Problem sheet 1

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1. Find the mean of the distribution whose M.G.F is $\frac{0.4e^t}{1-0.6e^t}$.
2. Find the mean of a random variable X if $f(x) = \begin{cases} ke^{-x}; x > 0 \\ 0; otherwise \end{cases}$.
3. Verify that the function $P(x)$ defined by $P(x) = \frac{3}{4}(\frac{1}{4})^x ; x = 0,1,2 \ldots$ is a PMF of a discrete random variable X.
4. If a random variable X has a Moment Generating Function $M_X(t) = \frac{2}{2-t}$, determine the variance of X.
5. A test engineer discovered that the cumulative distribution function of the lifetime equipment (in years) is given by $F_X(x) = \begin{cases} 0 \quad ; x < 0 \\ 1 - e^{-\frac{x}{5}} ; 0 \leq x < \infty \end{cases}$. What is the expected lifetime of the equipment?
6. Let X be a continuous random variable with p.d.f $f(x) = cx^2 , 0 < x < 1$, then find the value of $'c'$.
7. Let X be a continuous random variable with p.d.f $f(x) = \frac{1}{3}e^{\frac{-x}{3}} , x > 0$, then find its mean.
8. Let X be a continuous random variable with p.d.f $f(x) = \begin{cases} x \quad ; 0 \leq x \leq 1 \\ 2 - x ; 1 \leq x \leq 2 \end{cases}$, then find its cumulative distribution function of X.
9. Let X be a discrete random variable with p.m.f $p(x) = \frac{3}{4}\left(\frac{1}{4}\right)^{x-1}$, where $x = 1,2,3, \ldots$ then find its mean, variance and $p(x > 4/x > 2)$.
10. The probability density function of continuous random variable X is given by $f(x) = \begin{cases} 2e^{-2x}; x \geq 0 \\ 0; x < 0 \end{cases}$. Find (i) $E(X)$ and $Var(X)$, (ii) $P(X \leq 3)$.

11. A random variable X has the p.m.f

x	1	2	3	4	5	6	7
$p(x)$	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 +$

Find the value of k, cdf of X and if $P(X \leq k) > \frac{1}{2}$, then find the minimum value of k.

12. Assume that the length of a phone call in minutes is an **exponential** random variable X with parameter $\lambda = \frac{1}{10}$. If someone arrives at a phone booth just before you arrive, find the probability that you will have to wait (a) less than 5 minutes, and (b) between 5 and 10 minutes.

13. You are taking a multiple choice quiz that consists of five questions. Each question has four possible answers, only one of which is correct. To complete the quiz, you randomly guess the answer to each question. Find the probability of guessing (a) exactly three answers correctly, (b) atleast three answers correctly, and (c) less than three answers correctly.

14. A survey indicates that for trip to the supermarket, a shopper spends an average of 45 minutes with a standard deviation of 12 minutes in the store. The lengths of time spent in the store are **normally distributed** and are represented by the variable X. A shopper enters the store. (a) Find the probability that the shopper will be in the store for each interval given below. (b) Interprete your answer if 200 shoppers enter the store. How many shoppers would you expect to be in the store for each interval of time listed below?

(i) Between 24 and 54 minutes

(ii) More than 39 minutes.

15. A book of 2021 pages contains 2021 mistakes. Find the probability that there are at least 4 mistakes in randomly selected pages.

16. Suppose the cumulative distribution function of the random variable X is

$$F(x) = \begin{cases} 0 & ; x < -2 \\ 0.25x + 0.5; & -2 \leq x \leq 2 \\ 1 & ; x \geq 2 \end{cases}$$. Determine $P(X < 1.8)$, $P(X > -1.5)$, $P(X < -2)$.

17. A surgical technique is performed on seven patients. The results say that there is a 70% chance of success. Find the probability that the surgery is successful for exactly five patients, at least five patients and less than five patients.

18. The amounts a soft drink machine is designed to dispense for each drink are **normally distributed**, with mean of 12 fluid ounces and a standard deviation of 0.2 fluid ounces. A drink is randomly selected. Find the probability that the drink is less than 11.9 fluid ounces, between 11.8 and 11.9 fluid ounces, and more than 12.3 fluid ounces.

19. In a shooting competition two competitors A and B are allowed to shoot independently until each has hits his own target. They have probabilities $\frac{3}{5}, \frac{5}{7}$ of hitting the targets at each shot respectively. Find the probability that B will require more shots than A.

20. Suppose that the life of an industrial lamp (in thousands of hours) is **exponentially distributed** with mean life of 3000 hours. Find the probability that (a) the lamp will last more than mean life, (b) the lamp will last between 2000 and 3000 hours (c) the lamp will last another 1000 hours given that it has already lasted for 2500 hours.

21. The cumulative distribution function of a continuous random variable X is given by

$$F(x) = \begin{cases} 1 - e^{-2x} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$$

. Find the probability density function of X, $P(X > 2)$ and variance of X.

22. Let X be a continuous random variable whose pdf is $f(x) = \begin{cases} \frac{x^3}{4}, & 0 < x < c \\ 0, & otherwise \end{cases}$. What is the value of '$c$' that makes $f(x)$ a valid probability density function?

23. If X is **uniformly distributed** in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. Find the probability density function of $Y = tanX$.

24. It is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are inoculated, find the probability that

   (i) None contract the disease.

   (ii) Fewer than 2 contract the disease.

   (iii) More than 3 contract the disease.

25. Assume that X is continuous random variable with the following pdf

$$f(x) = \begin{cases} 0 & , x < 0.5 \\ ke^{-2(x-0.5)} & , x \geq 0.5 \end{cases}$$

Find the value of k and the cdf of X. Also $P(X \leq 1.5)$, $P(1.2 < X < 2.4)$.

26. Let X be a discrete random variable whose cdf is given by

$$F(x) = \begin{cases} 0 & , if\ x < -3 \\ 1/6 & , if\ -3 \leq x < 6 \\ 1/2 & , if\ 6 \leq x < 10 \\ 1 & , if\ x \geq 10 \end{cases}$$

Find $P(X \leq 4)$, $P(-5 < X \leq 4)$ and probability mass function of X.

27. Suppose that the service life (in hours) of a semiconductor device is a random variable having the **Weibull distribution** with $\alpha = 0.025$ and $\beta = 0.5$. What is the probability that such a device will still be in operating condition after 4000 hours? Also find its mean and variance.

28. A random variable $X \sim Unif(-3,3)$.Compute (i)$P(|X| < 2)$, (ii) $E(X)$,(iii)$P(|X-2| < 2)$.

29. A bank manager has learnt that the length of time the customers have to wait for being attended by the teller is **normally distributed** with mean time of 5minutes and standard deviation of 0.8minute. Find the probability that a customer has to wait (i) for more than 3.5minutes, (ii) between 3.4minutes and 6.2minutes.

30. Find the M.G.F of the random variable 'x' whose p.d.f is $f(x) = \begin{cases} \frac{x}{4}e^{-\frac{x}{2}} & ; if\ x > 0 \\ 0 & ; otherwise \end{cases}$

31. If a random variable 'x' has continuous cdf given by $f(x) = \begin{cases} 0 & if\ \ x \leq 0 \\ c(1-e^{-x}) & if\ \ x > 0 \end{cases}$

Then find (i) c (ii) $1 \leq x \leq 2$.

32. Given the random variable 'x' with the probability distribution $f(x) = \begin{cases} 2x & ;\ 0 < x < 1 \\ 0 & ; otherwise \end{cases}$

Find the probability distribution of $y = 8x^3$.

33. After correcting the proof upto 50 pages of a book, the proof reader found that there are three errors per 5 pages. Use **Poisson distribution** and estimate the number of pages with 0, 1, 2, 3 errors and more than 3 errors in a book of 100 pages.

34. In a certain town, 20% samples of the population are literate. Assume that 200 investigator each take samples of 10 individuals to see whether they are literate. How many investigation would you expect to report that 3 people or less are literate in the sample?

35. The time (in hours) required to repair a machine is **exponentially distributed** with parameter $\lambda = 1/2$. What is the probability that the repair time exceeds 2 hours? What is the conditional probability that the repair time takes atleast 10 hours given that its duration exceeds 9 hours?

36. A candidate applying for driving licence has the probability of 0.8 in passing the road test in a given trial. What is the probability that he will pass the test (i) on the fourth trial, (ii) in less than four trials.

37. A bus arrives every 20 minutes, at a specified stop, beginning at 6.40 a.m. and continuing until 8.40 a.m. A passenger arrives randomly between 7.00 a.m. and 7.30 a.m. What is the probability that the passenger has to wait for more than 5 minutes for a bus?

38. A system has a component whose time to failure is exponentially distributed with parameter $\lambda = \frac{1}{6}$. If 6 such components are installed in different systems, what is the probability that at least 2 are still working at the end of 9 years?

39. The life of a certain type of car is a random variable X having Weibull distribution with parameter $\beta = 2$. Find the value of the parameter α, given that the probability that the life of the car exceeds 5 years is $e^{-0.25}$, Also find mean and variance.

40. The daily consumption of milk in excess of 20,000 liters is approximately distributed as Gamma variable with parameter $k = 2, \lambda = \frac{1}{10000}$. If the city has a daily stock of 30,000 litres on a given day, find the probability that the stock is insufficient.

41. In a newly constructed township, 2000 electrical lamps are installed with an average the life of the lamps follows normal distribution, find (i) the number of lamps expected to fail during the first 700 hours, (ii) in what period of burning hours 10% of the lamps fail.

42. The mean yield for one acre plots is 662 kgs with S.D 32. Assuming normal distribution, how many acre plots in a batch of 1000 plots would you expect to yield (i) over 700 kgs, (ii) below 650 kgs, (iii) what is the lowest yield of the best 100 plots?

43. If X has uniform distribution in $(-a, a)$, where $a > 0$, find a such that

$P(|X| < 1) = P(|X| > 1)$.

44. If X is a random variable with normal distribution and $\mu = 1,\ \sigma = 2$.

Find $P(|X - 2| < \frac{1}{2} / X > 0$.

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