

Development of an open-source calibration framework for superconducting qubits



Master's degree in Physics

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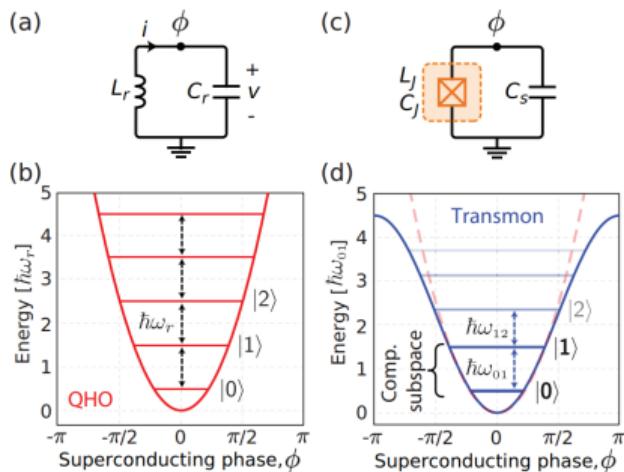
1. Superconducting qubits
2. Quantum Middleware
3. Average Clifford gate fidelity optimization
4. New features in Qibo
5. Conclusions & Outlooks

Superconducting qubits

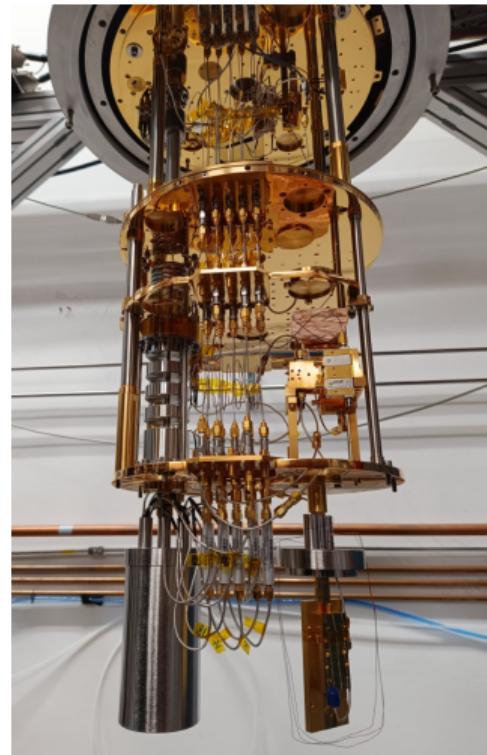
Qubit: two level quantum system

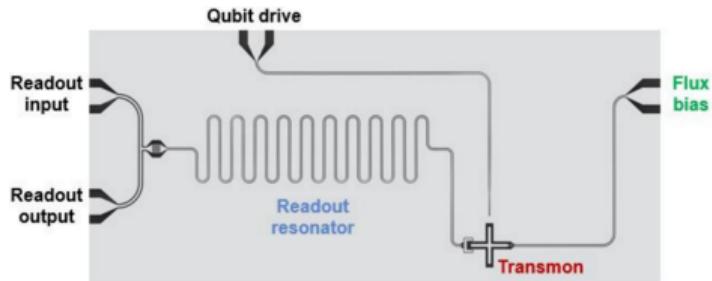
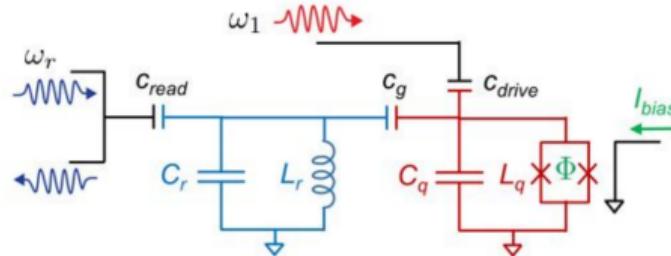
Qubit: two level quantum system

Superconducting qubits: use Josephson Junctions to build anharmonic oscillators



DOI: 10.1109/MAP.2022.3176593

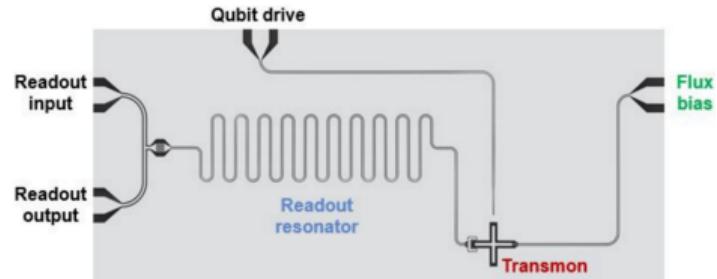
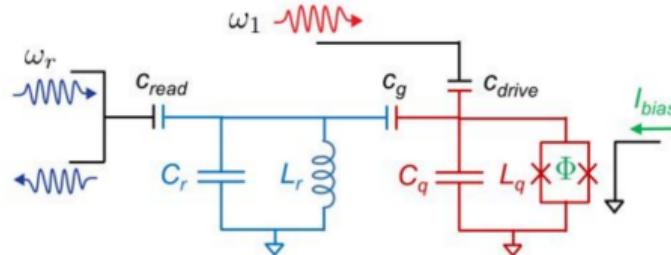




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Microwave pulse applied to the qubit

$$V_d(t) = A\varepsilon(t) \sin(\omega_d t + \alpha);$$



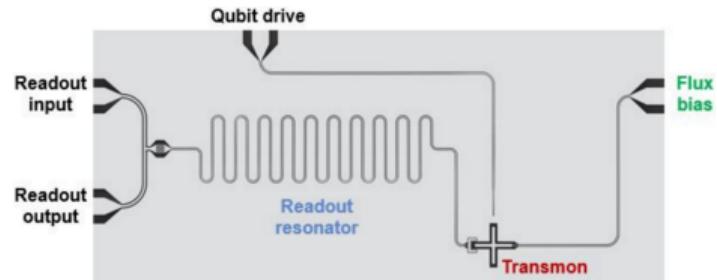
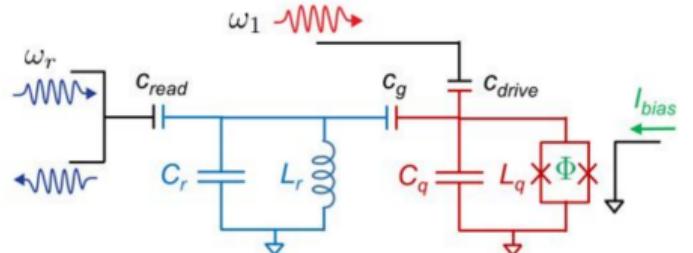
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Qubit - electric field Hamiltonian (RWA)

$$\hat{H} = -\frac{\hbar(\omega_q - \omega_d)}{2}\hat{\sigma}_z + \frac{\hbar\Omega}{2}\varepsilon(t)(\hat{\sigma}_x \cos \alpha + \hat{\sigma}_y \sin \alpha);$$



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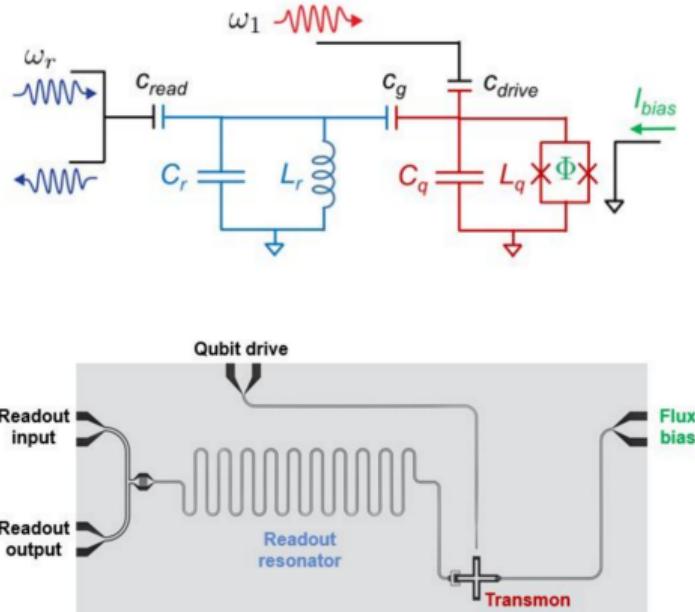
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Qubit evolution under microwave pulse:

$$R_{\hat{n}(\alpha)}(\theta) = e^{-\frac{i}{2}\hat{n}(\alpha) \cdot \vec{\sigma}\theta} = e^{-\frac{i}{2}(\hat{\sigma}_x \cos \alpha + \hat{\sigma}_y \sin \alpha)\theta},$$

where $\theta = \Omega \int_0^{+\infty} \varepsilon(t') dt'$.



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Quantum Middleware

Superconducting qubit calibration

Circuit: sequence of quantum gates applied to qubits to perform a computation.

Circuit with
logical gates

Superconducting qubit calibration

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Gates: abstract unitary operations (logical or native).



We need to calibrate *only* native gates.

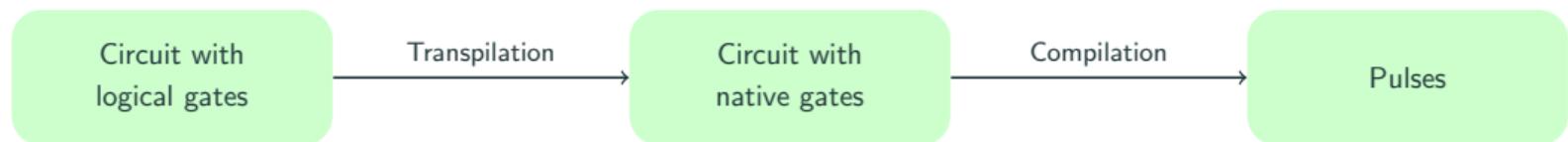
Native single qubit gates on superconducting qubits: $R_X(\pi)$, $R_X(\frac{\pi}{2})$, $R_Z(\theta)$.

Superconducting qubit calibration

Circuit: sequence of quantum gates applied to qubits to perform a computation.

Gates: abstract unitary operations (logical or native).

Pulses: physical signal (control field) implementing a gate.

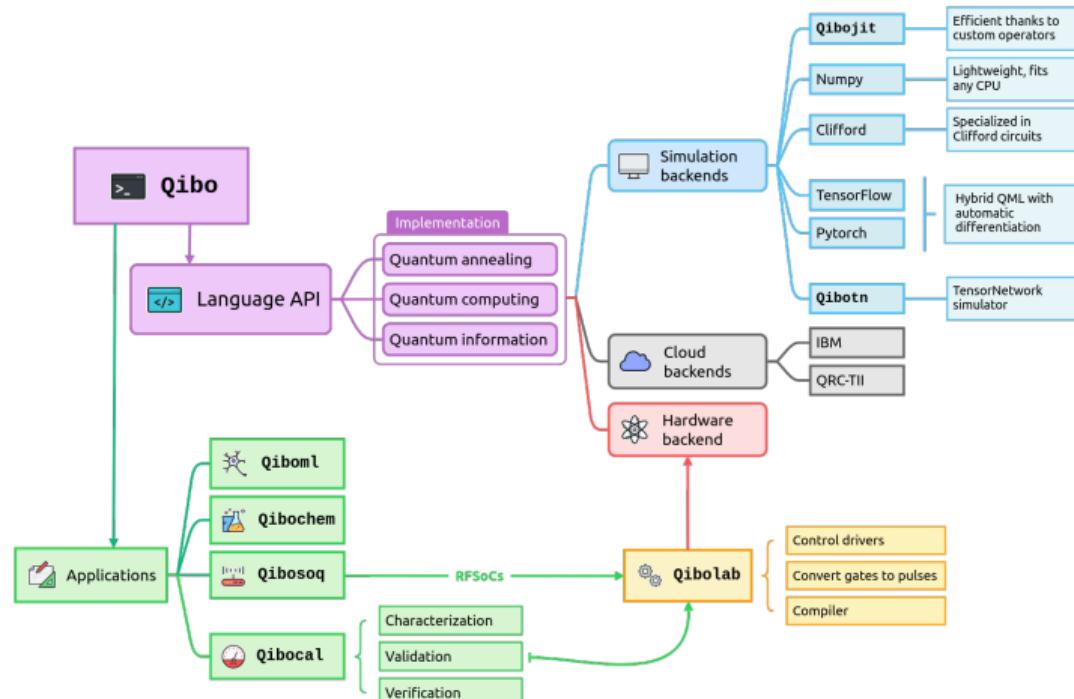


We need to calibrate *only* native gates.

Native single qubit gates on superconducting qubits: $R_X(\pi)$, $R_X(\frac{\pi}{2})$, $R_Z(\theta)$.

For each pulse we calibrate frequency, duration, power, shape.

Qibo: a modular and open-source framework for quantum computing, control and calibration.



<https://qibo.science/>

Average Clifford gate fidelity optimization

Randomized benchmarking estimates average gate fidelity by applying random sequences of Clifford gates followed by an inverting gate.

Randomization with Clifford gates provides a depolarized noise channel:

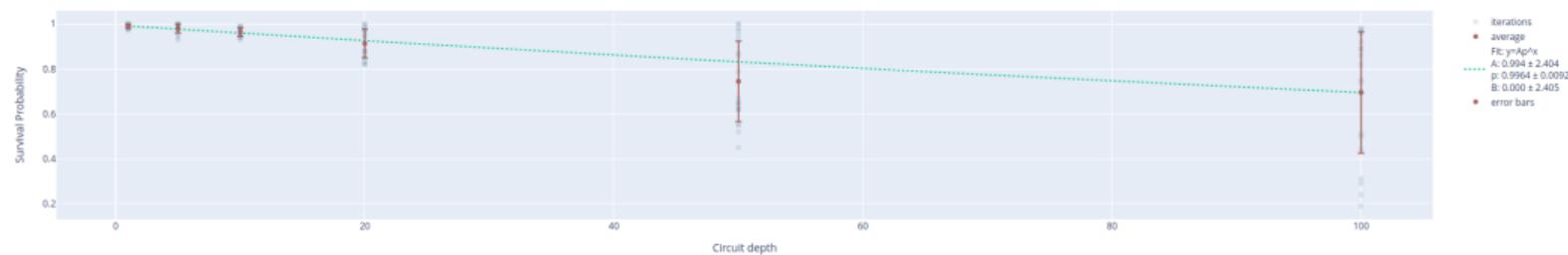
$$\rho \rightarrow d \frac{\mathbb{I}}{2} + (1 - d)\rho \text{ where } d \text{ is the depolarization rate.}$$

Standard Randomized Benchmarking protocol:

1. Initialize the system in the ground state
2. For each sequence length m draw a sequence of Clifford group elements
3. Calculate the inverse gate
4. Measure complete sequence
5. Repeat the process for multiple sequences of the same length while varying the length

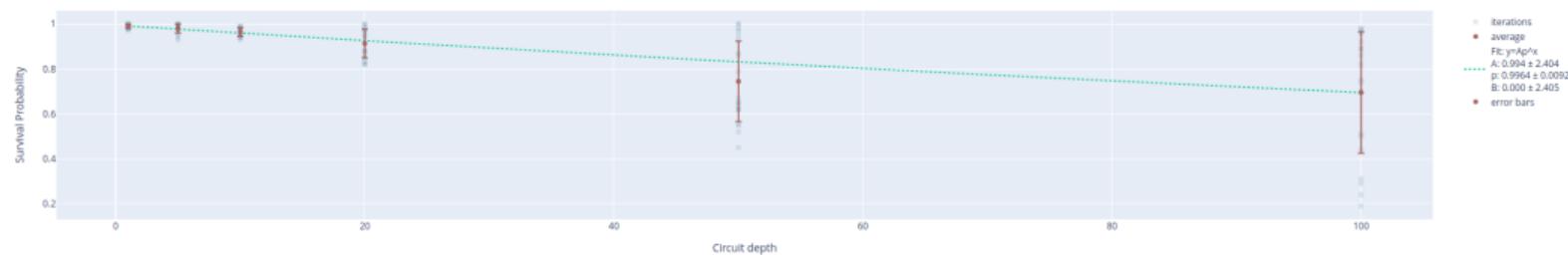
The survival probability decays exponentially with the number of Clifford gates
 $F(m) = Ap^m + B$ where $1 - p$ is the depolarization rate.

From p we can extract the average error per Clifford gate.



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Can we optimize the average Clifford gate fidelity to automate $R_X(\pi)$ gate recalibration?

Test closed-loop optimization with modern optimization libraries.

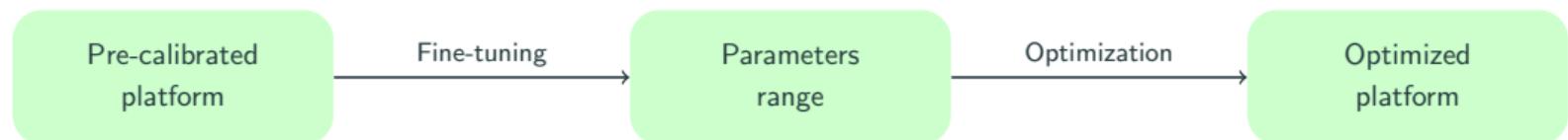
Test closed-loop optimization with modern optimization libraries.

Pre-calibrated
platform

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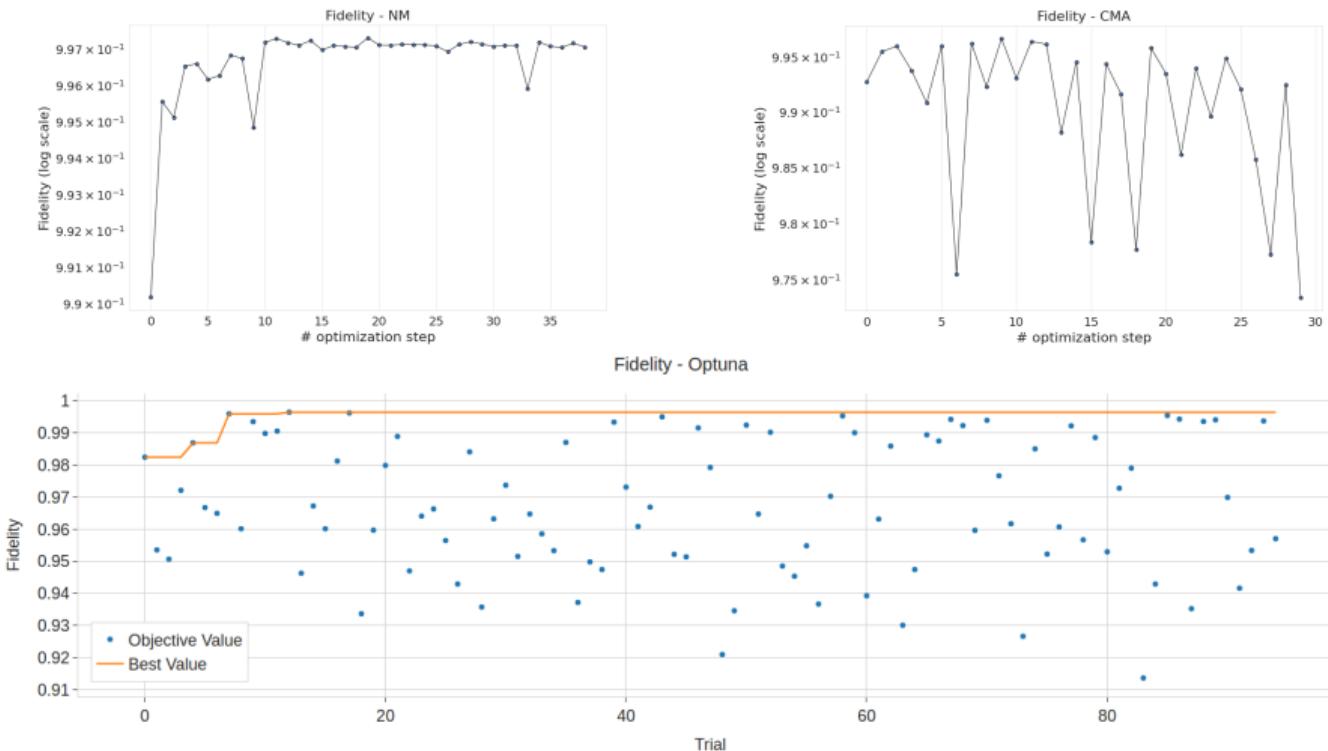
Optimization parameters:

Pulse amplitude

Pulse frequency

Pulse shape (through β DRAG parameter)

Average Clifford gate Fidelity



New features in Qibo

Improving Qubit Control for Calibration

Single qubit gates:

Qibo already implement many calibration routines for $R_X(\pi)$.

We can try to reduce residual errors by adding calibrating $R_X(\pi/2)$ as native gate.

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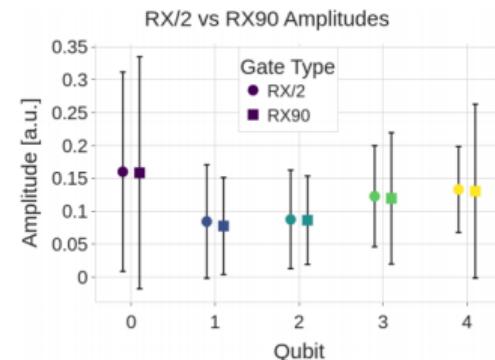
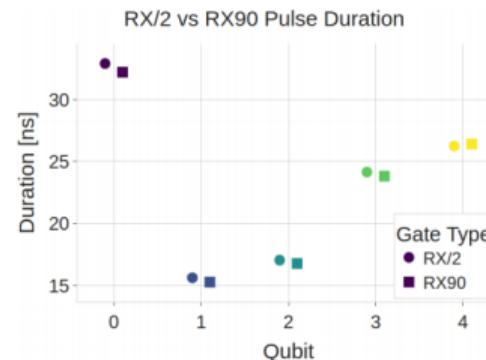
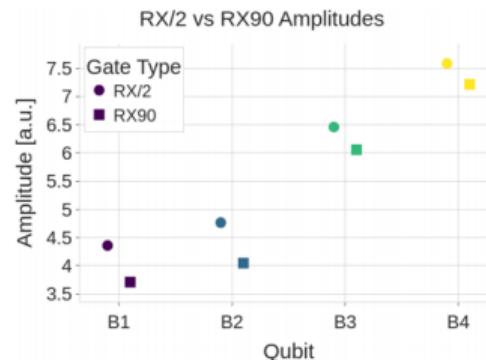
Problem: there may be nonlinearities and distortions in signals that degradete the $R_X(\frac{\pi}{2})$ fidelity.

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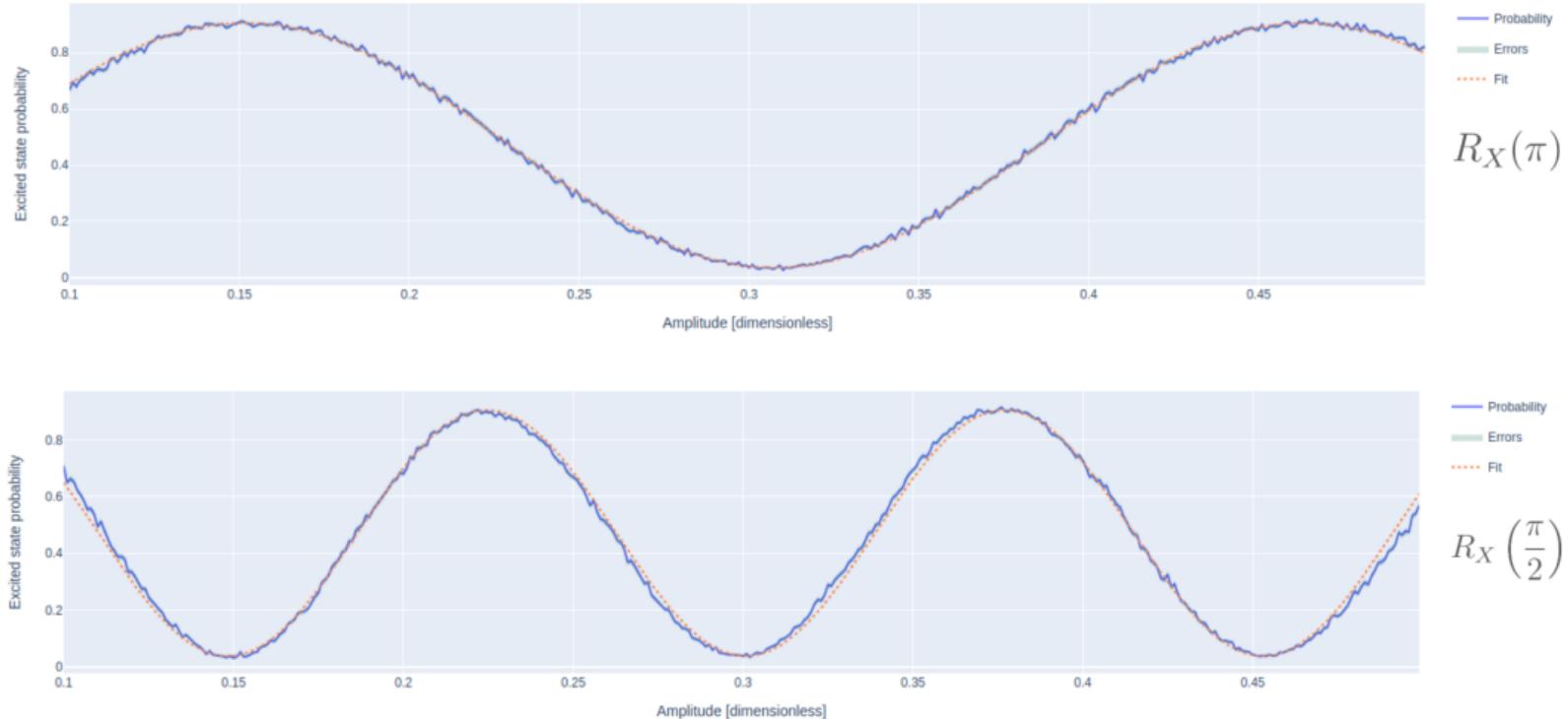
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Rabi amplitude experiment



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Two qubit gates:

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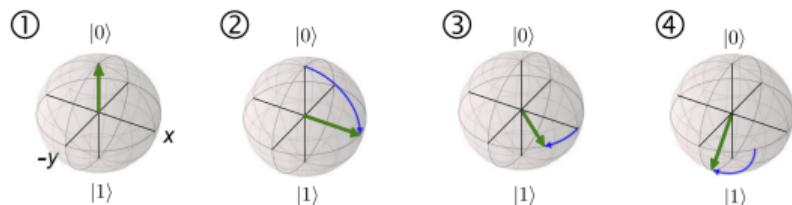
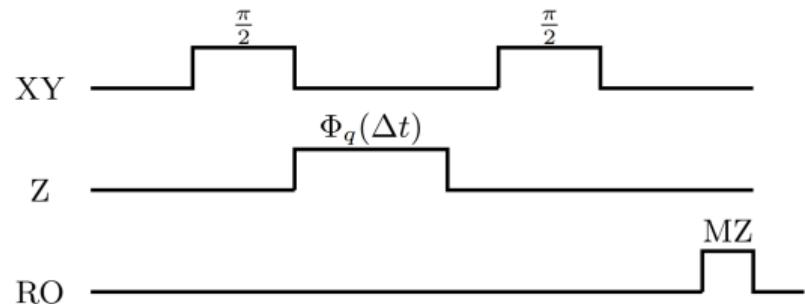
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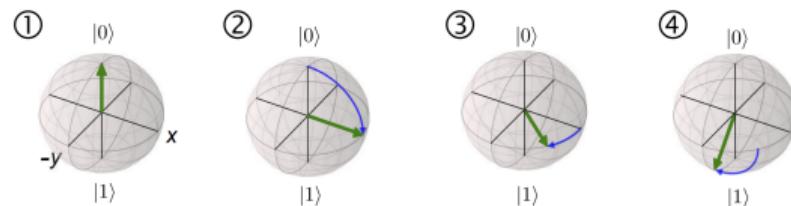
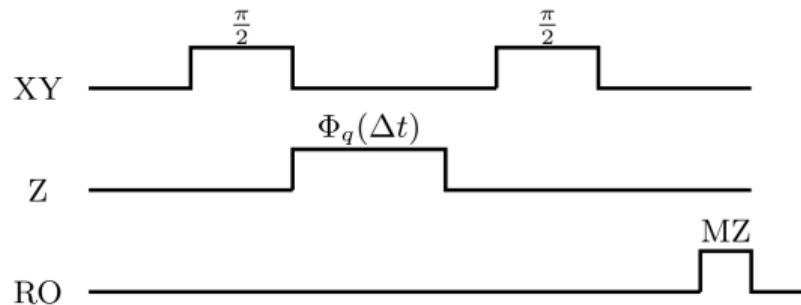
Add Cryoscope portocol to improve control over flux pulses.



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Transmon frequency dependence on magnetic flux:

$$f_q(\Phi_q) \approx \left(\sqrt{8E_J E_C} \left| \cos\left(\pi \frac{\Phi_q}{\Phi_0}\right) \right| \right).$$

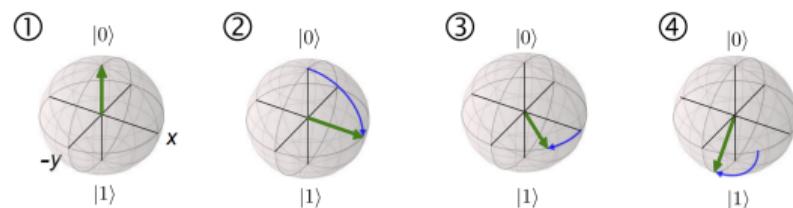
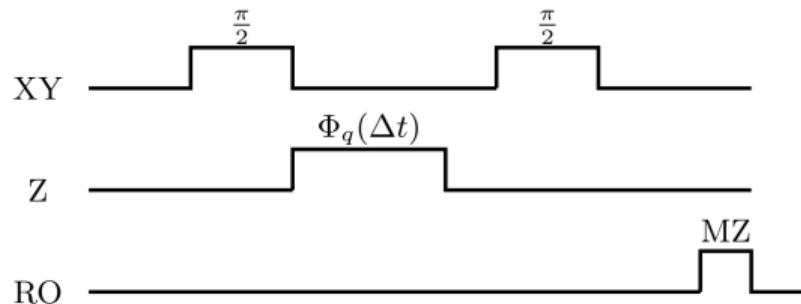


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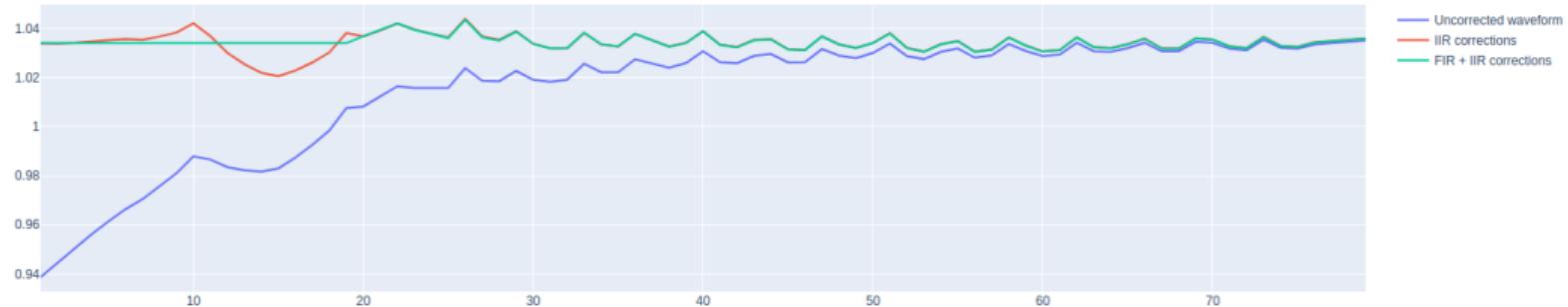
$$f_q(\Phi_q) \approx \left(\sqrt{8E_J E_C} \left| \cos\left(\pi \frac{\Phi_q}{\Phi_0}\right) \right| \right).$$

Detuning as function of the flux pulse:

$$\Delta f_q = \frac{\varphi_{\tau+\Delta\tau} - \varphi_\tau}{2\pi} \approx \frac{1}{\Delta\tau} \int_{\tau}^{\tau+\Delta\tau} \Delta f_q(\Phi_{q,\tau+\Delta\tau}(t)).$$

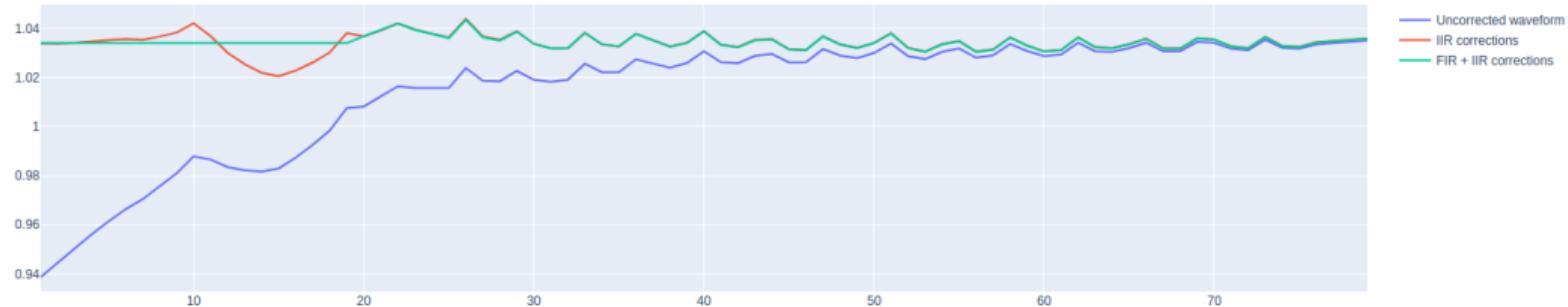


Filter determination



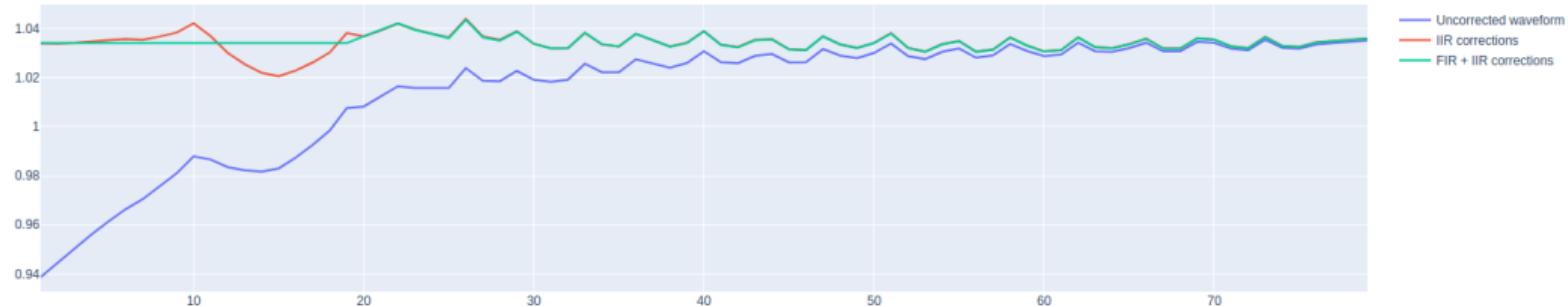
1. Determine exponential correction

Filter determination



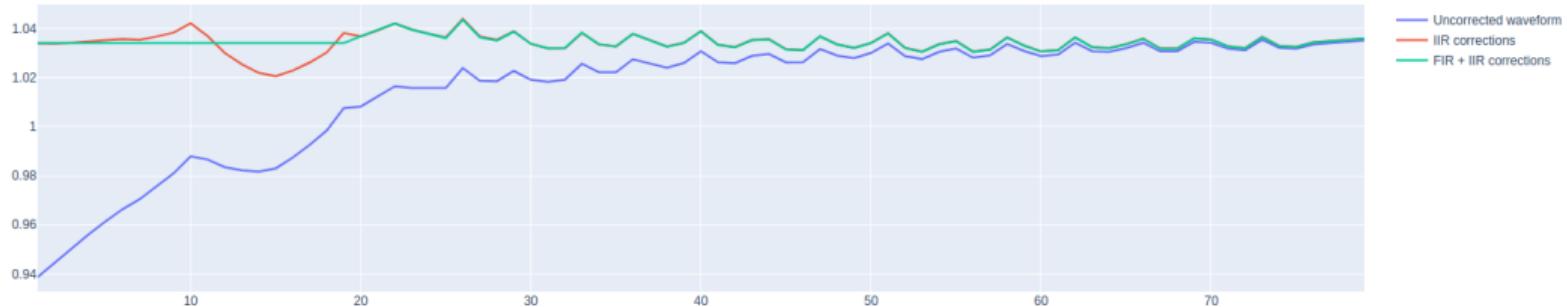
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2. Obtain coefficients for the Infinite Impulse Response from exponential correction

Filter determination



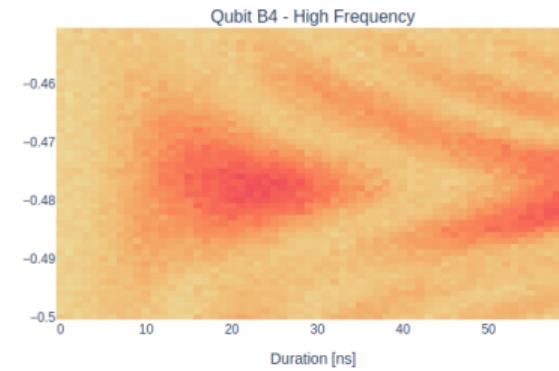
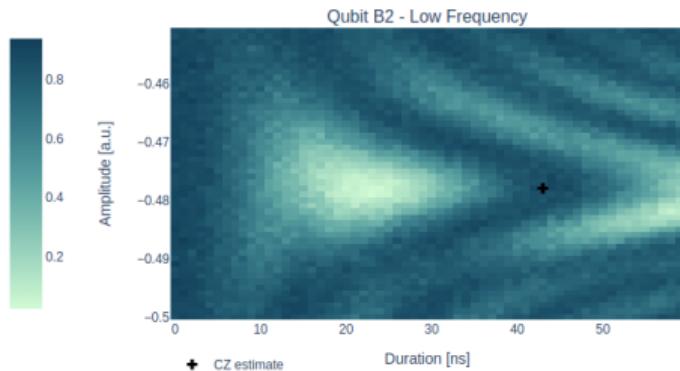
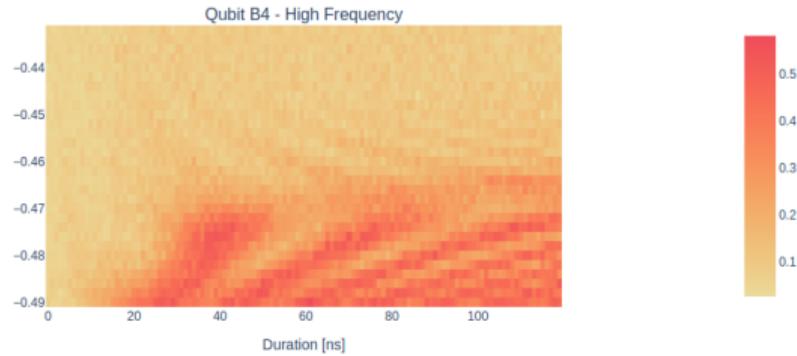
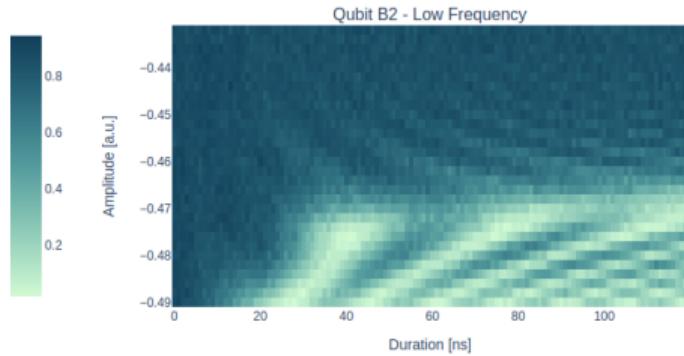
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3. Determine Finite Impulse Response coefficients

Filter determination



1. Determine exponential correction
2. Obtain coefficients for the Infinite Impulse Response from exponential correction
3. Determine Finite Impulse Response coefficients
4. Obtain coefficients for real-time correction

Impact of correction on chevron plots



Cryoscope routine



Qibocal · v0.2.2

Search

INTRODUCTION

Installation instructions

How to use Qibocal?

How to execute calibration protocols in Qibocal?

Minimal working example

GUIDES

Tutorials

Protocols

Time Of Flight (Readout)

Calibrate Discrimination Kernels

Resonator spectroscopy

Resonator punchout

Qubit spectroscopies

Rabi experiments

Ramsey experiments

`relaxation_time: float`

Wait time for the qubit to decohere back to the *gnd* state.

Example

A possible runcard to launch a Cryoscope experiment could be the following:

```
- id: cryoscope

operation: cryoscope
parameters:
    duration_max: 80
    duration_min: 1
    duration_step: 1
    flux_pulse_amplitude: 0.7
    relaxation_time: 50000
```

The expected output is the following:



Note

In the case where there are no filters the protocol will compute the FIR and the IIR filters. If the filters are already present the computation of the filters will be skipped and only the reconstructed waveform will be shown.

Requirements

• Single Shot Experiments

ON THIS PAGE
Cryoscope
Parameters
Example
Requirements

Conclusions & Outlooks

Gate recalibration by RB optimization

- ✓ We can always find parameters configuration with average Clifford gate fidelity > 99.5%.
- ✓ Does **not rely on manual tuning** by an operator.
- ✗ RB evaluation is **computationally expensive** (~ 30 minutes).
- ✗ More stable methods (eg. Nelder-Mead) require many cost function evaluation.
- ✗ Parameters drift makes optimization unstable.

Possible future work: **optimize RB parameters** to allow a faster and more reliable optimization.

Library additions

- ✓ Extended Qibolab and Qibocal libraries to support **native $R_X(\pi/2)$ gate** with dedicated calibration routines.
- ✓ Implemented and added the **Cryoscope** calibration experiment to Qibocal library to correct flux-pulse distortions (average NMSE improvement $\sim 70\%$).
- ✗ Study long-time distortions of the flux-pulse.

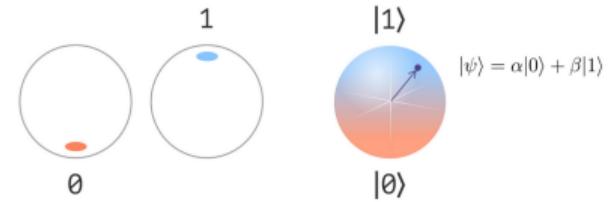
Possible future extensions:

- Readout optimization protocols
- Active qubit reset schemes
- Implement leakage mitigation strategies

Thank you

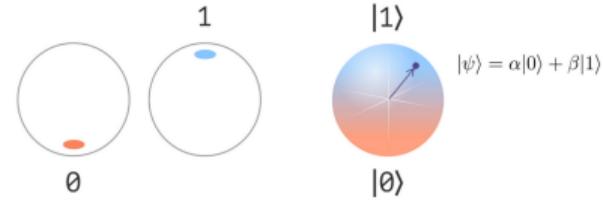
Backup slides

Quantum Computing, what for?



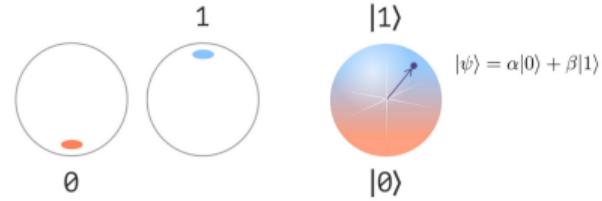
Quantum Computing, what for?

- Simulation of quantum system: classical computers can not efficiently simulate quantum states. Classical computers can usually simulate a ~ 30 qubits



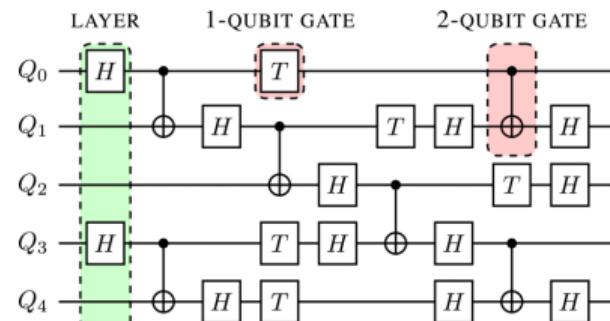
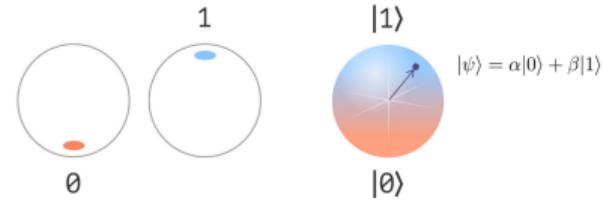
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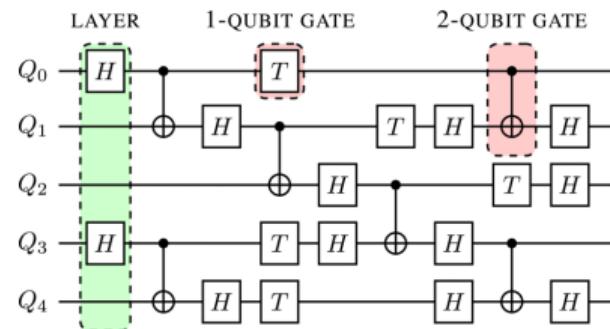
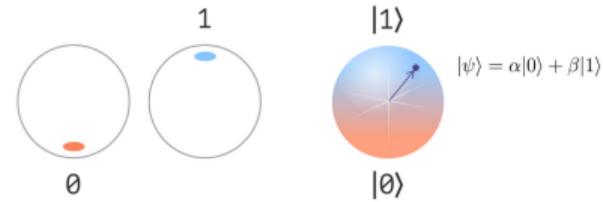
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- Gate-based circuits (eg. Shor, Grover, Deutsch)
- AI-inspired algorithms (VQE, autoencoders, classifiers ...)



Qubit platforms

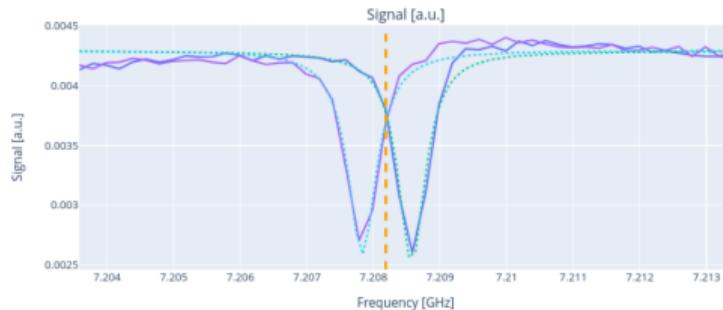
atoms	electron superconducting loops & controlled spin					photons			
vendors	trapped ions	cold atoms	quantum annealing	super-conducting	silicon	topological	photons		
 IONQ	 Honeywell	 PASQAL	 D-Wave The Quantum Computing Company	 Google	 amazon	 QUANTUM MOTION	 TURING	 Microsoft	
 AQT	 OXFORD IONICS	 ColdQuanta	 NEC	 intel	 IBM	 rigetti	 XANADU	 ORCA Computing	
 IQuEra	 eleQtron		 QILINJARO QUANTUM TECH	 OQC	 TUMQ	 equali.labs	 PsiQuantum	 TUNDRA SYSTEMS GLOBAL	
			 QUBIKON	 ALICE & BOB	 Nord quantum	bleximo	LightOn	QUANDELA	
				q-ctrl	ALYSSA	ARCHER	QUIX BardeenQ	University of BRISTOL	
labs (*)	IQST Institut für Quantenoptik und Quantentechnologie	CNRS Institut National de la Recherche Scientifique	Stanford University	ceasat	cnrs	ceasat	cnrs	ceasat	cnrs
	MIT	KIT Karlsruhe Institute of Technology	HARVARD UNIVERSITY	MIT	KIT Karlsruhe Institute of Technology	JÜLICH Forschungszentrum	MCQST	QuTech	University of BRISTOL
	IQI Institut für Quanteninformation	universität Innsbruck	PennState	ETH zürich	UCSB	UMassAmherst	MCQST	University of OXFORD	TU Delft
	NIST National Institute of Standards and Technology	GUTENBERG UNIVERSITÄT	EPFL	ETH zürich	Yale	HRL LABORATORIES	HARVARD UNIVERSITY	Niels Bohr Institutet	THE UNIVERSITY OF SYDNEY
	HARVARD UNIVERSITY	US UNIVERSITY OF SINGAPORE	The Ohio State University	Berkeley	Yale	PRIMER	PRINCETON UNIVERSITY	THE UNIVERSITY OF CHICAGO	東京大学 THE UNIVERSITY OF TOKYO

Qubit - resonator Hamiltonian:

$$\hat{H} = \hbar\omega_r \hat{a}\hat{a}^\dagger - \frac{\hbar\omega_{01}}{2} \hat{\sigma}_z + \hbar g (\hat{\sigma}^+ \hat{a} + \hat{\sigma}^- \hat{a}^\dagger);$$

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Dispersive regime ($g \ll \omega_q - \omega_r$):

$$\hat{H}_{disp} = \hbar(\omega_r - \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} - \frac{\hbar}{2} (\omega_{01} + \chi) \hat{\sigma}_z;$$

dispersive shift: $\chi = \frac{g^2}{\Delta}$,

$$\Delta = \omega_q - \omega_r.$$

State readout

[arXiv:cond-mat/0407325]

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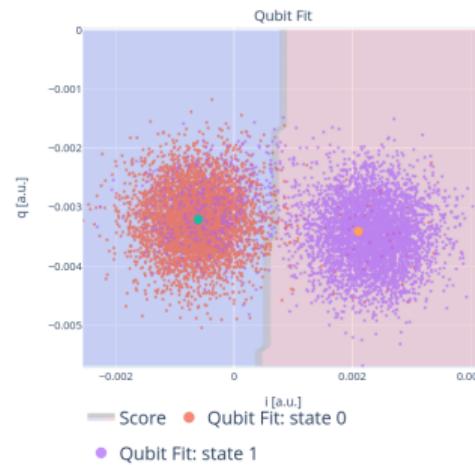
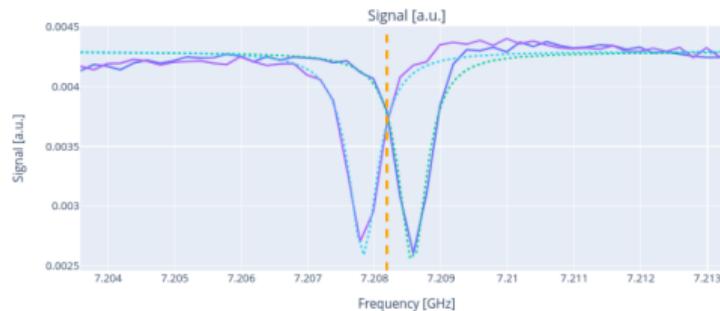
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Calibration workflow & assessment

Procedure:

1. Resonator characterization
2. Qubit characterization
3. Gate calibration
4. Gate set characterization

Metrics example:

- readout & assignment fidelity
- relaxation time T_1
- decoherence time T_2
- gate fidelity

Calibration workflow & assessment

Main steps:

1. Resonator characterization
2. Qubit characterization
3. Gate calibration
4. Gate set characterization

Calibration quality metrics:

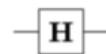
- readout & assignment fidelity
- relaxation time T_1
- decoherence time T_2
- gate fidelity

Qubit	Readout Fidelity	Assignment Fidelity	T1 [μs]	T2 [μs]	Gate infidelity ($\cdot 10^{-3}$)
D1	0.876	0.938	26.4 ± 0.4	13.0 ± 2.0	27.0 ± 21.0
D2	0.945	0.973	14.9 ± 0.1	18.0 ± 10.0	20.5 ± 6.2
D3	0.905	0.952	23.6 ± 0.3	28.0 ± 12.0	7.0 ± 18
D4	0.929	0.964	20.6 ± 0.2	38.0 ± 5.2	4.4 ± 4.8
B2	0.902	0.951	17.5 ± 0.2	27.5 ± 5.2	18.0 ± 16

Clifford gates

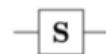
- Special subset of quantum gates that map Pauli operators to Pauli operators under conjugation
- Clifford gates group is generated by H , S , ($CNOT$) gates
- Quantum circuits that consist of only Clifford gates can be efficiently simulated with a classical computer
(Gottesman–Knill theorem)

Hadamard (H)



$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Phase (S, P)



$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

Controlled Not (CNOT, CX)

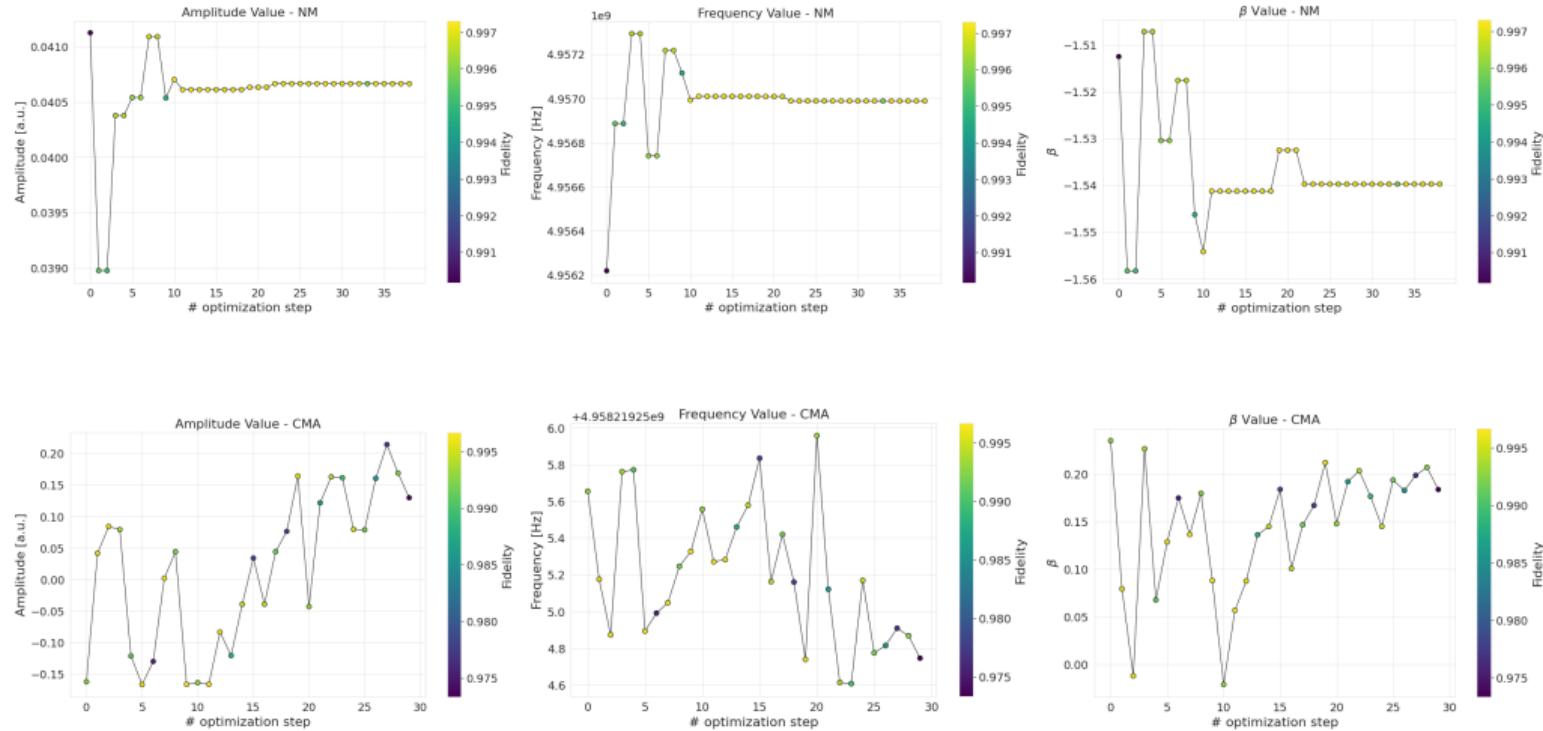


$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

RB optimization summary

Analysis Name	Iterations	Time [s]	Fidelity Best	Amplitude Best [a.u.]	Frequency Best [$\times 10^9$ Hz]	β Best
Nelder_Mead	40	2892	0.99731	0.04063	4.95701	-1.53253
init_simplex_1	40	3096	0.99533	0.04069	4.95820	-
init_simplex_2	40	3128	0.99564	0.04131	4.95817	-
init_simplex_3	40	3190	0.99554	0.04058	4.95819	-
SLSQP	15	3873	0.99518	0.04058	4.95822	-0.00115
CMA-ES	30	4536	0.99665	-0.16634	4.95822	0.08802
optuna_30	30	763	0.99583	0.28059	4.95601	2.75519
optuna_100	100	2257	0.99645	0.29026	4.955279	-1.46504
optuna_1000	1000	21811	0.99847	-0.00051	4.961836	0.2222
optuna no β	1000	21820	0.99846	0.04104	4.957374	-

Parameters evolution

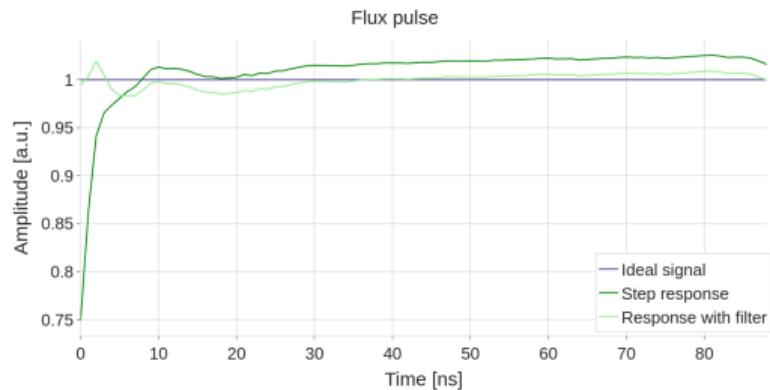
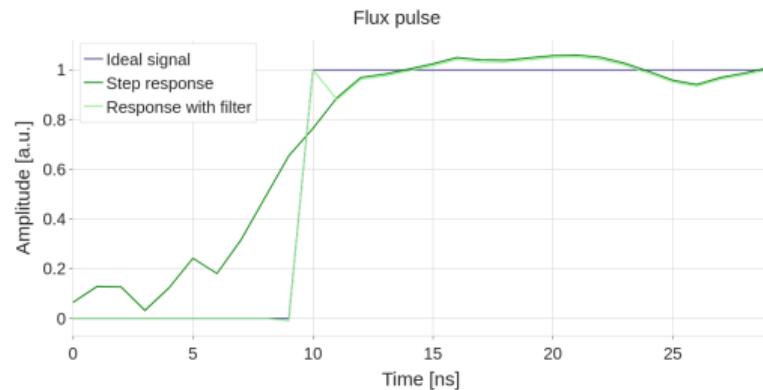


Filter determination details

Compensate distortion using a digital IIR filter based on an inverted exponential model

$$s_{\text{corr}}(t) = \frac{s(t)}{g(1 - Ae^{-t/\tau})}$$

parameters g , A , and τ are estimated via least-squares minimization using `scipy.optimize`



Filter determination details

Digital IIR filter implemented in control hardware using discretized coefficients.

$$a_0 y[n] = \sum_{i=0}^N b_i x[n-i] - \sum_{i=1}^M a_i y[n-i],$$

Filter coefficients derived from fitted parameters (1st order IIR):

$$a_0 = 1,$$

where $\alpha = 1 - \exp\left(-\frac{1}{f_s \cdot \tau(1+A)}\right)$ and

$$a_1 = -(1 - \alpha),$$

$$b_0 = 1 - k + k\alpha,$$

$$b_1 = -(1 - k)(1 - \alpha),$$

$$k = \begin{cases} \frac{A}{(1+A)(1-\alpha)}, & \text{if } A < 0, \\ \frac{A}{1+A-\alpha}, & \text{if } A \geq 0, \end{cases}$$

FIR filters:

$$y[n] = \sum_{i=0}^N b_i x[n-i],$$

FIR filter coefficients are optimized using CMA-ES to minimize the average relative deviation between the FIR-filtered IIR-corrected signal and the ideal step response.

Gate decomposition - single-qubit gates

Euler's theorem

Any single-qubit unitary $U \in SU(2)$ can be decomposed (up to global phase) as

$$U = e^{i\alpha} R_z(\phi) R_x(\theta) R_z(\lambda)$$

Hadamard gate decomposition:

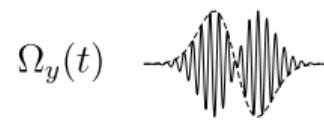
$$H = R_z(\pi) R_x\left(\frac{\pi}{2}\right) R_z(0)$$

DRAG pulse

Pulse shaping technique designed to suppress leakage and phase errors.

Drag pulse

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Standard gaussian pulse on first quadrature + pulse proportional to the gaussian derivative on second quadrature

ISBN: 9780198787488

$$\Omega_x(t) = \Omega e^{-t^2/2\sigma^2},$$

$$\Omega_y(t) = \lambda \eta \frac{d}{dt} \Omega_x(t).$$

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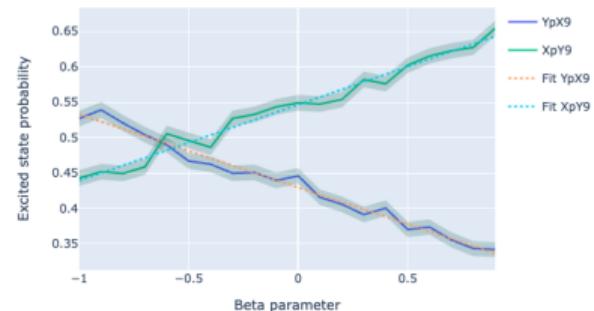
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A $R_z(\theta)$ gate is applied by shifting the phase of subsequent drive pulses by $-\theta$.