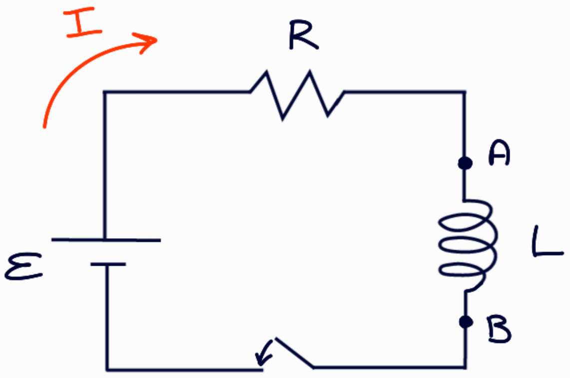


$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{tR}{L}}\right)$$

$$V_A - V_B = \varepsilon\, e^{-\frac{tR}{L}}$$

Per $t \approx 0$: $\quad I \approx 0$

$$\Delta V_L = V_B - V_A \approx -\varepsilon$$

Ai capi dell'induttore compare una differenza di potenziale che si oppone alla variazione della corrente.
Subito dopo la chiusura del circuito l'induttore si comporta come un circuito aperto.

Nota: in generale, se il circuito è tale che a $t<0$ vi era una corrente nell'induttanza, la corrente resterà tale subito dopo la chiusura del circuito

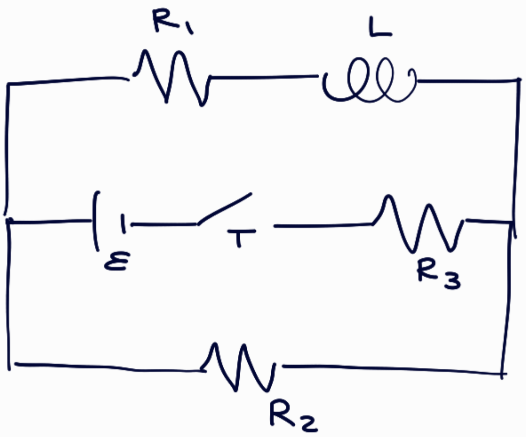
Per $t \rightarrow +\infty$: $\quad I \rightarrow \frac{\varepsilon}{R}$

$$\Delta V_L \rightarrow 0$$

Alla stazionarietà l'induttore si comporta come un corto circuito.

Energia immagazzinata nell'induttore: $\quad U_L = \frac{1}{2} L I^2$

Esercizio 1



$R_1 = 6\,\Omega$

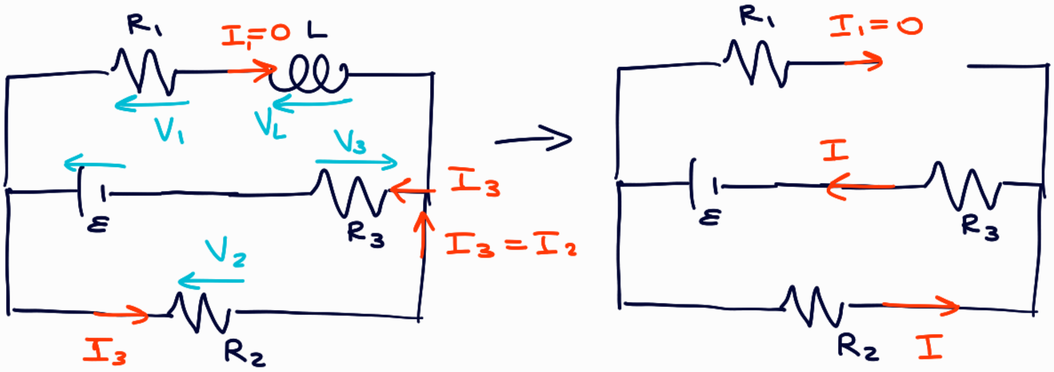
$R_2 = 3\,\Omega$

$R_3 = 2\,\Omega$

$\varepsilon = 12\,V$

correnti nelle resistenze e V_L a $t=0^+$ e a $t \to +\infty$

• $t = 0^+$:



$\boxed{I_1 = 0}$

(L diventa circuito aperto)

$$\varepsilon - V_2 - V_3 = 0$$

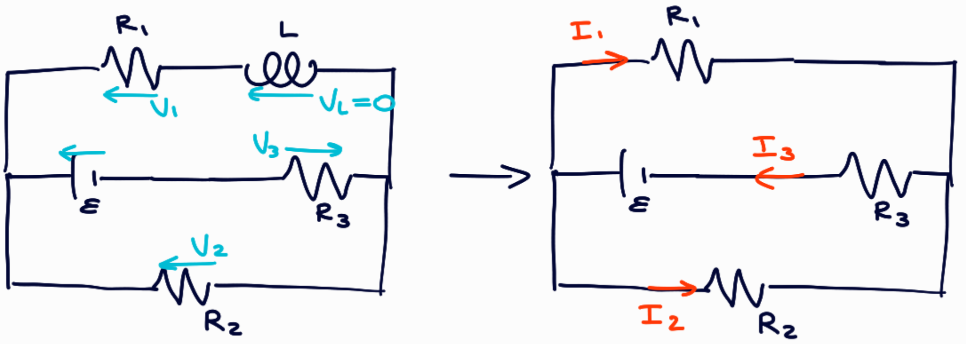
$$\varepsilon - I_2 R_2 - I_3 R_3 = 0 \qquad \text{ma } \boxed{I_2 = I_3 = I}$$

$$\varepsilon - I(R_2 + R_3) = 0$$

$$\boxed{I} = \frac{\varepsilon}{R_2 + R_3} = \boxed{2{,}4 \ A}$$

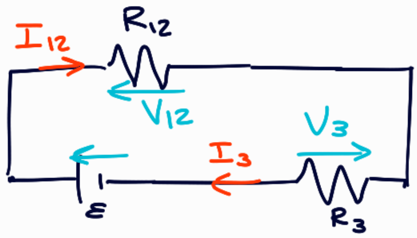
$$\boxed{V_L} = \varepsilon - \cancel{V_1}^{\,0} - V_3 = \varepsilon - I R_3 = \boxed{7{,}2 \ V}$$

• $t \to +\infty$: L → corto circuito → $\boxed{V_L = 0}$



$R_1 \,//\, R_2$: $R_{12} = 2A$

$\longrightarrow$



$$R_{12} = R_1 \parallel R_2 = 2\,\Omega$$

$$\mathcal{E} - V_{12} - V_3 = 0$$

$$\mathcal{E} - I_{12} R_{12} - I_3 R_3 = 0 \qquad \text{me} \quad I_{12} = I_3$$

$$\mathcal{E} - I_3 (R_{12} + R_3) = 0$$

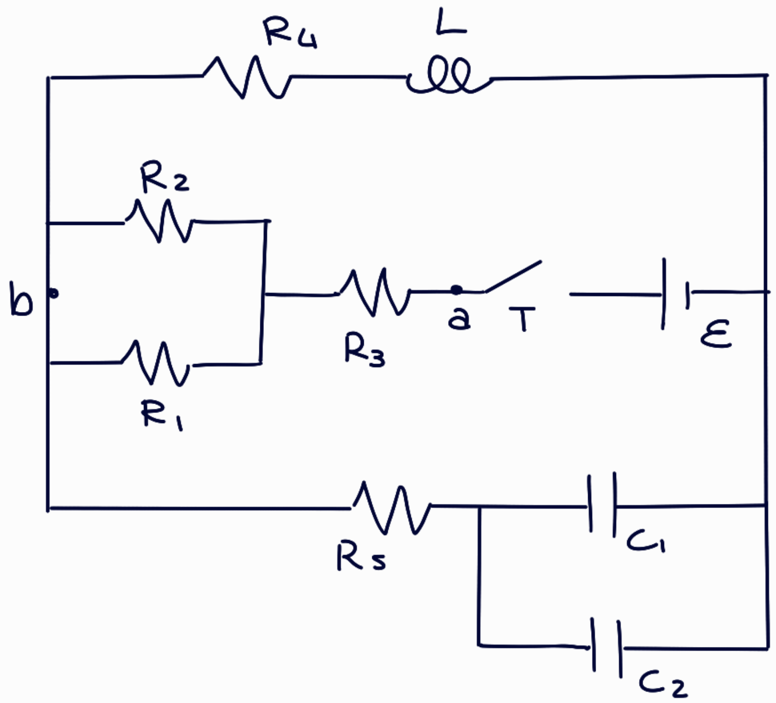
$$\boxed{I_3} = \frac{\mathcal{E}}{R_{12} + R_3} = \boxed{3\text{ A}}$$

$$V_{12} = V_1 = V_2 = R_{12} \cdot I_{12} = R_{12} \cdot I_3 = 6\text{V}$$

$$\Rightarrow \boxed{I_1} = \frac{V_1}{R_1} = \boxed{1\text{ A}}$$

$$\boxed{I_2} = \frac{V_2}{R_2} = \boxed{2\text{A}}$$

Esercizio 2



$R_1 = 1\ \Omega$

$R_2 = 2\ \Omega$

$R_3 = R_4 = R_5 = 3\ \Omega$

$\varepsilon = 10\ V$

$L = 1\ \mu H$

$C_1 = 0{,}5\ \mu F$

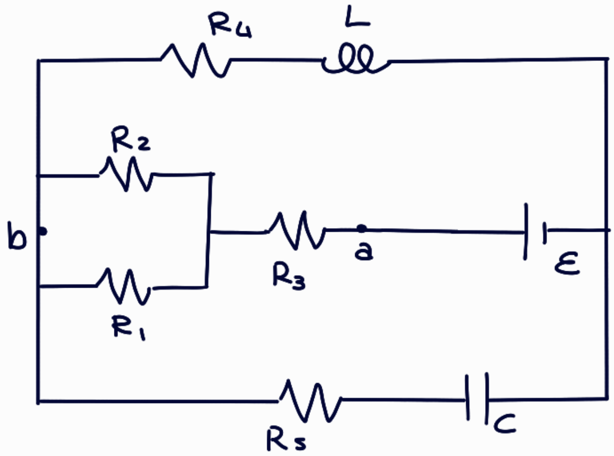
$C_2 = 1{,}5\ \mu F$

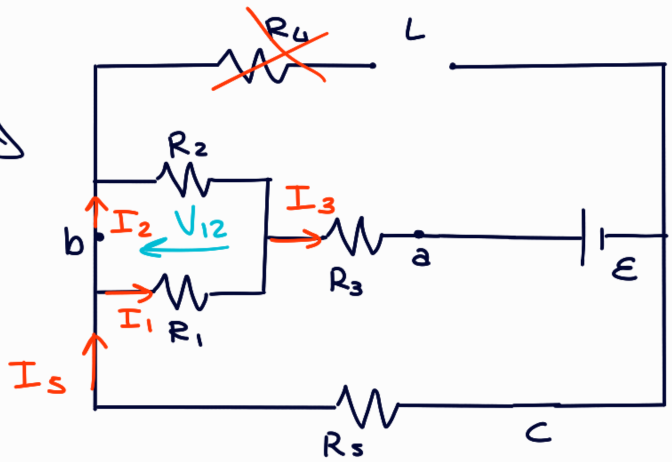
calcolare la corrente che circola nelle resistenze R_1 e R_2 subito dopo la chiusura del circuito in condizioni di stazionarietà

Per $t = 0^+$: $L \approx$ circuito aperto

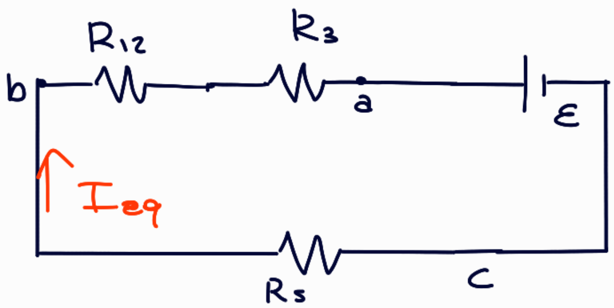
$C \approx$ corto circuito

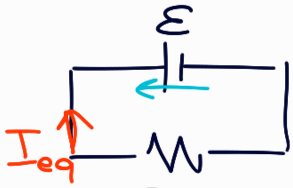
con $C = C_1 + C_2 = 2\ \mu F$





$R_{12} = R_1 \| R_2 = \frac{2}{3}\ \Omega$





$R_{eq} = R_{12} + R_3 + R_5 = \frac{20}{3}\ \Omega$

$$V_{eq} = \mathcal{E} \implies I_{eq} = I_5 = I_3 = I_{12} = \frac{\mathcal{E}}{R_{eq}} = 1{,}5\ A$$

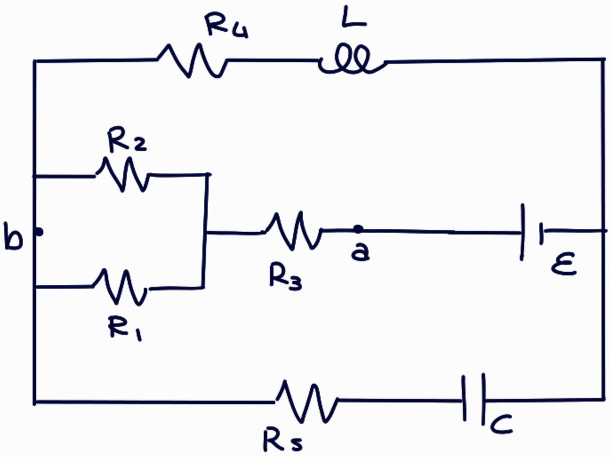
$$V_{12} = V_1 = V_2 = I_{12} \cdot R_{12} = 1\ V$$

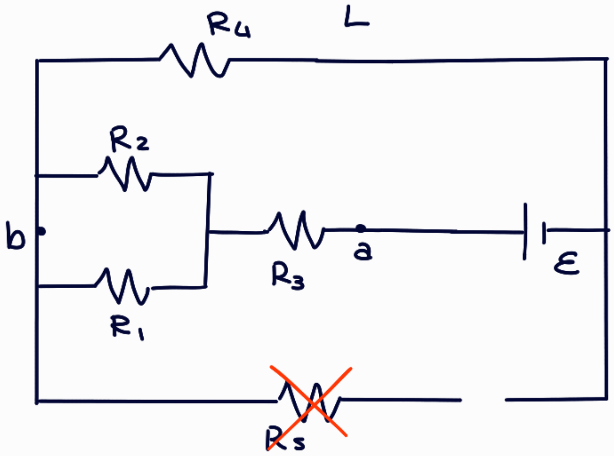
$$I_1 = \frac{V_{12}}{R_1} = 1\ A \qquad I_2 = \frac{V_{12}}{R_2} = 0{,}5\ A$$

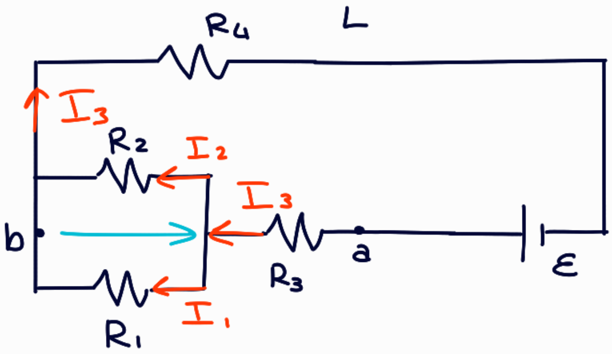
Per $t \to +\infty$: $L \approx$ corto circuito

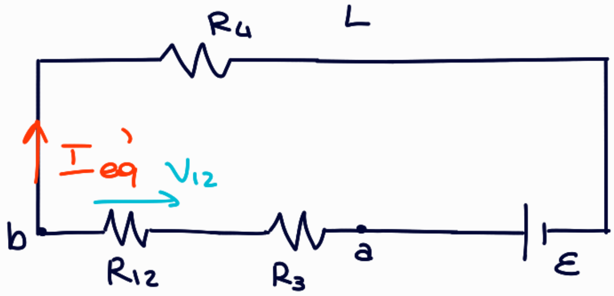
$C \approx$ circuito aperto

con $C = C_1 + C_2 = 2\,\mu F$

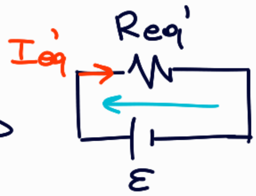








$$R_{12} = R_1 \parallel R_2 = \frac{2}{3}\,\Omega$$



$$R'_{eq} = R_{12} + R_3 + R_4 = R_{eq} = \frac{20}{3}\,\Omega$$

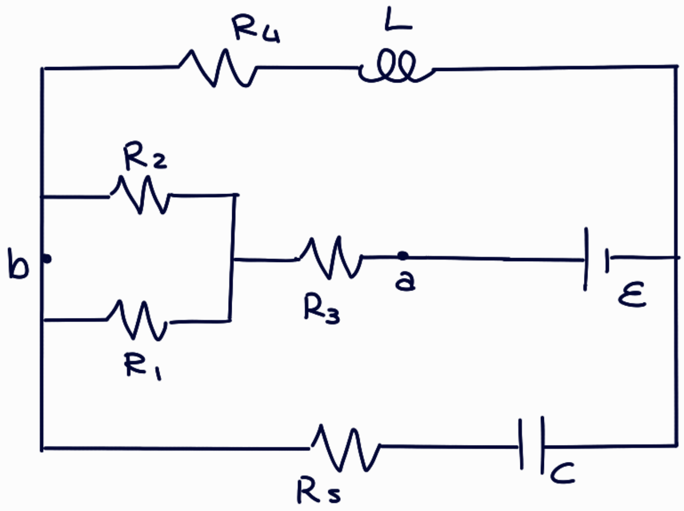
$$I'_{eq} = I_{12} = I_3 = I_4 = \frac{\mathcal{E}}{R_{eq}} = I'_{eq} = 1{,}5\ A$$

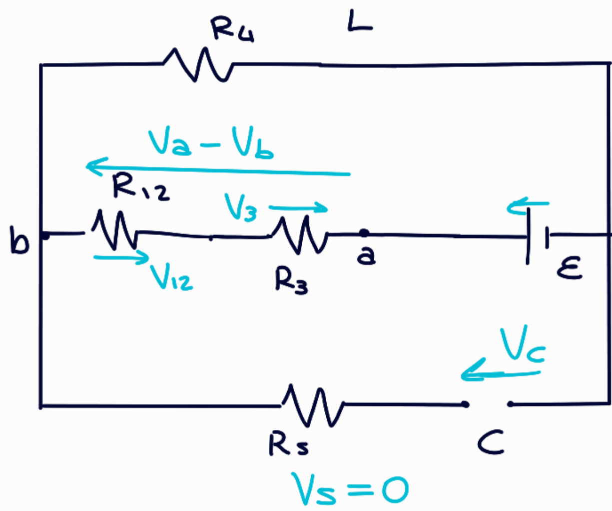
$$V_{12} = I_{12} \cdot R_{12} = 1V$$

$$I_1 = \frac{V_{12}}{R_1} = 1\ A \qquad I_2 = \frac{V_{12}}{R_2} = 0{,}5\ A$$

Sempre in condizioni stazionarie, calcolare:

- Differenza di potenziale $V_a - V_b$
- Energia accumulata nell'induttanza
- Energia accumulata in ciascun condensatore





$$V_a - V_b = V_{12} + V_3 = I_{12}R_{12} + I_3R_3 = 5,5 \text{ V}$$

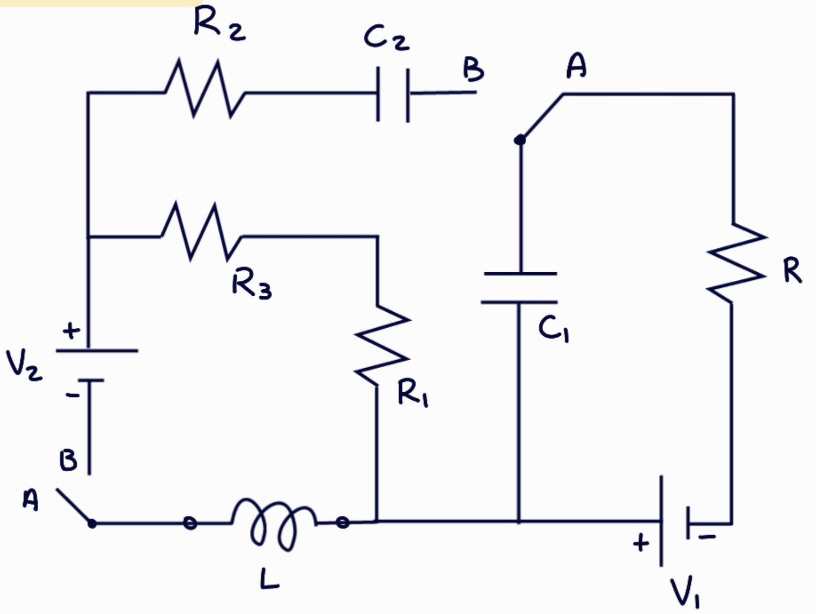
$$U_L = \frac{1}{2} L I_4^2 = 1,125 \ \mu\text{J}$$

$$\Delta V_c = \mathcal{E} - (V_a - V_b) = 4,5 \text{ V}$$

$$U_{C1} = \frac{1}{2} C_1 V_c^2 = 5,1 \ \mu\text{J}$$

$$U_{C2} = \frac{1}{2} C_2 V_c^2 = 15,2 \ \mu\text{J}$$

Esercizio 3



$R_1 = 2\,\Omega$

$R_2 = 4\,\Omega$

$R_3 = 6\,\Omega$

$V_1 = V_2 = 14\,V$

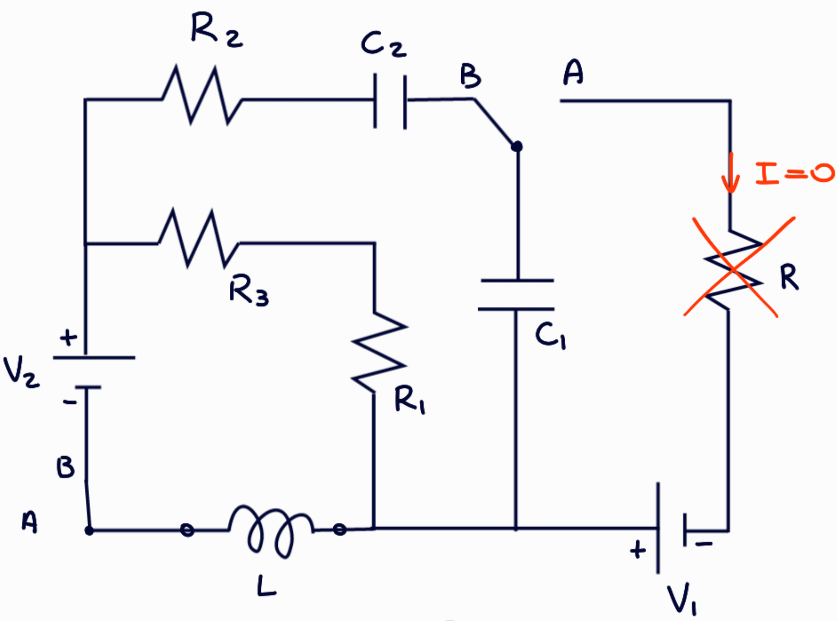
$L = 3\,\mu H$

$C_1 = 0{,}5\,\mu F$

$C_2 = 2\,\mu F$

I due interruttori commutanto a $t=0$ da A a B

- Calcolare corrente in R_1, R_2, R_3 a $t = 0^+$

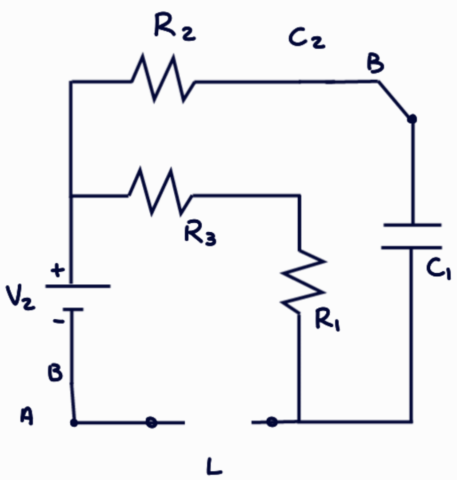


$\left.\begin{array}{l} C_1 \text{ carico} \\ C_2 \text{ scarico} \\ L \text{ scarico} \end{array}\right\} t = 0^-$

$C_2 \approx$ corto circuito

$L \approx$ circuito aperto

↓



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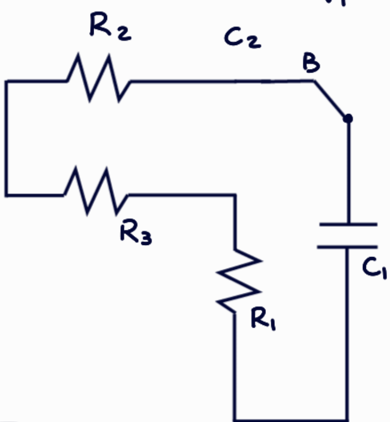

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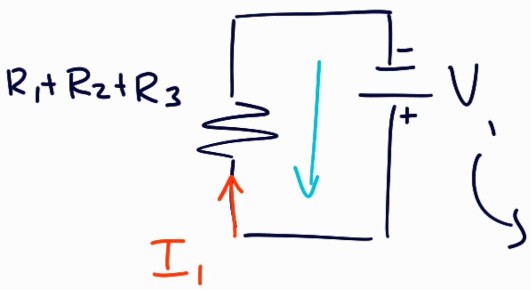


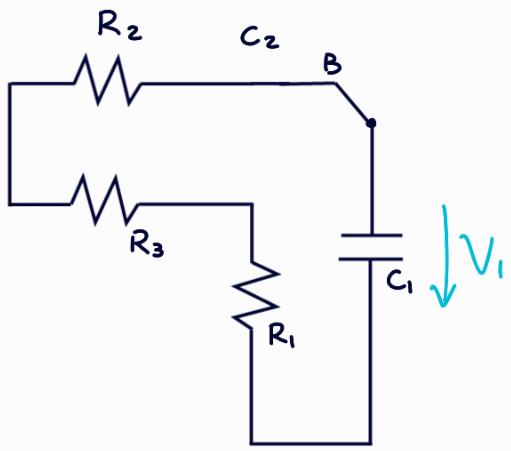
$V_{C_1} = V_1$

$$\boxed{I_1} = \frac{V_1}{R_1+R_2+R_3} = \boxed{1{,}2\ A}$$

$$\boxed{I_3 = I_2}$$

(with $I_1 = I_3$ indicated by "||" beneath)

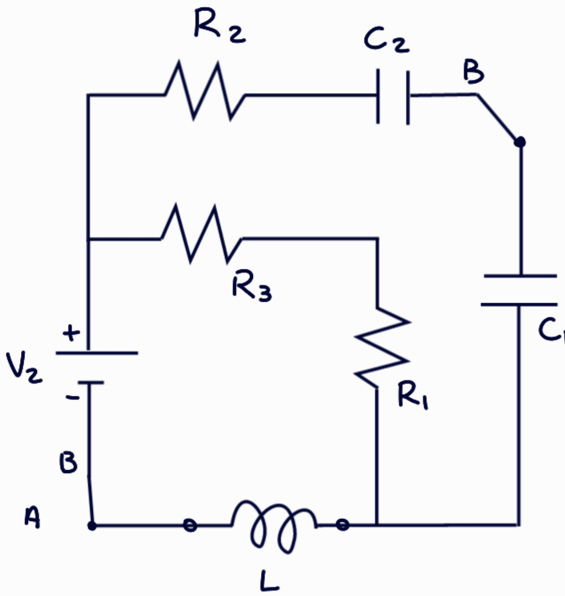
- Calcolare carica di C_1 e C_2 a $t=0^+$

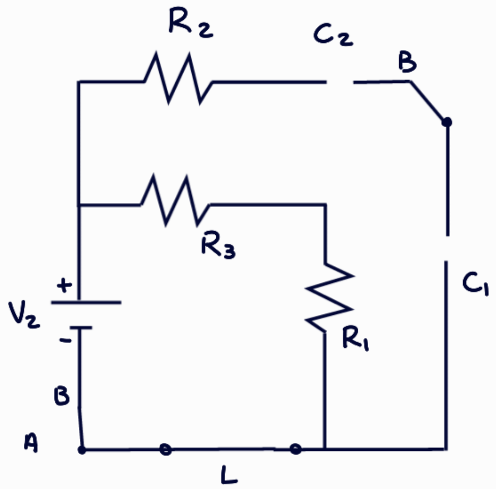


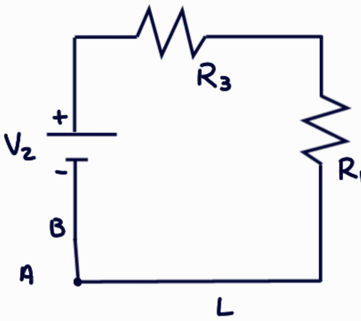
$$V_{C_2}=0 \implies Q_{C_2}=0$$

$$V_{C_1}=V_1 \implies Q_{C_1}=C_1V_1=7\,\mu C$$

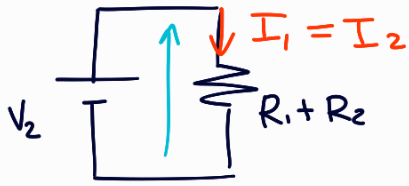
- Calcolare corrente in R_1, R_2, R_3 a $t \to +\infty$





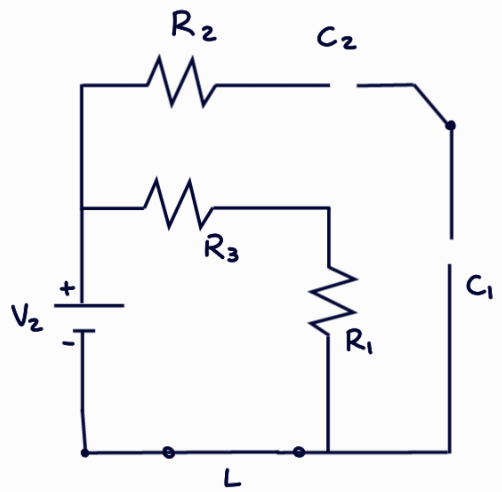


R_2 e' in serie con C_2 che si comporta come un circuito aperto $\implies I_2=0$



$$I_1=I_3=\frac{V_2}{R_1+R_3}=1{,}8\ \text{A}$$

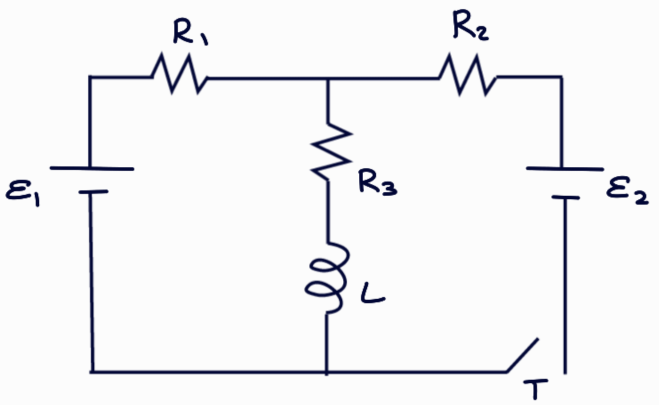
- Calcolare carica di C_1 e C_2 a $t \to +\infty$



$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = 0{,}4\ \mu F$$

$$V_{12} = V_2 \implies Q_{C_{12}} = Q_{C_1} = Q_{C_2} = C_{12} \cdot V_2 = 5{,}6\ \mu C$$

Esercizio 4



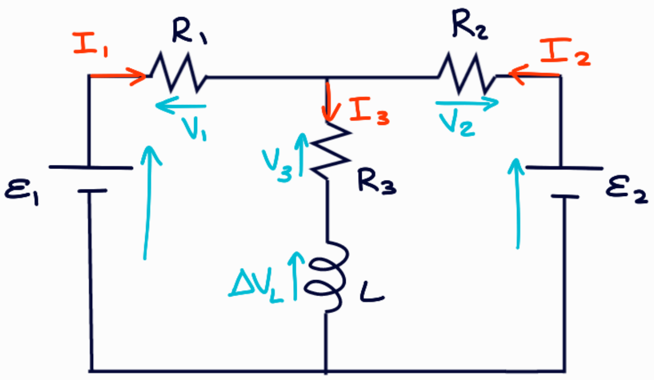
$R_1 = R_2 = R_3 = 1\ \Omega$

$\varepsilon_1 = 6V$

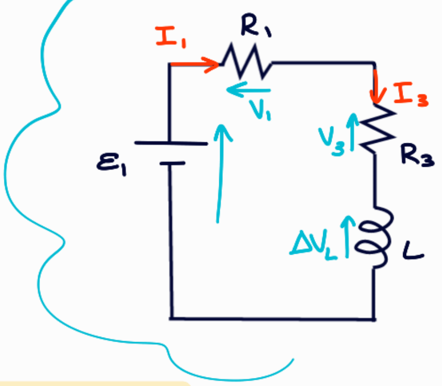
$\varepsilon_2 = 12\ V$

A $t=0$ chiudo T

- Calcolare I_1, I_2, I_3 a $t=0^+$



Per $t<0$:



$$I_1 = I_3 = I_L = \frac{\varepsilon_1}{R_1+R_2} = 3A$$

I_L NON VARIA $\rightarrow$ $I_L = I_3 = 3A$

$I_1 + I_2 - I_3 = 0$

$\Rightarrow I_1 + I_2 = 3A$

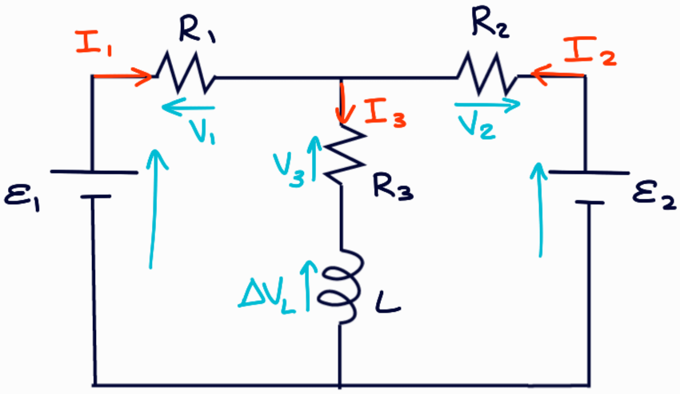
Kirchhoff sulla maglia esterna:

$$\begin{cases} \varepsilon_1 - I_1 R_1 + I_2 R_2 - \varepsilon_2 = 0 \\ I_1 = I_3 - I_2 \end{cases}$$

$$\begin{cases} \varepsilon_1 - R_1 I_3 + I_2(R_1+R_2) - \varepsilon_2 = 0 \\ \rightarrow \end{cases}$$

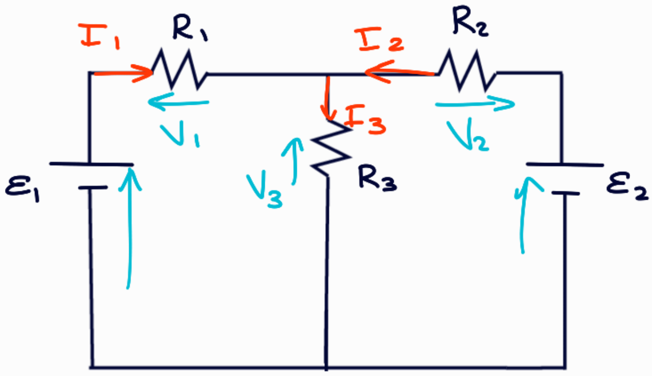
$$\begin{cases} I_2 = \dfrac{\varepsilon_2 - \varepsilon_1 + I_3 R_1}{R_1+R_2} = 4,5\ A \\ I_1 = -1,5\ A \end{cases}$$

- Calcolare I_1, I_2, I_3 a $t \to +\infty$



$\Delta V_L = 0 \to L \approx$ corto circuito

$\to$



Legge di kirchhoff

- sulla maglia SX :

$$\varepsilon_1 - V_1 - V_3 = 0$$

- sulla maglia dX :

$$\varepsilon_2 - V_2 - V_3 = 0$$

- al nodo A : $I_1 + I_2 = I_3$

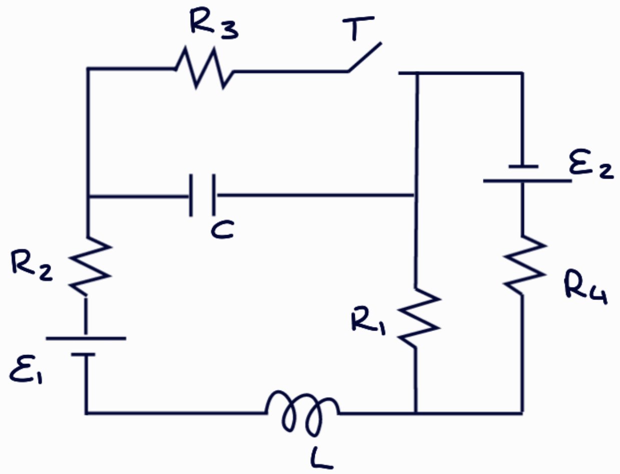
con $R_1 = R_2 = R_3 = R$

$$\Rightarrow \begin{cases} \varepsilon_1 - I_1 R - I_3 R = 0 \\ \varepsilon_2 - I_2 R - I_3 R = 0 \\ I_1 + I_2 = I_3 \end{cases} \quad \begin{cases} \varepsilon_1 - R(I_1 + I_1 + I_2) = 0 \\ \varepsilon_2 - R(I_2 + I_1 + I_2) = 0 \\ \to \end{cases}$$

$$\begin{cases} R(2I_1 + I_2) = \varepsilon_1 \\ R(2I_2 + I_1) = \varepsilon_2 \\ \to \end{cases} \quad \begin{cases} RI_2 = \varepsilon_1 - 2RI_1 \\ 2(\varepsilon_1 - 2RI_1) + RI_1 = \varepsilon_2 \\ \to \end{cases}$$

$$\begin{cases} \to \\ 2\varepsilon_1 - 3RI_1 = \varepsilon_2 \\ \to \end{cases} \quad \begin{cases} I_2 = \dfrac{-\varepsilon_1 + 2\varepsilon_2}{3R} \\ I_1 = \dfrac{2\varepsilon_1 - \varepsilon_2}{3R} \\ \to \end{cases} \quad \begin{cases} I_2 = 6\,A \\ I_1 = 0 \\ I_3 = 6\,A \end{cases}$$

Esercizio 5



$R_1 = 2\,\Omega$

$R_2 = 4\,\Omega$

$R_3 = 6\,\Omega$

$R_4 = 8\,\Omega$

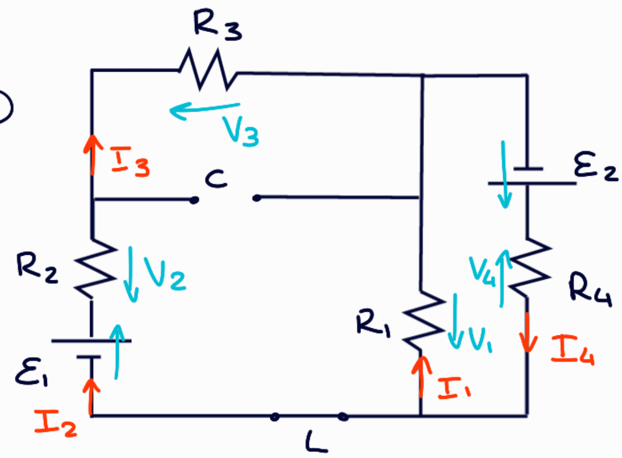
$\mathcal{E}_1 = \mathcal{E}_2 = 12\,V$

A $t=0$ apro T

Calcolare le corrente nelle resistenze e nell'induttore e la differenza di potenziale ai capi del condensatore:

① a $t=0^-$ ② a $t=0^+$ ③ a $t \to +\infty$

①



C e L sono collegati a $\mathcal{E}_1$ e $\mathcal{E}_2$ da molto tempo:

L ≈ corto circuito

C ≈ circuito aperto

$I_2 = I_3$ perché R_2 serie R_3

Leggi di Kirchhoff:

$$\begin{cases} \mathcal{E}_1 - I_2R_2 - I_2R_3 + I_1R_1 = 0 \\ \mathcal{E}_2 - I_4R_4 - I_1R_1 = 0 \\ I_4 = I_1 + I_2 \end{cases}$$

ma $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}$

$$\begin{cases} \mathcal{E} - I_2(R_2+R_3) + I_1R_1 = 0 \\ \mathcal{E} - I_1(R_1+R_4) - I_2R_4 = 0 \\ \rightarrow \end{cases}$$

$$\begin{cases} I_1 = \dfrac{I_2(R_2+R_3) - \mathcal{E}}{R_1} \\ \mathcal{E} - \dfrac{I_2(R_2+R_3) - \mathcal{E}}{R_1}(R_1+R_4) - I_2R_4 = 0 \\ \rightarrow \end{cases}$$

$$\begin{cases} \rightarrow \\ \mathcal{E} R_1 - I_2(R_2+R_3)(R_1+R_4) + \mathcal{E}(R_1+R_4) - I_2 R_4 R_1 = 0 \\ \rightarrow \end{cases}$$

$$\begin{cases} \rightarrow \\ -I_2(R_1R_2 + R_1R_3 + R_2R_4 + R_3R_4 + R_1R_4) + \mathcal{E}(2R_1 + R_4) = 0 \\ \rightarrow \end{cases}$$

$$\begin{cases} \rightarrow \\ I_2 = \dfrac{\mathcal{E}(2R_1+R_4)}{R_1R_2 + R_1R_3 + R_2R_4 + R_3R_4 + R_1R_4} = \dfrac{36}{29}\,A \\ \rightarrow \end{cases}$$

$$I_1 = \frac{I_2(R_2+R_3) - \mathcal{E}}{R_1} = \frac{6}{29}\,A$$

$I_2 = \frac{36}{29}\,A$

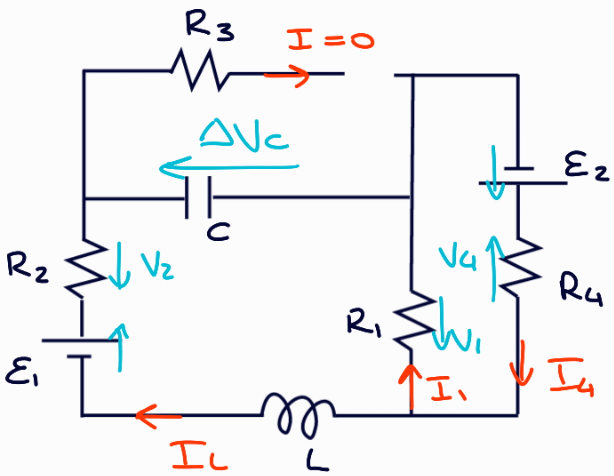
$I_3 = \frac{36}{29}\,A$

$I_4 = \frac{42}{29}\,A$

$I_L = I_2 = \frac{36}{29}\,A$

$\Delta V_c = V_3 = I_3 R_3 = 7{,}45\ V$

② $t = 0^+$



$I_3 = 0A$

I_L e ΔV_c non cambiano

$\Rightarrow I_L = I_2 = \frac{36}{29}\,A$

$\Delta V_c = 7{,}45\ V$

$I_4 = I_1 + I_2$

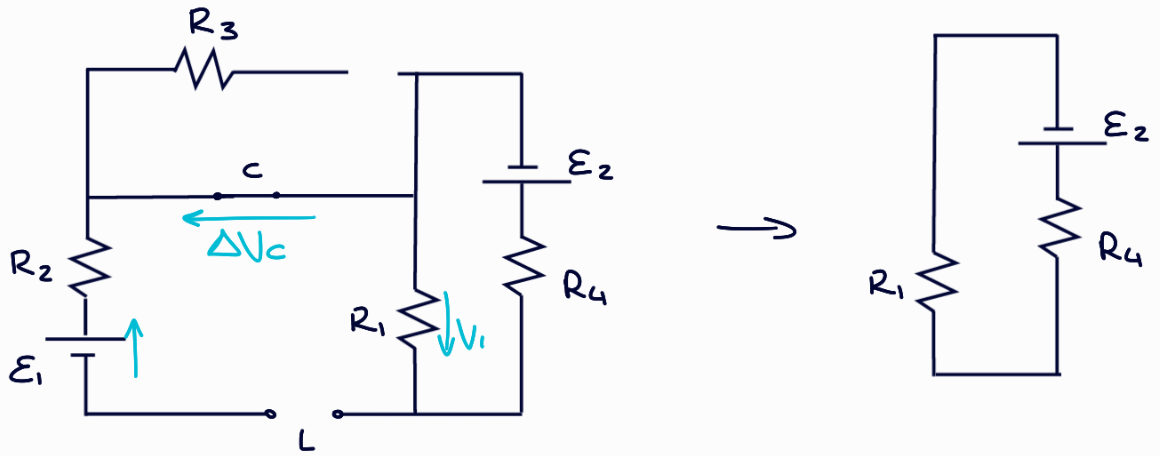
$\mathcal{E}_2 - I_4 R_4 - I_1 R_1 = 0$

$\Rightarrow \mathcal{E}_2 - I_1(R_1 + R_4) - I_2 R_4 = 0$

$$\Rightarrow \quad \boxed{I_1} = \frac{\mathcal{E}_2 - I_2 R_4}{R_1 + R_4} = \boxed{\frac{6}{29}\ A}$$

$$\boxed{I_4 = \frac{42}{29}\ A}$$

③ $t \to +\infty$:



$$\boxed{I_3 = 0\ A}$$

$$\boxed{I_L = 0\ A} \qquad \boxed{I_2 = 0\ A}$$

$$\mathcal{E}_2 - I_4 R_4 - I_1 R_1 = 0 \qquad \text{ma} \qquad I_1 = I_4$$

$$I_1 (R_1 + R_4) = \mathcal{E}_2 \quad \Rightarrow \quad \boxed{I_1} = \frac{\mathcal{E}_2}{R_1 + R_4} = \boxed{\frac{6}{5}\ A}$$

$$\boxed{I_4 = \frac{6}{5}\ A}$$

$$\boxed{\Delta V_c} = \mathcal{E}_1 - V_1 = \mathcal{E}_1 + I_1 R_1 = \boxed{14,4\ V}$$