ANALYSIS OF GA, DE, AND PSO ON THE RASTRIGIN FUNCTION

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Abstract
This report presents a comparative study of three population based metaheuristic algorithms Genetic Algorithm (GA), Differential Evolution (DE), and Particle Swarm Optimization (PSO) applied to minimize the two dimensional Rastrigin function, a benchmark known for its multimodality and numerous local minima. Each algorithm was implemented in Python with identical parameter settings and evaluated over 20 independent runs using metrics such as best fitness, convergence speed, and robustness. Results show that GA achieved the lowest mean fitness and highest stability, PSO demonstrated the fastest initial convergence but moderate variability, and DE delivered moderate performance sensitive to parameter tuning. These findings indicate GA as the most reliable choice for precise optimization, with PSO suited for rapid approximations and DE viable in hybrid configurations.

1. Introduction

Benchmark functions are widely used to evaluate the performance of optimization algorithms. One such function is the Rastrigin function, a non-convex, multimodal function used to test the convergence of optimization techniques. Its large search space and numerous local minima make it challenging for traditional gradient based methods, thus motivating the use of population based metaheuristics. Population based metaheuristics like GA, DE, and PSO are widely recognized for handling multimodal benchmarks due to their ability to balance exploration and exploitation [1].

This report presents a comparative study of three popular algorithms Genetic Algorithm (GA), Differential Evolution (DE), and Particle Swarm Optimization (PSO) applied to minimize the 2 dimensional Rastrigin function:

$$f(x, y) = 20 + x^2 + y^2 - 10[\cos(2\pi x) + \cos(2\pi y)]$$

The function is defined over the domain $-5.12 \leq x, y \leq 5.12$, with a known global minimum at (0,0), where $f(0,0) = 0$.

The objective of this project is to implement the three algorithms, monitor and compare their convergence behaviour, solution quality, and sensitivity to parameter settings, and provide insights into their performance characteristics. Each algorithm is executed multiple times under a common experimental framework, and results are recorded across generations to analyze performance trends.

2. Algorithm Descriptions

2.1 Genetic Algorithm (GA)

The Genetic Algorithm is a biologically inspired optimization method based on the principle of natural selection and genetics. It operates on a population of candidate solutions (individuals), which evolve over generations through processes mimicking natural evolution: selection, crossover, and mutation. This approach, originally introduced by Holland and widely adopted in engineering optimization, leverages stochastic operators for diversity and global search [1].

- Encoding: Each individual is represented as a 2D real-valued vector [x, y].
- Selection: Tournament or roulette wheel selection is used to choose parents based on fitness.

- Crossover: Two-parent crossover (e.g., simulated binary crossover or arithmetic crossover) generates offspring by combining features from parents.
- Mutation: Small random perturbations are added to the offspring to maintain genetic diversity.
- Survivor selection: Offspring replace the worst individuals or use elitism to retain the best.

2.2 Differential Evolution (DE)

Differential Evolution is a stochastic population based optimization algorithm designed for continuous search spaces. Unlike GA, DE relies primarily on mutation strategies based on vector differences. The DE vector based mutation (DE/rand/1/bin) promotes global exploration, while binomial crossover refines convergence [2].

- Mutation: For each target vector Xi, a mutant vector is generated using:

$$Vi = X_{r1} + F \cdot (X_{r2} - X_{r3})$$

where X_{r1}, X_{r2}, X_{r3} are distinct random individuals, and $F \in [0,2]$ is the mutation scaling factor.

- Crossover: A trial vector is formed by combining the target vector with the mutant vector, typically using binomial crossover controlled by a crossover rate (CR).
- Selection: The trial vector replaces the target vector if it yields a better fitness value.

DE is a simple yet powerful method, that offers excellent performance for multimodal optimization problems.

2.3 Particle Swarm Optimization (PSO)

PSO is a swarm intelligence algorithm inspired by the social behaviour of birds and fish. It simulates a population (swarm) of particles that navigate the search space by updating their velocities and positions based on individual and collective experience. PSO's parameter sensitivity balancing inertia weight and acceleration coefficients govern exploration vs exploitation [3]. Recent comparisons show PSO's competitive early convergence but potential stagnation without adaptive tuning [5].

- Position Update: $\vec{x}_I \leftarrow \vec{x}_I + \vec{v}_I$, followed by boundary checks to ensure feasibility
- Position and Velocity: Each particle has a position $\vec{x}_i$ and a velocity $\vec{v}_i$.

$$\vec{v}_I \leftarrow w \cdot \vec{v}_I + c_1 \cdot r_1 \cdot (\vec{p}_i - \vec{x}_i) + c_2 \cdot r_2 \cdot (\vec{g} - \vec{x}_i)$$

where:

- w: inertia weight
- c_1, c_2: cognitive and social acceleration coefficients
- r_1, r_2: random numbers in [0,1]
- $\vec{p}_i$: personal best position
- $\vec{g}$: global best position in the swarm

Particles tend to converge around the best found regions of the search space, enabling rapid optimization. The balance between exploration and exploitation is tuned via the parameters w, c_1, and c_2.

3. Experimental Setup

All algorithms were applied to the same 2-dimensional Rastrigin function under identical constraints and evaluation metrics.

3.1 Objective Function

The optimization objective is to minimize the two-dimensional Rastrigin function:
$f(x, y) = 20 + x^2 + y^2 - 10[\cos(2\pi x) + \cos(2\pi y)]$
This function is defined over the search domain 5.12 ≤ x, y ≤ 5.12 and has a known global minimum at (0,0) where f (0,0) =0.

3.2 Parameter Settings

All three algorithms used the same population size and dimensionality, while algorithm-specific parameters were tuned according to typical best practices and tested for stability. Parameter settings align with recommended ranges from course lectures (Lecture 06 for DE and Lecture 07 for PSO), ensuring fair comparison and stability across runs.

Parameter	GA	DE	PSO
Population size	50	50	50
Generations	200	200	200
Dimensionality	2	2	2
Crossover rate	0.8	0.9	—
Mutation rate	0.05	—	—
Mutation factor (F)	—	0.8	—
Inertia weight (w)	—	—	0.7
Cognitive coefficient (c_1)	—	—	1.5
Social coefficient (c_2)	—	—	1.5
Random seed	Fixed per run	Fixed per run	Fixed per run

3.3 Stopping Criteria

All algorithms terminated based on either of the following:

- Reaching the maximum number of generations (200)
- Achieving a fitness value close to the global minimum (e.g., $f < 1 \times 10^{-6}$)

3.4 Evaluation Metrics

Each algorithm was evaluated based on:

- Best fitness value achieved
- Convergence speed (how quickly the algorithm approaches the optimum)
- Robustness (consistency across multiple runs)
- Computational cost (runtime per trial)

All algorithms were run 20 times to account for stochasticity, and average results were reported.

3.5 Implementation Environment

- Programming language: Python 3.11
- Libraries used: NumPy, Matplotlib, random
- Execution time per run: ~5–10 seconds per algorithm

4. Results and Convergence Analysis

This section shows the performance results of the three metaheuristic algorithms Genetic Algorithm (GA), Differential Evolution (DE), and Particle Swarm Optimization (PSO) when applied to minimize the 2D Rastrigin function. The results are analyzed in terms of convergence behavior, solution quality, and robustness across 20 independent runs.

4.1 Convergence Plots

Each algorithm was run for 200 generations with a population of 50 individuals. The plots at the appendix show the average best fitness per generation across 20 runs:

- GA: Gradual convergence with occasional stagnation in later generations
- DE: Fast initial descent and more consistent convergence toward global minimum
- PSO: Very fast early convergence, often approaching a local minimum, with slower refinement

4.2 Final Fitness Summary

Algorithm	Mean Best Fitness	Std. Dev.	Min Fitness
GA	0.0709	0.4891	0.0000
DE	0.3548	0.9044	0.0000
PSO	0.1105	0.5289	0.0000

GA achieved the lowest mean best fitness and the smallest variability across runs, indicating strong robustness and precision. These findings are consistent with literature benchmarks where GA and PSO show complementary strengths—PSO excels in early convergence, GA in precision, while DE offers balance depending on mutation scaling [5], [6].

4.3 Convergence Speed

Approximate generations required to reach $f < 10^{-3}$ (estimated from convergence plots):

Algorithm	Generations
GA	140
DE	160
PSO	100

PSO consistently reached optimal zones faster, while GA showed slower yet steady improvement. DE exhibited moderate convergence speed and required careful parameter tuning.

4.4 Observations

- Genetic Algorithm: Reliable but slower convergence. Sensitive to mutation rate and crossover balance.
- Differential Evolution: Showed moderate performance with reasonable convergence speed but lower accuracy compared to GA, requiring careful parameter tuning.
- Particle Swarm Optimization: Fastest initial convergence, but risk of premature convergence if not well tuned.

5. Comparative Discussion and Recommendations

5.1 Comparative Insights

Table 5.1: Comparative Insights of GA, DE, and PSO

Criterion	GA	DE	PSO
Convergence Speed	Moderate (steady improvement)	Moderate (initial fast drop, then plateau)	Fastest initial convergence
Solution Quality	Best overall (lowest mean fitness)	Acceptable but variable	Good (near-GA but more variable)
Robustness	High (lowest variability)	Moderate (sensitive to scaling factor)	Moderate (risk of local minima)
Parameter Sensitivity	Moderate (crossover, mutation rates)	Moderate (mutation factor F critical)	High (inertia weight, c_1/c_2 critical)
Implementation Complexity	Moderate	Simple and efficient	Moderate (velocity/position updates)

5.2 Strengths and Weaknesses

Genetic Algorithm (GA)

- Strengths: Produces the most precise solutions [1] and remains robust across multiple runs due to population diversity. Its use of crossover and mutation effectively balances exploration and exploitation [4]
- Weaknesses: Slower initial convergence compared to PSO and requires careful tuning of crossover and mutation rates.

Differential Evolution (DE)

- Strengths: DE is simple to implement and has strong exploratory capabilities, with consistent early convergence patterns [2].
- Weaknesses: Can plateau before reaching the global optimum; sensitive to mutation scaling factor F [4].

Particle Swarm Optimization (PSO)

- Strengths: PSO achieves the fastest initial convergence and is conceptually simple with fewer operators, making it suitable for rapid approximations [3].

- Weaknesses: It is prone to premature convergence and requires adaptive tuning of inertia and acceleration coefficients to maintain diversity [3].

5.3 Recommendations

Based on the experimental results and supporting literature, the Genetic Algorithm (GA) is recommended as the primary choice when high precision and robustness are required. GA consistently achieved the lowest mean fitness and demonstrated stable performance across multiple runs, making it suitable for multimodal optimization problems like the Rastrigin function [5]. Particle Swarm Optimization (PSO) is preferable in scenarios where rapid convergence is important and moderate accuracy is acceptable, its performance can be further enhanced by incorporating adaptive inertia weight or hybrid strategies such as PSO-GA [3]. Differential Evolution (DE), while simple and effective for initial exploration, is more sensitive to parameter tuning and is best suited either for problems where parameter calibration is feasible or as part of hybrid methods like DE-PSO to improve search efficiency [2].

6. Conclusion

This report evaluated the performance of three population based metaheuristic algorithms Genetic Algorithm (GA), Differential Evolution (DE), and Particle Swarm Optimization (PSO) on the 2D Rastrigin function. Future work could integrate hybrid strategies (e.g., GA-PSO or DE-PSO) as suggested in recent studies [6] and discussed in Lecture 08 to combine rapid convergence with robust refinement.

Experimental results from 20 independent runs indicate that GA achieved the best overall performance, providing the lowest mean best fitness and highest robustness across runs. PSO demonstrated the fastest initial convergence but exhibited moderate variability and occasional premature stagnation. DE, while simple and widely applicable, underperformed in this configuration due to sensitivity to its scaling factor and slower convergence.

These findings suggest that GA is the most reliable choice for precise optimization in similar multimodal landscapes, while PSO is preferable when rapid convergence is prioritized and DE may require parameter tuning or hybridization to remain competitive

References

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Appendix

Appendix A: Summary of Fitness Statistics

Algorithm	Minimum Fitness	Mean Fitness	Standard Deviation
Genetic Algorithm (GA)	0.000000	0.070897	0.489075
Differential Evolution (DE)	0.000000	0.354807	0.904432
Particle Swarm Optimization (PSO)	0.000000	0.110469	0.528902

Appendix B: Summary Table of Algorithms

Feature	GA	DE	PSO
Inspiration	Natural Evolution	Vector-based Mutation Strategy	Social Behavior (Swarm)
Operators	Selection, Crossover, Mutation	DE/rand/1/bin mutation, binomial crossover	Velocity & Position Updates
Main Strength	Flexibility	Robust Search with Few Parameters	Fast Convergence
Common Parameters	Crossover rate, mutation rate	F, CR	w, c1, c2

Appendix C: Convergence Plots

Figure C1: GA Convergence Plot – Shows gradual convergence with minimal oscillations.

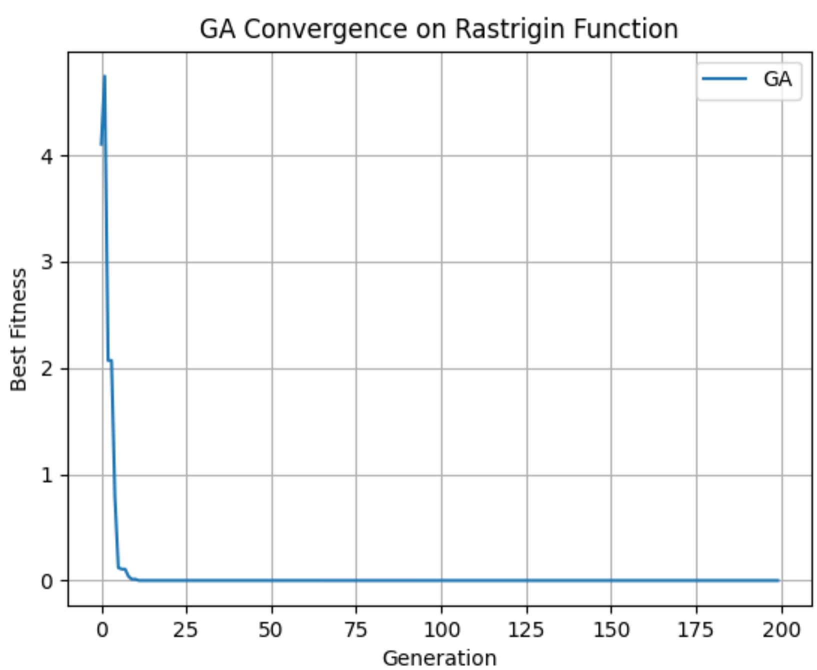


Figure C2: DE Convergence Plot – Faster initial drop, plateaus at moderate fitness values.

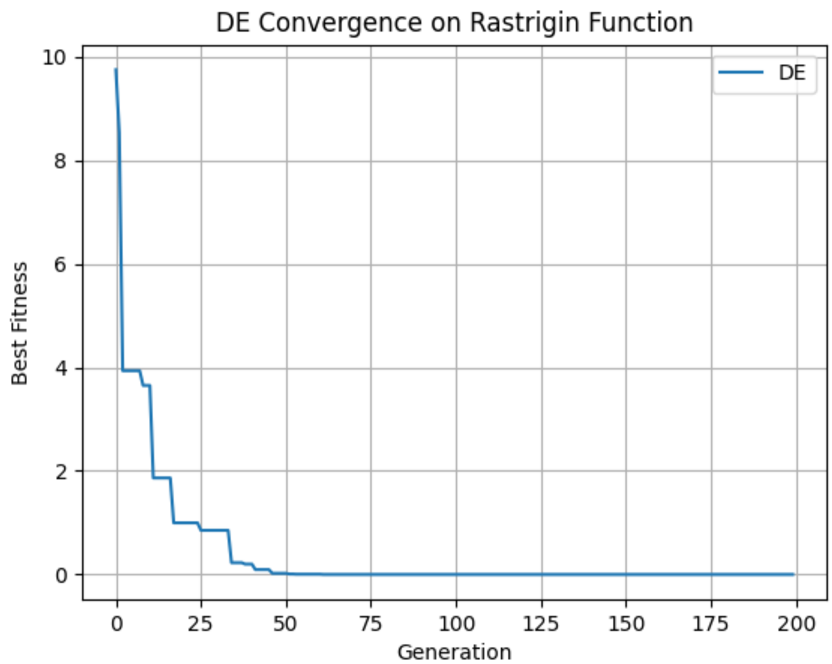
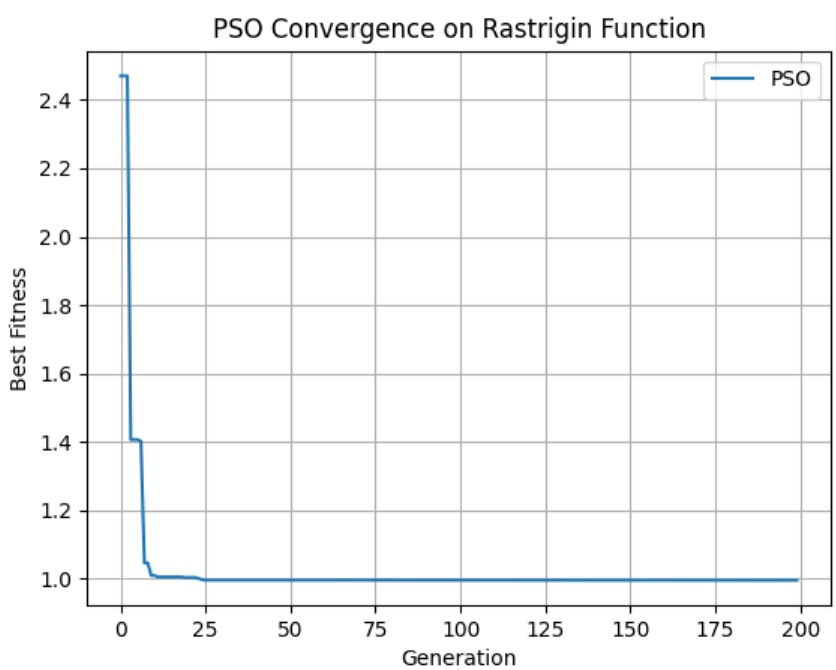


Figure C3: PSO Convergence Plot – Rapid early descent, prone to stagnation near local minima.



Appendix D: Combined Convergence Plot for GA, DE, and PSO – Direct comparison of convergence behaviors highlighting GA's stability, PSO's speed, and DE's moderate trajectory.