Task 1 - Finding the equation of a line

1: Find the equation of the line given $(2, 3)$ *and* $(5, 12)$
$m = \frac{y_2 - y_1}{x_2 - x_1}$
-> $m = \frac{12-3}{5-2}$
-> $m = \frac{9}{3} = 3$

$y = mx + c$
-> $3 = 3 \times 2 + c$
-> $3 = 6 + c$
-> $c = -3$

Final equation: $y = 3x - 3$

Verification
Testing with $(5, 12)$
$y = 3(5) - 3$
-> $y = 15 - 3$
-> $y = 12$

Testing with $(2, 3)$
$3 = 3x - 3$
-> $6 = 3x$
-> $x = 2$

2: Explain how you checked that your answer was correct
I took the known points $(2, 3) \& (5, 12)$ and substituted either the x or y value to check if it got the corresponding x/y value.
Another method would be to plot them out.

Task 2 - Simultaneous Equations - graphical interpretation

Set 1

$$\begin{cases} x + 4y = 2 \\ 4x + 8y = 7 \end{cases}$$

Answers

Equation	x-intercept	y-intercept	Point of Intersection	Solutions
$x + 4y = 2$	2	$\frac{1}{2}$	-	-
$4x + 8y = 7$	$\frac{7}{4}$	$\frac{7}{8}$	-	-
System	-	-	$(\frac{3}{2}, \frac{1}{8})$	One unique

Working out

Finding the point of intersection

Finding x

$x + 4y = 2$

-> times 2: $2x + 8y = 4$

$4x + 8y = 7$

$2x + 8y = 4$

minus

$4x + 8y = 7$

-> $-2x = -3$

-> $x = \frac{-3}{-2} = \frac{3}{2}$

Finding y

$\frac{3}{2} + 4y = 2$

-> $4y = \frac{4}{2} - \frac{3}{2}$

-> $4y = \frac{1}{2}$

-> $y = \frac{1}{8}$

Solution

$(\frac{3}{2}, \frac{1}{8})$

x- and y-intercepts

x-intercepts

$x + 4y = 2$

-> $x + 4(0) = 2$

-> $x\text{-intercept} = 2$

$4x + 8y = 7$

-> $4x + 8(0) = 7$

-> $x\text{-intercept} = \frac{7}{4}$

y-intercepts

$x + 4y = 2$

-> $0 + 4y = 2$

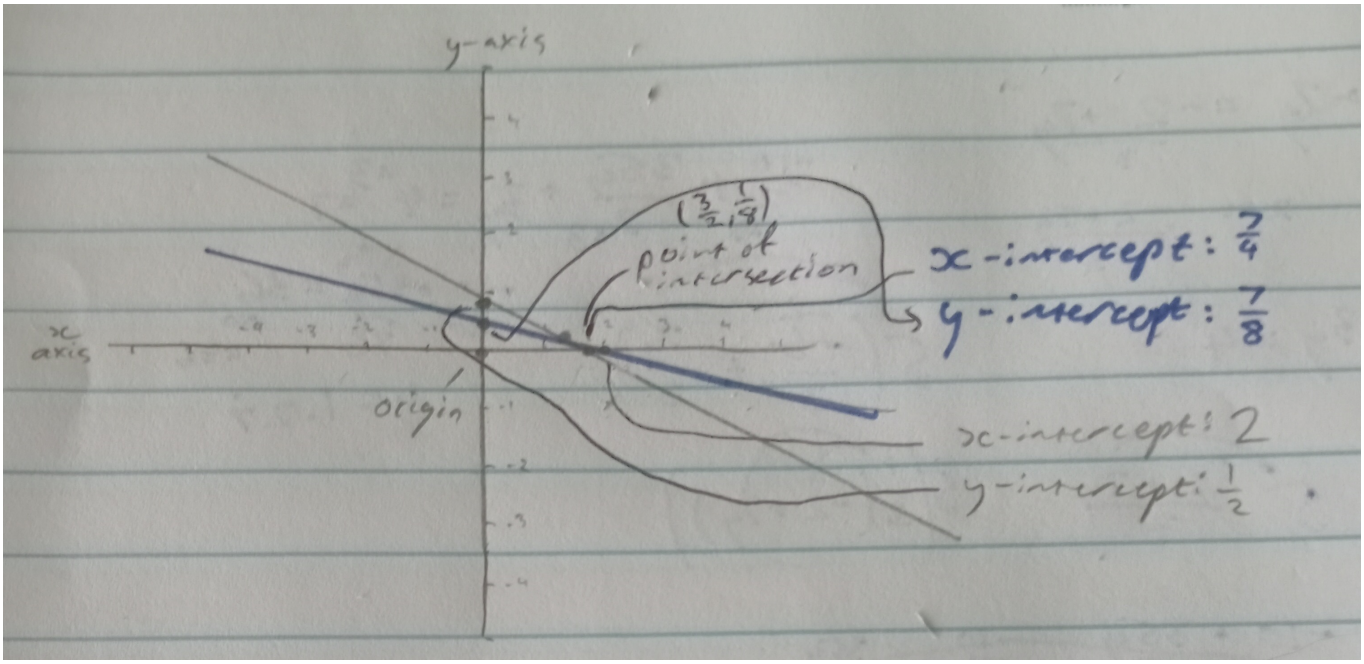
-> $y\text{-intercept} = \frac{2}{4} = \frac{1}{2}$

$4x + 8y = 7$

-> $4(0) + 8y = 7$

-> $y\text{-intercept} = \frac{7}{8}$

Graph



As visible, the two equations intersect at a point. This intersection is the sole solution. There are no more solutions as the lines diverge from each other after the intersect.

Set 2

$$\begin{cases} 3x - 6y = 9 \\ 8y - 4x = -12 \end{cases}$$

Answers

Equation	x-intercept	y-intercept	Point of Intersection	Solutions
$3x - 6y = 9$	3	$-\frac{3}{2}$	-	-
$8y - 4x = -12$	3	$-\frac{3}{2}$	-	-
System	-	-	Infinite points	Infinite solutions

Working out

$3x - 6y = 9$
-> times 4: $12x - 24y = 36$
$8y - 4x = -12$
-> times 3: $-12x + 24y = -36$

$12x - 24y = 36$
plus
$-12x + 24y = -36$
-> $0x + 0y = 0$

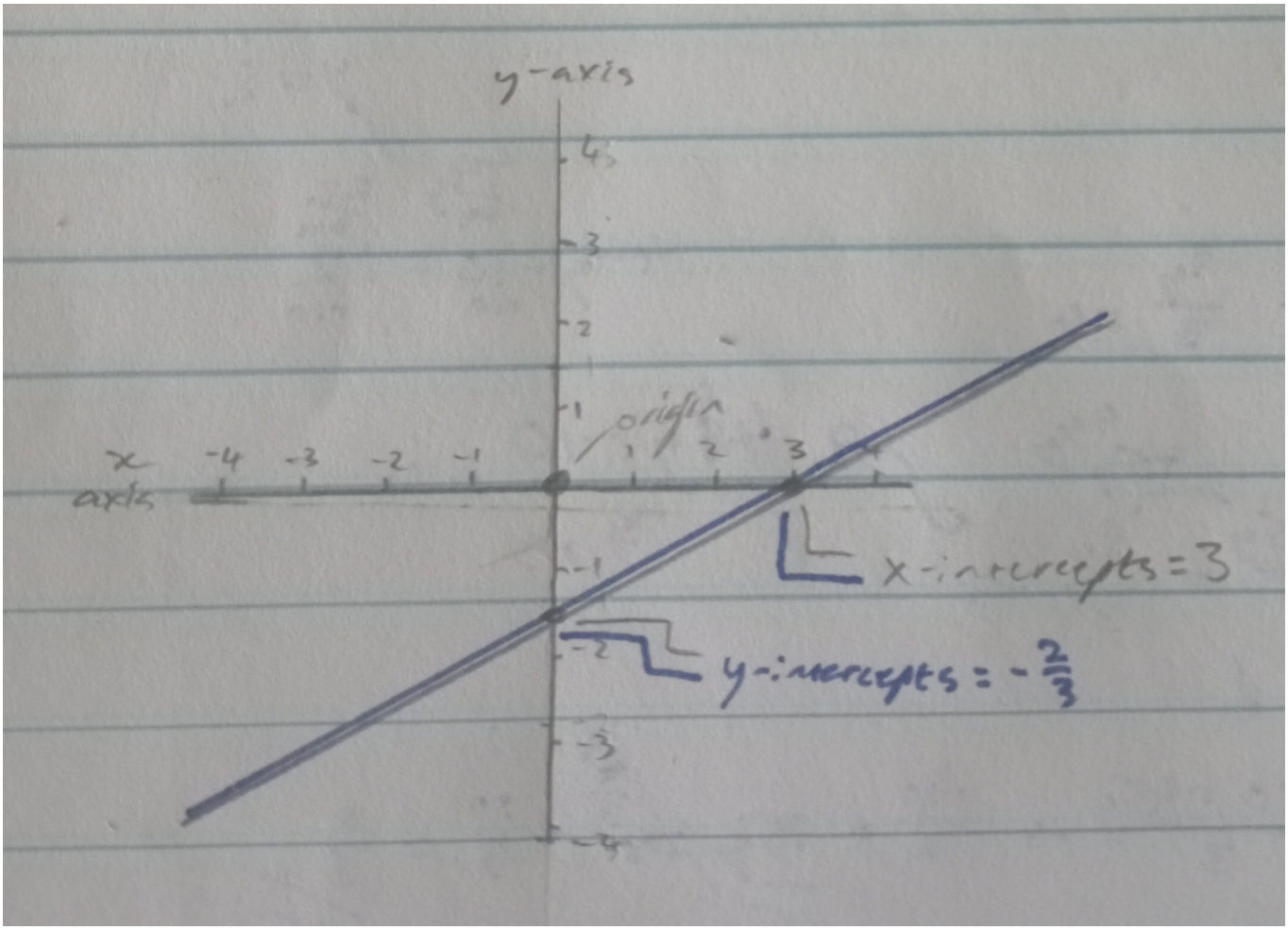
There are infinite solutions/points of intersection.

x-intercept (identical in both equations)
$3x - 6y = 9$

-> $3x - 6(0) = 9$
-> $x = \frac{9}{3} = 3$

y-intercept (identical in both equations as equations are identical)
$3x - 6y = 9$
-> $3(0) - 6y = 9$
-> $y = -\frac{9}{6} = -\frac{3}{2}$

Graph



The lines are identical to each other, meaning there are no unique solutions but infinite solutions as there are infinite points of intersection. There will always be infinite solutions as identical lines/equations will never separate.

Task 3: Matrices

\1. Shift the point $(5, 6)$ using $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$
Point: $(5 - 1, 6 + 1) = (4, 7)$

\2. Shift the point $(2, 7)$ using $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$
Point: $(2 - 1, 7 + 1) = (1, 8)$

\3. Find the line from these new points and compare to task 1 line

$y = mx + c$

-> $m = \frac{7-6}{2-5} = -\frac{1}{3}$

Finding c

Using $(5, 6)$

$6 = -\frac{1}{3} \times 5 + c$

-> $6 = -\frac{5}{3} + c$

-> $6 + \frac{5}{3} = c$

-> $\frac{18}{3} + \frac{5}{3} = c$

-> $c = \frac{23}{3}$

Final equation

$y = -\frac{1}{3}x + \frac{23}{3}$

Task 1 line: $y = 3x - 3$

Comparing the task 1 and task 3 points, the gradients are:
$\frac{-1}{3}$ and $\frac{9}{3}$. The task 1 gradient veers slightly negative (downwards) as x increases while the task 2 gradient increases by an extra 10 thirds to create a positive gradient, where every x means a dramatic increase of y