1 Product rule and quotient rule

1.1 Combinations of functions

1. (i) is clearly a quotient ($\sin x$ and divided by $x+1$).

   (ii) xe^x, Product of functions (x multiplied by e^x).

   (iii) is a function of a function (the first set of operations on x being the function $(3x^2+5x-8)$, followed by the ln function).

   (iv) is a function of a function (the first set of operations on x being the function (x^3-4), followed by raising to the power 5)

   (v) is a function of a function (the first set of operations on x being the function (x^2-4x+5), followed by the cosine function).

2. (i) Product of functions.

   (ii) Function of a function.

   (iii) Quotient of a functions.

   (iv) Function of a function.

   (v) Function of a function .

1.2 Product rule

1. (i) $y=(2x-1)\sin x$, $\frac{dy}{dx}=(2x-1)'\sin x+(2x-1)(\sin x)'=2\sin x+(2x-1)\cos x$.

   (ii) $y=(3x^2-6x+1)e^x$, $\frac{dy}{dx}=(3x^2-6x+1)'e^x+(3x^2-6x+1)(e^x)'=(6x-6)e^x+(3x^2-6x+1)e^x=e^x(6x-6+3x^2-6x+1)=e^x(3x^2-5)$.

   (iii) $(6x+5)\ln x$, $\frac{dy}{dx}=(6x+5)'\ln x+(6x+5)(\ln x)'=6\ln x+\frac{6x+5}{x}$.

   (iv) $y=x^2\cos x$, $\frac{dy}{dx}=(x^2)'\cos x+x^2(\cos x)'=2x\cos x-x^2\sin x$.

   (v) $y=x\sin(2x)$, $\frac{dy}{dx}=x'\sin(2x)+x[\sin(2x)]'=\sin(2x)+x\cos(2x)\times 2=\sin(2x)+2x\cos(2x)$.

   (vi) $y=e^{3x}\cos x$, $\frac{dy}{dx}=(e^{3x})'\cos x+e^{3x}(\cos x)'=3e^{3x}\cos x-e^{3x}\sin x$.

2. (i) $y=x^3\sin x$, $\frac{dy}{dx}=(x^3)'\sin x+x^3(\sin x)'=3x^2\sin x+x^3\cos x$.

(ii) $y = (6x^2 - 12x + 5)e^x$, $\frac{dy}{dx} = (6x^2 - 12x + 5)'e^x + (6x^2 - 12x + 5)(e^x)' = (12x - 12)e^x + (6x^2 - 12x + 5)e^x = e^x(6x^2 - 7)$.

(iii) $(7x - 4)\ln x$, $\frac{dy}{dx} = (7x - 4)' \ln x + (7x - 4)(\ln x)' = 7\ln x + (7x - 4) \times \frac{1}{x} = 7\ln x + \frac{x}{7x-4}$.

(iv) $y = (2x^2 + 3)\cos x$, $\frac{dy}{dx} = (2x^2 + 3)' \cos x + (2x^2 + 3)(\cos x)' = 4x\cos x - (2x^2 + 3)\sin x$.

(v) $y = x\cos(4x)$, $\frac{dy}{dx} = x' \cos(4x) + x(\cos(4x))' = \cos(4x) - x\sin(4x) \times 4 = \cos(4x) - 4x\sin(4x)$.

(vi) $y = e^{-x}\sin x$, $\frac{dy}{dx} = (e^{-x})' \sin x + e^{-x}(\sin x)' = -e^{-x}\sin x + e^{-x}\cos x = e^{-x}(\cos x - \sin x)$.

1.3 Quotient rule

1. (i) $y = \frac{2x-3}{2x+7}$, $\frac{dy}{dx} = \frac{(2x-3)'(2x+7)-(2x-3)(2x+7)'}{(2x+7)^2} = \frac{2(2x+7)-(2x-3)2}{(2x+7)^2} = \frac{4x+14-4x+6}{(2x+7)^2} = \boxed{\frac{20}{(2x+7)^2}}$.

   (ii) $y = \frac{e^{4x}}{x+6}$, $\frac{dy}{dx} = \frac{(e^{4x})'(x+6)-e^{4x}(x+6)'}{(x+6)^2} = \frac{4x^{4x}(x+6)-e^{4x}}{(x+6)^2}$

   (iii) $y = \frac{x^2-4}{2x^2+1}$, $\frac{dy}{dx} = \frac{(x^2-4)'(2x^2+1)-(x^2-4)(2x^2+1)'}{(2x^2+1)^2} = \frac{2x(2x^2+1)-(x^2-4)4x}{(2x^2+1)^2} = \frac{8x^3+2x-4x^3+16x}{(2x^2+1)^2} = \frac{4x^3+18x}{(2x^2+1)^2}$.

   (iv) $y = \frac{\ln x}{x^2}$, $\frac{dy}{dx} = \frac{(\ln x)'x^2-(\ln x)(x^2)'}{(x^2)^2} = \frac{\frac{x^2}{x}-2x\ln x}{x^4} = \frac{x(1-2\ln x)}{x^4} = \frac{1-2\ln x}{x^3}$.

   (v) $y = \frac{\sin(2x)}{x}$, $\frac{dy}{dx} = \frac{(\sin(2x))'x+\sin(2x)(x)'}{x^2} = \frac{2x\cos x+\sin(2x)}{x^2}$.

   (vi) $y = \frac{\sin x}{\cos x}$, $\frac{dy}{dx} = \frac{(\sin x)'\cos x-\sin x(\cos x)'}{cos^2x} = \frac{cos^2x+\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$.

2. (i) $y = \frac{3x-7}{7x-2}$, $\frac{dy}{dx} = \frac{3(7x-2)-(3x-7)7}{(7x-2)^2} = \frac{21x-6-21x+49}{(7x-2)^2} = \boxed{\frac{42}{(7x-2)^2}}$.

   (ii) $y = \frac{e^{2x}}{4x+1}$, $\frac{dy}{dx} = \frac{2e^{2x}(4x+1)-e^{2x}\times 4}{(4x+1)^2} = \frac{2(4x+1)e^{2x}-4e^{2x}}{(4x+1)^2} = \frac{e^{2x}(8x-2)}{(4x+1)^2}$.

   (iii) $y = \frac{2x^2+3}{x^2+1}$, $\frac{dy}{dx} = \frac{4x(x^2+1)-(2x^2+3)2x}{(x^2+1)^2} = \frac{4x^3+4x-4x^3-6x}{(x^2+1)^2} = \frac{-2x}{x^2+1)^2}$.

   (iv) $y = \frac{\ln x}{2x^3}$, $\frac{dy}{dx} = \frac{\frac{1}{x}2x^3-\ln x(6x^2)}{(2x^3)^2} = \frac{2x^2-6x^2\ln x}{4x^6} = \frac{2x^2(1-3\ln x)}{4x^6} = \frac{1-3\ln x}{2x^4}$

   (v) $y = \frac{\sin(5x)}{x}$, $\frac{dy}{dx} = \frac{5\cos(5x)\times x-\sin(5x)}{x^2} = \frac{5x\sin(5x)-\sin(5x)}{x^2}$.

   (vi) $y = \frac{\cos x}{\sin x}$, $\frac{dy}{dx} = \frac{-\sin x\sin x-\cos x\cos x}{\sin^2 x} = \frac{-(\cos^2 x+\sin^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x}$.