Linear programming

5000/25   cons... /1000   Graphical method

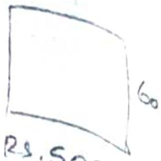
object function   Mathematical

Decision variables

Constraints

feasible solution

optional solution



Rs. 50,000

(x) chair → 500

(y) Table → 2500

region $x \le y$

line x, y

plain x, y, z

~~Maneo~~ Maximize $Z = 50x + 50y$

S.t [Subject to]

$x + y \le 60$

$500x + 2500y \le 50{,}000$

$\div 500$

$x + 5y \le 100$

and

$x, y \ge 0$

⇓

graphical method

Solution

$x + y \le 60$

$\boxed{x + y = 60}$

x	0	60
y	60	0

$x + 5y \le 100$

$\boxed{x + 5y = 100}$

x	0	100
y	20	0

$$
\begin{aligned}
x + y &= 60 \\
\overset{(-)}{x} \overset{(-)}{+} 5y &= \overset{(-)}{100} \\
\hline
4y &= 40
\end{aligned}
$$

$\boxed{y = 10}$

$x + 10 = 60$

$\boxed{x = 50}$

A co-operative society of farmers has 50 hecter of land of to grow two crops x and y.

The profit from crops x and y, per hecter are RS. 10,500 and RS, 9000 respectively.

To control weeds a liquid herbicides as to be used for crops x and y at rates of 20 litres and 10 litres per hecter. further no more than 800 litres of herbicides should be used in order to protect fish and wildlife using a pond. which collects drienagefrom this land.

How much ~~of~~ land should be allocated for each crop so as to maximize the total ~~propert~~ profit of society.

Solution:

Crop x (x)	10,500 / hectare	20l / hectare
crop y (y)	9000 / hectare	10l / hectare
		≤ 800

Soln:

The objectives function MAX $Z = 10500x + 9000y$

st

$20x + 10y \leq 800$

$x + y \leq 50$

and

$x, y \geq 0$

Consider

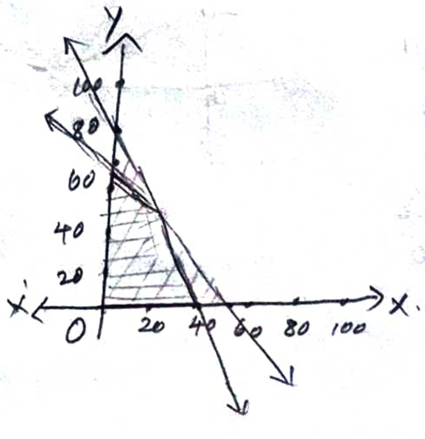
$20x + 10y = 800$

$2x + y = 80$

x	0	40
y	80	0

$x + y = 50$

x	0	50
y	50	0



A person whishes to mix two types of food P and Q. In such a way that the vitamin contain of mixture contains atleast 8 units of vitamin A and 11 unit of vitamin B. food P cost ₹ 60. per kg and food Q cost ₹ 80/kg. food P contains 3 unit/kg of vitamin A and 5 unit/kg of vitamin B. While food Q contain 4 unit/kg of vitamin A and 2 unit/kg of vitamin B. Determine the minimum cost of the mixture.

	vit A	vit B	cost
food P	3	5	60/kg
food Q	4	2	80/kg
	atleast 8	altleast 11	

The objective function min $z = 60x + 80y$

st

$3x + 4y \geq 8$

$5x + 2y \geq 11$

and $x, y \geq 0$

Consider:

$3x + 4y \geq 8$

(0,0) $3x + 4y = 8$

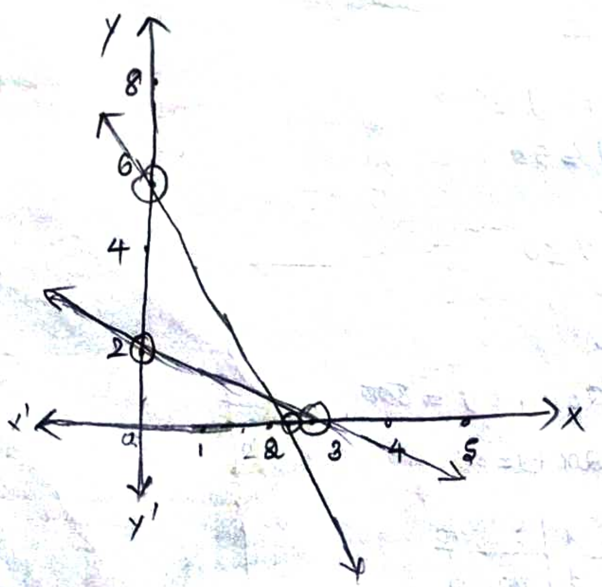
x	0	8/3
y	2	0

2.6 2.2 20 11-8 5 6/5 6.6 11/10 2

$5x + 2y \geq 11$

$5x + 2y = 11$

x	0	11/5
y	11/2	0



one kind of cake required 200gram of flour and 25gram of 25g of fat as another kind of cake require 100g of flour 50g of fat. find the max num of cakes which can be made from 5kg of flour. & 1kg of fat. Assuming that there is no shorbage of the other ingredients used for making the cake.

		flour	fat
cake I	x	200g	25g
cake II	y	100g	50g
Available		5kg	1kg

The objective function

$Max\ Z = x + y$

St

$200x + 100y \leq 5000$

$25x + 50y \leq 1000$

and

$x, y \geq 0$

consider:

$Max\ Z = x + y$

Simplex method:

$$\text{Max (or) Min } z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

s.t

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le = \ge b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le = \ge b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le = \ge b_m$$

and

$$x_1, x_2 \dots x_n \ge 0$$

$$c = (c_1, c_2, c_3 \dots c_n), \quad x = (x_1, x_2, x_3 \dots x_n)$$

$$\text{Max (or) Min } z = cx$$

s.t

$$x_1 \ \ x_2 \le 0$$

$$x_2 \le 0$$

(*: exam)

$$Ax \le = \ge B \text{ and } x \ge 0$$

Simplex 2face

$$\min z = -\max(-z)$$

(Canonical H/w)

1) By using simplex method

$$\text{MAX } z = 4x_1 + 10x_2 \Big\} \text{2 variable}$$

s.t

$$\left.\begin{aligned} 2x_1 + x_2 &\le 50 \\ 2x_1 + 5x_2 &\le 100 \\ 2x_1 + 3x_2 &\le 90 \end{aligned}\right\} \text{3 constrain}$$

$2x_1 + x_2 + s_1 = 50$
$2x_1 + 5x_2 + s_2 = 100$
$2x_1 + 3x_2 + s_3 = 90$
$x_1, x_2, s_1, s_2 \ge 0$
$z = 4x_1 + 10x_2 + 0s_1 + 0s_2$

and

$$x_1, x_2 \ge 0$$

Solution:

The standard from the LPP is

$$\text{Max } z = 4x_1 + 10x_2 + 0s_1 + 0s_2 + 0s_3$$

s.t

$$2x_1 + x_2 + s_1 = 50$$

$$2x_1 + 5x_2 + s_2 = 100$$

$$2x_1 + 3x_2 + s_3 = 90$$

and $x_1, x_2, s_1, s_2, s_3 \ge 0$

Table: 1

		c_j	4	10	0	0	0	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	θ = min ratio
0	s_1	50	2	1	1	0	0	50/1 = 50
0	s_2	100	2	5	0	1	0	100/5 = 20 ←
0	s_3	90	2	3	0	0	1	90/3 = 30

new s_3 = (90 2 3 0 0 1)
-3(20 2/5 1 0 1/5 0)
60 6/5 3 0 3/5 0
30 4/5 0 0 −3/5 1

s_1 row = (50 2 1 1 0 0)
-1(20 2/5 1 0 1/5 0)
30 8/5 0 1 −1/5 0

(−4 −10 0 0 0)
−(−10)(2/5 1 0 1/5 0)
4 10 0 2 0
0 0 0 2 0

Table:

C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3
0	s_1	30	8/5	0	1	−1/5	0
10	x_2	20	2/5	1	0	1/5	0
0	s_3	30	4/5	0	0	−3/5	1
$z_j - c_j$			0	0	0	2	0

Since all $z_j - c_j \ge 0$

This solution is optimal

$z_{max} = 200$ when $x_1 = 0$; $x_2 = 50$

2) Min $z = x_1 - 3x_2 + 2x_3$

s.t

$3x_1 - x_2 + 2x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

and

x_1, x_2, x_3

Solution : Max $w = -x_1 + 3x_2 - 2x_3$

min $z = -\max(-z) = -\max z^*$

The standard from the LPP

is $\max z^* = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$

s.t

$3x_1 - x_2 + 2x_3 + s_1 = 7$

$-2x_1 + 4x_2 + s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

and

$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$

Table : 1

		c_j	-1	3	-2	0	0	0	θ = min ratio
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	
0	s_1	7	3	-1	2	1	0	0	–
0	s_2	12	-2	4	0	0	1	0	$\frac{12}{4} = 3$ ←
0	s_3	10	-4	3	8	0	0	1	$\frac{10}{3} = 3.33$
$z_j^* - c_j$			1	-3 ↑	2	0	0	0	

Table : 2

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	min ratio
0	s_1	10	5/2	0	2	1	1/4	0	$10/5/2 = 4$ ←
3	x_2	3	-1/2	1	0	0	1/4	0	–
0	s_3	1	-5/2	0	8	0	-3/4	1	–
$z_j^* - c_j$			-1/2 ↑	0	2	0	3/4	0	

$z_j = -\frac{3}{2}$ 3 0 0 $\frac{3}{4}$ 0

+1 -3 +2 0 0 0

$-\frac{1}{2}$ 0 2 0 $\frac{3}{4}$ 0

$3 \times \frac{2}{} - 6$

$1 \times \frac{2}{5}$

Table 3:

Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
x_1	4	1	0	4/5	2/5	1/10	0
x_2	5	0	1	2/5	1/5	9/10	1
s_3	11	0	0	10	1	-1/2	1
$z_j - c_j$		0	0	$\frac{12}{5}$	$\frac{1}{5}$	$\frac{4}{5}$	0

Since all $z_j^* - c_j \ge 0$

This solution is optimal

Max $z^* = -4 + 3(5) \Rightarrow 11$

$x_1 = 4$

$x_2 = 5$

$x_3 = 0$

min $z = -11$

2)

$0 - (-1 \times 0)$

$0 - \left(\frac{1}{10} + \frac{9}{10}\right)$

$0 - \left(\frac{1}{10}\right)$

$3 \times \frac{3}{10}$

$\frac{9}{10}$

3) Max $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

S.t

$2x_1 + x_2 + 5x_3 + 6x_4 \le 20$

$3x_1 + x_2 + 3x_3 + 25x_4 \le 24$

$7x_1 + x_4 \le 70$

and $x_1, x_2, x_3, x_4 \ge 0$

Solution:

The standard form of given LPP is,

$\max z = 15x_1 + 6x_2 + 9x_3 + 2x_4 + 0s_1 + 0s_2 + 0s_3$

St

$2x_1 + x_2 + 5x_3 + 6x_4 + s_1 = 20$

$3x_1 + x_2 + 3x_3 + 25x_4 + s_2 = 24$

$7x_1 + x_4 + s_3 = 70$

and $x_1, x_2, x_3, x_4, s_1, s_2, s_3 > 0$

Table: 1

		c_j	15	6	9	2	0	0	0	
CB	y_B	x_B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	min ratio
0	s_1	20	2	1	5	6	1	0	0	$\frac{20}{2} = 10$
0	s_2	24	3	1	3	25	0	1	0	$\frac{24}{3} = 8 \leftarrow$
0	s_3	70	7	0	0	1	0	0	1	$70/7 = 10$
	$z_j - c_j$		-15	-6	-9	-2	0	0	0	

↑

Table: 2

CB	y_B	x_B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	θ
0	s_1	4	0	$1/3$	3	$-32/3$	1	$-\frac{2}{3}$	0	12 ←
15	x_1	8	1	$1/3$	1	$25/3$	0	$1/3$	0	24
0	s_3	14	0	$-7/3$	-7	$-172/3$	0	$-7/3$	1	-
	$z_j - c_j$		0	-1	6	123	0	5	0	

Table: 3

C_B	y_B	x_B	x_1	x_2	x_3	x_4	s_1	s_2	s_3
6	x_2	12	0	1	9	-32	3	-2	0
15	x_1	4	1	0	-2	19	-1	1	0
0	s_3	42	0	0	14	-132	7	-7	1
	$z_j - c_j$		0	0	15	91	3	3	0

4) Max $z = 20x_1 + 6x_2 + 8x_3$

St

$8x_1 + 2x_2 + 3x_3 \le 250$

$4x_1 + 3x_2 \le 150$

$2x_1 + x_3 \le 50$

and $x_1, x_1, x_3 \ge 0$

4) answer.

Solution:

The strandard form of given LPP is,

$max = 20x_1 + 6x_2 + 8x_3 + 0S_1 + 0S_2 + 0S_3$

s.t

$8x_1 + 2x_2 + 3x_3 + S_1 = 250$

$4x_1 + 3x_2 + S_2 = 150$

$2x_1 + x_3 + S_3 = 50$

and $x_1, x_2, x_3, S_1, S_2, S_3 > 0$

Table: 1

	Cj		20	6	8	0	0	0		
CB	YB	XB	x_1	x_2	x_3	S_1	S_2	S_3	R	θ
0	S_1	250	8	2	3	1	0	0	250/8	= 31.2
0	S_2	150	4	3	0	0	1	0	150/4	= 37.5
0	S_3	50	2	0	1	0	0	1	50/2	= 25 ←
	$z_j - c_j$		-20	-6	-8	0	0	0		

↑

Table: 2

	Cj		20	6	8	0	0	0	
CB	YB	XB	x_1	x_2	x_3	S_1	S_2	S_3	θ
0	S_1	50	0	2	-1	1	0	4	25
0	S_2	50	0	3	-2	0	1	-2	16.6 ←
20	x_1	25	01	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	-
	$z_j - c_j$		0	-6	2	0	0	10	

↑

S_1 old

250 8 2 3 1 0 0

$8 \times n \Rightarrow$ 200 8 0 4 0 0 4

(-) (-) (-) (-) (-) (-) (-)

50 0 2 -1 1 0 -4

S_2 old

150 4 3 0 0 1 0

$4 \times n \Rightarrow$ 100 4 0 2 0 0 2

(-) (-) (-) (-) (-) (-) (-)

50 0 3 -2 0 1 -2

Table: 3

		Cj	20	6	8	0	0	0	
CB	YB	XB	x_1	x_2	x_3	S_1	S_2	S_3	θ
0	S_1								
6	x_2	$\frac{50}{3}$	0	1	$-\frac{2}{3}$	0	$\frac{1}{3}$	$-\frac{2}{3}$	
20	x_1								

S_1 old

50 0 2 -1 1 0 4

$2 \times n \Rightarrow$ $\frac{100}{3}$ 0 2 $-\frac{4}{3}$ 0 $\frac{2}{3}$ $-\frac{4}{3}$

(-) (-) (-) (-) (-) (-) (-)

$\frac{50}{3}$ 0 0 $\frac{1}{3}$ 1 $-\frac{2}{3}$

$-\frac{100}{3} + 50$

$\frac{-100+150}{3}$

$\frac{50}{3}$

$\frac{4}{3} - 1$

$\frac{4-3}{3}$

$\frac{4}{3} + 4$

$\frac{4+12}{3}$

$\frac{16}{3}$

29/11/23
wednesday

Minimize $z = -2x_1 - x_2$

(s.t)

$x_1 + x_2 \geq 2$ and

$x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

Solution:

$\text{Max } z^* = 2x_1 + x_2$

(s.t)

$x_1 + x_2 - s_1 + R_1 = 2$

$x_1 + x_2 + s_2 = 4$

and

$x_1, x_2, s_1, s_2, R_1 \geq 0$

Phase - I

The auxiliary LPP is,

$\boxed{\text{Max } z^* = -R_1}$

s.t

Table : 1

C_B	Y_B	X_B	x_1 (cj: 0)	x_2 (0)	s_1 (0)	s_2 (0)	R_1 (−1)	min
−1	R_1	2	1	1	−1	0	1	2 ←
0	s_2	4	1	1	0	1	0	4
	$z_j - c_j$		−1	−1	1	0	0	

any one (−1) ↑

Table : 2

C_B	Y_B	X_B	x_1 (cj: 0)	x_2 (0)	s_1 (0)	s_2 (0)	R_1 (−1)
0	x_1	2	1	1	−1	0	1
0	s_2	2	0	0	1	1	−1
	$z_j - c_j$		0	0	0	0	+1

s_2 old

4 1 1 0 1 0

1×n 2 1 1 −1 0 1

(−) (−) (−) (−) (−) (−)

2 0 0 1 1 −1

Phase : II

Table : 1

C_B	Y_B	X_B	x_1 (cj: 2)	x_2 (1)	s_1 (0)	s_2 (0)	min
2	x_1	2	1	1	−1	0	−
0	s_1	2	0	0	1	1	2 ←
	$z_j - c_j$		0	1	−2 ↑	0	

Table : 2

C_B	Y_B	X_B	x_1 (cj: 2)	x_2 (1)	s_1 (0)	s_2 (0)
2	x_1	4	1	1	0	1
0	s_1	2	0	0	1	1
	$z_j - c_j$		0	1	0	2

x_1 old

2 1 1 −1 0

−1×n ⇒ −2 0 0 +1 −1

(−) (−) (−) (+) (−)

4 1 1 0 1

All $z_j - c_j \geq 0$

This is the optimal solution

$\text{Max } z' = 2(4) + 0 = 8$

when $x_1 = 4, x_2 = 0$

$\min z = -\text{Max } z'$

when ⇒ $x_1 = 4, x_2 = 0$

Transportation :

	D_1	D_2	D_3	...	D_n
S_1	x_{11} c_{11}	x_{12} c_{12}			x_{1n} c_{1n}
S_2	x_{21} c_{21}				
S_3	x_{31} c_{31}				
⋮					
S_m	x_{m1} c_{m1}				x_{mn} c_{mn}

$c \rightarrow$ cost unit

$x \rightarrow$ no of unit

Minimize $z = c_{11}x_{11} + c_{12}(x_{12}) + \dots c_{mn}x_{mn}$

$$= \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij} x_{ij}$$

(Standard Diviation)

$x_{11} + x_{12} + \dots\dots + x_{1n} = b_1$

$\sum_{j=1}^{n} x_j = b_j \dots\dots \quad i = 1, 2 \dots m$

$\sum_{i=1}^{m} x_{ij} = a_j \quad j = 1, 2 \dots n$

and $x_{ij} \geq 0$ $\quad i = 1, 2, \ldots m$

$\quad j = 1, 2, \ldots n$

Definition:

i) A set of non negative values x_{ij} where i varies 1 to m and j varies from 1 to n that satisfies the constraints this called a feasable solution to the Trans-polation problem (T.P) The Balan-ced Transpotation

ii) Problem has always feasable solution. A feasable solution to a $m \times n$ transpotation problem, that contain no more than $(m+n-1)$ non negative alloca-tions is called basic feasable solution to the transpolation Problem.

iii) A Basic feasable solution to a $m \times n$ ~~re~~ transpotation problem to said to non de-generate if it contains exactly $(m+n-1)$ non negative allocations.

iv) A Basic feasable solution that contains less than $(m+n-1)$ non negative allocation is said to be de-generate.

v) A Basic feasable solution is said to be optimal solution, if it minimizes the total trans-potation cost.

Methods to find Basic feasable solution (BFS)

1. North west corner rule (NWC)
2. Least cost method (LCM)
3. Vogel's apparximation method (VAM)

	A	B	C	D	E	supply
P	$\boxed{3}$ 2	$\boxed{1}$ 11	10	3	7	~~4~~ 1
Q	1	$\boxed{2}$ 4	$\boxed{4}$ 7	$\boxed{2}$ 2	1	~~8~~ 6 2
R	3	9	4	$\boxed{3}$ 8	$\boxed{6}$ 12	~~9~~ 6
	~~3~~	~~3~~ 2	~~4~~	~~5~~ 3	6.	

i) [NORTH WEST]

Demand

Here $\Sigma c_j = \Sigma b_j$ $\therefore$ This given TP is balanced.

IBFS

The transpartable Cost

$= (3 \times 2) + (1 \times 11) + (2 \times 4) + (4 \times 7) + (2 \times 2) + (3 \times 8) + (6 \times 12)$

$= 6 + 11 + 8 + 28 + 4 + 24 + 72$

$= 153$

ii) LCM

	A	B	C	D	E	
P	2	11	10	$\boxed{4}$ 3	7	~~4~~
Q	$\boxed{3}$ 1	4	7	2	$\boxed{5}$ 1	~~8~~ ~~3~~
R	3	$\boxed{3}$ 9	$\boxed{4}$ 4	$\boxed{1}$ 8	$\boxed{1}$ 12	~~9~~ 5 4
	~~3~~	3	~~4~~	~~5~~ 1	~~6~~ 1	

$= (3 \times 1) + (4 \times 3) + (5 \times 1) + (3 \times 9) + (4 \times 4) + (1 \times 8) + (1 \times 12)$

$= 3 + 12 + 5 + 27 + 16 + 8 + 12$

$= 83$

iii) VAM

	A	B	C	D	E		
P	2	11	10	3	7	4	(7-3) 4
Q	1	3 4	7	2	1	5 8	(1-1) 0
R	3	9	4	8	12	9	(4-3) 1
	3	3	4	5	6		
	(9-1)	(9-4)	(7-4)	(3-2)	(7-1)		
	1	5	3	1	6		

2) (H/W)

	D_1	D_2	D_3	D_4	
S_1	20 1	10 2	1	4	30 10
S_2	3	30 3	20 2	1	50 20
S_3	4	2	10 5	10 9	20 10
	20	40 30	30 10	10	

i) North west

The Transportable cost

$= (20\times1)+(10\times2)+(30\times3)+(20\times2)$
$+(10\times5)+(10\times9)$

$= 20+20+90+40+50+90$

$= 310$

ii) LCM

	D_1	D_2	D_3	D_4	
S_1	1	2	30 1	4	30
S_2	20 3	20 3	2	10 1	50 40 20
S_3	4	20 2	5	9	20
	20	40 20	30 30	10	

$= (30\times1)+(20\times3)+(20\times3)+(10\times1)+(20\times2)$

$= 30+60+60+10+20$

$= 180$

iii)

	D_1	D_2	D_3	D_4		
S_1	1	2	1	4	30	(1-1)=0
S_2	3	3	2	10 1	40	(2-1)=1
S_3	4	2	5	9	20	(4-2)=2
	20	40	30	10		
	(3-1)	(2-2)	(8-1)	(4-1)		
	2	0	1	3		

1	2	1	30	(1-1) = 0
3	3	2	40	(3-2) = 1
4	2	20 5	20	(4-2) = 2
20	40	30		
(3-1)	(2-2)	(2-1)		
2	0	1		

20 1	2	1	30 10	0
3	3	2	40	1
20	40	10		
2	1	1		

2	1	10	1
3	2	40 30	1
40	10		
1	1		

$\Rightarrow (10\times1)+(20\times5)+(20\times1)+(10\times2)+$
$(30\times3)+(10\times2)$

$\Rightarrow 10+100+20+20+90+20$
$= 260$

iii) Balance :

	A	C	D	E		
P	2	10	3	7	4	(3-2) 1
Q	1	7	2	5 1	5	(1-1) 0
R	3	4	8	12	9	(4-3) 1
	3	4	5	6 1		
	(1-2)	(7-4)	(3-2)	(7-1)		
	1	3	1	6		

	A	C	D	E		
P	2	10	3	7	4	1
R	3	4 4	8	12	9 5	1
	3	4	5	1		
	1	6	5	4		

2	3	7	4	1
3	5 8	12	5	5
3	5	1		
1	5	5		

3 2	1 7

$= (5\times1)+(4\times4)+(5\times8)+(3\times2)+(1\times7)$
$= 5+16+40+6+7$
$= 74$

1)

	D_1	D_2	D_3	D_4		
S_1	21	16	25	$\boxed{11}$ 13	11	(16-13)
S_2	17	18	14	23	13	(18-14)
S_3	32	27	18	41	19	(27-18)
	6	10	12	~~15~~ 4		
	(21-17)	(18-16)	(18-14)	(23-13)		

	D_1	D_2	D_3	D_4		
S_2	17	18	14	$\boxed{4}$ 23	9 ~~13~~	(17-14) 3
S_3	32	27	18	41	19	(27-18) 9
	6	10	12	4		
	(32-17)	(27-18)	(18-14)	(41-23)		
	15	9	4	18		

$\boxed{6}$ 17	18	14	3 ~~9~~	(1)
32	27	18	19	(9)
6	10	12		
15	9	4		

$\boxed{3}$ 18	14	3	(4)
27	18	19	(9)
10 7	12		
(9)	(4)		

$\boxed{7}$ 27	$\boxed{12}$ 18	19 7
7	12	

21	16	25	$\boxed{11}$ 13
$\boxed{6}$ 17	$\boxed{3}$ 18	14	$\boxed{4}$ 23
32	$\boxed{7}$ 27	$\boxed{12}$ 18	41

143 + 102 + 54 + 92 + 189 + 216

= 796

Since no of allocations = 6

$m+n-1 = 3+4-1 = 6$

∴ The solution is non-degenerate.

21 (14)	16 (8)	25 (26)	$\boxed{11}$ 13	$u_1 = -10$
$\boxed{6}$ 17	$\boxed{3}$ 18	14 (0)	$\boxed{4}$ 23	$u_2 = 0$
32 (6)	$\boxed{7}$ 27	$\boxed{12}$ 18	(9) 41	$u_3 = 9$
$v_1 = 17$	$v_2 = 18$	$v_3 = 9$	$v_4 = 23$	

for allocates cells

$C_{ij} = u_i + v_j$

for non-allocates cells

$\Delta_{ij} = d_{ij} = C_{ij} - (u_i + v_j)$

Since all $d_{ij} > 0$

∴ This solution is optimal & unique.

∴ The total TP cost = 796

$\Sigma a_i = \Sigma b_j = 10$

given TP is balanced

ans = 29

VAM. The IBFS is (not unique)

2)

	P	Q	R	
A	7	3	2	12
B	2	$\boxed{1}$ 1	3	3
C	3	4	6	5
	4	1	5	
	1	2	1	

7	$\boxed{2}$ 2	2	(5)
2	3	3	(1)
3	6	5	(3)
4	~~5~~ 3		
(1)	(1)		

2	3	2
$\boxed{4}$ 3	6	5
4	3	
(1)	(3)	

$\boxed{2}$ 3	2
6	1

$\boxed{1}$ 6	1

do answer is

7	3	$\boxed{2}$ 2
2	$\boxed{1}$ 1	$\boxed{2}$ 3
$\boxed{4}$ 3	4	$\boxed{1}$ 6

$= (2\times2) + (1\times1) + (2\times3) + (4\times3) + (1\times6)$

$= 4 + 1 + 6 + 12 + 6$

$= 29$

7 (8)	3 (3)	$\boxed{2}$ 2	$u_1 = -1$
2 (2)	$\boxed{1}$ 1	$\boxed{2}$ 3	$u_2 = 0$
$\boxed{4}$ 3	4 (0)	$\boxed{1}$ 6.	$u_3 =$
$v_1 =$	$v_2 = 1$	$v_3 = 3$	

(8) 7	(3) 3	$\boxed{2}$ 2	$u_1 = -1$
(2) 2	$\boxed{1}$ 1	$\boxed{2}$ 3	$u_2 = 0$
$\boxed{4}$ 3	(0) 4	$\boxed{1}$ 6	$u_3 = 3$
$v_1 = 0$	$v_2 = 1$	$v_3 = 3$	

3)

	W1	W2	W3	W4	W5		
F1	4	1	2	6	9	~~150~~ 100	(2-1) 1
F2	6	4	3	5	7	120	(4-3) 1
F3	5	$\boxed{50}$ 2	6	4	8	~~120~~ 70	(4-2) 2 ←
	40	50	70	90	90		
	(5-4) 1	(2-1) 1	(3-2) 1	(5-4) 1	(8-7) 1		

$\boxed{40}$ 4	$\boxed{10}$ 1	$\boxed{50}$ 2	6	9	$u_1 = -1$
$\boxed{0}$ 6	4	$\boxed{20}$ 3	$\boxed{10}$ 5	$\boxed{70}$ 7	$u_2 = 0$
5	$\boxed{40}$ 2	6	$\boxed{80}$ 4	8	$u_3 = 0$
$v_1 = 6$	$v_2 = 2$	$v_3 = 3$	$v_4 = 5$	$v_5 = 7$	

4	$\boxed{70}$ 2	6	9	~~100~~ 30	(4-2) 2 ←
6	3	5	7	120	(5-3) 2
5	6	4	8	70	(6-4) 2
40	70	90	90		
(5-4) 1	(3-2) 1	(5-4) 1	(8-7) 1		

4)

$\boxed{10}$ 10	20	5	7	10
13	$\boxed{20}$ 9	12	8	20
4	$\boxed{30}$ 5	7	9	30
14	$\boxed{10}$ 7	$\boxed{20}$ 1	$\boxed{10}$ 0	40
$\boxed{50}$ 3	$\boxed{E}$ 12	5	19	50
60	60	20	10	

$\boxed{30}$ 4	6	9	30 (2) ←
6	5	7	120 (1)
5	4	8	70 (1)
~~10~~ 40	90	90	
(1)	(1)	(1)	

$\boxed{10}$ 6	5	7	~~120~~ 110 (1)
5	4	8	70 (1)
10	90	90	
(1)	(1)	(1)	

5	7	110 (2)
$\boxed{70}$ 4	8	70 (4) ←
~~90~~ 20	90	
(1)	(1)	

$\boxed{20}$ 5	$\boxed{90}$ 7	~~110~~ 90
20	90	

$= (50\times2) + (70\times2) + (30\times4) + (10\times6) + (70\times4)$
$+ (20\times5) + (90\times7) \Rightarrow 1430$

$\boxed{30}$ 4 +θ 40	⓪ 1	$\boxed{70}$ 2 60 −θ	③ 6	④ 9	$u_1 = -2$
$\boxed{10}$ 6 0 −θ	① 4	(−1) 3 10+θ	$\boxed{20}$ 5	$\boxed{90}$ 7	$u_2 = 0$
⓪ 5	$\boxed{50}$ 2	③ 6	$\boxed{70}$ 4	② 8	$u_3 = -1$
$v_1 = 6$	$v_2 = 3$	$v_3 = 4$	$v_4 = 5$	$v_5 = 7$	

$\boxed{30}$ 4	⓪ 1	$\boxed{70}$ 2	③ 6	④ 9	$u_1 =$
$\boxed{10}$ 6	① 4	(−1) 3	$\boxed{20}$ 5	$\boxed{90}$ 7	$u_2 = 0$
⓪ 5	$\boxed{50}$ 2	③ 6	$\boxed{70}$ 4	② 8	$u_3 = -1$
$v_1 = 5$	$v_2 = 3$	$v_3 = 4$	$v_4 = 5$	$v_5 = 7$	

$\boxed{40}$ 4	(−1) +θ 1 10	$\boxed{60}$ 2	③ 6	③ 9	$u_1 = -1$
$\boxed{0}$ 6	① 4	$\boxed{10}$ 3 20 +θ	$\boxed{20}$ 5 −θ 10	$\boxed{90}$ 7	$u_2 = 0$
⓪ 5	$\boxed{50}$ 2 40	④ 6	$\boxed{70}$ 4 +θ 80	② 8	$u_3 = -1$
$v_1 = 5$	$v_2 = 3$	$v_3 = 3$	$v_4 = 5$	$v_5 = 7$	

9	12	9	6	9	10	5	$(6-9)=3$
7	3	7	7	5	$\boxed{2}$ 5	$\cancel{6}$ 4	$(5-3)=2$
6	5	9	11	3	11	2	$(5-3)=2$
6	8	11	2	2	10	9	$(6-2)=4$
4	4	6	2	4	2		
$(6-6)$ 0	$(5-3)$ 2	$(9-7)$ 2	$(6-2)$ 4	$(3-2)$ 1	$(10-5)$ 5		

9	12	$\boxed{4}$ 9	6	9	10	$u_1=0$
7	$\boxed{4}$ 3	7	7	5	$\boxed{2}$ 5	$u_2=$
$\boxed{1}$ 6	5	$\boxed{2}$ 9	11	3	11	$u_3=0$
$\boxed{3}$ 6	8	11	$\boxed{2}$ 2	$\boxed{4}$ 2	10	$u_4=0$
$v_1=6$	$v_2=$	$v_3=9$	$v_4=2$	$v_5=2$	$v_6=$	

9	12	9	6	9	5 (3)
7	3	7	7	5	4 (2)
6	5	9	11	3	2 (2)
6	8	11	$\boxed{2}$ 2	2	$\cancel{9}$ 7 (4)
4	4	6	2	4	
(0)	(2)	(2)	(4)	(1)	

9	12	9	9	5 (0)
7	3	7	5	4 (2)
6	5	9	3	2 (2)
6	8	11	$\boxed{4}$ 2	$\cancel{7}$ 3 (4)
4	4	6	4	
(0)	(2)	(2)	(1)	

9	12	9	5 (0)
7	$\boxed{4}$ 3	7	4 (4)
6	5	9	2 (1)
6	8	11	3 (2)
4	4	6	
(0)	(2)	(2)	

9	9	5 (0)
6	9	2 (3)
$\boxed{3}$ 6	11	3 (5)
$\cancel{4}$ 1	6	
(0)	(0)	

9	9	$\cancel{5}$ 4 (0)
$\boxed{1}$ 6	9	2 (3)
1	6	
(3)	(0)	

9	4
$\boxed{2}$ 9	2
$\cancel{6}$ 4	

$\boxed{4}$ 9	$\cancel{4}$
4	

$(2\times5)+(2\times2)+(4\times2)+(4\times3)+$
$(3\times6)+(1\times6)+(2\times9)+(4\times9)$
$=10+4+8+12+18+6+18+36 \Rightarrow 112$

Assignment problem.

	m_1	m_2	m_3	.. m_n
J_1	C_{11} x_{11}	C_{12} x_{12}	C_{13}, ...	......
J_2				
J_3				
⋮				
J_n				

$$\text{Min } z = \sum_{j=1}^{n} \sum_{i=1}^{n} C_{ij} x_{ij}$$

s.t

$$\sum_{j=1}^{n} x_{ij} = 1 \; ; \; i = 1, 2 \ldots n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \; ; \; j = 1, 2 \ldots n$$

and

$$x_{ij} = 0 \text{ (or) } 1$$

Hungarian method:

Consider the problem of assigning 5 jobs to 5 machines the assignment cost are given as follows,

① $$\begin{array}{c|ccccc} & m_1 & m_2 & m_3 & m_4 & m_5 \\ \hline J_1 & 8 & 4 & 2 & 6 & 1 \\ J_2 & 0 & 9 & 5 & 5 & 4 \\ J_3 & 3 & 8 & 9 & 2 & 6 \\ J_4 & 4 & 3 & 1 & 0 & 3 \\ J_5 & 9 & 5 & 8 & 9 & 5 \end{array}$$

Step: 1

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix}$$

Step: 2

$$\begin{matrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & & & \end{matrix}$$

$$\begin{pmatrix} 7 & 3 & \cancel{0} & 5 & \boxed{0} \\ \boxed{0} & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & \boxed{0} & 4 \\ 4 & 3 & \boxed{0} & \cancel{0} & 3 \\ 4 & \boxed{0} & 2 & 4 & \cancel{0} \end{pmatrix}$$

$J_1 \rightarrow M_5$

$J_2 \rightarrow M_1$ Const = 1+0+2+1+5

$J_3 \rightarrow M_4$ = 9

$J_4 \rightarrow M_3$

$J_5 \rightarrow M_2$

② $$\begin{array}{c|cccc} & m_1 & m_2 & m_3 & m_4 \\ \hline J_1 & 5 & 7 & 11 & 6 \\ J_2 & 8 & 5 & 9 & 6 \\ J_3 & 4 & 7 & 10 & 7 \\ J_4 & 10 & 4 & 8 & 3 \end{array}$$

Step: 1

$$\begin{pmatrix} 0 & 2 & 6 & 1 \\ 3 & 0 & 4 & 1 \\ 0 & 3 & 6 & 3 \\ 7 & 1 & 5 & 0 \end{pmatrix}$$

Step: 2

$$\begin{pmatrix} \boxed{0} & 2 & 2 & 1 \\ 3 & \boxed{0} & \cancel{0} & 1 \\ \cancel{0} & 3 & 2 & 3 \\ 7 & 1 & 1 & \boxed{0} \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 1 & 0 \\ 4 & 0 & 0 & 1 \\ 0 & 2 & 1 & 2 \\ 8 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 5 & \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & 1 & \boxed{0} & 2 \\ 8 & \cancel{0} & \cancel{0} & \boxed{0} \end{pmatrix}$$

$J_1 \rightarrow m_1$

$J_2 \rightarrow m_2$

$J_3 \rightarrow m_3$

$J_4 \rightarrow m_4$

Const = 5+5+10+3

= 23

3)
$$\begin{array}{c|cccc} & m_1 & m_2 & m_3 & m_4 \\ \hline J_1 & 10 & 5 & 13 & 15 \\ J_2 & 3 & 9 & 18 & 3 \\ J_3 & 10 & 7 & 3 & 2 \\ J_4 & 5 & 11 & 9 & 7 \end{array}$$

Step 1:

$$\begin{pmatrix} 5 & 0 & 8 & 10 \\ 0 & 6 & 15 & 0 \\ 8 & 5 & 1 & 0 \\ 0 & 6 & 4 & 2 \end{pmatrix}$$

Step 2:

$$\begin{pmatrix} 5 & \boxed{0} & 7 & 10 \\ 5 & 6 & 14 & \boxed{0} \\ 8 & 5 & \boxed{0} & \cancel{0} \\ \boxed{0} & 6 & 3 & 2 \end{pmatrix}$$

$J_1 \to M_2$

$J_2 \to M_4$

$J_3 \to M_3$

$J_4 \to M_1$

Const = 5+3+3+5
= 16

4)
$$\begin{array}{c|cccc} & m_1 & m_2 & m_3 & m_4 \\ \hline J_1 & 16 & 10 & 14 & 11 \\ J_2 & 14 & 11 & 15 & 15 \\ J_3 & 15 & 15 & 13 & 12 \\ J_4 & 13 & 12 & 14 & 15 \end{array}$$

Step 1:

$$\begin{pmatrix} 0 & 6 & 2 & 5 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 2 & 3 & 1 & 0 \end{pmatrix}$$

Step 1:

$$\begin{pmatrix} 0 & 6 & 2 & 5 \\ 2 & 5 & 1 & 1 \\ 1 & 1 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

Step 1:

$$\begin{pmatrix} \boxed{0} & 6 & 2 & 5 \\ 1 & 4 & \boxed{0} & \cancel{0} \\ \cancel{0} & \boxed{0} & 2 & 3 \\ 2 & 3 & 1 & \boxed{0} \end{pmatrix}$$

$J_1 \to m_1$

$J_2 \to m_3$

$J_3 \to m_2$

$J_4 \to m_4$

Max const = 16+15+15+15
= 61

5)
$$\begin{array}{c|ccccc} & m_1 & m_2 & m_3 & m_4 & m_5 \\ \hline J_1 & 10 & 11 & 4 & 2 & 8 \\ J_2 & 7 & 11 & 10 & 14 & 12 \\ J_3 & 5 & 6 & 9 & 12 & 14 \\ J_4 & 13 & 15 & 11 & 10 & 7 \\ J_5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

here we add duplicate row for (5×5) matrix

Step 1:

$$\begin{pmatrix} 8 & 9 & 2 & \boxed{0} & 6 \\ \boxed{0} & 4 & 3 & 7 & 5 \\ \cancel{0} & (1) & 4 & 7 & 9 \\ 6 & 9 & 4 & 3 & \boxed{0} \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

$$\begin{pmatrix} 8 & 8 & 1 & \boxed{0} & 6 \\ \boxed{0} & 3 & 2 & 7 & 5 \\ \cancel{0} & \boxed{0} & 3 & 7 & 9 \\ 6 & 7 & 3 & 3 & \boxed{0} \\ \cancel{0} & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

$J_1 \to m_4$

$J_2 \to m_1$

$J_3 \to m_2$

$J_4 \to m_5$

$J_5 \to m_3$

Const = 2+7+6+7
= 22

$$
6)\quad \begin{array}{c} \\ J_1 \\ J_2 \\ J_3 \end{array}
\begin{array}{c}
\begin{array}{cccc} m_1 & m_2 & m_3 & m_4 \end{array} \\
\begin{pmatrix} 12 & 9 & 12 & 9 \\ 15 & - & 13 & 20 \\ 4 & 8 & 10 & 6 \end{pmatrix}
\end{array}
$$

Step: 1

$$
\begin{pmatrix} 12 & 9 & 12 & 9 \\ 15 & \alpha & 13 & 20 \\ 4 & 8 & 10 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \text{duplicate}
$$

$$
\begin{pmatrix} 3 & \boxed{0} & 3 & \cancel{0} \\ 2 & \alpha & \boxed{0} & 7 \\ \boxed{0} & 4 & 6 & 2 \\ \cancel{0} & \cancel{0} & \cancel{0} & \boxed{0} \end{pmatrix} \rightarrow \text{dought}
$$

$J_1 \rightarrow m_2$

$J_2 \rightarrow m_3$

$J_3 \rightarrow m_1$

$J_4 \rightarrow m_4$

Const $= 9 + 15 + 4$

$= 26$

$$
7)\quad \begin{pmatrix} 12 & 3 & 6 & - & 5 & 8 \\ 4 & 11 & - & 5 & - & 3 \\ 8 & 2 & 10 & 9 & 7 & 5 \\ - & 7 & 8 & 6 & 12 & 10 \\ 5 & 8 & 9 & 4 & 6 & - \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \text{duplicate}
$$

Step: 1

$$
\begin{pmatrix} 12 & 3 & 6 & \infty & 5 & 8 \\ 4 & 11 & \infty & 5 & \infty & 3 \\ 8 & 2 & 10 & 9 & 7 & 5 \\ \infty & 7 & 8 & 6 & 12 & 10 \\ 5 & 8 & 9 & 4 & 6 & \infty \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}
$$

$$
\begin{pmatrix} 9 & \boxed{0} & 3 & \infty & 2 & 5 \\ (1) & 8 & \infty & 2 & \infty & \boxed{0} \\ 6 & \cancel{0} & 8 & 7 & 5 & 3 \\ \infty & 1 & 2 & \boxed{0} & 6 & 4 \\ 1 & 4 & 5 & 0 & 2 & \infty \\ \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}
$$

Min choose (or) Max choose (do)

$$
\begin{bmatrix} 8 & \boxed{0} & 2 & \infty & 1 & 5 \\ \cancel{0} & 8 & \infty & 2 & \infty & \boxed{0} \\ 5 & \cancel{0} & 7 & 7 & 4 & 3 \\ \infty & 1 & (1) & \boxed{0} & 5 & 4 \\ \boxed{0} & 4 & 4 & \cancel{0} & 1 & \infty \\ \cancel{0} & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{bmatrix}
$$

$$
\begin{bmatrix} 8 & 0 & 2 & \infty & 1 & 5 \\ \cancel{0} & 8 & \infty & 2 & \infty & 0 \\ 5 & \cancel{0} & 7 & 7 & 4 & 3 \\ \infty & 1 & 1 & 0 & 5 & 4 \\ 0 & 4 & 4 & \cancel{0} & 1 & \infty \\ \cancel{0} & 1 & 0 & 1 & \cancel{0} & 1 \end{bmatrix}
$$

$$
\begin{bmatrix} 8 & \cancel{0} & 1 & \infty & 0 & 5 \\ & 8 & \infty & 2 & \infty & 0 \\ \cancel{0} & & & & & 3 \\ 5 & 0 & 6 & 7 & 3 & \\ \infty & 1 & 0 & 0 & 4 & 4 \\ 0 & 4 & 3 & 0 & 0 & \infty \\ 1 & 2 & 0 & 2 & 0 & 2 \end{bmatrix}
$$

$$
\begin{bmatrix} 8 & 0 & 2 & \infty & 1 & 5 \\ \cancel{0} & 8 & \infty & 2 & \infty & 0 \\ 6 & \cancel{0} & 7 & 7 & 4 & 3 \\ \infty & & 1 & 0 & 5 & 4 \\ 0 & 4 & 4 & \cancel{0} & 1 & \infty \\ \cancel{0} & 1 & 0 & 1 & \cancel{0} & 1 \end{bmatrix}
$$

$$
\begin{bmatrix} 9 & 0 & 3 & \infty & & \\ 1 & 0 & \infty & 2 & \infty & 0 \\ 6 & \cancel{0} & 8 & 7 & 5 & 3 \\ \infty & & 2 & 0 & 6 & 4 \\ 1 & 4 & 5 & \cancel{0} & 2 & \\ 0 & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{bmatrix}
$$

Infinite fourier transform:

$$F(s) = F(f(x)) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx = F(s)$$

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds = F^{-1}(f(s))$$

fourier Cosine Transform:

$$F[f(x)] = \sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} f(x)\cos sx\, dx.$$

$$f(x) = \sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} F(s)\cos sx\, ds.$$

fourier sine Transform:

$$F[f(x)] = \sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} f(x)\sin sx\, dx.$$

$$f(x) = \sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} F(s)\sin sx\, ds.$$

Properties :- show proof

① $F(s)$ & $G(s)$, $f(x)$ & $g(x)$

$$F(af(x)) + bg(x)) = a\, F(s) + b\, G(s)$$

$F(af(x) + bg(x))$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} [\underbrace{a f(x) + bg(x)}_{f(x)}]\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} b\, g(x)\, e^{isx}\, dx$$

$= a\, F(s) + b\, G(s)$

2) shifting Theorem:

if $F(f(x)) = F(s)$, then

$$F[f(x-a)] = e^{ias} F(s)$$

3) Change of scale property: (extra multiply)

If $F[f(x)] = F(s)$, then

$$F(F(ax)) = \frac{1}{|a|} F\left(\frac{s}{a}\right), \text{ where } a \neq 0$$

4) $F[e^{iax} f(x)] = F(s+a)$

5) $F(f(x)\cos ax) = \frac{1}{2}[F(s-a) + F(s+a)]$

6) If $F(f(x)) = F(s)$ then

$$F(x^n f(x)) = (-i)^n \frac{d^n}{ds^n} F(s)$$

7) $F(f'(x)) = -is\, F(s)$ if $f(x) \to 0$ as $x \to \pm\infty$

2) $F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$

$$F[f(x-a)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x-a)\, e^{isx}\, dx$$

put $x - a = t \Rightarrow x = a + t$

$dx = dt$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{is(a+t)}\, dt$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, (e^{isa})\, e^{ist}\, dt$$

$$= e^{isa}\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{ist}\, dt\right)$$

$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dx$$

$= e^{isa} \cdot F(s)$

3) $F[f(ax)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(ax)\, e^{isx}\, dx$

$t = ax$

$dt = a\,dx$

$\frac{dt}{a} = dx$

If $a > 0$

$$F(f(ax)) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{is(t/a)} \left(\frac{dt}{a}\right)$$

$$= \frac{1}{a}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{i(\frac{s}{a})t}\, dt$$

$$= \frac{1}{a} F\left(\frac{s}{a}\right)$$

Similarly:

If $a<0$,

$$F(f(ax)) = \frac{1}{-a} F\left(\frac{s}{a}\right)$$

Q 1) Find the fourier transform:

$$f(x) = \begin{cases} x & \text{for } |x| \le a \\ 0 & \text{for } |x|>a. \end{cases}$$

$$= \begin{cases} x & ; -a \le x \le a \\ 0 & ; |x| > a \end{cases}$$

$u=(1-x^2)$, $u' = 0-2x$, $dv = e^{isx}dx$, $v = \frac{e^{isx}}{is}$, $u''=-2$, $v''=\frac{e^{is}}{is}$

Solution:

$$F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx.$$

$\int u\,dv = uv - \int v\,du$ $= \frac{1}{\sqrt{2\pi}}\int_{-a}^{a} x\, e^{isx}\, dx$ $\quad u=x \quad dv = e^{isx}dx$, $u'=1 \quad v=\frac{e^{isx}}{is}$

$\int uv = uv - u'v_1 + u''v_2 - \dots$

$$= \frac{1}{\sqrt{2\pi}}\left[x\left(\frac{e^{isx}}{is}\right) - 1\left(\frac{e^{isx}}{-s^2}\right)\right]_{-a}^{a} \qquad (i^2=-1)$$

I - Inverse
L - log
A - Algebra
T - Trignometry
E - Exponense

$$= \frac{1}{\sqrt{2\pi}}\left[\left\{a\left(\frac{e^{isa}}{is}\right) + \frac{e^{isa}}{s^2}\right\} - \left\{\frac{-a e^{-isa}}{is} + \frac{e^{-isa}}{s^2}\right\}\right]$$

$\frac{2}{\sqrt{2\pi}}$

$$= \frac{1}{\sqrt{2\pi}}\left[\left(\frac{a e^{isa}}{is} + \frac{a e^{-isa}}{is}\right) + \left(\frac{e^{isa}}{s^2} - \frac{e^{-isa}}{s^2}\right)\right]$$

$$= \frac{1}{\sqrt{2\pi}}\left[\frac{a}{is}\left(e^{isa} + e^{-isa}\right) + \frac{1}{s^2}\left(e^{isa} - e^{-isa}\right)\right]$$

$$= \frac{1}{\sqrt{2\pi}}\left[\frac{a}{is}\, 2\cos x + \frac{1}{s^2}\, 2i\sin sa\right]$$

$$\boxed{\begin{array}{ll} e^{i\theta} = \cos\theta + i\sin\theta & e^{i\theta} + e^{-i\theta} = 2\cos\theta \\ e^{-i\theta} = \cos\theta - i\sin\theta & e^{i\theta} - e^{-i\theta} = 2i\sin\theta \end{array}}$$

2) find the boolean transform of

$$f(x) = \begin{cases} 1-x^2 & ; |x|<1 \\ 0 & ; |x|>1 \end{cases}$$

hence evaluate $\int_0^{\infty}\left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}\,dx$

Solution:

$$F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-1}^{1} (1-x^2)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\left[(1-x)^2\left(\frac{e^{isx}}{is}\right) - (-2x)\left(\frac{e^{isx}}{i^2 s^2}\right) + (-2)\left(\frac{e^{isx}}{i^3 s^3}\right)\right]_{-1}^{1}$$

$$= \frac{1}{\sqrt{2\pi}}\left[\left\{0 + 2\frac{e^{is}}{-s^2} - 2\frac{e^{is}}{-is^3}\right\} - \left\{0 - 2\frac{e^{-is}}{-s^2} - 2\frac{e^{is}}{-is^3}\right\}\right]$$

$$= \frac{1}{\sqrt{2\pi}}\left[-\frac{2e^{is}}{s^2} - \frac{2e^{is}}{s^2} + \frac{2e^{is}}{is^3} - \frac{2e^{-is}}{is^3}\right]$$

$$= \frac{2}{\sqrt{2\pi}}\left[-\frac{1}{s^2}\left(e^{is} + e^{-is}\right) + \frac{1}{is^3}\left(e^{is} - e^{-is}\right)\right]$$

$$= \sqrt{\frac{2}{\pi}}\left[-\frac{1}{s^2}\, 2\cos s + \frac{1}{s^3}\, 2i\sin s\right]$$

$$= \frac{4}{\sqrt{2\pi}}\left[\frac{\sin s}{s^3} - \frac{s\cos s}{s^3}\right]$$

$$= \frac{4}{\sqrt{2\pi}}\left[\frac{\sin s - s\cos s}{s^3}\right]$$

$$= -\frac{4}{\sqrt{2\pi}}\left[\frac{s\cos s - \sin s}{s^3}\right]$$

$e^{i\theta} = \cos\theta + i\sin\theta$
$e^{-i\theta} = \cos\theta - i\sin\theta$
$2\cos\theta$ $\quad 2\sin\theta$ $\quad sa = \theta$ $\quad \cos\theta$

even func : $f(-x) = f(x)$

odd func : $f(-x) = -f(x)$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds$$

$$1-x^2 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{-4}{\sqrt{2\pi}} \left(\frac{s\cos s - \sin s}{s^3} \right) e^{-isx} ds$$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$$= \frac{-4}{2\pi} \int_{-\infty}^{\infty} \left(\frac{s\cos s - \sin s}{s^3} \right) e^{-isx} ds \Rightarrow (\cos sx - i\sin sx)$$

$$\frac{s\cos s - \sin s}{s^3}$$

$$= 1-x^2 = \frac{-2}{\pi} \cdot 2 \int_{0}^{\infty} \frac{s\cos s - \sin s}{s^3} (\cos sx - i\sin sx) ds$$

even fn. $\Rightarrow \dfrac{-s\cos(-s) - \sin(-s)}{(-s)^3}$

equal the real parts.

$f(-x) = -f(x)$.

$$= \frac{-s\cos s + \sin s}{-s^3}$$

$$1-x^2 = \frac{-4}{\pi} \int_{0}^{\infty} \frac{s\cos s - \sin s}{s^3} \cos sx \, ds$$

$$= \frac{s\cos s - \sin s}{s^3}$$

$\boxed{x = \frac{1}{2}}$

$\cos(-sx) = \cos sx$

2) Find the fourier transform of

$$f(x) = \begin{cases} 1-x^2 ; & |x|<1 \\ 0 ; & |x| \geq 1 \end{cases}$$

evaluate $\displaystyle\int_{0}^{\infty} \frac{s\cos s - \sin s}{s^3} \cos\frac{s}{2} \, ds$.

$u = 1-x^2$ $\quad dv = e^{isx}$

$v = \dfrac{e^{isx}}{is}$

$u' = -2x$ $\quad v_1 = \dfrac{e^{isx}}{-s^2}$

$u'' = -2$ $\quad v_2 = \dfrac{e^{isx}}{i^3 s^3}$

$\int u\,dv = uv - u'v_1 + u''v_2 - \ldots$

Solution:

$$f(x) = \begin{cases} 1-x^2 ; & -1 < x < 1 \\ 0 ; & |x| \geq 1 \end{cases}$$

$$= \frac{1}{\sqrt{2\pi}} \left[(1-x^2) \cdot \frac{e^{isx}}{is} + 2x \frac{e^{isx}}{-s^2} - 2\frac{e^{isx}}{-is^3} \right]_{-1}^{1}$$

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[(1-x^2) \frac{e^{isx}}{is} - \frac{2x e^{isx}}{s^2} + 2\frac{e^{isx}}{is^3} \right]_{-1}^{1}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2) e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}}\left[0 - \frac{2e^{is}}{s^2} + \frac{2e^{is}}{is^3} - 0 - \frac{2^{-ps x}}{s^2}\right]$$

formula filling:

$$F[F(x)] = \frac{-4}{2\sqrt{\pi}}\left[\frac{s\cos s - \sin s}{s^3}\right]$$

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds$$

$$1-x^2 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{-4}{2\sqrt{\pi}}\left(\frac{s\cos s - \sin s}{s^3}\right) e^{-isx}\, ds$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{-4}{\sqrt{2\pi}}\left(\frac{s\cos s - \sin s}{s^3}\right)$$

$$(\cos sx - \underbrace{i\sin sx}_{\text{imaginary part}})\, ds$$

we need the real part alone,

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{-4}{\sqrt{2\pi}}\left(\frac{s\cos s - \sin s}{s^3}\right)\cos sx\, ds.$$

since it is an even function,

$$= \frac{1}{\sqrt{2\pi}}\, 2\int_{0}^{\infty} \frac{-4}{\sqrt{2\pi}}\left(\frac{s\cos s - \sin s}{s^3}\right)\cos sx\, ds.$$

$$= \frac{-4\times 2}{2\pi}\int_0^{\infty}\left(\frac{s\cos s - \sin s}{s^3}\right)\cos sx\, ds$$

at $x = 1/2$

$$1 - x^2 = \frac{-4}{\pi}\int_0^{\infty} \frac{s\cos s - \sin s}{s^3}\cos\frac{s}{2}\, ds.$$

$$= -\frac{\pi}{4}\left(1 - \frac{1}{4}\right) = -\frac{3\pi}{16}$$

$$F[f(x)\cos x] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cos x\, e^{-isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{-isx}\left(\frac{e^{iax} + e^{-iax}}{2}\right) dx$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{-isx}\cdot e^{iax}\, dx + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{-isx}\cdot e^{-iax}\, dx\right]$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{-i(s-a)x}\, dx + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{-i(s+a)x}\, dx\right]$$

$$= \frac{1}{2}\left[F(s-a) + F(s+a)\right]$$

Identity

If $F_c(s), G_c(s)$ $\quad f(x), g(x)$

i) $\int_0^{\infty} f(x)g(x)\, dx = \int_0^{\infty} F_c(s)\cdot G_c(s)\, ds$

ii) $\int_0^{\infty} f(x)g(x)\, dx = \int_0^{\infty} F_s(s)\cdot G_s(s)\, ds$

iii) $\int_0^{\infty} |f(x)|^2 dx = \int_0^{\infty} |F_c(s)|^2 ds = \int_0^{\infty} |F_s(s)|^2 ds$

- $\int_0^{\infty} \frac{x\sin \alpha x}{a^2+x^2}\, dx = \frac{\pi}{2} e^{-\alpha a}$
- $\int_0^{\infty} \frac{\cos \alpha x}{a^2+x^2}\, dx = \frac{\pi}{2} e^{-\alpha a}$

$$\frac{x}{a^2+x^2} \qquad \frac{1}{a^2+x^2}$$

$$F_s(f(x)) = \sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} f(x)\sin sx\, dx$$

$$= \sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} \frac{x}{a^2+x^2}\sin sx\, dx$$

$$= \sqrt{\frac{2}{\pi}}\cdot\frac{\pi}{2} e^{-sa}$$

$$= \sqrt{\frac{\pi}{2}}\, e^{-sa}$$

$$F_c(f(x)) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\cos sx\, dx$$

$$= \sqrt{\frac{2}{\pi}}\left(\int_0^{\infty} \frac{1}{a^2+x^2}\cos sx\, dx\right)$$

$$= \sqrt{\frac{2}{\pi}}\cdot\frac{\pi}{2a} e^{-sa}$$

$$= \sqrt{\frac{\pi}{2}}\cdot\frac{1}{a} e^{-sa}$$

Using above partial parseval's Identity

$f(2) = \int_0^\infty \frac{dx}{(a^2+x^2)^2}$ & $\int_0^\infty \frac{x^2}{a^2+x^2} dx$

$F_s(e^{-ax}) = \sqrt{\frac{2}{\pi}} \frac{s}{a^2+s^2} \Rightarrow$

$f(x) \qquad f(s)$

Parseval's identity

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds$$

$$\int_0^\infty (e^{-ax})^2 dx = \int_0^\infty \frac{2}{\pi} \frac{s^2}{(a^2+s^2)^2} ds$$

$$\int_0^\infty e^{-2ax} dx = \int_0^\infty \frac{2}{\pi} \frac{s^2}{(a^2+s^2)^2} ds$$

$$\int_0^\infty \frac{s^2}{(a^2+s^2)^2} ds = \frac{\pi}{2} \left[\frac{e^{-2ax}}{-2a}\right]_0^\infty$$

$$= \frac{\pi}{-4a}[0-1]$$

$$= \frac{\pi}{4a}$$

$$\therefore \int_0^\infty \frac{x^2}{(a^2+x^2)^2} dx = \frac{\pi}{4a}.$$

$$\int_0^\infty (e^{-ax})^2 dx = \int_0^\infty \frac{2}{\pi} \frac{a^2}{(a^2+s^2)^2} ds$$

$$\int_0^\infty \frac{ds}{(a^2+x^2)^2} = \frac{\pi}{2a^2} \left[\frac{e^{-2ax}}{-2a}\right]_0^\infty$$

$$= \frac{\pi}{-4a^3}[0-1]$$

$$= \frac{\pi}{4a^3}$$

evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using fourier transform.

$f(x) = e^{-ax}, \; g(x) = e^{-bx}$

Since $F_c(e^{-ax}) = \sqrt{\frac{2}{\pi}} \frac{a}{a^2+s^2}$

$G_c(e^{-bx}) = \sqrt{\frac{2}{\pi}} \frac{b}{b^2+s^2}$

$\therefore$ By property,

$$\int_0^\infty F_c(s) \cdot G_c(s) ds = \int_0^\infty f(x) \cdot g(x) dx.$$

$$\int_0^\infty \sqrt{\frac{2}{\pi}} \cdot \frac{a}{a^2+s^2} \cdot \sqrt{\frac{2}{\pi}} \frac{b}{b^2+s^2} ds = \int_0^\infty e^{-(a+b)x} dx$$

$$\int_0^\infty \frac{ds}{(a^2+s^2)(b^2+s^2)} = \frac{\pi}{2ab} \left[\frac{e^{-(a+b)x}}{-(a+b)}\right]_0^\infty$$

$$= \frac{\pi}{-2ab(a+b)} (0-1)$$

$$= \frac{\pi}{2ab(a+b)}$$

pde

consider 2nd order partial differential equation (PDE),

$$AU_{xx} + BU_{xy} + CU_{yy} + DU_x + EU_y + F = 0$$

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + F = 0$$

If $B^2 - 4AC = 0$ then the pde is parabolic

If $B^2 - 4AC < 0$ then the pde is elliptic

If $B^2 - 4AC > 0$ then the pde is hyperbolic

(Consider the problem find the nature of above

classify the following PDE.

1) $U_{xx} + 2U_{xy} + U_{yy} = 0$

2) $x^2 f_{xx} + (1-y)^2 f_{yy} = 0$

3) $U_{xx} + 4U_{xy} + (x^2+4y^2) U_{yy} = \sin(x+y)$

4) $(x+1)U_{xx} - 2(x+2) U_{xy} + (x+3)U_{yy} = 0$

5) $x f_{xx} + y f_{yy} = 0, \; x > 0, y > 0.$

Solution :

1) $A=1,\ B=2,\ C=1$

$B^2-4AC = (2)^2-4(1)(1)=0$

$\therefore$ parabolic

2) $A=x^2,\ B=0,\ C=1-y^2$

$B^2-4AC = 0-4(x^2)(1-y^2)$

$= -4x^2(1-y^2)$

i) If $x=0$ (or) $y=\pm 1$

then the pde is parabolic

ii) If $x\neq 0,\ -1<y<1$

$B^2-4AC<0$

$\therefore$ Elliptic

iii) If $x\neq 0,\ y<-1$ or $y>1$

then $B^2-4AC>0$,

$\therefore$ hyperbolic

3) $A=1,\ B=4,\ C=x^2+4y^2$

$B^2-4AC = 16-4(x^2+4y^2)$

If $B^2-4AC=0$

$16-4(x^2+4y^2)=0$

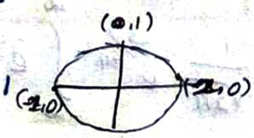
$4(x^2+4y^2)=16$

$\boxed{x^2+4y^2=4}$

$\div 4$

$\frac{x^2}{4}+y^2=1$

(Parabolic)



If $x^2+4y^2<4$

then

$B^2-4AC>0$ (hyperbolic)

If $x^2+4y^2>4$

$B^2-4AC<0$

(elliptic)

4) $A=x+1,\ B=-2(x+2),\ C=x+3$

$B^2-4AC = 4(x+2)^2-4(x+1)(x+3)$

$=4(x^2+4+4x)-4(x^2+3x+x+3)$

$=4x^2+16+16x-4x^2-12x-4x-12$

$= \cancel{4x^2+4-8x}$ $16-12$

$=4>0$

$\therefore$ hyperbolic

5) $A=x,\ B=0,\ C=y$

$B^2-4AC = 0-4xy$

$= -(4xy)<0$

$\therefore$ ellipse.

note:

1) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial^2 u}{\partial t^2}$ (1D wave equation)

$u_{xx} = \frac{1}{\alpha^2}u_{tt}$

$\alpha^2 u_{xx} - u_{tt} = 0$

$A=\alpha^2,\ B=0,\ C=-1$

$B^2-4AC = 0-4(\alpha^2)(-1)$

$=4\alpha^2>0$

hyperbolic

2) $\frac{\partial u}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t^2}$

$\alpha^2 u_{xx} = u_t$

$\alpha^2 u_{xx} - u_t = 0$

$A=\alpha^2,\ B=0,\ C=0$

$B^2-4AC = 0-4(\alpha^2)(0)$

$=0$

parabolic

3) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ (Laplace equation)

$u_{xx}+u_{yy}=0$

$A=1,\ B=0,\ C=1$

$B^2-4AC = 0-4(1)(1) = -4<0$

elliptic

4) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f(x,y)$ (poisson eqn)

$u_{xx}+u_{yy}=f(x,y)$

$A=1,\ B=0,\ C=1$

continue ... prepare

$$f(x) = \begin{cases} 1 \ ; |x| < a \\ 0 \ ; |x| > a \end{cases}$$

hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.

$$F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} (1) e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{isx}}{is} \right]_{-a}^{a}$$

$$= \frac{1}{\sqrt{2\pi}} \left[\left(\frac{e^{isa}}{is} \right) - \left(\frac{e^{-isa}}{is} \right) \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{1}{is} \left(e^{isa} - e^{-isa} \right) \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{1}{\cancel{i}s} (2\cancel{i} \sin sa) \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{1}{s} (2 \sin sa) \right]$$

$$F(f(x)) = \sqrt{\frac{2}{\pi}} \left[\frac{\sin sa}{s} \right] = F(s)$$

$$\boxed{\because f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds}$$

$$1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}} \left(\frac{\sin sa}{s} \right) e^{-isx} ds$$

$$1 = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin sa}{s} (\cos sx - i \sin sx) ds$$

5) S.T the fourier transform $e^{-x^2/2}$ is $e^{-s^2/2}$ by finding the fourier transform of $e^{-a^2x^2}$

$f_{(x)} = e^{-a^2x^2}, a>0$

Solution:

$$F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2} \cdot e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2 + isx} dx$$

Consider, $-a^2x^2 + isx = -a^2 \left(x^2 - \frac{isx}{a^2} \right)$

$$= -a^2 \left(x^2 + \frac{isx}{a^2} + \frac{i^2s^2}{4a^4} - \frac{i^2s^2}{4a^4} \right)$$

$$= -a^2 \left[\left(x - \frac{is}{2a^2} \right)^2 + \frac{s^2}{4a^4} \right]$$

$$F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2 \left[\left(x - \frac{is}{2a^2} \right)^2 + \frac{s^2}{4a^4} \right]} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2 \left(x - \frac{is}{2a^2} \right)^2} e^{-\frac{s^2}{4a^2}} dx$$

$$= \frac{1}{\sqrt{2\pi}} e^{-s^2/4a^2} \int_{-\infty}^{\infty} e^{-a^2 \left(x - \frac{is}{2a^2} \right)^2} dx.$$

put $t = x - \frac{is}{2a^2}$

$dt = dx$

$$F(f(x)) = \frac{1}{\sqrt{2\pi}} e^{-\frac{s^2}{4a^2}} \int_{-\infty}^{\infty} e^{-a^2t^2} dt$$

put $at = v$; $dt = \frac{dv}{a}$

$a\,dt = dv$

$$F(f(x)) = \frac{1}{\sqrt{2\pi}} e^{-\frac{s^2}{4a^2}} \int_{-\infty}^{\infty} e^{-v^2} \frac{dv}{a}$$

$$= \frac{1}{\sqrt{2\pi}\, a} e^{-\frac{s^2}{4a^2}} \int_{-\infty}^{\infty} e^{-v^2} dv$$

$$= \frac{2}{\sqrt{2}\sqrt{\pi}\, a} e^{-\frac{s^2}{4a^2}} \left(\int_{0}^{\infty} e^{v^2} dv \right)$$

$$= \frac{\sqrt{2}}{\sqrt{\pi}}$$

convolution Theorem:

The convalution of two function $f(x)$ & $g(x)$ is

$$f(x) * g(x) = \frac{1}{\sqrt{2\pi}} \int f(t) g(x-t)\, dt$$

The fourier transform of the convolution of $f(x)$ and $g(x)$ is the product of their fourier transform

$$F[f(x) * g(x)] = f(s) \cdot G(s)$$

Parseral's identity:

If $f(s)$ is fourier transform of $f(x)$ then,

$$\boxed{\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds} \rightarrow \text{apply logic}$$

using parsral's identity proof that,

$$\boxed{\int_{0}^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}} \rightarrow \text{proof (answer)}$$

$$\boxed{\text{If } f(x) = \begin{cases} 1 & ; |x| < a \\ 0 & ; |x| > a \end{cases}} \rightarrow \text{condition}$$

$$\boxed{F[f(x)] = \sqrt{\frac{2}{\pi}} \frac{\sin as}{s}} \rightarrow \text{given value} = f(s)$$

$$= \frac{2}{\sqrt{2\pi}} \int_0 (1-x) \cos x\, dx.$$

$$= \sqrt{\frac{2}{\pi}} \left[(1-x)\left(\frac{\sin sx}{s}\right) - (-1)\left(\frac{-\cos x}{s^2}\right) \right]_0^1$$

$$= \sqrt{\frac{2}{\pi}} \left[\left[0 - \frac{\cos s}{s^2}\right] - \left[0 - \frac{1}{s^2}\right] \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{1}{s^2} - \frac{\cos s}{s^2} \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{1-\cos s}{s^2} \right] = \sqrt{\frac{2}{\pi}} \left[\frac{2\sin^2\frac{s}{2}}{s^2} \right]$$

By parseral's identity

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds$$

$$\int_{-1}^{1} (1-|x|)^2 dx = \int_{-\infty}^{\infty} \left(\sqrt{\frac{2}{\pi}}\, 2\, \frac{\sin^2 s/2}{s} \right)^2 ds$$

$$2\int_0^1 (1-x)^2 dx = \frac{8}{\pi} \int_{-\infty}^{\infty} \left(\frac{\sin^2\frac{s}{2}}{s^2} \right)^2 ds$$

$$2\int_0^1 (1+x^2-2x)\, dx = \frac{16}{\pi} \int_0^{\infty} \left(\frac{\sin^2 s/2}{s^2} \right)^2 ds$$

$$\left[x + \frac{x^3}{3} - 2\frac{x^2}{2} \right]_0^1 = \frac{8}{\pi} \int_0^{\infty} \frac{\sin^4\frac{s}{2}}{s^4} ds$$

$$\left[\left(1 + \frac{1}{3} - 1\right) - (0) \right]$$

put $t = \frac{s}{2}$

$$\cancel{16}^{2}\, \frac{1}{3} \frac{\pi}{8} = \int_0^{\infty} \frac{\sin^4 t}{s^4} ds$$

By Parseral's identity:

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds$$

$$\int_{-a}^{a} |1|^2 dx = \int_{-\infty}^{\infty} \left| \sqrt{\frac{2}{\pi}} \frac{\sin as}{s} \right|^2 ds$$

$$\int_{-a}^{a} dx = \frac{2}{\pi} \int_{-\infty}^{\infty} \left(\frac{\sin as}{s} \right)^2 ds$$

$$[a - (-a)] = \frac{2}{\pi} \int_{-\infty}^{\infty} \left(\frac{\sin as}{s} \right)^2 ds$$

$$2a = \frac{2}{\pi} \int_{-\infty}^{\infty} \left(\frac{\sin as}{s} \right)^2 ds$$

$$\pi a = \int_{-\infty}^{\infty} \left(\frac{\sin as}{s} \right)^2 ds$$

$$\pi a = 2 \int_0^{\infty} \left(\frac{\sin as}{s} \right)^2 ds$$

doughto

Put $a = 1$

$$\frac{\pi}{2} = \int_0^{\infty} \left(\frac{\sin s}{s} \right)^2 ds$$

$$\boxed{\frac{\pi}{2} = \int_0^{\infty} \left(\frac{\sin t}{t} \right)^2 dt} \rightarrow \text{Prooved}$$

$$f(x) = \begin{cases} 1-|x| \; ; & f|x|<1 \quad -1 \le x < 1 \\ 0 \; ; & 0 \text{therwise} \end{cases}$$

$$\int_0^\infty \frac{\sin^4 t}{t^4}\, dt \qquad |x| = \begin{cases} x \; ; & x \geq 0 \\ -x \; ; & x<0 \end{cases}$$

Solution :

$$F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} ((1-|x|)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-|x|)\,(\cos x + i \sin x)\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-|x|) \cos x\, dx + \frac{i}{\sqrt{2\pi}} \int_{-1}^{1} (1-|x|) \sin x\, dx$$

$$= \frac{2}{\sqrt{2\pi}} \int_0^1 (1-x) \cos x\, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[(1-x) \left(\sin \right. \right.$$

Maths

$u(x,t)$

pts

1) $F\left[\frac{\partial u}{\partial x}\right] = (i\alpha)\, u(\alpha,t)$

2) $F\left[\frac{\partial^2 u}{\partial x^2}\right] = (i\alpha)^2 u(\alpha,t)$

(1 does tan)

3) $F\left[\frac{\partial^n u}{\partial x^n}\right] = (-i\alpha)^n u(\alpha,t)$

4) $F\left[\frac{\partial u}{\partial t}\right] = \frac{d}{dt} u(\alpha,t)$

5) $F\left[\frac{\partial^2 u}{\partial t^2}\right] = \frac{d^2}{dt^2} u(\alpha,t)$

6) $F_s\left[\frac{\partial^2 u}{\partial x^2}\right] = \sqrt{\frac{2}{\pi}}\, \alpha\, u(x,t)\Big|_{x=0} - \alpha^2 F_s\left[u(x,t); x\to\alpha\right]$

$= \sqrt{\frac{2}{\pi}}\, \alpha\, u(0,t) - \alpha^2 u_s(\alpha,t)$

7) $F_c\left[\frac{\partial^2 u}{\partial x^2}\right] = -\sqrt{\frac{2}{\pi}}\, \alpha\, u_x(0,t) - \alpha^2 u_c(\alpha,t)$

Derived from $u(x,t)$

$u|_{x=0} \quad u(0,t) \to \sin$

$\frac{\partial u}{\partial x}\Big|_{x=0} \quad u_x(0,t) \to \cos$

Solve the heat conduction equation given by Pde : $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$; $-\infty < x < \infty,\ t \geq 0$

$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$; $-\infty < x < \infty$, $t \geq 0$.

Subject to : BC's : $u(x,t)$ and $u_x(x,t)$ both $\to 0$ as $(x) \to \infty$

Boundary condition

IC : $u(x,0) = f(x)$: $-\infty < x < \infty$

Initial condition

Solution :

$i^2 = -1$

$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$

Taking fourier transform both sides

$k F\left[\frac{\partial^2 u}{\partial x^2}\right] = F\left[\frac{\partial u}{\partial t}\right]$

$k(-i\alpha)^2 u(\alpha,t) = \frac{d}{dt} u(\alpha,t)$

$-k\alpha^2 u(\alpha,t) = \frac{d}{dt} u(\alpha,t)$

$\frac{d}{dt} u(\alpha,t) + k\alpha^2 u(\alpha,t) = 0$

$(D + k\alpha^2)\, u(\alpha,t) = 0$

The A.E is } Auxiliary or Arithmetic equation

$m + k\alpha^2 = 0$

$\boxed{m = -k\alpha^2}$

$C.F = Ae^{-k\alpha^2 t}$

P.I $= \frac{1}{f(0)} e^{kx}$

$= \frac{1}{f(x)}$

$P.I = 0$

$\therefore u(\alpha,t) = Ae^{-k\alpha^2 t} \to (1)$

$(D^2 + D + k)y = x$

$m^2 + m + k = 0$

$m = m_1, m_2$

$C.F = Ae^{m_1 x} + Be^{m_2 x}$

$C.F = (Ax+B)e^{mx}$

$C.F = e^{\alpha x}$

$[A\cos px + B\sin px]$

IC : $u(x,0) = f(x)$

$F[u(x,0)] = F(f(x))$

$u(\alpha,0) = F(\alpha)$

$Ae^{-kx^2(0)} = F(\alpha)$

$\boxed{A = F(\alpha)}$

$u(\alpha,t) = F(\alpha)e^{-k\alpha^2 t}$

Applying Inverse Fourier transform.

$F^{-1}[u(\alpha,t)] = F^{-1}\left[F(\alpha)e^{-k\alpha^2 t}\right]$

$= F^{-1}[F(\alpha) \cdot G(\alpha)]$ where $G(\alpha) = e^{-kx^2 t}$

$= f(x) * g(x)$

where $f(x) = F^{-1}(F(\alpha))$

$g(x) = F^{-1}[G(\alpha)]$

$f * g = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(\alpha) g(x-\alpha)\, dx$.

$g(x) = F^{-1}[G(\alpha)]$

$= F^{-1}\left[e^{-kx^2 t}\right]$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-kx^2 t} e^{-i\alpha t}\, dx$.

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(k\alpha^2 + i\alpha)t}\, dx$.

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty}$

Consider:

$\int_{-\infty}^{\infty} e^{-ax^2 - 2bx}\, dx = \int_{-\infty}^{\infty} e^{-a\left[x^2 + \frac{2bx}{a}\right]} dx$

$= \int_{-\infty}^{\infty} e^{-a\left[x^2 + \frac{2b}{a}x + \left(\frac{b}{a}\right)^2 - \left(\frac{b}{a}\right)^2\right]} dx$.

$= \int_{-\infty}^{\infty} e^{-a\left[x + \frac{b}{a}\right]^2} \cdot e^{a\left(\frac{b^2}{a^2}\right)} dx$

Solve the heat conduction equation described by

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad 0<x<\infty,\ t\geq 0$$

BC's : $u(0,t) = u_0$: $t\geq 0$

IC : $u(x,0) = 0$: $0<x<\infty$

u & $\frac{\partial u}{\partial x}$ both tends to $\to 0$ as $x\to\infty$

Solution :

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

Taking fourier sine tranform on both side,

$$k\,F_s\left[\frac{\partial^2 u}{\partial x^2}\right] = F_s\left[\frac{\partial u}{\partial t}\right]$$

$$k\left[\sqrt{\frac{2}{\pi}}\,\alpha u(0,t) - \alpha^2 U_s(\alpha,t)\right] = \frac{d}{dt}U_s(\alpha,t)$$

$$k\left[\sqrt{\frac{2}{\pi}}\,\alpha u_0 - \alpha^2 U_s(\alpha,t)\right] = \frac{d}{dt}U_s(\alpha,t)$$

$$k\alpha\sqrt{\frac{2}{\pi}}\,u_0 - k\alpha^2 U_s = \frac{d}{dt}U_s$$

$$\Rightarrow \frac{d}{dt}U_s + k\alpha^2 U_s = k\alpha u_0\sqrt{\frac{2}{\pi}}$$

$$(D + k\alpha^2)U_s = k\alpha U_0\sqrt{\frac{2}{\pi}}$$

The A.E is

$$m + k\alpha^2 = 0$$

$$m = -k\alpha^2$$

$$\boxed{C.F = A e^{-k\alpha^2 t}}$$

$$P.I = \frac{1}{D+k\alpha^2}\,k\alpha u_0\sqrt{\frac{2}{\pi}}$$

$$= \frac{1}{D+k\alpha^2}\,k\alpha U_0\sqrt{\frac{2}{\pi}}\,e^{at}$$

$$= \frac{1}{k\alpha^2}\,k\alpha u_0\sqrt{\frac{2}{\pi}} = \sqrt{\frac{2}{\pi}}\,\frac{U_0}{\alpha}$$

The general soln is

$$U_s(\alpha,t) = C.F + P.I$$

$$U_s(\alpha,t) = A e^{-k\alpha^2 t} + \sqrt{\frac{2}{\pi}}\,\frac{u_0}{\alpha}$$

IC : $U(x,0) = 0$

$$U_s(\alpha,0) = 0$$

$$U_s(\alpha,0) = A e^{0} + \sqrt{\frac{2}{\pi}}\cdot\frac{u_0}{\alpha}$$

$$0 = A + \sqrt{\frac{2}{\pi}}\,\frac{u_0}{\alpha}$$

$$\boxed{A = -\sqrt{\frac{2}{\pi}}\,\frac{u_0}{\alpha}}$$

$$\therefore U_s(\alpha,t) = \sqrt{\frac{2}{\pi}}\,\frac{u_0}{\alpha} - \sqrt{\frac{2}{\pi}}\,\frac{u_x}{\alpha}e^{-k\alpha^2 t}$$

$$U_s(\alpha,t) = \sqrt{\frac{2}{\pi}}\,\frac{u_0}{\alpha}\left[1 - e^{-k\alpha^2 t}\right]$$

Applying Inverse fourier since transform, we get

$$u(x,t) = \sqrt{\frac{2}{\pi}}\int_0^\infty \sqrt{\frac{2}{\pi}}\,\frac{u_0}{\alpha}\left[1 - e^{-k\alpha^2 t}\right]\sin\alpha x\,d\alpha$$

$$= \frac{2}{\pi}\,\frac{u}{a}\int_0^\infty \left(1 - e^{-k\alpha^2 t}\right)\sin\alpha x\,d\alpha$$

find the solution by given rule.

2) $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$

wave equation

$$\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$$

The possible solution of wave equation are

1. $y = (Ae^{\lambda x} + Be^{-\lambda x})(Ce^{\lambda ct} + De^{-\lambda ct})$ exponential
   capital

2. $y = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda ct + D\sin\lambda ct)$ trignamentary

3. $y = (Ax+B)(Ct+D)$ algebric

where A, B, C, D are arbitary constants

boundary condition $= y(0,t)=0 \Rightarrow$ when $x=0, y=0$
initial condition $=$ $y(l,t)=0$ when $x=l, y=0$

6/4/24

9) Solve the following problem described by PDE

$U_{xx} + U_{yy} = 0$, $-\infty < x < \infty$, $y > 0$

BC : $U_y(x,0) = f(x)$; $-\infty < x < \infty$

u is bounded on $y \to \infty$

u and $\frac{\partial u}{\partial x}$ both variables as $|x| \to \infty$

Solution

Let in define a function

$$\phi(x,y) = u_y(x,y) = \frac{\partial}{\partial y} u(x,y)$$

$$\phi_{xx} + \phi_{yy} = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}$$

$$= \frac{\partial^2}{\partial x^2}\left[\frac{\partial}{\partial y} u(x,y)\right] + \frac{\partial^2}{\partial y^2}\left[\frac{\partial}{\partial y} u(x,y)\right]$$

$$= \frac{\partial}{\partial y}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

$$= \frac{\partial}{\partial y}(0)$$

$$\phi_{xx} + \phi_{yy} = 0$$

$\therefore$ PDE : $\nabla^2\phi = 0$ (or) $\phi_{xx} + \phi_{yy} = 0$

BC : $\phi(x,0) = f(x)$ $-\infty < x < \infty$

By using previous problem,

$$\phi(x,y) = \frac{y}{\pi}\int_{-\infty}^{\infty} f(\varepsilon)\frac{1}{y^2+(\varepsilon-x)^2}\,d\varepsilon$$

$$\boxed{u_y(x,y) = \phi(x,y)}$$

Then,

$$u(x,y) = \int\left[\frac{y}{\pi}\int_{-\infty}^{\infty} f(\varepsilon)\frac{1}{y^2+(\varepsilon-x)^2}\,d\varepsilon\right]dy$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} f(\varepsilon)\,d\varepsilon \int\frac{y\,dy}{y^2+(\varepsilon-x)^2}$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} f(\varepsilon)\,d\varepsilon\,\frac{1}{2}\int\frac{2y\,dy}{y^2+(\varepsilon-x)^2}$$

$$u(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\varepsilon)\,d\varepsilon\left[\ln\left[y^2+(\varepsilon-x)^2\right]\right] + c$$

---

8) using fourier tranform, solve the PDE

$U_{xx} + U_{yy} = 0$; $-\infty < x < \infty$, $y > 0$

BC : $u(x,0) = f(x)$ $-\infty < x < \infty$

$u \to 0$ as $r \to \infty$ when $r = \sqrt{x^2+y^2}$

$$f(x) = \begin{cases} T_0 , & |x| < b \\ 0 , & |x| > b \end{cases}$$

Solution

By using (8) problem,

$$u(x,y) = \frac{y}{\pi}\int_{-\infty}^{\infty} f(\varepsilon)\frac{1}{y^2+(\varepsilon-x)^2}\,d\varepsilon$$

$$f(\varepsilon) = \begin{cases} T_0 \; ; |\varepsilon| < b & -b < \varepsilon < b \\ 0 \; ; |\varepsilon| > b \end{cases}$$

$$u(x,y) = \frac{y}{\pi}\int_{-\infty}^{\infty} \frac{f(\varepsilon)\,dx}{y^2+(\varepsilon-x)^2}$$

$$= \frac{y}{\pi}\int_{-b} \frac{T_0}{y^2+(\varepsilon-x)^2}\,d\varepsilon$$

$$= \frac{yT_0}{\pi}\int_{-b}^{b} \frac{d\varepsilon}{y^2+(\varepsilon-x)^2}$$

Let $t = \varepsilon - x$ | when $\varepsilon = -b$, $t = -b-x = -(b+x)$

$dt = d\varepsilon$ | $\varepsilon = b$, $t = b-x$.

$$u(x,y) = \frac{yT_0}{\pi}\int_{-(b+x)}^{b-x} \frac{dt}{y^2+t^2}$$

$$= \frac{yT_0}{\pi}\left[\frac{1}{y}\tan^{-1}\left(\frac{t}{y}\right)\right]_{(b+x)}^{(b-x)}$$

$$= \frac{T_0}{\pi}\left[\left\{\tan^{-1}\left(\frac{b-x}{y}\right) - \tan^{-1}\left(\frac{-b+x}{y}\right)\right\}\right]$$

$$= \frac{T_0}{\pi}\left[\tan^{-1}\left(\frac{b-x}{y}\right) + \tan^{-1}\left(\frac{b+x}{y}\right)\right]$$

$$= \frac{T_0}{\pi}\left[\tan^{-1}\left[\frac{\left(\frac{b-x}{y}\right)+\left(\frac{b+x}{y}\right)}{1-\frac{(b-x)}{y}\frac{(b+x)}{y}}\right]\right]$$

$$u(x,y) = \frac{T_0}{\pi}\tan^{-1}\left(\frac{2b/y}{\frac{y^2-(b^2-x^2)}{y^2}}\right)$$

$$= \frac{T_0}{\pi}\tan^{-1}\left(\frac{2by}{y^2+x^2-b^2}\right)$$

8) using fourier transform for the PDE

$$U_{xx} + U_{yy} = -xe^{-x^2} \; ; -\infty < x < \infty,\; y > 0$$

BC : $u(x,0) = 0 \quad -\infty < x < \infty$

$u(x,y) \to 0$ as $y \to \infty$

S.T $u(\alpha,y) = (1-e^{-|\alpha|y})\frac{i}{2\sqrt{2}\alpha}e^{\frac{-\alpha^2}{4}}$

Solution:

$$U_{xx} + U_{yy} = -xe^{-x^2}$$

Taking F.T as both sides,

$$F[U_{xx}] + F(U_{yy}) = F[-xe^{-x^2}]$$

$$(-i\alpha)^2 u(x,y) + \frac{d^2}{dy^2}u(x,y) = F[-xe^{-x^2}] \quad \text{—(1)}$$

now,

$$F(-xe^{-x^2}] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} -xe^{-x^2}e^{i\alpha x}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} -x.e^{-x^2+i\alpha x}\,dx$$

consider,

$$-x^2+i\alpha x = -[x^2 - i\alpha x]$$

$$= -\left[x^2 - i\alpha x + \left(\frac{i\alpha}{2}\right)^2 - \left(\frac{i\alpha}{2}\right)^2\right]$$

$$= -\left[\left(x-\frac{i\alpha}{2}\right)^2 + \left(\frac{\alpha}{2}\right)^2\right]$$

$$= -\left(x-\frac{i\alpha}{2}\right)^2 - \left(\frac{\alpha}{2}\right)^2$$

$$F[-xe^{-x^2}] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} -x.e^{-(x-\frac{i\alpha}{2})^2}.e^{-\alpha^2/4}\,dx$$

$$= \frac{-e^{-\alpha^2/4}}{\sqrt{2}}\int_{-\infty}^{\infty} x\,e$$

Solution:

max $z = 5x_1 + 8x_2$

S.t $= 3x_1 + 2x_2 - S_1 + R_1 = 3$

$x_1 + 4x_2 - S_2 + R_2 = 4$

$x_1 + x_2 + S_3 = 5$

$x_1, x_2, S_1, S_2, S_3, R_1, R_2 \geq 0$

Phase 1

The new auxillary LPP is

max $z^* = -R_1 - R_2$

st

$3x$

Determine the temperature Distribution in semi infinite medium $x \geq 0$, when the end $x=0$ is maintain at zero temperature and the initial temperature is $f(x)$.

The given problem is described by PDE

$$\frac{du}{dt} = k\frac{\partial^2 u}{\partial x^2} ; \quad 0 < x < \infty$$

BC: $u(0,t) = 0 ; \ t > 0$

IC: $u(x,0) = f(x) ; \ 0 < x < \infty$

Solution:

$$\boxed{k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}}$$

why, we sine

Taking fourier Since transform on both sides, we get

$$kF_S\left[\frac{\partial^2 u}{\partial x^2}\right] = F_S\left[\frac{\partial u}{\partial t}\right]$$

$$k\left[\sqrt{\frac{2}{\pi}}\,\alpha u(0,t) - \alpha^2 U_s(\alpha,t)\right] = \frac{d}{dt}U_s(\alpha,t)$$

$$-k\alpha^2 U_s(\alpha,t) = \frac{d}{dt}U_s(\alpha,t)$$

$$\frac{d}{dt}U_s(\alpha,t) + k\alpha^2 U_s(\alpha,t) = 0$$

$$(D + k\alpha^2)U_s(\alpha,t) = 0$$

The A.E is

$m + k\alpha^2 = 0$

$m = -k\alpha^2$

$\therefore$ C.F $= Ae^{-k\alpha^2 t}$

P.I $= 0$

The general soln, is

$U_s(\alpha,t) = C.F + P.I$

$$U_s(\alpha,t) = Ae^{-k\alpha^2 t} \longrightarrow (1)$$

IC: $U(x,0) = f(x)$

$U_s(\alpha,0) = F(\alpha)$

At $t=0$

$(1) \Rightarrow U_s(\alpha,0) = Ae^0$

$$\boxed{F(\alpha) = A}$$

$U_s(\alpha,t) = F(\alpha) \cdot e^{-k\alpha^2 t}$

Taking Inverse fourier Since transform, we get

$$u(x,t) = \sqrt{\frac{2}{\pi}}\int_0^\infty F(\alpha)e^{-k\alpha^2 t}\sin\alpha x\, d\alpha$$

4) Using fourier transform method, ~~fou~~ (sine, cosine, or general) we solve $\boxed{U_{xx} = a^{-2} U_t}$; $0 < x < \infty,\ t > 0$

BC : $U_x(0,t) = 0$; $t > 0$

IC : $U(x,0) = f(x)$; $0 < x < \infty$

$$\left(\frac{1}{a^2}\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}\right)$$

Solution

$$\frac{\partial^2 u}{\partial x^2} = a^{-2}\frac{\partial u}{\partial t}$$

Taking fourier cosine transform on both sides, we get

$$F_c\left[\frac{\partial^2 u}{\partial x^2}\right] = a^{-2} F_c\left[\frac{\partial u}{\partial t}\right]$$

$$-\sqrt{\frac{2}{\pi}}\,\alpha\, u_x(0,t) - \alpha^2 u_c(\alpha,t) = a^{-2}\frac{d}{dt}U_c(\alpha,t)$$

$$-\alpha^2 U_c(\alpha,t) = \frac{1}{a^2}\frac{d}{dt}U_c(\alpha,t)$$

$$\frac{d}{dt}U_c(\alpha,t) + a^2\alpha^2 U_c(\alpha,t) = 0$$

$$(D + a^2\alpha^2)\,u_c(\alpha,t) = 0$$

The A.E is

$m + a^2\alpha^2 = 0$

$m = -a^2\alpha^2$

$\therefore C.F = Ae^{-a^2\alpha^2 t}$

$PI = 0$

the general solution is

$U_c(\alpha,t) = CF + PI$

$U_c(\alpha,t) = Ae^{-a^2\alpha^2 t} \rightarrow (1)$

IC : $U(x,0) = f(x)$

$u_c(\alpha,0) = F(\alpha)$

At, $t = 0$

$(1) \Rightarrow U_c(\alpha,0) = Ae^{0}$

$F(\alpha) = A$

$\therefore U_c(\alpha,t) = F(\alpha)e^{-a}$

Taking inverse fourier ~~write~~ transform, we get

$$U(x,t) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} F(\alpha)\, e^{-a^2\alpha^2 t}\cos\alpha x\, d\alpha$$

5) Compute the displacement $u(x,t)$ of an infinite string using the method of fourier transform given that the string is initially at rest and the initial displacement is $f(x)$, $-\infty < x < \infty$

Displacement of an infinite string is generated by the PDE.

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

IC : $u_t(x,0)$; $-\infty < x < \infty$

$u(x,0) = f(x)$:

Solution:

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

Taking fourier transform on both sides we go

$$F\left[\frac{\partial^2 u}{\partial t^2}\right] = c^2 F\left[\frac{\partial^2 u}{\partial x^2}\right]$$

$$\frac{d^2}{dt^2}u(\alpha,t) = c^2\left[(-i\alpha)^2 u(\alpha,t)\right]$$

$$= -c^2\alpha^2 u(\alpha,t)$$

$$\frac{d^2}{dt^2}u(\alpha,t) + c^2\alpha^2 u(\alpha,t) = 0$$

$$(D^2 + c^2\alpha^2)\,u(\alpha,t) = 0$$

The A.E is

$m^2 + c^2\alpha^2 = 0$

$m^2 = -c^2\alpha^2$

$m = \pm\sqrt{-c^2\alpha^2}$

$m = \pm ic\alpha$

~~$m = \alpha \pm i\beta$~~

~~$CF = e^{\alpha x}[A\cos\beta x + B\sin\beta x]$~~

~~$y = f(x)$~~

$C.F = e^{0t}[A\cos(c\alpha t) + B\sin(c\alpha t)]$

$C.F = A\cos(\alpha ct) + B\sin(\alpha ct)$

$PI = 0$

The general solution is

$u(\alpha,t) = C.F + P.I$

$u(\alpha,t) = A\cos(\alpha ct) + B\sin(\alpha ct) \rightarrow (1)$

IC: $u(x,0)=f(x)$

$u(\alpha,0)=F(\alpha)$

when $t=0$

① $\Rightarrow u(\alpha,0)=A(\cos 0+B\sin 0)$

$\boxed{F(\alpha)=A}$

$\left(\begin{array}{l}\cos 0=1\\ \sin 0=0\\ \sin n\pi=0\end{array}\right)$

IC: $u_t(x,0)=0 \Rightarrow u_t(\alpha,0)=0$

Difference ① with respect to 't'

$u_t(\alpha,t)=-A\alpha c\sin(\alpha ct)$
$\quad +B\alpha c\cdot\cos(\alpha ct) \rightarrow$ ②

$\therefore u(\alpha,t)=F(\alpha)\cdot\cos(\alpha ct)$

Taking I.F.T (Inverse fourier Transform)

$$u(x,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}F(\alpha)\cos(\alpha ct)\,e^{-i\alpha x}\,dx$$

$\rightarrow$ when $t=0$,

$2\Rightarrow u_t(\alpha,0)=-A\alpha c\sin 0-B\alpha c\cos 0$

$0=B\alpha c$

$B=0$

6) Solve the Boundary Value problem (B.V.P) in half plane $y>0$. described by the PDE.

$u_{xx}+u_{yy}=0$; $-\infty<x<\infty$, $y>0$

$u(x,0)=f(x)$; $-\infty<x<\infty$

u is bounded as $y=\infty$, u & $\frac{du}{dx}$ vanish as $|x|\rightarrow\infty$

Solution:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Taking fourier Transform on both sides.

$$F\left[\frac{\partial^2 u}{\partial x^2}\right]+F\left[\frac{\partial^2 u}{\partial y^2}\right]=0$$

$$(-i\alpha)^2 u(\alpha,y)+\frac{d^2}{dy^2}u(\alpha,y)=0$$

$$-\alpha^2 u(\alpha,y)+\frac{d^2}{dy^2}u(\alpha,y)=0$$

$$\frac{d^2}{dy^2}u(\alpha,y)-\alpha^2 u(\alpha,y)=0$$

$$(D^2-\alpha^2)\,u(\alpha,y)=0$$

The A.E is

$m^2-\alpha^2=0$, $m^2=\alpha^2$, $m=\pm\alpha$

C.F $=Ae^{\alpha y}+Be^{-\alpha y}$

$=(A+B)\,e^{-|\alpha|y}$

$=(\text{Constant})\,e^{-|\alpha|y} \longrightarrow$ ①

B.C is $u(x,0)=f(x)$

$u(\alpha,0)=F(\alpha)$

when $y=0$,

① $\Rightarrow u(\alpha,0)=(\text{constant})\,e^{-|\alpha|0}$

$F(\alpha)=\text{Constant}$

$u(\alpha,y)=F(\alpha)\cdot e^{-|\alpha|y}$

Apply Inverse fourier Transform

$$u(x,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}F(\alpha)\,e^{-|\alpha|y}\,e^{-i\alpha y}\,dx.$$

~~$F(\alpha)=\frac{1}{\sqrt{2\pi}}$~~

~~$f(x)=$ some function~~

$$F(\alpha)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)\,e^{i\alpha x}\,dx$$

$$F(\alpha)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(\xi)\,e^{i\xi x}\,d\xi,$$

$$u(x,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(\xi)\,e^{i\xi x}\,d\xi$$

continue. $e^{-|\alpha|y}\,e^{-i\alpha x}\,d\alpha$

$$u(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(\xi)\,d\xi\int_{-\infty}^{\infty}e^{i\xi x}\,e^{-|\alpha|y}\,e^{-i\alpha x}\,d\alpha$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(\xi)\,d\xi\int_{-\infty}^{\infty}e^{-|\alpha|y}\,e^{i\xi x}\,e^{-i\alpha x}\,d\alpha.$$

Consider

$$\int_{-\infty}^{\infty}e^{-|\alpha|y}\,e^{i\xi x}\,e^{-i\alpha x}\,dx=\int_{-\infty}^{0}e^{\alpha y}\,e^{i\xi x}\,e^{-i\alpha x}\,d\alpha$$
$$+\int_{0}^{\infty}e^{-\alpha y}\,e^{i\xi x}\,e^{-i\alpha x}\,d\alpha$$

$$=\int_{0}^{\infty}e^{-\alpha(y+i(x-\xi))}\,d\alpha+\int_{-\infty}^{0}e^{\alpha[y+i(\xi-x)]}\,d\alpha$$

$$=\int_{-\infty}^{0}e^{-\alpha(y+i(x-\xi))}\,d\alpha.$$
$$+\int_{-\infty}^{0}e^{\alpha[y+i(\xi-x)]}\,d\alpha$$

$$= \left[\frac{e^{-\infty[y+i(x-\xi)]}}{-[y+i(x-\xi)]}\right]_{-\infty}^{0} + \left[\frac{e^{\infty[y+i(\xi-x)]}}{y+i(\xi-x)}\right]_{0}^{\infty}$$

$$= \left[0 + \frac{1}{y+i(x-\xi)}\right] + \left[\frac{1}{y+i(\xi-x)} - 0\right]$$

$$= \frac{1}{y-i(\xi-x)} + \frac{1}{y+i(\xi-x)}$$

$$= \frac{2y}{y^2+(\xi-x)^2}$$

$(a+b)+(a-b)=2a$

$(a+b)(a-b)=a^2-b^2$

$$\therefore \boxed{u(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\xi)\cdot\frac{2y}{y^2+(\xi-x)^2}\,d\xi}$$

Assignment problem;

① Solve using F.T method.

$a^2 u_{xx} = u_t \;;\; 0<x<\infty,\; t>0$

BC: $u(0,t)=f(t) \;;\; \left.\begin{array}{l} u(x,t)\to 0 \\ u_x(x,t)\to 0 \end{array}\right\}$ as $x\to\infty$

IC: $u(x,0)=0 \;;\; 0<x<\infty$

② $u_{tt} = c^2 u_{xx} \;;\; 0<x<\infty$

IC: $u(x,0)=f(x)$

$u_t(x,0)=g(x)$