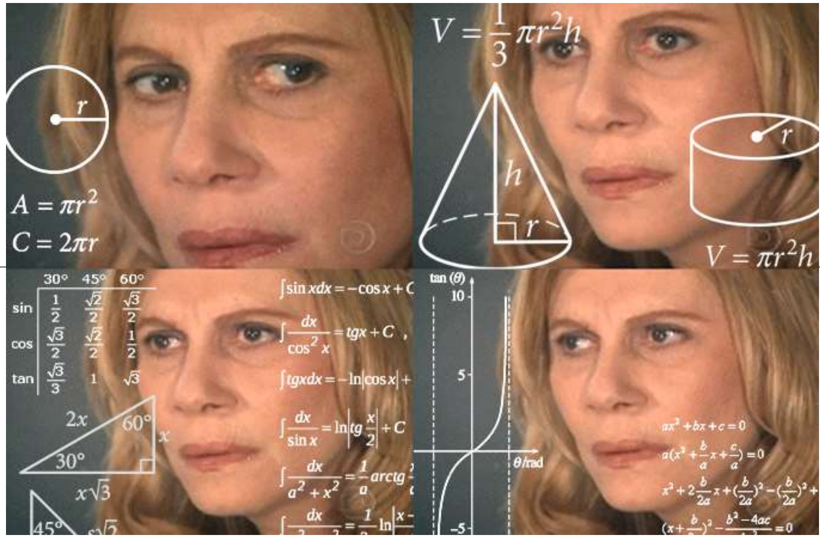
How to:

Binomial theorem



Binomial Theorem in Genetics

- Calculate the chance of obtaining a certain assortment of phenotypes in a group of offspring
- Can only be applied when there are only two possible phenotypes
- You must know the genotypes of the parents so you can calculate the probability of their offspring having a particular phenotype, via a Punnett square

Binomial Theorem equation

$$\frac{n!}{x!(n-x)!} p^x q^{(n-x)}$$

Where: n = number of events (e.g. total number of offspring)
x = number of phenotype 1
n – x = number of phenotype 2
p = probability of phenotype 1 occurring by chance
q = probability of phenotype 2 occurring by chance

! = factorial – multiply that number by all the whole numbers lower than it

Applying the binomial theorem

$$\frac{n!}{x!(n-x)!}p^{x}q^{(n-x)}$$

Always match the probability with the correct number of phenotypes associated with it.

Example 1

- You breed an individual with genotype hh with an individual that has genotype Hh. They have 6 offspring. What is the probability that 4 offspring will have the dominant phenotype **Hairy** and 2 will have the recessive phenotype **Bald**?
- First draw a punnet square of the possible offspring **genotypes** arising from those individuals:

	h	**h**
H	Hh	Hh
h	hh	hh

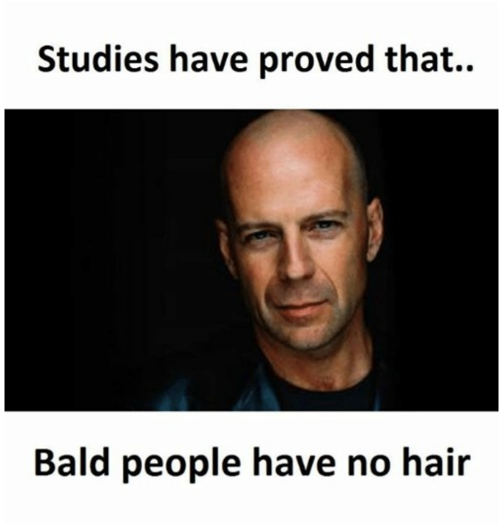
Example 1

- Use the punnet square to calculate the probability of each **phenotype** occurring:

	h	h
H	Hh	Hh
h	hh	hh

Hairy = $^{2}/_{4}$ = 0.5

Bald = $^{2}/_{4}$ = 0.5



Example 1

- Use the probability of each **phenotype** occurring and apply to the binomial theorem:

$$\frac{n!}{x!(n-x)!}p^x q^{(n-x)}$$

q = Hairy = 0.5

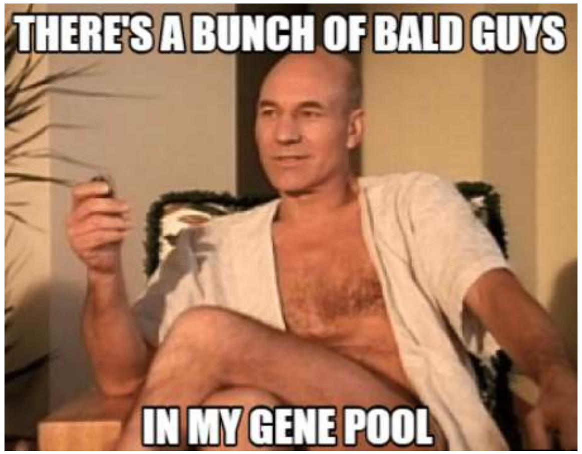
p = Bald = 0.5

- You breed an individual with genotype hh with an individual that has genotype Hh. They have 6 offspring. What is the probability that 4 offspring will have the dominant phenotype **Hairy** and 2 will have the recessive phenotype **Bald**?

$$\frac{6!}{2!(6-2)!}0.5^2 0.5^{(6-2)} \Rightarrow \left(\frac{(6\times5\times4\times3\times2\times1)}{(2\times1)(4\times3\times2\times1)}\right)(0.5^2)(0.5^4) = 0.2344 = 23.44\%$$

Example 2

- You breed an individual with genotype hh with an individual that has genotype hh. They have 6 offspring. What is the probability that 1 offspring will have the dominant phenotype **Hairy** and 5 will have the recessive phenotype **Bald**?
- First draw a punnet square of the possible offspring **genotypes** arising from those individuals:



	h	**h**
h	hh	hh
h	hh	hh

Example 2

- Use the punnet square to calculate the probability of each **phenotype** occurring:

	h	h
h	hh	hh
h	hh	hh

Hairy = $^{0}/_{4}$ = 0

Bald = $^{4}/_{4}$ = 1

Example 2

- Use the probability of each **phenotype** occurring and apply to the binomial theorem:

$$\frac{n!}{x!(n-x)!}p^x q^{(n-x)}$$

q = Hairy = 0

p = Bald = 1

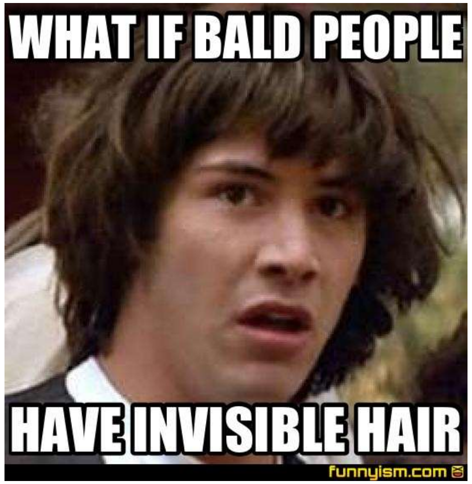
- You breed an individual with genotype hh with an individual that has genotype Hh. They have 6 offspring. What is the probability that 1 offspring will have the dominant phenotype **Hairy** and 5 will have the recessive phenotype **Bald**?

$$\frac{6!}{5!(6-5)!}1^6 0^{(6-5)} \Rightarrow \left(\frac{(6\times5\times4\times3\times2\times1)}{(5\times4\times3\times2\times1)(1)}\right)(1^5)(0^1) = 0$$

- The chance of having all bald offspring is 100%

Example 3

- You breed an individual with genotype Hh with an individual that has genotype Hh. They have 6 offspring. What is the probability that 6 offspring will have the dominant phenotype **Hairy** and 0 will have the recessive phenotype **Bald**?
- First draw a punnet square of the possible offspring **genotypes** arising from those individuals:



	H	**h**
H	HH	Hh
h	Hh	hh

Example 3

- Use the punnet square to calculate the probability of each **phenotype** occurring:

	H	h
H	HH	Hh
h	Hh	hh

Hairy = $^{3}/_{4}$ = 0.75

Bald = $^{1}/_{4}$ = 0.25

Example 3

- Use the probability of each **phenotype** occurring and apply to the binomial theorem:

$$\frac{n!}{x!(n-x)!}p^{x}q^{(n-x)}$$

q = Hairy = 0.75

p = Bald = 0.25

- You breed an individual with genotype hh with an individual that has genotype Hh. They have 6 offspring. What is the probability that 6 offspring will have the dominant phenotype **Hairy** and 0 will have the recessive phenotype **Bald**?

$$\frac{6!}{0!(6-0)!}0.25^{0}0.75^{(6-0)} \Rightarrow \left(\frac{(6\times5\times4\times3\times2\times1)}{(1)(6\times5\times4\times3\times2\times1)}\right)(0.25^{0})(0.75^{6}) = 0.178 = 17.8\%$$

Example 3 – Is that maths correct?

- $\frac{6!}{0!(6-0)!}0.25^{0}0.75^{(6-0)}$ WITHOUT CALCULATOR: $\left(\frac{(6\times5\times4\times3\times2\times1)}{(0)(6\times5\times4\times3\times2\times1)}\right)(0.25^{0})(0.75^{6})$ = ERROR

- With Calculator:
  - 0! = 1
  - 0.25^{0} = 1

$$\frac{6!}{0!(6-0)!}0.25^{0}0.75^{(6-0)} = 0.178 = 17.8\%$$

- Mathematical reasoning: 0! & x^{0} = 1

- YouTube video explanations (mathematical tricks):
  - Why 0! = 1 < https://www.youtube.com/watch?v=Mfk_L4Nx2ZI&t >
  - Why x to the power of 0 = 1 < https://www.youtube.com/watch?v=EwlMSnMiJvc&t= >