9.3D

Due to an excessive amount of rain, the village dam is filling at a rate of $310(11+3t)$ litres an hour $(t \geq 0)$. The mayor is planning to evacuate the village before the dam overflows, flooding the valley. The amount of water currently in the dam is $3,090,000$ litres and the mayor wants to know how many hours it will take for the dam to be completely full. The dam is 60 metres long, 20 metres wide and 10 metres deep. You can assume that the villagers are not using any of the dam water and there is no water loss through evaporation.

Information we have
$310(11+3t)$ litres an hour $(t \geq 0)$

- Currently $3,090,000$ litres at (presumably) hour zero
- Dam is $60 \times 20 \times 10$ metres

1) What is the maximum capacity of the dam in litres?

Hint: $1000L = 1\text{m}^3$

The dam is $60 \times 20 \times 10$ metres $= 12,000\text{m}^3$.
Since $1\text{m}^3 = 1000L$,
-> $12,000\text{m}^3 \times 1000L = 12,000,000L$

The maximum capacity of the dam is 12 million litres.

2) Find the function that gives the number of litres in the dam at time t hours.

We are at hour zero and the dam is at $3,090,000$ litres full.

Since the equation $\frac{L}{t} = 310(11+3t)$ gives the litres/hour (litres/t in this) when what we want is only the litres given a time, that must mean it detects the change in gradient and therefore is a derivative of an equation. It is likely not a double-derivative as the omniscient mayor has not mentioned any acceleration of the rainfall (and there is no extra constant).

$310(11+3t)$, $t \geq 0$
-> $\int(3410+930t)dt$
-> $\frac{3410}{1}t^{0+1} + \frac{930}{2}t^{1+1} + C$
$= 3410t + 465t^2 + C$

If we evaluate this integral for $t = 0$...
-> $3410(0) + 465(0)^2 + C$
-> $0 + 0 + C$
$f(0) = C$
All we get is C. Hour zero should have the starting $3,090,000$ litres, so C must be exactly that. Now the equation gives the correct answer.

Final equation to find L
$L = 465t^2 + 3410t + 3,090,000,\ t \geq 0$

3) The village mayor wants to have the town evacuated before the dam is full. At what time t would the dam be full?

Litres-given-time equation: $L = 465t^2 + 3410t + 3,090,000$
Max capacity: $12,000,000L$

Since we want to find the time, we plug the maximum capacity we found earlier into the equation and turn the equation into a quadratic:
$12,000,000 = 465t^2 + 3410t + 3,090,000$
-> $0 = 465t^2 + 3410t + 3,090,000 - 12,000,000$
-> $0 = 465t^2 + 3410t - 8,910,000$
-> $t = \frac{-(3410) \pm \sqrt{(3410)^2 - 4(465)(-8{,}910{,}00)}}{2(465)}$
-> $t = \frac{-3410 \pm \sqrt{11{,}628{,}100 + 16{,}572{,}600{,}000}}{930}$
-> $t = \frac{-3410 \pm \sqrt{16{,}584{,}228{,}100}}{930}$
$t^+ = \frac{-3410 + \sqrt{16{,}584{,}228{,}100}}{930} \approx 134.81$ hours
$t^- = \frac{-3410 - \sqrt{16{,}584{,}228{,}100}}{930} \approx -142.14$ hours
We can discard negative times both as they don't make sense in this context and due to the parameter of $t \geq 0$.

So, the time it would take for the dam to entirely fill up would be ≈ 134.81 hours which is ≈ 5.62 days, or more precisely, $\frac{-3410 + \sqrt{16{,}584{,}228{,}100}}{930}$ hours.

In common language, it is approximately 5 days, 14 hours, 48 minutes, 22 seconds, and 319ms.

Days and hours: $\frac{134.80619}{24} = 5.6169245833$ days, 5 days and $134 \bmod 24 = 14$ hours.
Minutes: $0.806... \times 60 = 48$ minutes and $0.371..$
Seconds $0.371.. \times 60 = 22$ seconds and 0.319
Milliseconds: $0.319.. \times 1000 = 319$ milliseconds and $0.5338...$
....
533 microseconds
866 nanoseconds
523 picoseconds
502 femtoseconds
881 attoseconds
754 zeptoseconds
626 yoctoseconds
409 rontoseconds
56483154264857495 quectoseconds (10^{-30})
...and...
$7196715351536936...$ Planck time (10^{-44})

Or put in a more digestible format,
$5\text{d} : 14\text{h} : 48\text{m} : 22\text{s} : 319\text{ms} : 533\mu\text{s} : 866\text{ns} : 523\text{ps} : 502\text{fs} : 881\text{as} : 754\text{zs} : 626\text{ys} : 409\text{rs} :$

$564831542648574 95\mathrm{qs} : 7196715351536936..\,\mathrm{t_P}$

I hope this is a good enough approximation for the mayor. Imagine if I got this wrong after all that... the mayor would not be happy with me.