16/11/2023

UNIT - IV

LINEAR PROGRAMMING

Formulation of linear programming problem:

1. Objective function → Maximize profit (some situation minimize)
2. Decision variables → Eg: x, y
3. constraints
4. Feasible solution → x, y satisfies constraint
5. optimal solution → out of those x, y which gives objective function

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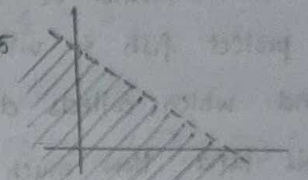
$x + y < 5$



Eg:

60 (chair + Table)

Rs : 50,000

	cost	profit
ⓧ chair	500	50 (1 chair)
ⓨ Table	2500	500 (1 table)

① Maximize $z = 50x + 50y$

subject to (② x, y)

③ $x + y \leq 60$

$500x + 2500y \leq 50000$

and

$x, y \geq 0$

(Linear : Degree = 1)

* If distinct equation → unique soln

soln: $x + y \leq 60$

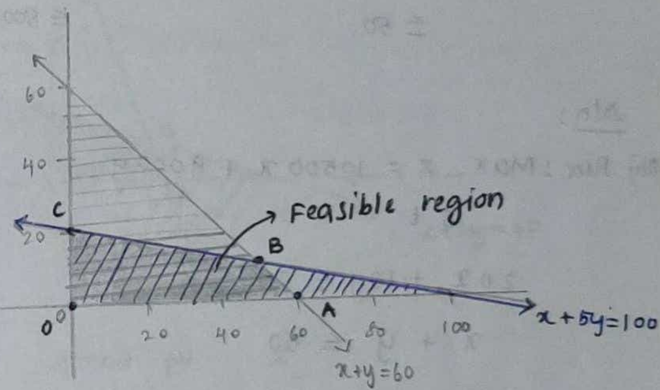
$x + y = 60$

x	0	60
y	60	0

$x + 5y \leq 100$

$x + 5y = 100$

x	0	100
y	20	0



corner pt	z value
O (0,0)	0
A (60,0)	3000
B (50,10)	7500
C (0,20)	$\boxed{10000}$ ← Max

$x + y = 60$

$x + 5y = 100$

$-4y = -40$

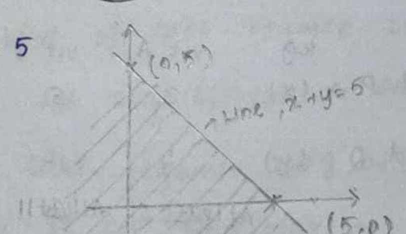
$y = 10$

$x = 50$

$\therefore Z_{max} = 10000$ at $x = 0, y = 20$

GRAPHICAL METHOD

$x + y = 5$



$x + y \leq 5$

↳ Region

$(0, 0) \leq 5$

* x, y - Non-ve constraints

⇓

1st Quadrant gives feasible solution

⊙ unbounded region

convex   concave

⊙ Bounded region

↓

corner pts - gives optimal solution

Q: A cooperative society of farmers has 50 hectares of land to grow two crops X & Y. The profit from crops X & Y per hectare are ₹10500 & ₹9000 respectively. To control weeds a liquid herbicides has to be used for crops X & Y at rates of 20L & 10L per hectare. Further no more than 800L of herbicide should be used in order to protect fish & wildlife using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximize the total profit of society.

	hectare	profit	Herbicide
Crop X	(x)	10500/hectare	20L/h
Crop Y	(y)	9000/hectare	10L/h
	≤ 50		$\le 800L$

Soln:

Obj Func: Max $z = 10500x + 9000y$

s.t

$20x + 10y \le 800$

$x + y \le 50$

and

$x, y \ge 0$

consider

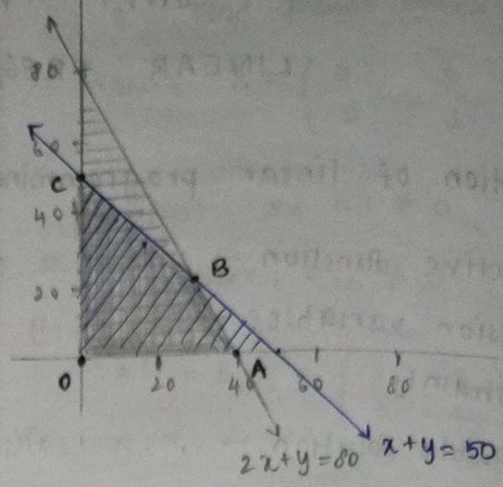
$20x + 10y = 800$

$2x + y = 80$

x	0	40
y	80	0

$x + y = 50$

x	0	50
y	50	0



corner pts	Z
O(0,0)	0
A(40,0)	420000
B(30,20)	$\boxed{495000}$
C(0,50)	450000

$2x+y=80$
$x+y=50$
$x=30$
$y=20$

$\therefore$ Max $Z = 495000$ at $x=30, y=20$

Q: A person wishes to mix two types of food P & Q in such a way that the vitamin contain of mixture contains atleast 8 units of vit A & 11 units of vit B. Food P costs ₹60/kg & Food Q costs ₹80/kg. Food P contains 3 units/kg of vit A & 5 units/kg of vit B. while Food Q contains 4 units/kg of vit A & 2 units/kg of vit B. Determine the minimize cost of mixture

	kg	vit A	vit B	cost
Food P	(x)	3	5	60
Food Q	(y)	4	2	80
		Atleast 8	Atleast 11	

- objective func min $z = 60x + 80y$

s.t

$3x + 4y \ge 8$

$5x + 2y \ge 11$

and $x, y \ge 0$

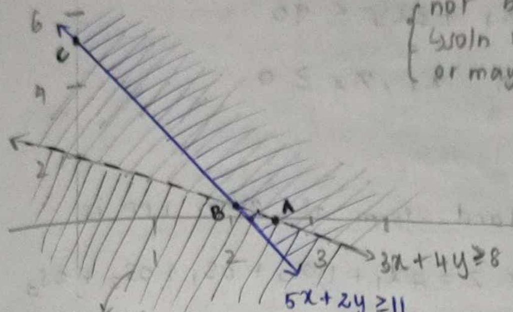
consider,

$3x + 4y \geq 8$ $\qquad$ $5x + 2y \geq 11$

$3x + 4y = 8$ $\qquad$ $5x + 2y = 11$

x	0	8/3
y	2	0

x	0	11/5
y	11/2	0



not bounded
soln may exist
or may not exist

$60x+80y < 160$
Not in feasible region

corner pts	Z
A $(0, 11/2)$	440
B $(2, 1/2)$	160
C $(8/3, 0)$	160

→ Min

$3x + 4y = 8$
$10x + 4y = 22$
$-7x = -14$
$x = 2$
$y = \frac{1}{2}$

∵ unbounded,

$60x + 80y < 160$

$3x + 4y < 8$

x	0	8/3
y	2	0

→ Not in feasible region

∴ $Z_{min} = 160$ at $x = 2$, $y = 1/2$

and $x = 8/3$, $y = 0$

Q: One kind of cake require 200g of flour and 25g of fat & another kind of cake require 100g of flour and 50g of fat. Find max no. of cake which can be made from 5kg of flour & 1kg of fat. Assuming that there is no shortage of other ingredients in making the cake

	no. of cake	flour	fat
cake 1	x	200g	25g
cake 2	y	100g	50g
Available		5kg	1kg

objective function

Max $Z = x + y$

s.t

$200x + 100y \leq 5000$

$25x + 50y \leq 1000$

and

$x, y \geq 0$

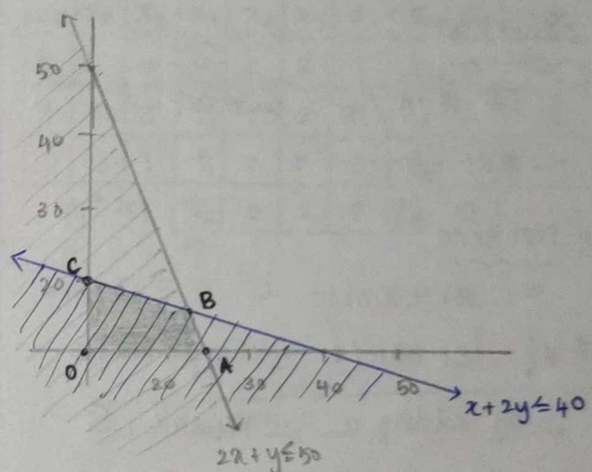
consider

$2x + y \leq 50$ $\qquad$ $25x + 50y \leq 1000$

$2x + y = 50$ $\qquad$ $x + 2y = 40$

x	0	25
y	50	0

x	0	40
y	20	0



corner pts	Z
O$(0, 0)$	0
A $(25, 0)$	25
B $(20, 10)$	30 ← Max
C $(0, 20)$	20

$2x + y = 50$
$2x + 4y = 80$
$-3y = -30$
$y = 10$
$x = 20$

∴ $Z_{max} = 30$ at $x = 20$, $y = 10$

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SIMPLEX METHOD

- For more than 2 parameters

Max (or) Min $z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

subject to

constraints

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le = \ge b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le = \ge b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le = \ge b_m$

and $x_1, x_2, \ldots, x_n \ge 0$

[n - variable
m - constraints
n - m = 0]

① $C = (c_1, c_2, \ldots, c_n)$

$X = (x_1, x_2, \ldots, x_n)$

* canonical
  - Less than / = constraint
  - RHS = +ve

* standard
  - = constraint

⇒ If less than / = → change to = by adding ______ variable, S

⇒ If greater than / = → change to = by subtracting ______ variable S

Max (or) Min $z = CX$

s.t

$AX \le = \ge B$

and $X \ge 0$

if Min(z) is given,

Min $z = -\text{Max}(-z)$

Q1: Solve the following LPP using simplex method: (HW: use graphical method)

Maximize $z = 4x_1 + 10x_2$

s.t

$2x_1 + x_2 \le 50$

$2x_1 + 5x_2 \le 100$

$2x_1 + 3x_2 \le 90$

and $x_1, x_2 \ge 0$

soln:

standard form of given LPP is

Max $Z = 4x_1 + 10x_2 + 0S_1 + 0S_2 + 0S_3$

s.t

$2x_1 + x_2 + S_1 = 50$

$2x_1 + 5x_2 + S_2 = 100$

$2x_1 + 3x_2 + S_3 = 90$

and $x_1, x_2, S_1, S_2, S_3 \ge 0$

[C_B = coeff of basic variable (X_B, Y_B)
C_j = coeff]

Table 1:

		C_j	4	10	0	0	0	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3	Min Ratio θ
0	S_1	50	2	1	1	0	0	$50/1 = 50$
0	S_2	100	2	5	0	1	0	$100/5 = 20$ ←
0	S_3	90	2	3	0	0	1	$90/3 = 30$
		$Z_j - C_j$	−4	−10	0	0	0	

($Z_j = C_B \times Y_B$)

① Most -ve

② pivot row (Min Ratio)

pivot element ③

New row = old row − (pivot column element) × (New pivot row)

Table 2:

C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3	
0	S_1	30	$8/5$	0	1	$-1/5$	0	
10	x_2	20	$2/5$	1	0	$1/5$	0	← new pivot row
0	S_3	30	$4/5$	0	0	$-3/5$	1	
		$Z_j - C_j$	0	0	0	2	0	

new s_1 row $= 50 \quad 2 \quad 1 \quad 1 \quad 0 \quad 0$

$-1\,(20 \quad 2/5 \quad 1 \quad 0 \quad 1/5 \quad 0)$

$30 \quad 8/5 \quad 0 \quad 1 \quad -1/5 \quad 0$

new s_2 row $= 90 \quad 2 \quad 3 \quad 0 \quad 0 \quad 1$

$-3\,(20 \quad 2/5 \quad 1 \quad 0 \quad 1/5 \quad 0)$

$30 \quad 4/5 \quad 0 \quad 0 \quad -3/5 \quad 1$

new $z_j - c_j = -4 \quad -10 \quad 0 \quad 0 \quad 0$

$-(-10)\,(2/5 \quad 1 \quad 0 \quad 1/5 \quad 0)$

$-4 \quad -10 \quad 0 \quad -2 \quad 0$

$0 \quad 0 \quad 0 \quad 2 \quad 0$

• $z_j - c_j \Rightarrow$ strictly optimal, unique soln
$\Downarrow$
greater than zero

since all $z_j - c_j \geq 0$

this soln is optimal

$\therefore Z_{max} = 4(0) + 10(20) = 200$

when $x_1 = 0, \quad x_2 = 20$

2. solve following LPP using simplex method:

$\text{Min } z = x_1 - 3x_2 + 2x_3$

s.t

$3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

and

$x_1, x_2, x_3 \geq 0$

soln:

$\text{Min } z = -\text{Max}(-z)$

$= -\text{Max } z^*$

The standard form of LPP is

$\text{Max } z^* = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$
(changed sign)

s.t

$3x_1 - x_2 + 2x_3 + s_1 = 7$

$-2x_1 + 4x_2 + s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

and $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Table 1:

		c_j	-1	3	-2	0	0	0	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	θ
0	s_1	7	3	-1	2	1	0	0	$7/-1 = -1 = -$
0	s_2	12	-2	4	0	0	1	0	$12/4 = 3$ ←
0	s_3	10	-4	3	8	0	0	1	$10/3 = 3.33$
	$z_j^* - c_j$		1	-3	2	0	0	0	
				↑					

Table 2:

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	θ
0	s_1	10	5/2	0	2	1	1/4	0	$10/5/2 = 4$ ←
3	x_2	3	-1/2	1	0	0	1/4	0	–
0	s_3	1	-5/2	0	8	0	-3/4	1	–
	$z_j^* - c_j$		-1/2	0	2	0	3/4	0	
			↑						

new $S1 = 7 \quad 3 \quad -1 \quad 2 \quad 1 \quad 0 \quad 0$

$-(-1)\,(3 \quad -1/2 \quad 1 \quad 0 \quad 0 \quad 1/4 \quad 0)$

$10 \quad 5/2 \quad 0 \quad 2 \quad 1 \quad 1/4 \quad 0$

new $S2 = 10 \quad -4 \quad 3 \quad 8 \quad 0 \quad 0 \quad 1$

$-3\,(3 \quad -1/2 \quad 1 \quad 0 \quad 0 \quad 1/4 \quad 0)$

$9 \quad -3/2 \quad 3 \quad 0 \quad 0 \quad 3/4 \quad 0$

$1 \quad -5/2 \quad 0 \quad 8 \quad 0 \quad -3/4 \quad 1$

new $z_j^* - c_j = 1 \quad -3 \quad 2 \quad 0 \quad 0 \quad 0$

$-(-3)\,(-1/2 \quad 1 \quad 0 \quad 0 \quad 1/4 \quad 0)$

$3/2 \quad -3 \quad 0 \quad 0 \quad -3/4 \quad 0$

$-1/2 \quad 0 \quad 2 \quad 0 \quad +3/4 \quad 0$

Table 3:

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
-1	x_1	4	1	0	4/5	2/5	1/10	0
3	x_2	5	0	1	2/5	1/5	3/10	0
0	s_3	11	0	0	10	1	-1/2	0
$z_j^* - c_j$			0	0	12/5	1/5	4/5	0

new x_2 = 5 -1/2 1 0 0 1/4 0

$-(\frac{-1}{2})(4 \quad 1 \quad 0 \quad \frac{4}{5} \quad \frac{2}{5} \quad \frac{1}{10} \quad 0)$

(-) -2 -1/2 0 -2/5 -1/5 -1/20 0

5 0 1 +2/5 1/5 3/10 0

new s_3 = 1 -5/2 0 8 0 -3/4 1

$-(-5/2)(4 \quad 1 \quad 0 \quad \frac{4}{5} \quad \frac{2}{5} \quad \frac{1}{10} \quad 0)$

(-) -10 -5/2 0 -2 -1 -1/4 0

11 0 0 10 1 -1/2 0

new $z_j^* - c_j$ = -1/2 0 2 0 3/4 0

$-(-1/2)(1 \quad 0 \quad \frac{4}{5} \quad \frac{2}{5} \quad \frac{1}{10} \quad 0)$

(-) -1/2 0 $\frac{-2}{5}$ $\frac{-1}{5}$ $\frac{-1}{20}$ 0

0 0 $\frac{12}{5}$ $\frac{1}{5}$ $\frac{4}{5}$ 0

∵ All $z_j^* - c_j \geq 0$

∴ The solution is optimal

Max $z^* = -4 + 3(5) - 2(0) = 11$

$x_1 = 4, \; x_2 = 5, \; x_3 = 0$

∴ Min $z = -11$

3. Solve the following LPP using simplex method

Max $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

s.t

$2x_1 + x_2 + 5x_3 + 6x_4 \leq 20$

$3x_1 + x_2 + 3x_3 + 25x_4 \leq 24$

$7x_1 + x_4 \leq 70$

and

$x_1, x_2, x_3, x_4 \geq 0$

Soln:

The standard form of LPP is

Max $z = 15x_1 + 6x_2 + 9x_3 + 2x_4 + 0s_1 + 0s_2 + 0s_3$

s.t

$2x_1 + x_2 + 5x_3 + 6x_4 + s_1 = 20$

$3x_1 + x_2 + 3x_3 + 25x_4 + s_2 = 24$

$7x_1 + x_4 + s_3 = 70$

and

$x_1, x_2, x_3, x_4, s_1, s_2, s_3 \geq 0$

Table 1:

		c_j	15	6	9	2	0	0	0	
C_B	Y_B	X_B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	θ
0	s_1	20	2	1	5	6	1	0	0	20/2 = 10
0	s_2	24	3	1	3	25	0	1	0	24/3 = 8 ←
0	s_3	70	7	0	0	1	0	0	1	70/7 = 10
$z_j - c_j$			-15 ↑	-6	-9	-2	0	0	0	

Table 2:

C_B	Y_B	X_B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	θ
0	s_1	4	0	1/3	3	-32/3	1	-2/3	0	4/(1/3) = 12 ←
15	x_1	8	1	1/3	1	25/3	0	1/3	0	8/(1/3) = 24
0	s_3	14	0	-7/3	-7	-172/3	0	-7/3	1	-
$z_j - c_j$			0	-1 ↑	6	123	0	5	0	

new s_1 = 20 2 1 5 6 1 0 0

$-2(8 \quad 1 \quad \frac{1}{3} \quad 1 \quad \frac{25}{3} \quad 0 \quad \frac{1}{3} \quad 0)$

(-) 16 2 2/3 2 50/3 0 2/3 0

4 0 $\frac{1}{3}$ 3 $\frac{-32}{3}$ 1 $\frac{-2}{3}$ 0

new s_3 = 70 7 0 0 1 0 0 1

$-7(8 \quad 1 \quad \frac{1}{3} \quad 1 \quad \frac{25}{3} \quad 0 \quad \frac{1}{3} \quad 0)$

(-) 56 7 7/3 7 $\frac{175}{3}$ 0 $\frac{7}{3}$ 0

14 0 $\frac{-7}{3}$ -7 $\frac{-172}{3}$ 0 $\frac{-7}{3}$ 1

new $z_j - c_j$ = -15 -6 -9 -2 0 0 0

$-(-15)(1 \quad \frac{1}{3} \quad 1 \quad \frac{25}{3} \quad 0 \quad \frac{1}{3} \quad 0)$

(-) -15 -5 -15 -125 0 -5 0

0 -1 6 123 0 5 0

Table 3 :

C_B	Y_B	X_B	x_1	x_2	x_3	x_4	s_1	s_2	s_3
6	x_2	12	0	1	9	-32	3	-2	0
15	x_1	4	1	0	-2	19	-1	1	0
0	s_3	42	0	0	14	-132	7	-7	1
$z_j - c_j$			0	0	15	91	3	3	0

new $x_1 = 8 \quad 1 \quad \frac{1}{3} \quad 1 \quad \frac{25}{3} \quad 0 \quad \frac{1}{3} \quad 0$

$-\frac{1}{3}\,(12 \quad 0 \quad 1 \quad 9 \quad -32 \quad 3 \quad -2 \quad 0)$

$(-) \quad 4 \quad 0 \quad \frac{1}{3} \quad 3 \quad \frac{-32}{3} \quad 1 \quad \frac{-2}{3} \quad 0$

$4 \quad 1 \quad 0 \quad -2 \quad 19 \quad -1 \quad 1 \quad 0$

new $s_3 = 14 \quad 0 \quad \frac{-7}{3} \quad -7 \quad \frac{-172}{3} \quad 0 \quad \frac{-7}{3} \quad 1$

$-\left(\frac{-7}{3}\right)(12 \quad 0 \quad 1 \quad 9 \quad -32 \quad 3 \quad -2 \quad 0)$

$(-) \quad -28 \quad 0 \quad \frac{-7}{3} \quad -21 \quad \frac{224}{3} \quad -7 \quad \frac{-14}{3} \quad 0$

$42 \quad 0 \quad 0 \quad 14 \quad -132 \quad 7 \quad -7 \quad 1$

new $z_j - c_j = 0 \quad -1 \quad 6 \quad 123 \quad 0 \quad 5 \quad 0$

$-(-1)(0 \quad 1 \quad 9 \quad -32 \quad 3 \quad -2 \quad 0)$

$0 \quad 0 \quad 15 \quad 91 \quad 3 \quad 3 \quad 0$

$\because$ All $z_j - c_j \geq 0$

$\therefore$ The solution is optimal

Max $Z = 15(4) + 6(12) + 0 = 132$

when $x_1 = 4,\ x_2 = 12,\ x_3 = 0,\ x_4 = 0$

4. Solve the following LPP using simplex method :

Max $Z = 20x_1 + 6x_2 + 8x_3$

s.t

$8x_1 + 2x_2 + 3x_3 \leq 250$

$4x_1 + 3x_2 \leq 150$

$2x_1 + x_3 \leq 50$

and

$x_1, x_2, x_3 \geq 0$

Soln :

The standard form of LPP is

Max $z = 20x_1 + 6x_2 + 8x_3 + 0s_1 + 0s_2 + 0s_3$

s.t

$8x_1 + 2x_2 + 3x_3 + s_1 = 250$

$4x_1 + 3x_2 + s_2 = 150$

$2x_1 + x_3 + s_3 = 50$

and

$x_1, x_2, x_3, x_4, s_1, s_2, s_3 \geq 0$

Table 1 :

		c_j	20	6	8	0	0	0	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	θ
0	s_1	250	8	2	3	1	0	0	$250/8 = 31.2$
0	s_2	150	4	3	0	0	1	0	$150/4 = 37.5$
0	s_3	50	2	0	1	0	0	1	$50/2 = 25$ ←
$z_j - c_j$			-20	-6	-8	0	0	0	
			↑						

Table 2 :

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	θ
0	s_1	50	0	2	-1	1	0	-4	$50/2 = 25$
0	s_2	50	0	3	-2	0	1	-2	$50/3 = 16.6$ ←
20	x_1	25	1	0	½	0	0	½	–
$z_j - c_j$			0	-6	2	0	0	10	
				↑					

new $s_1 = 250 \quad 8 \quad 2 \quad 3 \quad 1 \quad 0 \quad 0$

$-8\,(25 \quad 1 \quad 0 \quad \frac{1}{2} \quad 0 \quad 0 \quad \frac{1}{2})$

$(-) \quad 200 \quad 8 \quad 0 \quad 4 \quad 0 \quad 0 \quad 4$

$50 \quad 0 \quad 2 \quad -1 \quad 1 \quad 0 \quad -4$

new $s_2 = 150 \quad 4 \quad 3 \quad 0 \quad 0 \quad 1 \quad 0$

$-4\,(25 \quad 1 \quad 0 \quad \frac{1}{2} \quad 0 \quad 0 \quad \frac{1}{2})$

$(-) \quad 100 \quad 4 \quad 0 \quad 2 \quad 0 \quad 0 \quad 2$

$50 \quad 0 \quad 3 \quad -2 \quad 0 \quad 1 \quad -2$

new $z_j - c_j = -20 \quad -6 \quad -8 \quad 0 \quad 0 \quad 0$

$-(-20)\,(1 \quad 0 \quad \frac{1}{2} \quad 0 \quad 0 \quad \frac{1}{2})$

$(-) \quad -20 \quad 0 \quad -10 \quad 0 \quad 0 \quad -10$

$0 \quad -6 \quad 2 \quad 0 \quad 0 \quad +10$

Table 3:

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	θ	
0	s_1	$\frac{50}{3}$	0	0	$\frac{1}{3}$	1	$\frac{-2}{3}$	$\frac{-8}{3}$	$\frac{50}{3}/\frac{1}{3} = 50$	←
6	x_2	$\frac{50}{3}$	0	1	$\frac{-2}{3}$	0	$\frac{1}{3}$	$\frac{-2}{3}$	—	
20	x_1	25	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$25/\frac{1}{2} = 50$	←
$z_j - c_j$			0	0	-2 ↑	0	2	6		

new s_1 = 50 0 2 -1 1 0 -4

$-2\left(\frac{50}{3} \quad 0 \quad 1 \quad \frac{-2}{3} \quad 0 \quad \frac{1}{3} \quad \frac{-2}{3}\right)$

$(-) \quad \frac{100}{3} \quad 0 \quad 2 \quad \frac{-4}{3} \quad 0 \quad \frac{2}{3} \quad \frac{-4}{3}$

$\frac{50}{3} \quad 0 \quad 0 \quad \frac{1}{3} \quad 1 \quad \frac{-2}{3} \quad \frac{-8}{3}$

new x_1 = 25 1 0 $\frac{1}{2}$ 0 0 $\frac{1}{2}$

$-0\left(\frac{50}{3} \quad 0 \quad 1 \quad \frac{-2}{3} \quad 0 \quad \frac{1}{3} \quad \frac{-2}{3}\right)$

$25 \quad 1 \quad 0 \quad \frac{1}{2} \quad 0 \quad 0 \quad \frac{1}{2}$

new $z_j - c_j$ = 0 -6 2 0 0 10

$-(-6)\left(0 \quad 1 \quad \frac{-2}{3} \quad 0 \quad \frac{1}{3} \quad \frac{-2}{3}\right)$

$(-) \quad 0 \quad -6 \quad 4 \quad 0 \quad -2 \quad 4$

$0 \quad 0 \quad -2 \quad 0 \quad 2 \quad 6$

Table 4: X

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	θ	
8	x_3	50	0	0	1	3	-2	-8	—	
6	x_2	50	0	1	0	2	-1	-6	—	
20	x_1	0	1	0	0	$\frac{-3}{2}$	1	$9/2$	$0/\frac{9}{2} = 0$	← X
$z_j - c_j$			0	0	0	6	-2	-10 ↑		

new x_2 = $\frac{50}{3}$ 0 1 $\frac{-2}{3}$ 0 $\frac{1}{3}$ $-\frac{2}{3}$

$-\left(-\frac{2}{3}\right)(50 \quad 0 \quad 0 \quad 1 \quad 3 \quad -2 \quad -8)$

$(-) \quad \frac{-100}{3} \quad 0 \quad 0 \quad \frac{-2}{3} \quad -2 \quad \frac{4}{3} \quad \frac{16}{3}$

$50 \quad 0 \quad 1 \quad 0 \quad 2 \quad -1 \quad -6$

new x_1 = 25 1 0 $\frac{1}{2}$ 0 0 $\frac{1}{2}$

$-\frac{1}{2}(50 \quad 0 \quad 0 \quad 1 \quad 3 \quad -2 \quad -8)$

$(-) \quad 25 \quad 0 \quad 0 \quad 1/2 \quad 3/2 \quad -1 \quad -4$

$0 \quad 1 \quad 0 \quad 0 \quad \frac{-3}{2} \quad 1 \quad \frac{9}{2}$

new $z_j - c_j$ = 0 0 -2 0 2 6

$-(-2)(0 \quad 0 \quad 1 \quad 3 \quad -2 \quad -8)$

$(+) \quad 0 \quad 0 \quad 2 \quad 6 \quad -4 \quad -16$

$0 \quad 0 \quad 0 \quad 6 \quad -2 \quad -10$

Table 4:

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	0	$\frac{-2}{3}$	0	0	1	$\frac{-2}{3}$	-3
6	x_2	50	$4/3$	1	0	0	$\frac{1}{3}$	0
8	x_3	50	2	0	1	0	0	1
$z_j - c_j$			4	0	0	0	2	8

new s_1 = $\frac{50}{3}$ 0 0 $\frac{1}{3}$ 1 $\frac{-2}{3}$ $-\frac{8}{3}$

$-\frac{1}{3}(50 \quad 2 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1)$

$- \quad \frac{50}{3} \quad \frac{2}{3} \quad 0 \quad \frac{1}{3} \quad 0 \quad 0 \quad \frac{1}{3}$

$0 \quad \frac{-2}{3} \quad 0 \quad 0 \quad 1 \quad \frac{-2}{3} \quad -3$

new x_2 = $\frac{50}{3}$ 0 1 $\frac{-2}{3}$ 0 $\frac{1}{3}$ $-\frac{2}{3}$

$-\left(\frac{-2}{3}\right)(50 \quad 2 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1)$

$(-) \quad \frac{-100}{3} \quad 4/3 \quad 0 \quad \frac{-2}{3} \quad 0 \quad 0 \quad -\frac{2}{3}$

$50 \quad \frac{4}{3} \quad 1 \quad 0 \quad 0 \quad \frac{1}{3} \quad 0$

new $z_j - c_j$ = 0 0 -2 0 2 6

$-(-2)(2 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1)$

$(-) \quad -4 \quad 0 \quad -2 \quad 0 \quad 0 \quad -2$

$4 \quad 0 \quad 0 \quad 0 \quad 2 \quad 8$

$\because$ All $z_j - c_j \geq 0$

$\therefore$ The solution is optimal

when $x_1 = 0$,

$x_2 = 50$,

$x_3 = 50$

$Z_{max} = 20(0) + 6(50) + 8(50)$

$= 0 + 300 + 400$

$Z_{max} = 700$

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TWO PHASE METHOD

The two phase method is used to solve LPP where the artificial variables are involved.

Phase 1:

In this phase the simplex method is applied to a specially constructed axillary LPP lead to a final simplex table containing a basic feasible solution to the original problem.

STEP-1: Assign a cos(-1) to each artificial variables & cos(0) to all other variables in the objective function. Thus the new objective function is

$$\begin{cases} \text{Max/Min } z = \\ \quad \text{s.t} \\ \quad -S_1 \le \\ \quad +R_2 = \\ \quad -S_2 + R_2 \ge \\ \text{and} \\ \quad \ge 0 \end{cases}$$

$\underbrace{R_1, R_2}_{\text{Artificial variable}}$

Phase 1:

$$\boxed{\text{Max } z^* = -R_1 - R_2 \dots - R_n}$$

s.t

⋮

STEP-2: construct the axillary LPP in which the new objective function z^* is to be maximized subject to the given set of constraints

STEP-3: Solve the axillary LPP by simplex method until either of the following 3 possibilities arise:

(i) Max $z^* < 0$ and atleast one artificial variable appears in the optimum basis at the non zero level then the given LPP doesnot posses any feasible solution.

(ii) Max $z^* = 0$ and atleast one artificial variable appears in the optimum basis at zero level then proceed phase 2.

(iii) Max $z^* = 0$ and no artificial variable appears in the optimum basis then proceed phase 2.

Phase 2:

use the optimum basic feasible solution of phase 1 as the starting solution for the original LPP.

STEP-1: Assign the actual cost to the variable in the objective function & zero cost to every artificial variable that appears the basis at zero level

STEP-2: use simplex method to the modified simplex table obtained at the end of phase 1 till an optimum basic solution is obtained.

NOTE 1: In phase 1 the iterations are stopped as soon as the value of new objective function becomes zero.

NOTE 2: The new objective function is always maximization type. Regardless whether the original problem is of maximization / Minimization.

NOTE 3: Before starting phase 2 remove all artificial variables from the table which where non basic at the end of phase 1.

Q 1: Use two phase simplex method to solve: Max $Z = 5x_1 + 8x_2$

s.t

$3x_1 + 2x_2 \geq 3$

$x_1 + 4x_2 \geq 4$

$x_1 + x_2 \leq 5$

and $x_1, x_2 \geq 0$

Soln:

The standard form of given LPP is

Max $Z = 5x_1 + 8x_2 + 0S_1 + 0S_2 + 0S_3 + 0R_1 + 0R_2$

s.t

$3x_1 + 2x_2 - S_1 + R_1 = 3$

$x_1 + 4x_2 - S_2 + R_2 = 4$

$x_1 + x_2 + S_3 = 5$

and $x_1, x_2, S_1, S_2, S_3, R_1, R_2 \geq 0$

PHASE 1:

The new axillary LPP is

Max $Z^* = -R_1 - R_2$

s.t

$3x_1 + 2x_2 - S_1 + R_1 = 3$

$x_1 + 4x_2 - S_2 + R_2 = 4$

$x_1 + x_2 + S_3 = 5$

and $x_1, x_2, S_1, S_2, S_3, R_1, R_2 \geq 0$

Table 1:

		c_j	0	0	0	0	0	-1	-1	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3	R_1	R_2	θ
-1	R_1	3	3	2	-1	0	0	1	0	$3/2 = 1.5$
-1	R_2	4	1	4	0	-1	0	0	1	$4/4 = 1$ ←
0	S_3	5	1	1	0	0	1	0	0	$5/1 = 5$
$z_j^* - c_j$			-4	-6	1	1	0	0	0	
			$-3-1$	$-2-4$ ↑	$1+0$	$0+1$	0	$(-1-0)-(-1)=0$	$(0-1)-(-1)=$	

Table 2:

C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3	R_1	R_2	θ
-1	R_1	1	$\frac{5}{2}$	0	-1	$\frac{1}{2}$	0	1	$\frac{-1}{2}$	$2/5 = 0.4$ ←
0	x_2	1	$\frac{1}{4}$	1	0	$\frac{-1}{4}$	0	0	$\frac{1}{4}$	4
0	S_3	4	$\frac{3}{4}$	0	0	$\frac{1}{4}$	1	0	$\frac{-1}{4}$	$\frac{16}{3} = 5.3$
$z_j^* - c_j$			$-5/2$ ↑	0	1	$-1/2$	0	0	$3/2$	

new $R_1 \rightarrow$ 3   3   2   -1   0   0   1   0

$-2($ 1 $\frac{1}{4}$ 1   0 $\frac{-1}{4}$ 0   0 $\frac{1}{4}$

$-$ 2 $\frac{1}{2}$ 2   0 $-\frac{1}{2}$ 0   0 $\frac{1}{2}$

1   $\frac{5}{2}$ 0   -1 $\frac{1}{2}$ 0   1 $\frac{-1}{2}$

new $S_3 \rightarrow$ 5   1   1   0   0   1   0   0

$-1($ 1 $\frac{1}{4}$ 1   0 $\frac{-1}{4}$ 0   0 $\frac{1}{4})$

4   $\frac{3}{4}$ 0   0 $\frac{1}{4}$ 1   0 $\frac{-1}{4}$

new $z_j^* - c_j \rightarrow$ -4   -6   1   1   0   0   0

$-(-6)($ $\frac{1}{4}$ 1   0 $\frac{-1}{4}$ 0   0 $\frac{1}{4})$

(+) $3/2$ 6   0 $-3/2$ 0   0 $3/2$

$-5/2$ 0   1 $\frac{-1}{2}$ 0   0 $\frac{3}{2}$

Table 3:

C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3	R_1	R_2
0	x_1	$\frac{2}{5}$	1	0	$\frac{-2}{5}$	$\frac{1}{5}$	0	$\frac{2}{5}$	$\frac{-1}{5}$
0	x_2	$\frac{9}{10}$	0	1	$\frac{1}{10}$	$\frac{-3}{10}$	0	$\frac{-1}{10}$	$\frac{3}{10}$
0	S_3	$\frac{31}{10}$	0	0	$\frac{3}{10}$	$\frac{1}{10}$	1	$\frac{-3}{10}$	$\frac{-1}{10}$
$z_j^* - c_j$			0	0	0	0	0	1	1

→ Basic feasible Solution for original LPP

new $x_2 \rightarrow$ 1 $\frac{1}{4}$ 1   0 $\frac{-1}{4}$ 0   0 $\frac{1}{4}$

$-\frac{1}{4}($ $\frac{2}{5}$ 1   0 $\frac{-2}{5}$ $\frac{1}{5}$ 0 $\frac{2}{5}$ $\frac{-1}{5})$

(−) $\frac{1}{10}$ $\frac{1}{4}$ 0 $\frac{-1}{10}$ $\frac{1}{20}$ 0 $\frac{1}{10}$ $\frac{-1}{20}$

$\frac{9}{10}$ 0   1 $\frac{1}{10}$ $\frac{-3}{10}$ 0 $\frac{-1}{10}$ $\frac{3}{10}$

new $S_3 \to 4 \quad \frac{3}{4} \quad 0 \quad 0 \quad \frac{1}{4} \quad 1 \quad 0 \quad \frac{-1}{4}$

$-\frac{3}{4}\left(\frac{2}{5} \quad 1 \quad 0 \quad \frac{-2}{5} \quad \frac{1}{5} \quad 0 \quad \frac{2}{5} \quad \frac{-1}{5}\right)$

$(-) \quad \frac{3}{10} \quad \frac{3}{4} \quad 0 \quad \frac{-3}{10} \quad \frac{3}{20} \quad \frac{3}{10} \quad \frac{-3}{20}$

$\frac{37}{10} \quad 0 \quad 0 \quad \frac{3}{10} \quad \frac{1}{10} \quad 1 \quad \frac{-3}{10} \quad \frac{-1}{10}$

new $z_j^* - c_j \to \frac{-5}{2} \quad 0 \quad 1 \quad \frac{-1}{2} \quad 0 \quad 0 \quad \frac{3}{2}$

$-\left(-\frac{5}{2}\right)\left(1 \quad 0 \quad \frac{-2}{5} \quad \frac{1}{5} \quad 0 \quad \frac{2}{5} \quad \frac{-1}{5}\right)$

$(+) \quad \frac{5}{2} \quad 0 \quad -1 \quad \frac{1}{2} \quad 0 \quad 1 \quad \frac{-1}{2}$

$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1$

All $z_j - c_j \geq 0$

PHASE 2:

Table 1:

$c_j \quad 5 \quad 8 \quad 0 \quad 0 \quad 0$

C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3	θ
5	x_1	$\frac{2}{5}$	1	0	$\frac{-2}{5}$	$\frac{1}{5}$	0	$\frac{2/5}{1/5} = 2$ ←
8	x_2	$\frac{9}{10}$	0	1	$\frac{1}{10}$	$\frac{-3}{10}$	0	$\frac{9/10}{-3/10} = -$
0	S_3	$\frac{37}{10}$	0	0	$\frac{3}{10}$	$\frac{1}{10}$	1	$\frac{37/10}{1/10} = 37$
$z_j - c_j$			0	0	$\frac{-6}{5}$	$\frac{-7}{5}$	0	

$(5+0)-5 \qquad \frac{-10+4}{5}$ ↑

Table 2:

C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3	θ
0	S_2	2	5	0	-2	1	0	–
8	x_2	$3/2$	$3/2$	1	$-1/2$	0	0	–
0	S_3	$1/2$	$-1/2$	0	$1/2$	0	1	$\frac{1/2}{1/2} = 7$ ←
$z_j - c_j$			12	8	-4	0	0	

new $x_2 \to \frac{9}{10} \quad 0 \quad 1 \quad \frac{1}{10} \quad \frac{-3}{10} \quad 0$

$-\left(-\frac{3}{10}\right)\left(2 \quad 5 \quad 0 \quad -2 \quad 1 \quad 0\right)$

$\frac{-6}{10} \quad \frac{-3}{2} \quad 0 \quad \frac{6}{10} \quad \frac{-3}{10} \quad 0$

$\frac{3}{2} \quad \frac{3}{2} \quad 1 \quad \frac{-1}{2} \quad 0 \quad 0$

new $S_3 \to \frac{37}{10} \quad 0 \quad 0 \quad \frac{3}{10} \quad \frac{1}{10} \quad 1$

$-\frac{1}{10}\left(2 \quad 5 \quad 0 \quad -2 \quad 1 \quad 0\right)$

$(-) \quad \frac{2}{10} \quad \frac{5}{10} \quad 0 \quad \frac{-2}{10} \quad \frac{1}{10} \quad 0$

$\frac{7}{2} \quad \frac{-1}{2} \quad 0 \quad \frac{1}{2} \quad 0 \quad 1$

~~new $z_j - c_j \to 0 \quad 0 \quad -\frac{6}{5} \quad -\frac{7}{5} \quad 0$~~

~~$-\left(\frac{-7}{5}\right)\left(5 \quad 0 \quad -2 \quad 1 \quad 0\right)$~~

~~$(-) \quad 7$~~

Table 3:

C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3
0	S_2	16	3	0	0	1	4
8	x_2	5	1	1	0	0	1
0	S_1	7	-1	0	1	0	2
$z_j - c_j$			8	8	0	0	8

new $S_2 \to 2 \quad 5 \quad 0 \quad -2 \quad 1 \quad 0$

$-(-2)\left(7 \quad -1 \quad 0 \quad 1 \quad 0 \quad 2\right)$

$(-) \quad -14 \quad 2 \quad 0 \quad -2 \quad 0 \quad -4$

$16 \quad 3 \quad 0 \quad 0 \quad 1 \quad 4$

new $x_2 \to 3/2 \quad 3/2 \quad 1 \quad -1/2 \quad 0 \quad 0$

$-\left(-\frac{1}{2}\right)\left(7 \quad -1 \quad 0 \quad 1 \quad 0 \quad 2\right)$

$(-) \quad \frac{-7}{2} \quad \frac{1}{2} \quad 0 \quad \frac{-1}{2} \quad 0 \quad -1$

$5 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1$

$12 \quad 8 \quad -4 \quad 0 \quad 0$

$-(-4)\left(-1 \quad 0 \quad 1 \quad 0 \quad 2\right)$

$8 \quad 8 \quad 0 \quad 0 \quad 8$

$Z_{max} = 5(0) + 8(5) = 40$

when $x_1 = 0$, $x_2 = 5$

H/W: Max $Z = 2x_1 + x_2 + \frac{1}{4}x_3$

s.t

$4x_1 + 6x_2 + 3x_3 \leq 8$

$3x_1 - 6x_2 - 4x_3 \leq 1$

$2x_1 + 3x_2 - 5x_3 \geq 4$

and $x_1, x_2, x_3 \geq 0$

soln:

The standard form of given LPP is

Max $Z = 2x_1 + x_2 + \frac{1}{4}x_3 + 0S_1 + 0S_2 + 0S_3 + 0R_1$

s.t

$4x_1 + 6x_2 + 3x_3 + S_1 = 8$

$3x_1 - 6x_2 - 4x_3 + S_2 = 1$

$2x_1 + 3x_2 - 5x_3 - S_3 + R_1 = 4$

and

$x_1, x_2, x_3, S_1, S_2, S_3, R_1 \geq 0$

PHASE 1:

The new axillary LPP is

$$\max z^* = -R_1$$

s.t

⋮

Table 1:

C_j			0	0	0	0	0	0	-1	
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	R_1	θ
-1	R_1	4	2	3	-5	0	0	-1	1	$\frac{4}{3} = 1.3$ ←
0	S_1	8	4	6	3	1	0	0	0	$\frac{8}{6} = 1.3$
0	S_2	1	3	-6	-4	0	1	0	0	—
$Z_j^* - C_j$			-2	-3 ↑	5	0	0	1	-1	

Table 2:

C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	R_1	θ
0	x_2	$\frac{4}{3}$	$\frac{2}{3}$	1	$\frac{-5}{3}$	0	0	$\frac{-1}{3}$	$\frac{1}{3}$	
0	S_1	0	0	0	13	1	0	2	-2	
0	S_2	9	7	0	-14	0	1	-2	2	
$Z_j^* - C_j$			0	0	0	0	0	0	0	

new $S_1 \rightarrow$ 8 4 6 3 1 0 0 0

$-6\left(\frac{4}{3} \quad \frac{2}{3} \quad 1 \quad \frac{-5}{3} \quad 0 \quad 0 \quad \frac{-1}{3} \quad \frac{1}{3}\right)$

− 8 4 6 −10 0 0 −2 2

0 0 0 13 1 0 2 −2

new $S_2 \rightarrow$ 1 3 −6 −4 0 1 0 0

$-(-6)\left(\frac{4}{3} \quad \frac{2}{3} \quad 1 \quad \frac{-5}{3} \quad 0 \quad 0 \quad \frac{-1}{3} \quad \frac{1}{3}\right)$

(−) −8 −4 −6 10 0 0 2 −2

9 7 0 −14 0 1 −2 2

new $Z_j^* - C_j \rightarrow$ −2 −3 5 0 0 1 −1

$-(-3)\left(\frac{2}{3} \quad 1 \quad \frac{-5}{3} \quad 0 \quad 0 \quad \frac{-1}{3} \quad \frac{1}{3}\right)$

(−) −2 −3 5 0 0 1 −1

0 0 0 0 0 0 0

PHASE 2:

Table 1:

C_j			2	1	$\frac{1}{4}$	0	0	0	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	θ
1	x_2	$\frac{4}{3}$	$\frac{2}{3}$	1	$\frac{-5}{3}$	0	0	$\frac{-1}{3}$	—
0	s_1	0	0	0	13	1	0	2	0 ←
0	s_2	9	7	0	-14	0	1	-2	—
$Z_j - C_j$			$\frac{-2}{3}$ (−0.6)	0	$\frac{-23}{12}$ (−1.9) ↑	0	0	$\frac{-1}{3}$ (−0.3)	

Table 2:

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	
1	x_2	$4/3$	$2/3$	1	0	$5/39$	0	$1/13$	
$\frac{1}{4}$	x_3	0	0	0	1	$1/13$	0	$2/13$	
0	s_2	9	7	0	0	$\frac{14}{13}$	1	$\frac{2}{13}$	
$Z_j - C_j$			$2/3$	1					

new $x_2 \rightarrow$ $\frac{4}{3}$ $\frac{2}{3}$ 1 $-\frac{5}{3}$ 0 0 $\frac{-1}{3}$

$-\left(\frac{-5}{3}\right)\left(0 \quad 0 \quad 0 \quad 1 \quad \frac{1}{13} \quad 0 \quad \frac{2}{13}\right)$

0 0 0 $\frac{-5}{3}$ $\frac{-5}{39}$ 0 $-10/39$

$\frac{4}{3}$ $\frac{2}{3}$ 1 0 $\frac{5}{39}$ 0 $\cancel{\frac{11}{39}}$ $\frac{1}{13}$

new $s_2 \rightarrow$ 9 7 0 −14 0 1 −2

$-(-14)\left(0 \quad 0 \quad 0 \quad 1 \quad \frac{1}{13} \quad 0 \quad \frac{2}{13}\right)$

0 0 0 −14 $-14/13$ 0 $-28/13$

9 7 0 0 $\frac{14}{13}$ 1 $\frac{2}{13}$

$\text{Max } Z = 2(0) + \frac{4}{3} + \frac{1}{4}(0)$

$= \frac{4}{3}$

when $x_1 = 0$, $x_2 = \frac{4}{3}$, $x_3 = 0$

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Q: using simplex algorithm

$\text{Min } z = -2x_1 - x_2$

s.t

$x_1 + x_2 \geq 2$

$x_1 + x_2 \leq 4$

and $x_1, x_2 \geq 0$

soln:

the standard form is:

$\text{Max } z' = 2x_1 + x_2 \quad (-\text{Min } z)$

s.t

$x_1 + x_2 - S_1 + R_1 = 2$

$x_1 + x_2 + S_2 = 4$

and $x_1, x_2, S_1, S_2, R_1 \geq 0$

PHASE 1:

the auxillary LPP is

$\text{Max } z^* = -R_1$

s.t

⋮

Table 1:

		c_j	0	0	0	0	-1	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	R_1	θ
-1	R_1	2	1	1	-1	0	1	$2/1 = 2$ ←
0	S_2	4	1	1	0	1	0	$4/1 = 4$
$z_j^* - c_j$			-1 ↑	-1	1	0	0	

Table 2:

C_B	Y_B	X_B	x_1	x_2	S_1	S_2	R_1
0	x_1	2	1	1	-1	0	1
0	S_2	2	0	0	1	1	-1
$z_j^* - c_j$			0	0	0	0	1

new $S_2 \rightarrow$ 4 1 1 0 1 0

−1 (2 1 1 −1 0 1)

2 0 0 1 1 −1

new $z_j^* - c_j \rightarrow$ −1 −1 1 0 0

−(−1) (1 1 −1 0 1)

(−) −1 −1 1 0 −1

0 0 0 0 1

Since all $z_j^* - c_j \geq 0$

PHASE 2:

Table 1:

		c_j	2	1	0	0	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	Min ratio (θ)
2	x_1	2	1	+1	-1	0	−
0	S_2	2	0	0	1	1	$2/1 = 2$ ←
$z_j' - c_j$			0	0	-2 ↑	0	

Table 2:

		c_j	2	1	0	0
C_B	Y_B	X_B	x_1	x_2	S_1	S_2
2	x_1	4	1	1	0	1
0	S_1	2	0	0	1	1
$z_j' - c_j$			0	1	0	2

new $x_1 \rightarrow$ 2 1 1 −1 0

−(−1)(2 0 0 1 1)

(−) −2 0 0 −1 −1

4 1 1 0 1

$\because$ All $z_j' - c_j \geq 0$

$\therefore$ This is optimal solution

$\text{Max } z' = 2(4) + 0 = 8$

when $x_1 = 4, x_2 = 0$

$\text{Min } z = -\text{Max } z'$

$= -8$

when $x_1 = 4, x_2 = 0$

TRANSPORTATION

m - sources, n - destinations

	D_1	D_2	D_3	....	D_n	(Availability) Supply
S_1	x_{11} c_{11}	x_{12} c_{12}	x_{13} c_{13}	...	x_{1n} c_{1n}	b_1
S_2	x_{12} c_{21}	x_{22} c_{22}	x_{23} c_{23}	...	x_{2n} c_{2n}	b_2
S_3				...		:
:				...		:
S_m	x_{m1} c_{m1}	x_{m2} c_{m2}	x_{m3} c_{m3}	...	x_{mn} c_{mn}	b_m
Demand (Requirements)	a_1	a_2	a_3	...	a_n	

* Total transportation cost → Min

x_{11} - No. of units transported
c_{11} - cost for the unit
→ product gives total cost

* sum of supply = sum of demand

Minimize $Z = c_{11}x_{11} + c_{12}x_{12} + \dots + c_{mn}x_{mn}$

$$= \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij}\, x_{ij} \quad \begin{cases} i = 1 \text{ to } m \\ j = 1 \text{ to } n \end{cases}$$

s.t

$$\begin{cases} x_{11} + x_{12} + \dots + x_{1n} = b_1 \\ x_{11} + x_{21} + \dots + x_{m1} = a_1 \end{cases}$$

m+n constraints
$$\begin{cases} \sum_{j=1}^{n} x_{ij} = b_i \quad , i = 1, 2, 3, \dots, m \\ \sum_{i=1}^{m} x_{ij} = a_j \quad , j = 1, 2, 3, \dots, n \end{cases}$$

and $\underbrace{x_{ij} \geq 0}_{m \times n \text{ constraints}}$, $i = 1, 2, \dots, m$
$j = 1, 2, \dots n$

DEFINITION 1:

A set of non negative value x_{ij} where i varies 1 to m & j varies 1 to n that satisfies the constraints is called feasible soln to the transportation problem (TP)

NOTE

A balanced transportation problem will always have feasible soln. $(a_j = b_i)$

DEFINITION 2

A feasible soln to a m×n TP that contains no more than m+n-1 non negative allocations is called basic feasible solution to the transportation problem.

DEFINITION 3:

A basic feasible soln to a m×n TP is said to be non degenerate if it contains exactly m+n-1 non negative allocations.

DEFINITION 4:

A basic feasible soln that contains less than m+n-1 non negative allocations is said to be degenerate

DEFINITION 5:

A basic feasible soln is said to be optimal soln if it minimizes the total transportation cost.

Methods to find basic feasible soln (BFS)

1. North west corner rule (NWC)
2. Least cost Method (LCM)
3. Vogel's Approximation Method (VAM)

(BFS - satisfies constraints)

	D1	D2	D3	Availability
S1	NW			
S2				
S3				
Requirements				

→ Assign least to cells.

- NWC – North west allocate
- LCM – which cell has least cost
  ↳ Allocate that cell
- VAM – Row diff / column difference
  ↳ Diff b/w Least nos.
  ↓
  choose least column cell for allotment – from max row/col diff

Q ① Find the basic feasible soln for the following TP:

	A	B	C	D	E	Su
P	2	11	10	3	7	4
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
De	3	3	4	5	6	

- First check – balanced / not
  if not balanced add dummy rows/cols

soln $b_i = 4+8+9 = 21$

$a_j = 3+3+4+5+6 = 21$

Here $\Sigma a_j = \Sigma b_i$

∴ The given TP is balanced

(i) NWC

	A	B	C	D	E	
P	$\boxed{3}$ 2	$\boxed{1}$ 11	10	3	7	~~4~~ ~~1~~
Q	1	$\boxed{2}$ 4	$\boxed{4}$ 7	$\boxed{2}$ 2	1	8 ~~6~~ ~~2~~
R	3	9	4	$\boxed{3}$ 8	$\boxed{6}$ 12	9 ~~6~~
	~~3~~	~~3~~ 2	~~4~~	~~5~~ 3	~~6~~	

IBFS:

The total Transportation cost

$= 3\times2 + 1\times11 + 2\times4 + 4\times7 + 2\times2 + 3\times8 + 6\times12$

$= 153$

(ii) LCM

	2	11	10	$\boxed{4}$ 3	7	4
	$\boxed{3}$ 1	4	7	2	$\boxed{5}$ 1	8 ~~5~~
	3	$\boxed{3}$ 9	$\boxed{4}$ 4	$\boxed{1}$ 8	$\boxed{1}$ 12	9 ~~5~~ ~~4~~ X
	~~3~~	~~3~~	~~4~~	~~5~~ X	~~6~~ X	

Total TP cost $= 4\times3 + 3\times1 + 5\times1 + 3\times9 + 4\times4 + 1\times8 + 1\times12$

$= 83$

(iii) VAM

							Row diff
	2	11	10	$\boxed{4}$ 3	7	4	1 1 1 1 8←
	1	$\boxed{2}$ 4	7	2	$\boxed{6}$ 1	~~8~~ 2	0
	$\boxed{3}$ 3	$\boxed{1}$ 9	$\boxed{4}$ 4	$\boxed{1}$ 8	12	9 ~~6~~ 2 X	1 1 1 5← 1
	~~3~~	~~3~~	~~4~~	~~5~~ X	~~6~~		

col diff

1	5	3	1	6 ↑
1	5 ↑	3	1	
1	2	6 ↑	5	
1	2		5	
	2		5	

Total TP cost $= 4\times3 + 2\times4 + 6\times1 + 3\times3 + 1\times9 + 4\times4 + 1\times8$

$= 68$

⊛ VAM gives better basic feasible solution

② HW Find the BFS for following TP

	D_1	D_2	D_3	D_4	
S_1	1	2	1	4	30
S_2	3	3	2	1	50
S_3	4	2	5	9	20
	20	40	30	10	

$\Sigma a_j = 100$, $\Sigma b_i = 100$

∵ $\Sigma a_j = \Sigma b_i$

∴ The given TP is balanced

(i) NWC

[20] 1	[10] ~~2~~	1	4	~~30~~ ~~10~~
3	[30] 3	[20] ~~2~~	1	~~50~~ ~~20~~
4	2	[10] 5	[10] 9	~~20~~ ~~10~~
~~20~~	~~40~~ ~~30~~	~~30~~ ~~10~~	~~10~~	

Total TP cost = 20×1 + 10×2 + 30×3 + 20×2 + 10×5 + 10×9

= 310

(ii) LCM

[20] 1	2	[10] 1	4	~~30~~ ~~10~~
3	[20] 3	[20] 2	[10] 1	~~50~~ ~~40~~ ~~20~~
4	[20] 2	5	9	~~20~~
~~20~~	~~40~~ ~~20~~	~~30~~ ~~20~~	~~10~~	

Total TP cost = 20×1 + 10×1 + 20×3 + 20×2 + 10×1 + 20×2

= 180

(iii) VAM

[20] 1	2	[10] 1	4	~~30~~ ~~10~~	0	0	1	1 ←
3	[20] 3	[20] 2	[10] 1	~~50~~ ~~40~~ ~~20~~	1	1	1	1
4	[20] 2	5	9	~~20~~	2	2	3 ←	
~~20~~	~~40~~ ~~20~~	~~30~~ ~~20~~	~~10~~					
2	0	1	3 ↑					
2 ↑	0	1						
	0	1						
	1	1						

total TP cost = 20×1 + 10×1 + 20×3 + 20×2 + 10×1 + 20×2

= 180

02/12/2023

Q1: Solve the TP:
(Find the optimal solution)

	D_1	D_2	D_3	D_4	
S_1	21	16	25	13	11
S_2	17	18	14	23	13
S_3	32	27	18	41	19
	6	10	12	15	

$\because \sum a_j = \sum b_i$

$\therefore$ the given TP is balanced

21	16	25	[11] 13	~~11~~	3				
[6] 17	[3] 18	14	[4] 23	~~13~~ ~~9~~ 3	3	3	3	4	
32	[7] 27	[12] 18	41	~~19~~ 7	9	9	9	9	
~~6~~	~~10~~ 7	12	~~15~~ 4						
4	2	4	10 ↑						
15	9	4	18 ↑						
15 ↑	9	4							
	9 ↑	4							

$\therefore$ By using VAM, IBFS is

TP cost = (11×13) + (6×17) + (3×18) + (4×23) + (7×27) + (12×18)

= ₹796

If non-degenerate – Allocate epsilon in non allocated cell

$\because$ No. of allocation = 6 (m - row, n - col)

m + n − 1 = 3 + 4 − 1 = 6

$\because$ No. of allocation = m+n−1

$\therefore$ The soln is non degenerate

{ If no. of allocation < m+n−1 soln is degenerate }

OPTIMALITY TEST (MODI METHOD)

(14) 21	(8) 16	(26) 25	[11] 13	$u_1 = -10$
[6] 17	[3] 18	(5) 14	[4] 23	$u_2 = 0$
(6) 32	[7] 27	[12] 18	(9) 41	$u_3 = 9$
$v_1 = 17$	$v_2 = 18$	$v_3 = 9$	$v_4 = 23$	

most allocated row

* For allocated cells,

$$\boxed{c_{ij} = u_i + v_j}$$

Here, $u_2 = 0$

$c_{14} = u_1 + v_4 \Rightarrow u_1 = c_{14} - v_4 = -10$

$c_{21} = u_2 + v_1 \Rightarrow v_1 = 17 - 0 = 17$

$c_{22} = u_2 + v_2 \Rightarrow v_2 = 18 - 0 = 18$

$c_{24} = u_2 + v_4 \Rightarrow v_4 = 23 - 0 = 23$

$c_{32} = u_3 + v_2 \Rightarrow u_3 = 27 - 18 = 9$

$c_{33} = u_3 + v_3 \Rightarrow v_3 = 18 - 9 = 9$

* For non allocated cells,

$$\boxed{\Delta_{ij} = d_{ij} = c_{ij} - (u_i + v_j)}$$

- If all $d_{ij} > 0 \rightarrow$ optimal soln
  $d_{ij} < 0 \rightarrow$ Not optimal
- Max no. of allocated rows/cols $\Rightarrow u$ or $v = 0$ respectively

Here,

$\Delta_{11} = c_{11} - (u_1 + v_1)$

- $\Delta_{11} = 21 - (-10 + 17) = 14$
- $\Delta_{12} = 16 - (-10 + 18) = 8$
- $\Delta_{13} = 25 - (-10 + 9) = 26$
- $\Delta_{23} = 14 - (0 + 9) = 5$
- $\Delta_{31} = 32 - (9 + 17) = 6$
- $\Delta_{34} = 41 - (9 + 23) = 9$

$\because$ All $d_{ij} > 0$

$\therefore$ This soln is optimal & unique

$\therefore$ the total TP cost = ₹ 796

2. Obtain the optimal feasible soln.

Solve the TP:

	P	Q	R	
A	7	3	2	2
B	2	1	3	3
C	3	4	6	5
	4	1	5	

$\because \Sigma a_j = \Sigma b_i$

$\therefore$ The given TP is balanced

7	3	[2] 2	~~2~~	1	5 ←	
2	[1] 1	[2] 3	~~3~~ 2	1	1	1
[4] 3	4	[1] 6	~~5~~ 1	1	3	3 ←
~~4~~	~~1~~	~~5~~ 3				
1	2 ↑	1				
1		1				
1		3				

By using VAM, The IBFS is

Total TP cost $= 2 \times 2 + 1 \times 1 + 2 \times 3 + 4 \times 3 + 1 \times 6$

$= 29$

$\because$ No. of allocation $= 5$

$m + n - 1 = 3 + 3 - 1 = 5$

$\therefore$ The soln is non-degenerate

optimality test (MODI method)

(8) 7	(3) 3	[2] 2	$u_1 = 2$
(2) 2	[1] 1	[2] 3	$u_2 = 3$
[4] 3	(0) 4	[1] 6	$u_3 = 6$
$v_1 = -3$	$v_2 = -2$	$v_3 = 0$	

For allocated cells, $c_{ij} = u_i + v_j$

For nonallocated cells, $d_{ij} = c_{ij} - (u_i + v_j)$

$\because$ All $d_{ij} \geq 0$

$\therefore$ The soln is optimal but not unique

$\therefore$ total TP cost = ₹ 29

3. Solve the TP:

	W_1	W_2	W_3	W_4	W_5	
F_1	4	1	2	6	9	100
F_2	6	4	3	5	7	120
F_3	5	2	6	4	8	120
	40	50	70	90	90	

$\because \Sigma a_j = \Sigma b_i$

$\therefore$ TP is balanced

$\boxed{30}$ 4	1	$\boxed{70}$ 2	6	9	~~100~~ 30	1	2 ←	2 ←		
~~$\boxed{10}$~~ 6	4	3	$\boxed{90}$ 5	$\boxed{30}$ 7	~~120~~ 30	1	2	1	1 ← ~~2~~	
$\boxed{10}$ 5	$\boxed{50}$ 2	6	~~$\boxed{10}$~~ 4	$\boxed{60}$ 8	~~120~~ ~~70~~ 60	2 ←	1	1	1	~~3~~ 7 ←
~~40~~ ~~10~~	~~50~~	~~70~~	~~90~~	~~90~~						
1	1	1	1	1						
1		1	1	1						
1			1	1						
1			1	1 → same, Least supply allocated						
1			0	1						

By using VAM, IBFS is

TP cost $= 30 \times 4 + 70 \times 2 + 90 \times 5$
$+ 30 \times 7 + 10 \times 5 + 50 \times 2 + \cancel{70 \times 4}$ 60×8
$=$ ~~1430~~ 1550

$\because$ No. of allocations $= 7$

$m + n - 1 = 3 + 5 - 1 = 7$

$\therefore$ soln is non degenerate

Optimality test (MODI method)

$\boxed{30}$ 4	(0) 1	$\boxed{70}$ 2	(1) 6	(2) 9	$u_1 = -1$
(2) 6	(3) 4	(1) 3	$\boxed{90}$ 5	$\boxed{30}$ 7	$u_2 = -1$
$\boxed{10}$ 5	$\boxed{50}$ 2	(3) 6	(-2) 4	$\boxed{60}$ 8	$u_3 = 0$
$v_1 = 5$	$v_2 = 2$	$v_3 = 3$	$v_4 = 6$	$v_5 = 8$	

4. Solve the TP:

10	20	5	7	10
13	9	12	8	20
4	5	7	9	30
14	7	1	0	40
3	12	5	19	50
60	60	20	10	

$\Sigma a_j = \Sigma b_i \Rightarrow$ given TP is balanced

				Supply				
[10] 10	20	5	7	~~10~~	2	5	10	-
13	[20] 9	12	8	~~20~~	1	3	4	4
4	[30] 5	7	9	~~30~~	1	1	1	1
14	[10] 7	[20] 1	[10] 0	~~40~~ ~~30~~ ~~10~~	1	6	7	7
[50] 3	[ε] 12	5	19	~~50~~	2	2	9	9
~~60~~ ~~10~~	~~60~~ ~~30~~ ~~20~~	~~20~~	~~10~~					
1	2	4	7					
1	2	4	-					
1	2	-	-					
1	2	-	-					

By using VAM, the IBFS is

$TP = 10\times10 + 20\times9 + 30\times5 + 10\times7$

$+ 20\times1 + 10\times0 + 50\times3$

$= 670$

$\because$ No. of allocation $= 7$

$m+n-1 = 5+4-1 = 8 \quad (7 < 8)$

$\therefore$ The soln is degenerate

* Dummy allocation, $\epsilon \to 0$
  ↓
  Most no. of allocated rows / cols
  ⇓
  Non allocated - Least value
  (Here c_{52})

optimality test

[10] 10	(1) 20	(-8) ε 5	(-5) 7	$u_1 = 19$
(13) 13	[20] 9	(9) 12	(6) 8	$u_2 = 9$
(8) 4	[30] 5	(8) 7	(11) 9	$u_3 = 5$
(16) 14	[10] 7 (+)	[20] 1 (-)	[10] 10	$u_4 = 7$
[50] 3	[ε] 12 (-)	(-1) 5	(14) 19	$u_5 = 12$
$v_1 = -9$	$v_2 = 0$	$v_3 = -6$	$v_4 = -7$	

Most -ve: -8

(θ =

				u_i
[10] 10	(9) 20	[ε] 5	(3) 7	4
(5) 13	[20] 9	(9) 12	(6) 8	2
(0) 4	[30] 5	(8) 7	(11) 9	-2
(8) 14	[10] 7	[20] 1	[10] 0	0
[50] 3	(8) 12	(7) 5	(22) 19	-3
v_j: 6	7	1	0	

$\because$ All $d_{ij} \geq 0$

ϵ - positive nearest to zero

Total TP cost $= 10\times10 + 5\times\epsilon + 20\times9$

$+ 30\times5 + 10\times7 + 20\times1$

$+ 10\times0 + 50\times3$

$= ₹670$

HW:

4. Solve the following TP:

9	12	9	6	9	10	5
7	3	7	7	5	5	6
6	5	9	11	3	11	2
6	8	11	2	2	10	9
4	4	6	2	4	2	

5. Solve the following TP:

5	6	9	100
3	5	10	75
6	7	6	50
6	4	10	75
70	80	120	

4) soln:

9	12	[5] 9	6	9	10	5	3	3	0	0	0	0
7	[4] 3	7	7	5	[2] 5	~~6~~ ~~2~~	2	2	2	4		
[1] 6	5	[1] 9	11	3	11	~~2~~ 1	2	2	2	1	3	3
[3] 6	8	11	[2] 2	[4] 2	10	~~9~~ ~~7~~ 3	0	0	4	2	5	
~~4~~ 1	~~4~~	~~6~~	~~2~~	~~4~~	~~2~~							
0	2	2	4	1	5							
0	2	2	4	1								
0	2	2		1								
0	2	2										
0		0										
3		0										

13/12/2023

ASSIGNMENT PROBLEMS

- Assign a particular job to person or machine

	M_1	M_2	M_3	. . .		M_n
J_1	c_{11} x_{11}	c_{12} x_{12}	c_{13} x_{13}			c_{1n} x_{1n}
J_2	c_{21} x_{21}	c_{22} x_{22}	c_{23} x_{23}			c_{2n} x_{2n}
J_3						
⋮				c_{ij}		
J_n	c_{n1} x_{n1}	c_{n2} c_{n2}				c_{nn} x_{nn}

* No. of jobs = No. of machine

c_{11} = charge

⇒ c_{ij} : cost for doing i^{th} job by j^{th} machine

x_{11} → Decision variable

No. of jobs alloted

Minimize - Assignment problem.

$$\text{Min } Z = \sum_{j=1}^{n} \sum_{i=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad , i = 1 \text{ to } n$$

$$\sum_{i=1}^{n} x_{ij} = 1. \qquad , j = 1 \text{ to } n$$

and

$$x_{ij} = 0 \text{ (or) } 1$$

Hungarian Method:

- To find optimal solution

Q1) Consider the problem of assigning 5 jobs to 5 machines. The assignment cost are given as follows:

$$\begin{array}{c|ccccc} & M_1 & M_2 & M_3 & M_4 & M_5 \\ \hline J_1 & 8 & 4 & 2 & 6 & 1 \\ J_2 & 0 & 9 & 5 & 5 & 4 \\ J_3 & 3 & 8 & 9 & 2 & 6 \\ J_4 & 4 & 3 & 1 & 0 & 3 \\ J_5 & 9 & 5 & 8 & 9 & 5 \end{array}$$

- For each row - choose minimum cost & subtract from all other entries.
- same for column.

soln:

Step 1: (Row)

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix}$$

Step 2: (column)

$$\begin{array}{c} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \end{array} \begin{pmatrix} 7 & 3 & \cancel{0} & 5 & \boxed{0} \\ \boxed{0} & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & \boxed{0} & 4 \\ 4 & 3 & \boxed{0} & \cancel{0} & 3 \\ 4 & \boxed{0} & 2 & 4 & \cancel{0} \end{pmatrix}$$

If only one zero encircle & delete other rows

$J_1 \to M_5$, $J_2 \to M_1$, $J_3 \to M_4$

$J_4 \to M_3$, $J_5 \to M_2$

∴ cost = 1 + 0 + 2 + 1 + 5

= 9

Q2) 4 different jobs & 4 machines

$$\begin{array}{c|cccc} & M_1 & M_2 & M_3 & M_4 \\ \hline J_1 & 5 & 7 & 11 & 6 \\ J_2 & 8 & 5 & 9 & 6 \\ J_3 & 4 & 7 & 10 & 7 \\ J_4 & 10 & 4 & 8 & 3 \end{array}$$

step-1:

$$\begin{pmatrix} 0 & 2 & 6 & 1 \\ 3 & 0 & 4 & 1 \\ 0 & 3 & 6 & 3 \\ 7 & 1 & 5 & 0 \end{pmatrix}$$

step-2:

$$\begin{pmatrix} \boxed{0} & 2 & 2 & (1) \\ 3 & \boxed{0} & \cancel{0} & 1 \\ \cancel{0} & 3 & 2 & 3 \\ 7 & 1 & 1 & \boxed{0} \end{pmatrix}$$

- Minimum no. of horizontal & vertical line cover zero.
- Choose min cost from uncovered, add it to intersection, subtract it from uncovered.

$$\begin{pmatrix} \cancel{0} & 1 & (1) & \cancel{0} \\ 4 & \boxed{0} & \cancel{0} & 1 \\ \boxed{0} & 2 & 1 & 2 \\ 8 & 1 & 1 & \boxed{0} \end{pmatrix}$$

$$\begin{pmatrix} \boxed{0} & \cancel{0} & 0 & 0 \\ 5 & \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & 1 & \boxed{0} & 2 \\ 8 & \cancel{0} & \cancel{0} & \boxed{0} \end{pmatrix} \rightarrow$$ All zero entries → min cost from main problem

$J_1 \to M_1$, $J_2 \to M_2$, $J_3 \to M_3$, $J_4 \to M_4$

cost $= 5+5+10+3$

$= 23$

Q3)

$$\begin{array}{c|cccc} & M_1 & M_2 & M_3 & M_4 \\ \hline J_1 & 10 & 5 & 13 & 15 \\ J_2 & 3 & 9 & 18 & 3 \\ J_3 & 10 & 7 & 3 & 2 \\ J_4 & 5 & 11 & 9 & 7 \end{array}$$

$$\begin{pmatrix} 5 & 0 & 8 & 10 \\ 0 & 6 & 15 & 0 \\ 8 & 5 & 1 & 0 \\ 0 & 6 & 4 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 5 & \boxed{0} & 7 & 10 \\ \cancel{0} & 6 & 14 & \boxed{0} \\ 8 & 5 & \boxed{0} & \cancel{0} \\ \boxed{0} & 6 & 4 & 2 \end{pmatrix}$$

$J_1 \to M_2$, $J_2 \to M_4$, $J_3 \to M_3$, $J_4 \to M_1$

Assignment cost $= 5+3+3+5$

$= 16$

Q4) Find the maximum assigning cost

$$\begin{array}{c|cccc} & M_1 & M_2 & M_3 & M_4 \\ \hline J_1 & 16 & 10 & 14 & 11 \\ J_2 & 14 & 11 & 15 & 15 \\ J_3 & 15 & 15 & 13 & 12 \\ J_4 & 13 & 12 & 14 & 15 \end{array}$$

Max value subtract from all entries

$$\begin{pmatrix} 0 & 6 & 2 & 5 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 2 & 3 & 1 & 0 \end{pmatrix}$$

step 1:

$$\begin{pmatrix} \boxed{0} & 6 & 2 & 5 \\ 1 & 4 & \boxed{0} & \cancel{0} \\ \cancel{0} & \boxed{0} & 2 & 3 \\ 2 & 3 & 1 & \boxed{0} \end{pmatrix}$$

$J_1 \to M_1$, $J_2 \to M_3$, $J_3 \to M_2$, $J_4 \to M_4$

cost $= 16 + 15 + 15 + 15$

$= 61$

Q5) Find the optimal assignment of jobs to machines.

$$\begin{array}{c} \\ J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \end{array} \begin{array}{c} \begin{array}{ccccc} M_1 & M_2 & M_3 & M_4 & M_5 \end{array} \\ \begin{pmatrix} 10 & 11 & 4 & 2 & 8 \\ 7 & 11 & 10 & 14 & 12 \\ 5 & 6 & 9 & 12 & 14 \\ 13 & 15 & 11 & 10 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \to \text{Add zeroes.}$$

soln

Step-1:

$$\begin{pmatrix} 8 & 9 & 2 & \boxed{0} & 6 \\ \boxed{0} & 4 & 3 & 7 & 5 \\ \cancel{0} & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & \boxed{0} \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

$$\begin{pmatrix} 8 & 8 & 1 & \boxed{0} & 6 \\ \boxed{0} & 3 & 2 & 7 & 5 \\ \cancel{0} & \boxed{0} & 3 & 7 & 9 \\ 6 & 7 & 3 & 3 & \boxed{0} \\ 1 & \cancel{0} & \boxed{0} & 1 & 1 \end{pmatrix}$$

$J_1 \to M_3$, $J_2 \to M_1$, $J_3 \to M_2$, $J_4 \to M_5$.

cost $= 2 + 7 + 6 + 7$

$= 22$

Q6)

$$\begin{array}{c} \\ J_1 \\ J_2 \\ J_3 \end{array} \begin{array}{c} \begin{array}{cccc} M_1 & M_2 & M_3 & M_4 \end{array} \\ \begin{pmatrix} 12 & 9 & 12 & 9 \\ 15 & - & 13 & 20 \\ 4 & 8 & 10 & 6 \end{pmatrix} \end{array}$$

step 1:

$$\begin{pmatrix} 12 & 9 & 12 & 9 \\ 15 & \infty & 13 & 20 \\ 4 & 8 & 10 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

→ Never be zero

Max allotment

our choice

$$\begin{pmatrix} 3 & \boxed{0} & 3 & \cancel{0} \\ 2 & \infty & \boxed{0} & 7 \\ \boxed{0} & 4 & 6 & 2 \\ \cancel{0} & \cancel{0} & \cancel{0} & \boxed{0} \end{pmatrix}$$

$J_1 \to M_2$, $J_2 \to M_3$, $J_3 \to M_1$

cost $= 9 + 13 + 4$

$= 26$

Q7)

$$\begin{pmatrix} 12 & 3 & 6 & \infty & 5 & 8 \\ 4 & 11 & \infty & 5 & \infty & 3 \\ 8 & 2 & 10 & 9 & 7 & 5 \\ \infty & 7 & 8 & 6 & 12 & 10 \\ 5 & 8 & 9 & 4 & 6 & \infty \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

step 1:

$$\begin{pmatrix} 9 & \boxed{0} & 3 & \infty & 2 & 5 \\ 1 & 8 & \infty & 2 & \infty & \boxed{0} \\ 6 & \cancel{0} & 8 & 7 & 5 & 3 \\ \infty & 1 & 2 & \boxed{0} & 6 & 4 \\ 1 & 4 & 5 & \cancel{0} & 2 & \infty \\ \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

$$\begin{pmatrix} 8 & \boxed{0} & 2 & \infty & 1 & 5 \\ \cancel{0} & 8 & \infty & 2 & \infty & \boxed{0} \\ 5 & \cancel{0} & 7 & 7 & 4 & 3 \\ \infty & 1 & 1 & \boxed{0} & 5 & 4 \\ \boxed{0} & 4 & 4 & \cancel{0} & 1 & \infty \\ \cancel{0} & 1 & \boxed{0} & 1 & \cancel{0} & 1 \end{pmatrix}$$

$$\begin{pmatrix} 8 & \cancel{0} & 1 & \infty & \boxed{0} & 5 \\ \cancel{0} & 8 & \infty & 2 & \infty & \boxed{0} \\ 5 & \boxed{0} & 6 & 7 & 3 & 3 \\ \infty & 1 & \cancel{0} & \boxed{0} & 4 & 4 \\ \boxed{0} & 4 & 3 & \cancel{0} & \cancel{0} & \infty \\ 1 & 2 & \boxed{0} & 2 & \cancel{0} & 2 \end{pmatrix}$$

$J_1 \rightarrow M_5$

$J_2 \rightarrow M_6$

$J_3 \rightarrow M_2$

$J_4 \rightarrow M_4$

$J_5 \rightarrow M_1$

cost $= 5 + 3 + 2 + 6 + 5$

$= 21$