Improving distributed reasoning with privacy using tree decomposition

Vincent Armant

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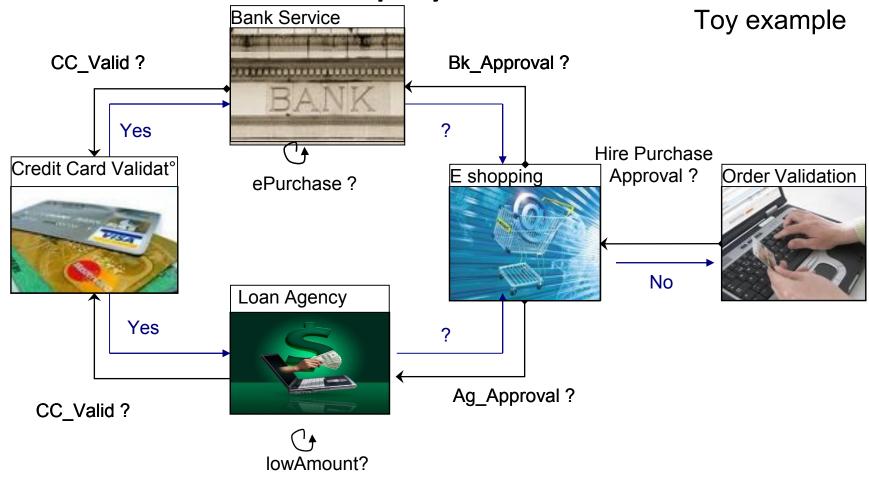




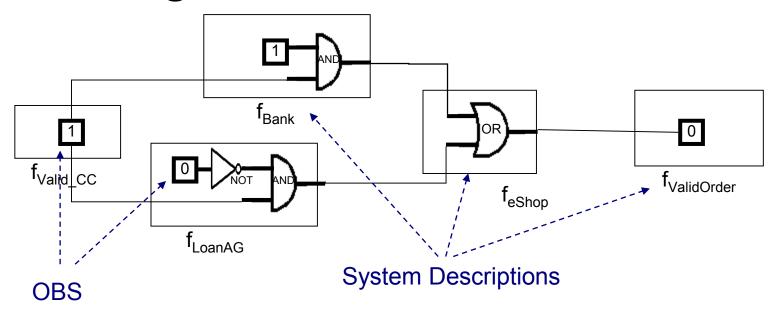
Outline

- Introduction
 - A distributed Reasoning Problem
 - Graphical Tree Decomposition
- Distributed tree decomposition
 Preserve network structure
 Keep local information local
- Centralized tree decomp. VS concurrent approaches
- Token elimination
- Experimental results on small-world graphs
- Conclusion / perspectives

Three times web-payment certification

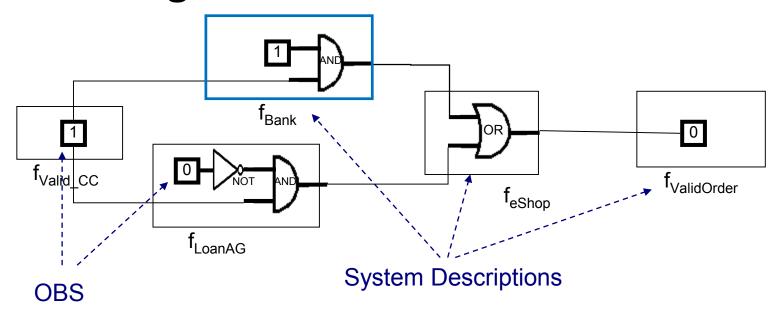


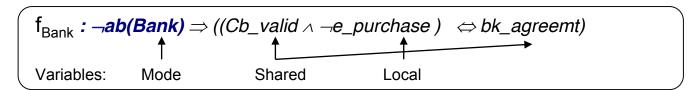
Modeling the behaviors



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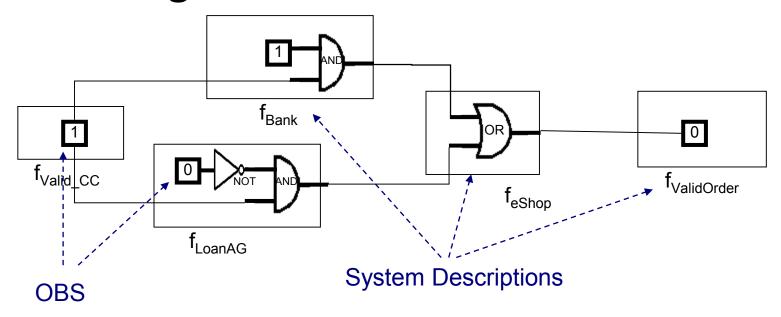
Modeling the behaviors

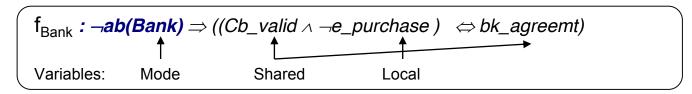




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Modeling the behaviors



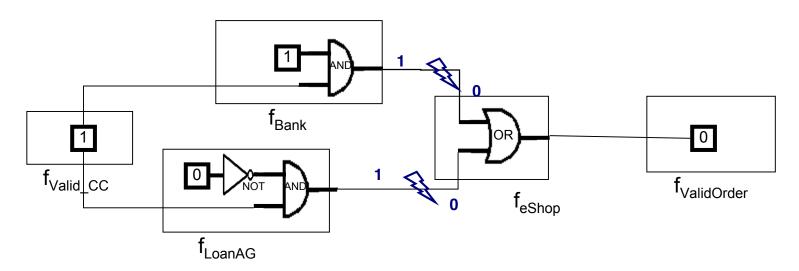


Global model: set of observations and local system descriptions

$$f_{global} = \Lambda f_i \Lambda_i OBS$$

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Minimal conflicts



Minimal Conflict:

are components that are together inconsistent with observations

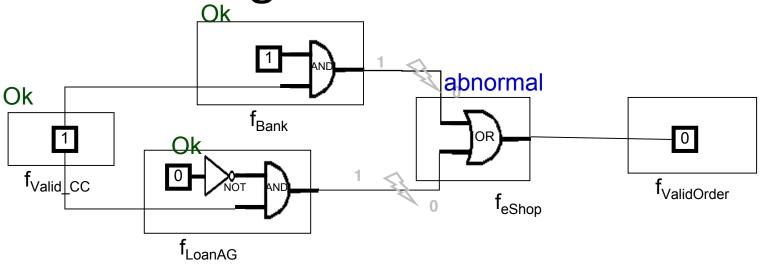
$$\wedge f \wedge OBS \neq C$$

s.t. \forall C' conflict, if C' \Rightarrow C then C' = C

$$C \subseteq AB$$
, $AB = \{ab1, ...,abn \}$

Example:

Minimal diagnoses



Minimal Diagnosis Δ :

Is a minimal explanation which cover all minimal conflicts

$$\land f \land OBS \land \Delta \land \overline{AB \backslash \Delta} \models \bot$$

s.t. $\forall \Delta'$ diagnosis, if $\Delta' \Rightarrow \Delta$ then $\Delta' = \Delta$

$$\Delta \subseteq F$$
, $F = \{ab1, ..., abn \}$

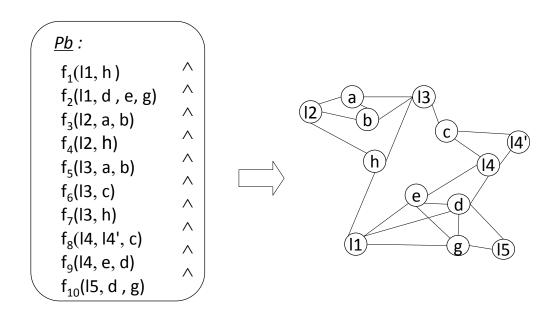
Example:

Challenge of distributed Reasoning

- Context : Distributed Algorithm
 - □ Each peer performs the same algorithm
 - ☐ A peer only know:
 - Its acquaintance
 - Its own description
 - □ A peer do not want to share some private knowledge
 - But must share any local knowledge that is "interesting" for the task
 - □ The network incrementally returns solutions (i.e. diagnoses)

How to solve efficiently a distributed reasoning problem?

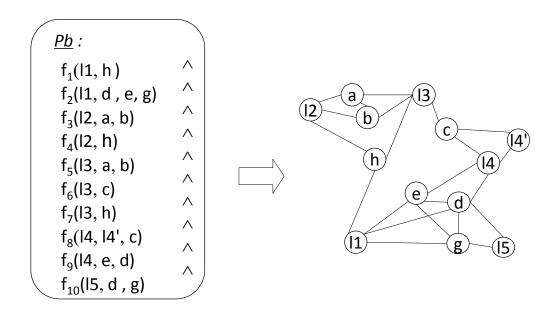
Primal graph



centralized problem description

Its primal graph

Primal graph



centralized problem

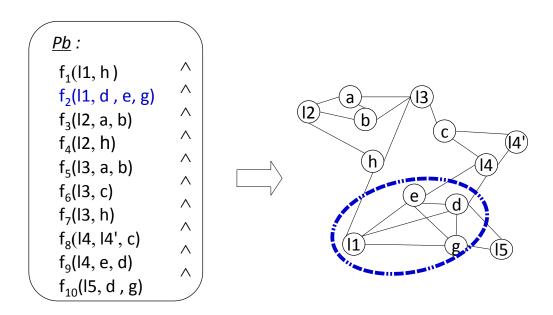
Generalization description

Its primal graph

Pb = join of databases relation (Primal Graph ~ Data Base Schema)
"A new approach to database logic Kuper,Vardi 1984"

Pb = Bayesian Inference (Primal Graph ~ Variable dependencies)

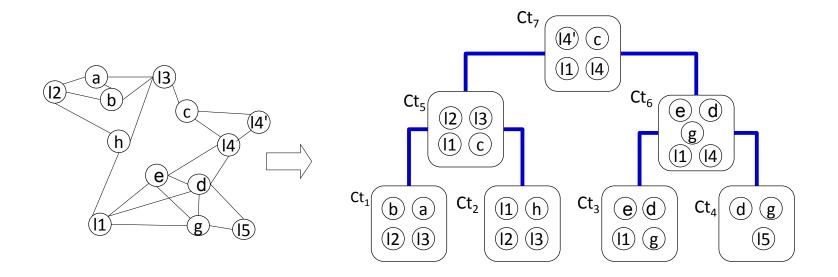
Primal graph



- Each variable labels exactly one node
- All variables contained in the scope of a formula in the problem description are neighbors in the primal graph

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Tree Decomposition



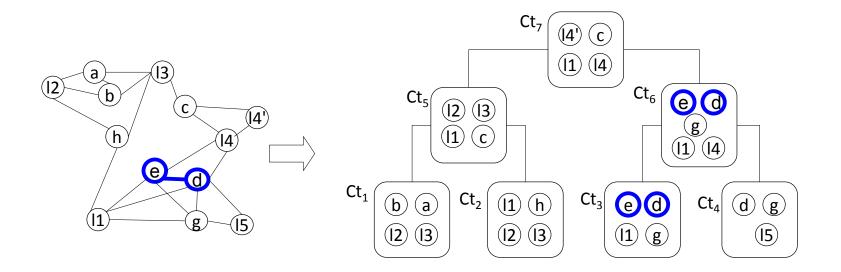
Primal graph

A tree decomposition

- 1) is a tree of clusters
- 2) preserves variables dependency
- 3) ensures running intersection

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Tree Decomposition



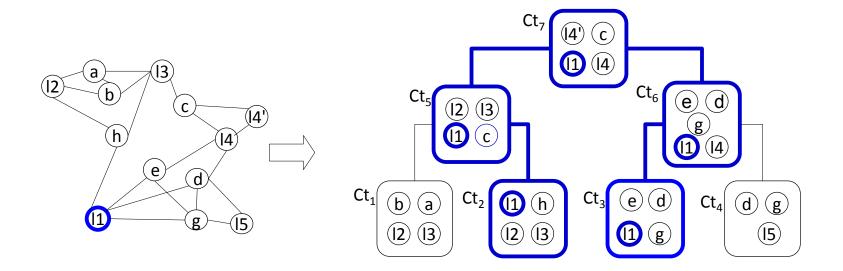
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Tree Decomposition

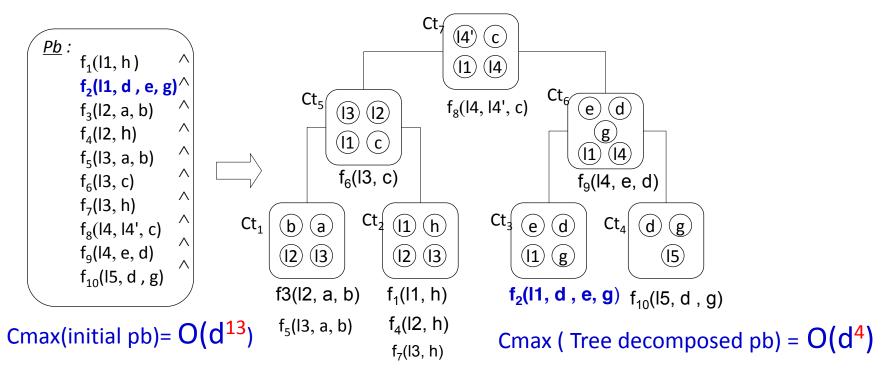


Primal graph

A tree decomposition

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Introduction Why is it useful?



1) Good points:

- divides the initial problem into sub-problems organized in a tree structure
- allows concurrent resolution and /or backtrack free search
- bounds time and space complexity by the size of the largest cluster (width) e.g. allows succinct representation (OBDD, MDD, DNNF, ..)

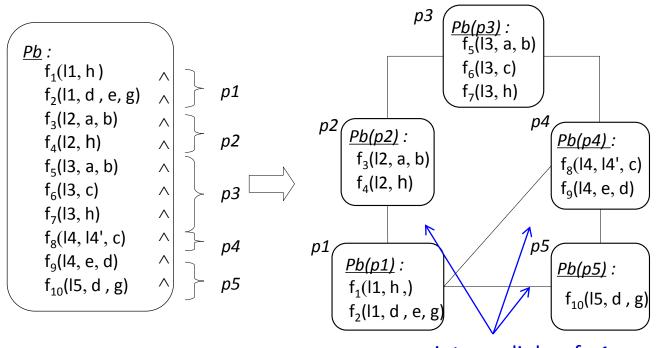
2) Limitations:

- finding an optimal tree-decomposition is NP-Hard

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Distributed system

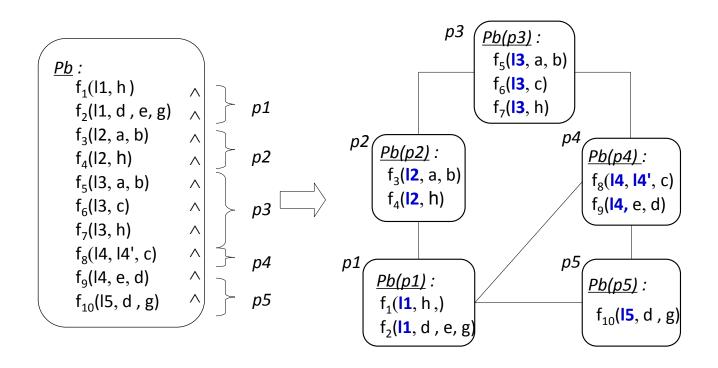


acquaintance links of p1

Initial problem setting is distributed among a set of peers

- 1) each peer can only interact with its neighbors by acquaintance links
- 2) local variables remain local

Distributed system

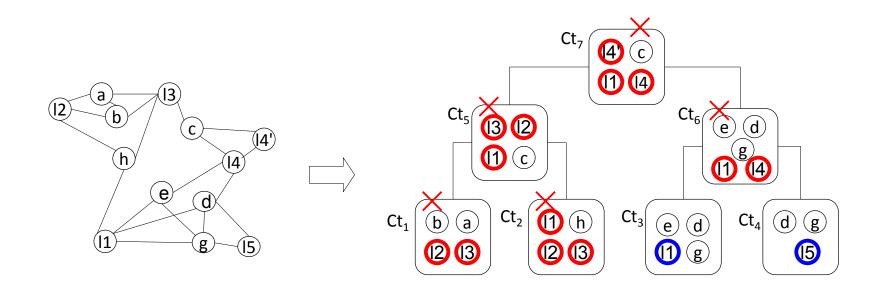


each « li » represents a local variable of pi

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Problematic: How to decompose a distributed system respecting privacy and the peer acquaintances?



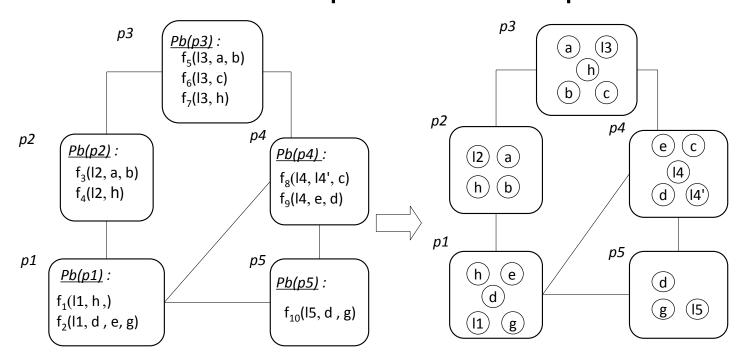
a primal graph

its tree decomposition

The classical notion of tree decomposition is not sufficient it does not respect the privacy of local variables it does not preserve the peer acquaintances

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Distributed Tree Decomposition Acquaintance Graph

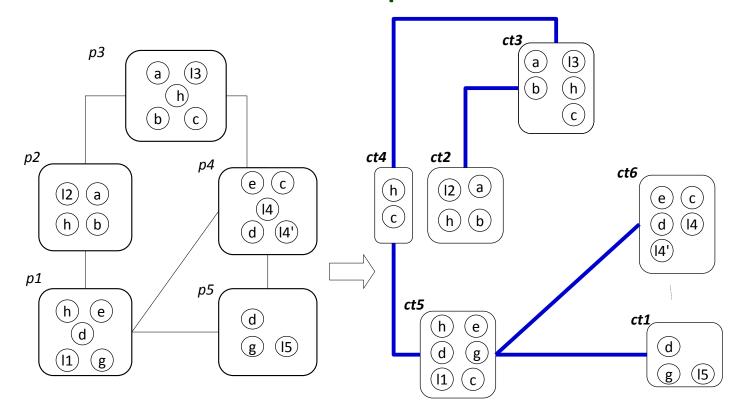


Distributed system

Acquaintance Graph G((P,V), ACQ)

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- 1) P represents the set of peers
- 2) V labels each peer by its set of variables
- 3) ACQ \subseteq P x P represents is acquaintance links

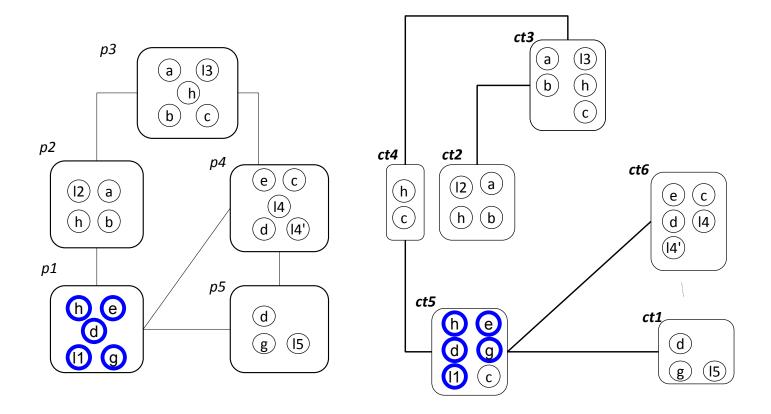


Acquaintance Graph

Distributed Tree Decomposition

22

- 1) is a tree of clusters
- 2) preserves the variables dependencies
- 3) respects the running intersection property
- 4) preserves the peers acquaintance
- 5) respectis the privacy of local variables

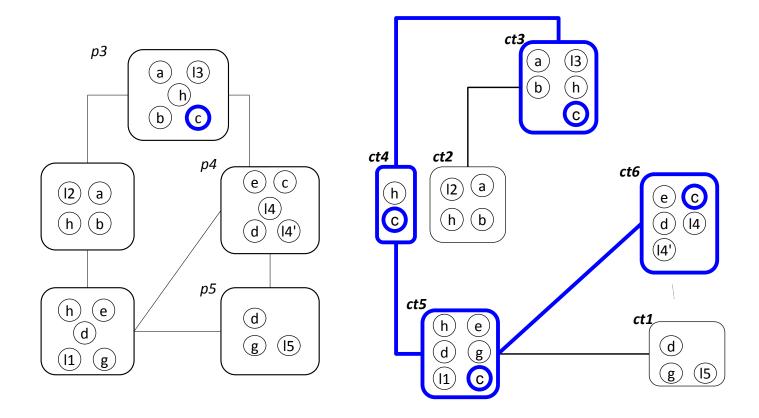


Acquaintance Graph

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23

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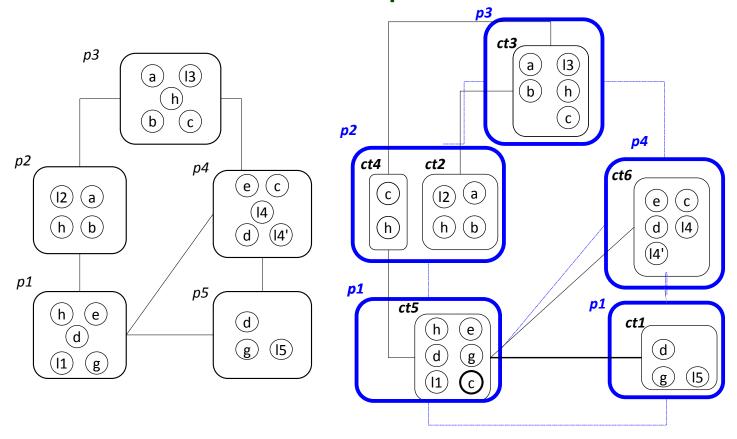


Acquaintance Graph

Distributed Tree Decomposition

24

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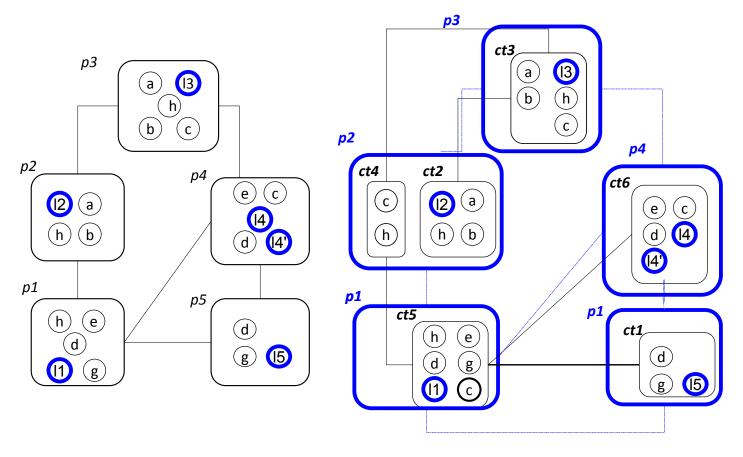
Acquaintance Graph

- -a cluster is created by one peer
- -2 neighboring clusters come from:
 - the same peer
 - neighboring peers

Distributed Tree Decomposition

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Acquaintance Graph

A local variable from pi can only appear in a cluster created by pi

Distributed Tree Decomposition

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Elimination process

What are the good tree decomposition techniques? Why?

Finding optimal Tree Decomposition \Leftrightarrow Finding optimal Elimination Order

It is always possible to build a TD from the clusters induced by Elimination order

Primal graph Elimination Clusters order While the graph is not empty 1) Choose a variable v 2) Add edges between unconnected neighbors 3) Create a cluster ($v \cup$ neighbors) 4) Eliminate v

Elimination process

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Primal graph

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Elimination

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Elimination

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Primal graph order While the graph is not empty 1) Choose a variable v **I**₅ 2) Add edges between unconnected 15 neighbors 4) Create a cluster ($v \cup$ neighbors) 3) Eliminate v

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Elimination

Clusters

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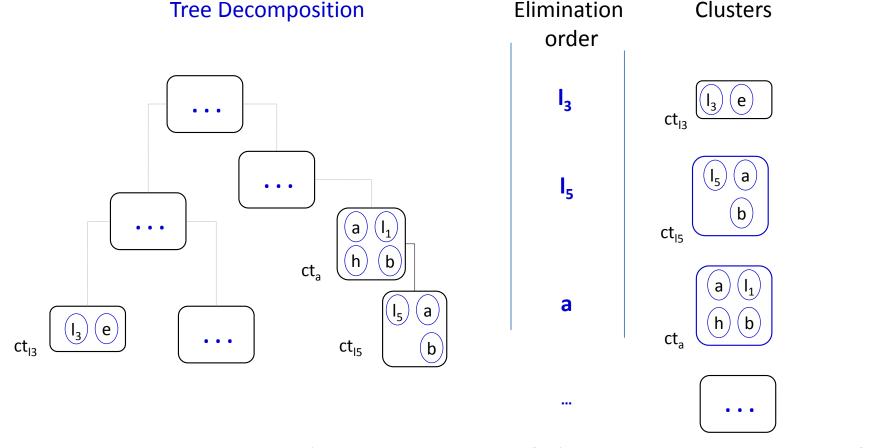
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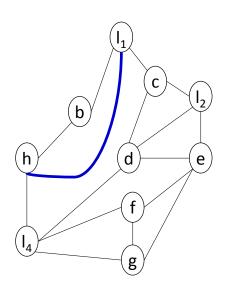
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Observation: The edge added between I1 and h will increase the size of the cluster induced I1 or h



Remark: If we add no edges → Perfect elimination

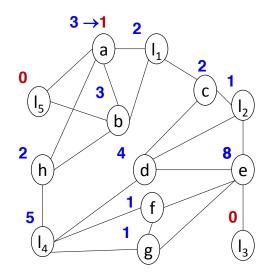


Heuristic: Eliminate first the variable that minimizes the number of additional edges : (Min Fill)



Pb: elimination order cannot be directly applied No privacy, No notion of acquaintance links

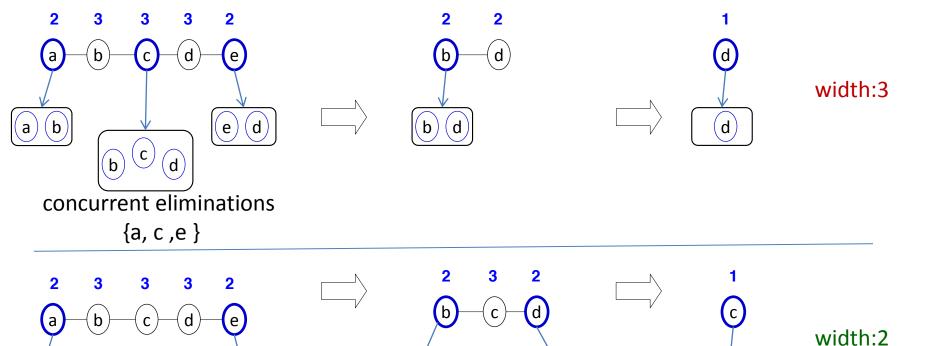
Idea: Weight each node by the quality of the clusters that the node will produce if it is the next to be eliminated



Lesson learn from distributed context

Intuition:

distributed settings can speed up the elimination process by concurrent eliminations



concurrent eliminations {a, e }

concurrent eliminations {b, d}

Concurrent eliminations can be bad for tree decomposition

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Token Elimination: Principle

- Distributed algorithm
 - Phase 1: Implicit building of a DTD
 - Elimination
 - Local elections and votes
 - Token passing
 - Phase 2: clusters reconnection (acquaintance property).

Heuristics:

- Min-Cluster: Each peer estimates the size of the cluster it will produce if it is the next to be eliminated.
- Min-Proj: Each peer estimates the size of additional variables it will add to the token if it is the next to be eliminated.

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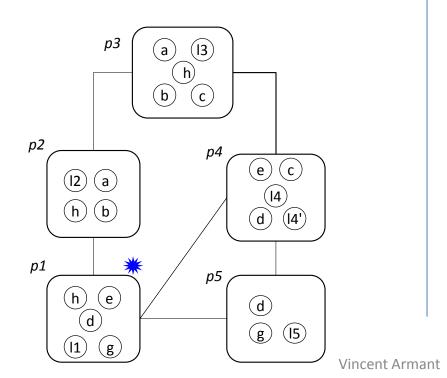
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Distributed algorithm

On going Distributed Tree Decomposition

p receives the token

- organizes a local election
- peers vote , p is a local minimal ?
 - . No: sends the token
 - . Yes: eliminates itself, creates a new cluster, adds shared variables to the token, reorganizes local election sends the token



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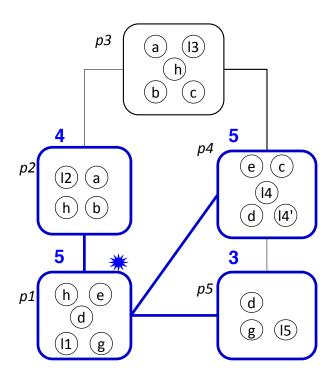
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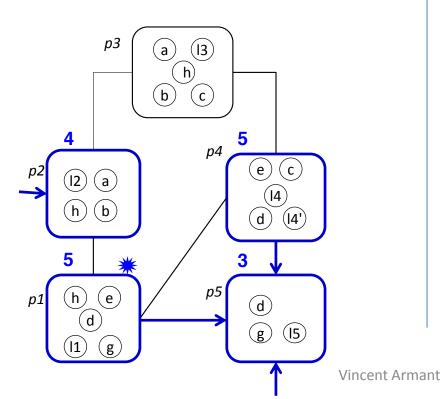


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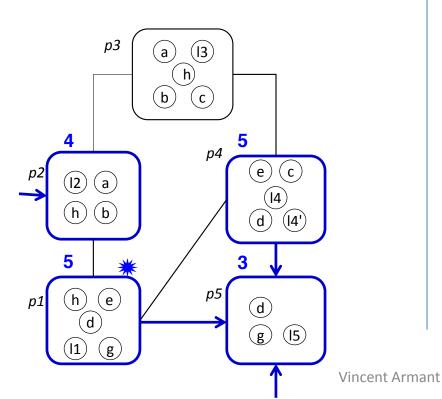
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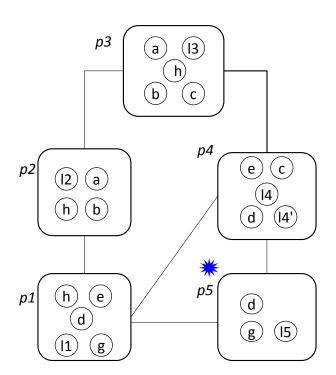
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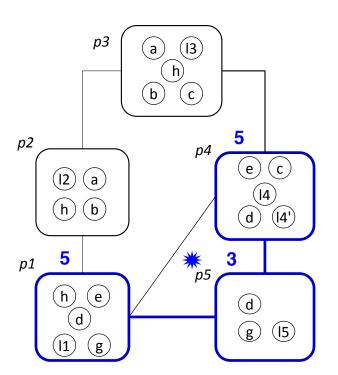
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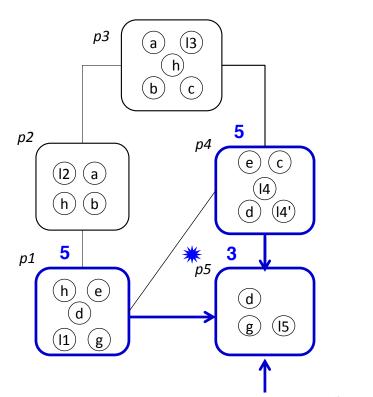


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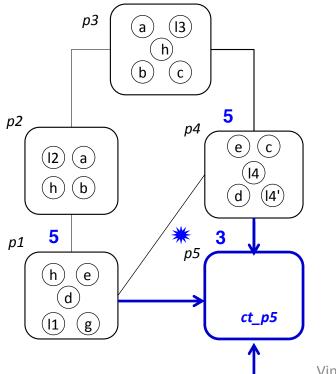


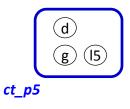
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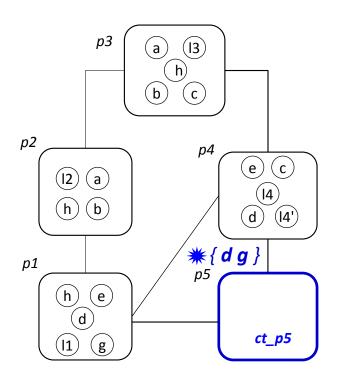
p5 creates the cluster for I5 (privacy)

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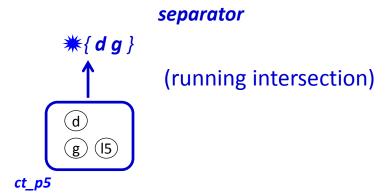
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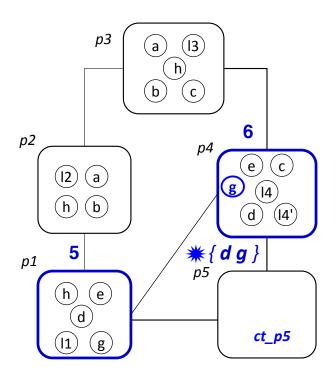
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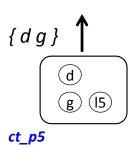
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reorganizes local election

peers vote and sends the token





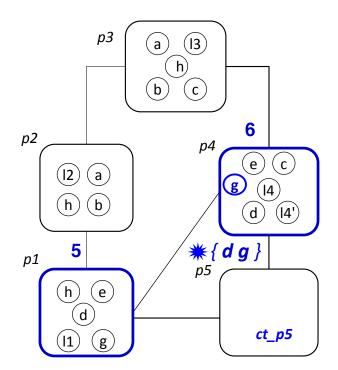
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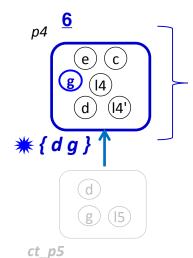
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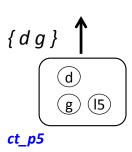
peers vote and sends the token



On going Distributed Tree Decomposition



If p4 is the next to be eliminated, it will produce a cluster of 6 variables



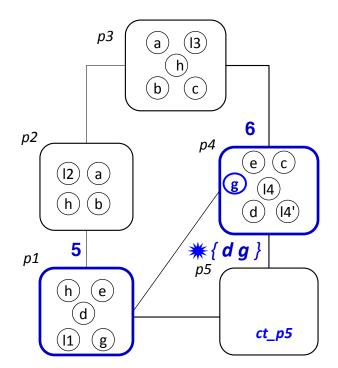
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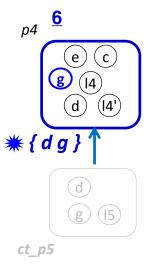
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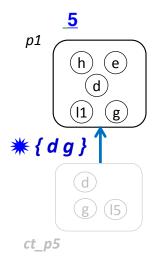
reorganizes local election

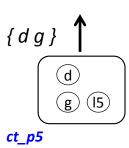
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On going Distributed Tree Decomposition







Distributed algorithm

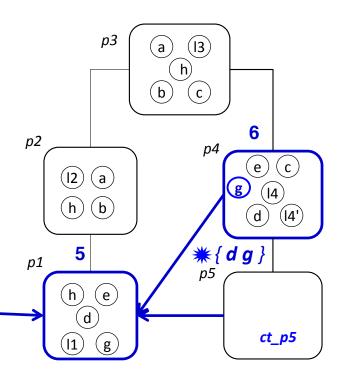
p receives the token

- organize a local election

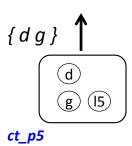
- peers vote, p is a local minimal?

- . No: sends the token
- . Yes: eliminates itself, creates a new cluster, adds shared variables to the token, reorganizes local election

peers vote and p sends the token



On going Distributed Tree Decomposition



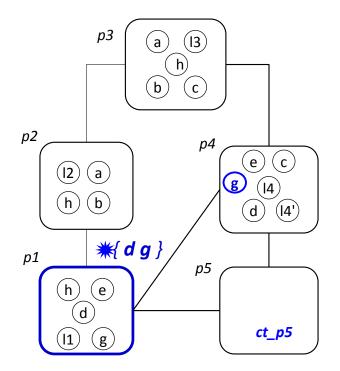
Distributed algorithm

Distributed digoritin

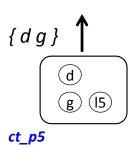
- organize a local election

p receives the token

- peers vote , p is a local minimal ?
 - . No: sends the token
 - . Yes: eliminates itself, creates a new cluster, adds shared variables to the token, reorganizes local election peers vote and p sends the token



On going Distributed Tree Decomposition

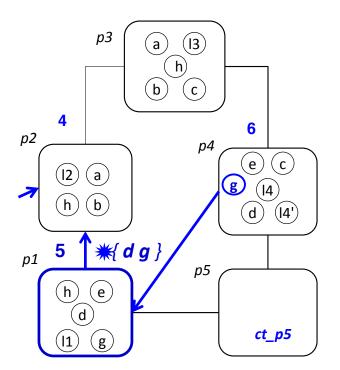


Distributed algorithm

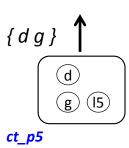
- p1 receives the token
- peers vote, p1 is a local minimal?
 - . No: sends the token

- organize a local election

. Yes: eliminates itself, creates a new cluster, adds shared variables to the token, reorganizes local election peers vote and p sends the token



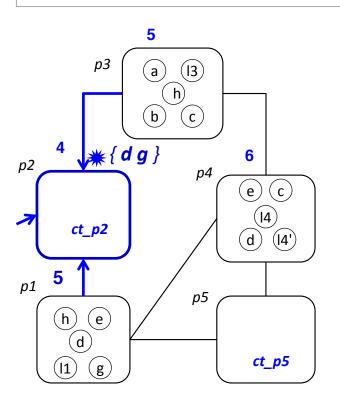
On going Distributed Tree Decomposition

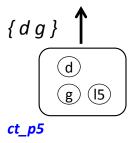


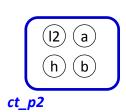
Distributed algorithm

p2 receives the token

- organize a local election
- peers vote, p2 is a local minimal?
 - . No: sends the token
 - Yes: eliminates itself, creates a new cluster, adds shared variables to the token, reorganizes local election peers vote and p sends the token



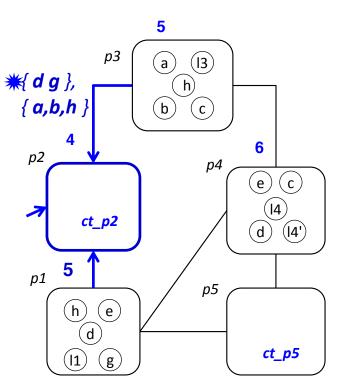


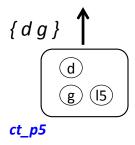


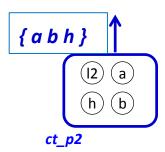
Distributed algorithm

p2 receives the token

- organize a local election
- peers vote, p2 is a local minimal?
 - . No: sends the token
 - Yes: eliminates itself, creates a new cluster, adds shared variables to the token, reorganizes local election peers vote and p sends the token



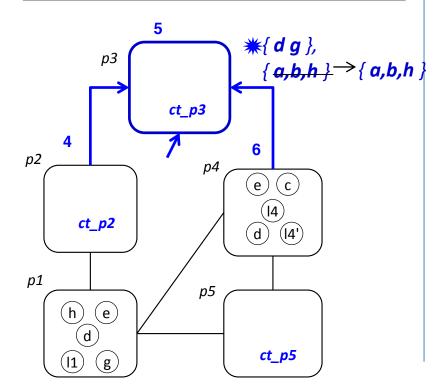


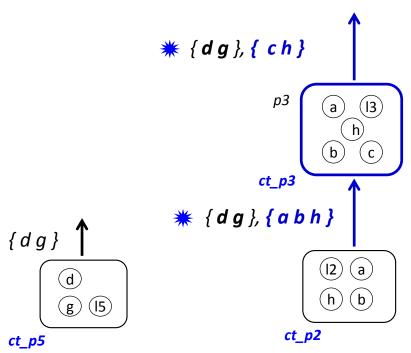


Distributed algorithm

p3 receives the token

- organize a local election
- peers vote, p3 is a local minimal?
 - . No: sends the token
 - Yes: eliminates itself, creates a new cluster, adds shared variables to the token, reorganizes local election peers vote and p sends the token

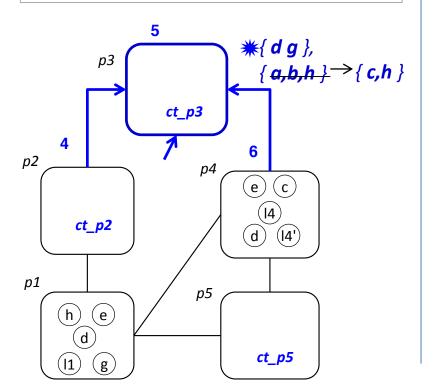


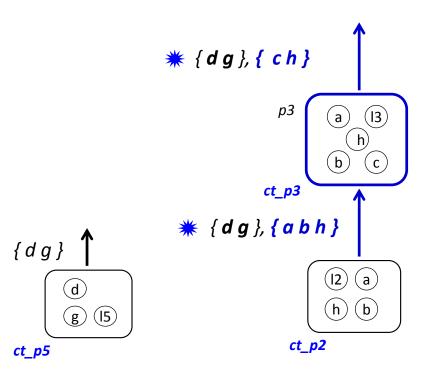


Distributed algorithm

p3 receives the token

- organize a local election
- peers vote, p3 is a local minimal?
 - . No: sends the token
 - Yes: eliminates itself, creates a new cluster, adds shared variables to the token, reorganizes local election peers vote and p sends the token

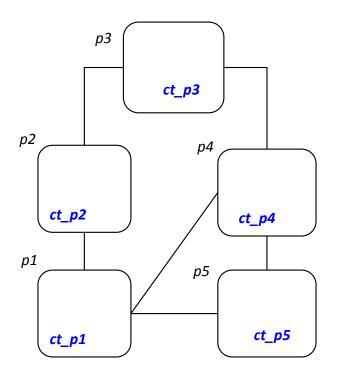


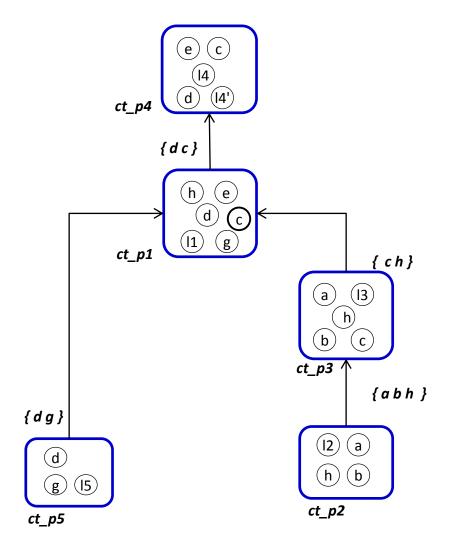


Distributed algorithm

p3 receives the token

- organize a local election
- peers vote, p3 is a local minimal?
 - . No: sends the token
 - Yes: eliminates itself, creates a new cluster, adds shared variables to the token, reorganizes local election peers vote and p sends the token

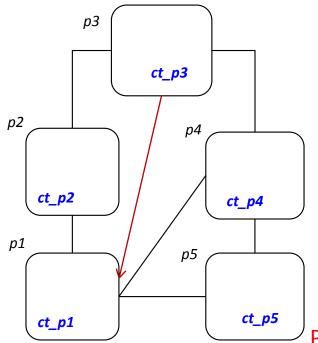




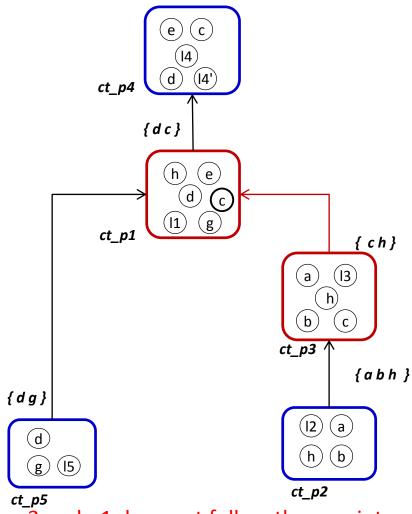
Distributed algorithm

p3 receives the token

- organize a local election
- peers vote, p3 is a local minimal?
 - . No: sends the token
 - Yes: eliminates itself, creates a new cluster, adds shared variables to the token, reorganizes local election peers vote and p sends the token



On going Distributed Tree Decomposition

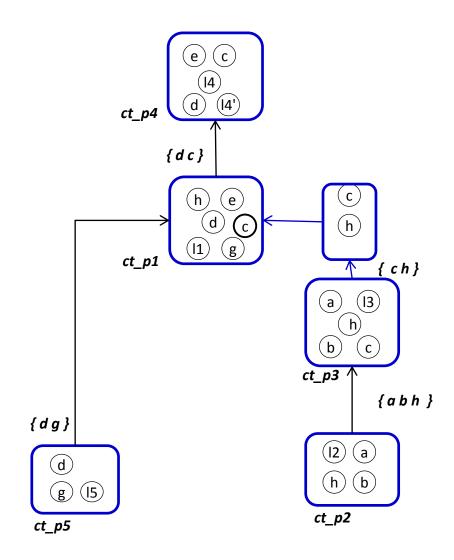


Pb: link between p3 and p1 does not follow the accointances

Distributed structured network

р3 (13) h **p2** p4 (<mark>14</mark>' р1 р5 е (d)

Final Distributed Tree Decomposition

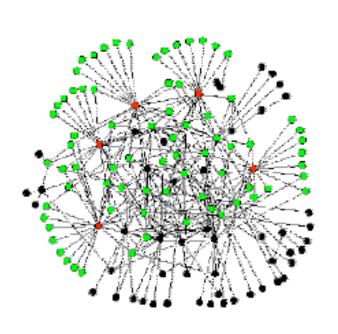


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Outline

- Preliminary: Tree Decomposition
- Problematic: How to decompose a distributed system respecting privacy and acquaintances
- Distributed Tree Decomposition
- Token Elimination
- Experimental results on small world graph
- Conclusion et perspectives

Barabasi et Albert (B.A.) graphs



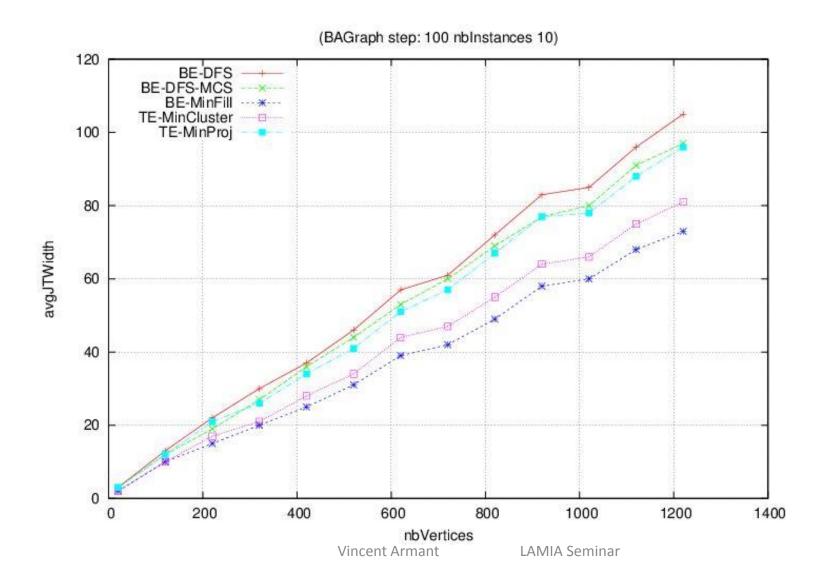
Properties

low average distance between 2 nodes

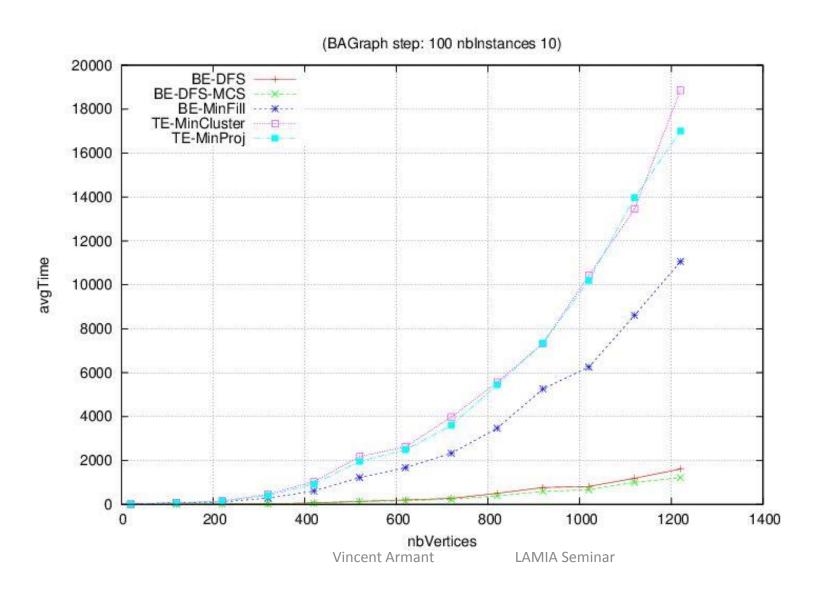
heterogeneity (degree distribution follows a power law)

 represents interaction graph of a lot of real world applications

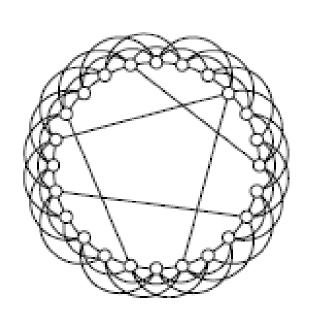
width of tree decomposed BA Graphs



CPU-Time of the tree decomposed BA Graphs



Watts et Strogatz (W.S.) graphs



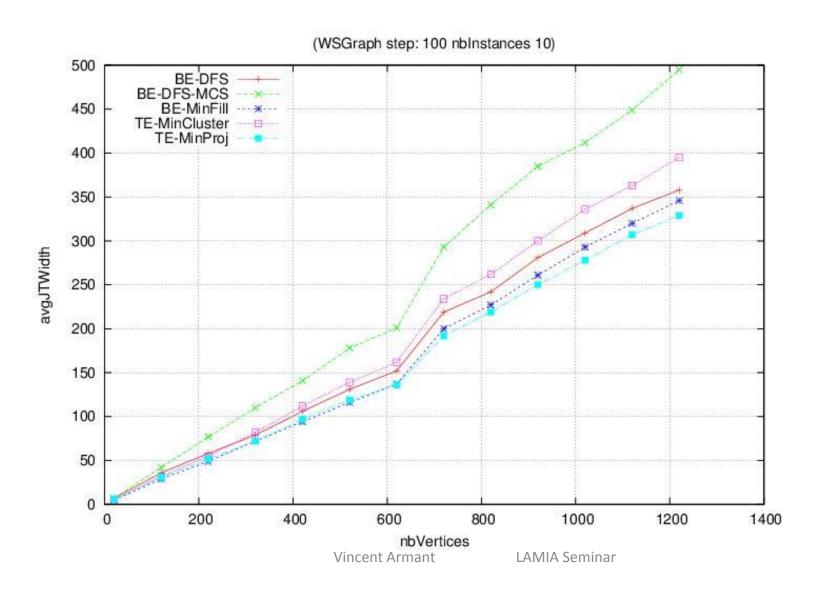
Properties

Short average distance between nodes

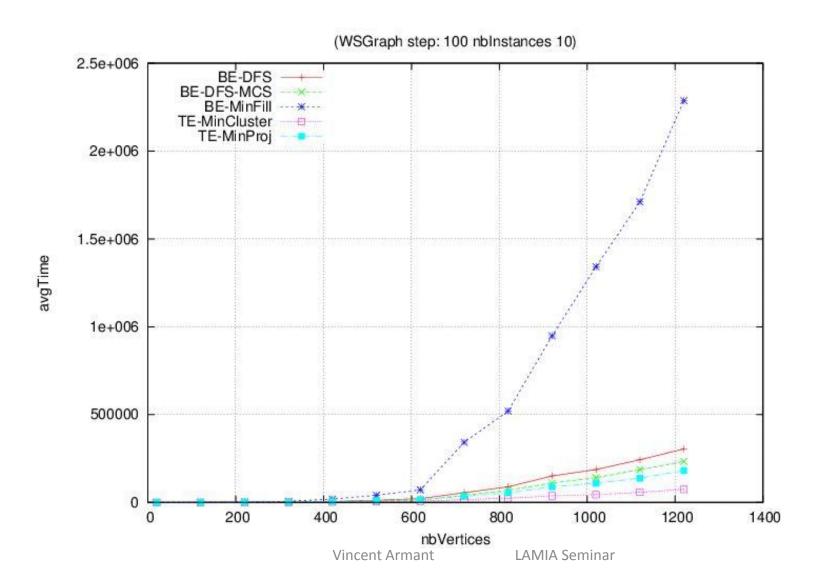
Homogenous (degree distribution follows Poisson law)

Represents some applications
 s.t. ISCAS circuits...

width of the tree decomposed WS Graphs



CPU time of the tree decomposed de WS graph



Conclusions

- Distributed Tree Decomposition respecting
 - privacy (main reason for distributed systems)
 - preserving network acquaintance
- Token Elimination relying
 - On elimination order
 - on votes, token passing
- Results: Token Elimination
 - outperforms classical distributed decomposition methods
 - is competitive with centralized methods

Thanks for your Attention ©

– Questions?

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