

It was later shown that it is possible to simplify this procedure even more, by restricting the unitaries to gates in the Clifford group [1] and by not requiring that the sequence is strictly self-inverting [4].

The fundamental principle of RB is the application of sequences of randomly selected quantum gates from the Clifford group $\mathcal{C}$ followed by an inversion gate which, in absence of noise, return the system to its initial state. For real systems, where noise is present, the observed survival probability provides an estimate of the avarage gate fidelity.

The standard RB protocols consist of the following steps:

1. Initialize the system in ground state $|0\rangle$
2. For each sequence-length m build a sequence of m randomly drawn Clifford gates $C_1, C_2, ..., C_m$
3. Determine the inverse gate $C_{m+1} = (C_m \circ ... \circ C_1)^{-1}$
4. Measure $C_{m+1} \circ C_m \circ ... \circ C_1 |0\rangle$

The process must be repeated for multiple sequence of the same length and with varying length.

In ideal systems without noise we should have

$$C_{m+1} \circ C_m \circ ... \circ C_1 |0\rangle = (C_m \circ ... \circ C_1)^{-1} \circ (C_m \circ ... \circ C_1) |0\rangle = |0\rangle \tag{3.1}$$

[1] unitary rotations mapping the group of Puali operators in itself

In realsystems, where noise is present, eq. **??** does not hold; instead randomization with Clifford gates behave as a depolarizing channel 1.2 with depolarization probability d. The survival probability of the initial state $|0\rangle$ for different sequence lengths follows the exponential decay model

$$F(m) = Ap^m + B, \tag{3.2}$$

where $1 - p$ is the rate of depolarization and A and B capture the state preparation and measurement error. Note that the exponential form arises naturally due to the assumption that each gate introduces independent noise.

The parameter p is directly related to the depolarization probability d through the avarage gate fidelity F which, for a depolarizing channel, is given by

$$F = 1 - \frac{d}{2^n - 1}. \tag{3.3}$$

For the details of the calculations to obtain eq. 3.3 see Since the decay rate satisfies $p = 2F - 1$ we can express

$$p = 1 - \frac{d}{2^n - 1} \tag{3.4}$$

Now we can derive the avarage error per Clifford gate $\epsilon_{Clifford}$

$$\epsilon_{Clifford} = 1 - F, \tag{3.5}$$

where F is the avarage gate fidelity. Sobstituting in 3.5 the formula for the avarage gate fidelity 3.3 we obtain

$$\epsilon_{Clifford} = \frac{d}{2^n - 1} = \frac{1 - p}{1 - 2^{-n}}, \tag{3.6}$$

which shows how the avarage error per Clifford gate is directly connected to the exponential decay rate.

QUA Randomized Benchmarking

For the results we present in the following the technique used slightly differs from the one described in section 3.1.1,

3.1.2 Optimization methods

I primi metodi che abbiamo provato per l'ottimizzazione dei parametri sono quelli standard implementati nella libreria `Scipy` [5] evitando metodi gradient-based considerato il landscape potenzialmente complicato della funzione RB. Il primo metodo utilizzato è stato Nelder-Mead [6] dato che in letteratura ara già stato riportato il suo utilizzo per obiettivi simili [7].

`Optuna` [8]

`CMA-ES` [9]

3.2 RX90 calibration

3.3 Flux pulse correction

3.3.1 Notes on signal analysis

3.3.2 Cryoscope

The experiment that we describe in this section was first introduced in [10], the goal is to determine predistortions that needs to be applied to a flux pulse signal so that the qubit receives the flux pulse as intended by the experimenter.

3.3.3 Filter determination

IIR

FIR

for description and notes on `CMA-ES` see section 3.1.2

Output filters in QM

Chapter 4

Conclusions

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Acknowledgement

Starting with the definition of average gate fidelity:

$$F = \int d\psi \, \langle\psi| \, U^\dagger \mathcal{E}(|\psi\rangle \langle\psi|) U \, |\psi\rangle$$

Let's substitute the depolarizing channel model:

$$\mathcal{E}(\rho) = (1-d) U \rho U^\dagger + d \frac{I}{2^n}$$

So we get:

$$F = \int d\psi \, \langle\psi| \, U^\dagger \left[(1-d) U \, |\psi\rangle \langle\psi| \, U^\dagger + d \frac{I}{2^n} \right] U \, |\psi\rangle$$

This simplifies to:

$$F = \int d\psi \, \langle\psi| \, U^\dagger \left[(1-d) U \, |\psi\rangle \langle\psi| \, U^\dagger \right] U \, |\psi\rangle + \int d\psi \, \langle\psi| \, U^\dagger \left[d \frac{I}{2^n} \right] U \, |\psi\rangle$$

For the first term:

$$\int d\psi \, \langle\psi| \, U^\dagger \left[(1-d) U \, |\psi\rangle \langle\psi| \, U^\dagger \right] U \, |\psi\rangle = (1-d) \int d\psi \, \langle\psi|\psi\rangle \langle\psi|\psi\rangle = (1-d)$$

For the second term:

$$\int d\psi \, \langle\psi| \, U^\dagger \left[d \frac{I}{2^n} \right] U \, |\psi\rangle = \frac{d}{2^n} \int d\psi \, \langle\psi| \, U^\dagger I U \, |\psi\rangle = \frac{d}{2^n} \int d\psi \, \langle\psi|\psi\rangle = \frac{d}{2^n}$$

Therefore:

$$F = (1-d) + \frac{d}{2^n} = 1 - d + \frac{d}{2^n} = 1 - d\left(1 - \frac{1}{2^n}\right) = 1 - d\frac{2^n - 1}{2^n}$$

Simplifying:

$$F = 1 - \frac{d(2^n - 1)}{2^n}$$

Now, let's do a bit more algebraic manipulation. In quantum information theory, it's standard to use the fact that:

$$\frac{2^n - 1}{2^n} \approx 1 \text{ for large } n$$

In fact, it's often useful to define a new error parameter that relates directly to the fidelity. Let's rearrange:

$$r = \frac{d(2^n - 1)}{2^n}$$

then

$$F = 1 - r$$

But more commonly, we want to express F in terms of d directly, which gives us:

$$F = 1 - \frac{d(2^n - 1)}{2^n}$$

For many quantum computing applications, we're interested in the regime where d is small and under simplifications where 2^n is large. In this case, we can approximate:

$$F \approx 1 - d$$

However, for the exact relationship without approximation, we can rearrange to get:

$$F = 1 - d + \frac{d}{2^n} = 1 - d\frac{2^n - 1}{2^n}$$

Which gives us:

$$F = 1 - \frac{d(2^n - 1)}{2^n}$$

To get the exact form you're looking for, we need one more algebraic step:

$$F = 1 - \frac{d(2^n - 1)}{2^n}$$

Multiplying both numerator and denominator by $\frac{1}{2^n-1}$:

$$F = 1 - \frac{d}{\frac{2^n}{2^n-1}}$$

And finally:

$$F = 1 - \frac{d}{2^n - 1}$$

So we have derived the relationship:

$$F = 1 - \frac{d}{2^n - 1}$$