SIT190 - MAGI - ALGEBRA - ONTRACK ASSESSMENT

TRIMESTER 1, 2024

Task 1: Simultaneous Equations

A linear equation in two variables is the equation of a line. A linear equation in three variables gives the equation of a plane.

In the case of two linear equations in two variables, you can have:

- a unique solution (e.g. $x = 5, y = 2$),
- infinitely many solutions (i.e. the lines are the same), or
- no solutions (i.e. the lines are parallel)

In the case of three linear equations in three variables, you can have:

- a unique solution (eg. $x = 3, y = 1, z = 5$)
- infinitely many solutions
- no solutions

(1) Solve the following set of simultaneous equations:

$$\begin{aligned} x + 2y + 3z &= 5 \\ 2x + 5y + 3z &= 3 \\ x \ \ + 8z &= 17 \end{aligned}$$

(2) The following set of simultaneous equations have infinitely many solutions. Solve for z and identify the line where the two planes intersect.

$$\begin{aligned} 3x + 5y - 4z &= 7 \\ -3x - 2y + 4z &= -1 \\ 6x + y - 8z &= -4 \end{aligned}$$

Task 2: Matrices

The point (x, y) can be represented as the matrix $\begin{bmatrix} x \\ y \end{bmatrix}$. In this task, we look at how matrix multiplication can be used to rotate a point (x, y) around the origin.

2.1 Give the 2×2 rotation matrix M such that $M \times \begin{bmatrix} x \\ y \end{bmatrix}$ gives the point rotated by θ degrees around the origin in an anticlockwise direction.

2.2 Find $M \times \begin{bmatrix} x \\ y \end{bmatrix}$ when $\theta = 180^o$ and explain what happens to the point (x, y) when this rotation is applied.

Task 3: Functions

(1) Compare the functions $y = t^2$, $y = (t+1)^2$ and $y = t^2 + 1$.
   - Give the range and domain of each function.
   - The graph $y = (t+1)^2$ is the graph $y = t^2$ shifted in which direction and by how many units?
   - The graph $y = t^2 + 1$ is the graph $y = t^2$ shifted in which direction and by how many units? ?
   - What is the function obtained by shifting the graph $y = t^2$ one unit down and one unit to the right?
   - What is the function obtained by shifting the graph $y = t^2$ up h units and k units to the right?

(2) Compare the functions $y = \ln(t)$, $y = \ln(t+1)$ and $y = \ln(t) + 1$.
   - Explain the effect of the "$+1$" in $y = \ln(t+1)$ and $y = \ln(t)+1$ respectively.

(3) Solve the following equations for x giving all solutions in the given domain.
   - $2\sin(2x + \pi/4) = \sqrt{2}$, $x \in [-\pi, \pi]$
   - $\sqrt{2}\cos(x) + 1 = 0$, $x \in [0, 2\pi]$

   (You may find it useful to sketch the graphs to ensure you find all solutions.)

(4) Briefly summarise what you have learned about functions in this task.

Submission

In order to complete this task, you must submit the following:

1.1 Solve the set of simultaneous equations showing all working.
1.2 Solve the set of simultaneous equations, identifying the correct value for z and the equation of the line, showing all working.
2.1 Give the matrix M
2.2 Provide all working for the matrix rotation of the point (x, y) when $\theta = 180$ degrees. Explain what the rotation does to the point.
3.1 Compare the functions by comparing their domains and ranges; and how the graph $y = x^2$ is shifted. Provide the function that shifts the function $y = x^2$ down one unit and one unit to the right. Provide the general function to shift $y = x^2$ up and/or to the right.
3.2 A comparison of the three log functions. You should draw on the comparisons used in the previous question.
3.3 All working and solutions in radians.
3.4 A concise summary of what you learned in this task.

Useful Resources

Although the unit material will help you complete these tasks, you will need to do some independent research. Here are some suggestions:

- Task 1:
  - You can use the elimination and substitution techniques for solving simultaneous equations for this task.
  - Look out for lines of the form $ax + by = c$ in Task 1.2.
- Task 2:

  - 'Google' some key words eg "rotation matrix". An image search might help you.
  - There is plenty of interesting material beyond the scope of this course. So if you are reading about normal vectors or parameterisation of lines then you are probably going past what this task requires (or just enjoying some extra reading).
- Task 3:
  - Sketching the functions may help you compare their behaviour. You can also create a table for some value of x.
  - Watch, Read and Think gives you some information on the graphs for logarithmic and trignometric functions.
  - "Essential Mathematical Methods 3 & 4" also discusses transformations and dilations of functions.

Further Thoughts

1. Further Thoughts

You have completed the tasks for this week. Reflect on how you might apply the new knowledge to your areas of interest and study.
Many recreational puzzles can be solved using mathematics. Here is an example. One of these equations is not of the form we have looked at this week, but you can use the substitution method to solve it.

Ruby is two times older than Sammi. Fourteen years ago, Ruby was four times older than Sammi. How old is Sammi now?

Note: This puzzle is **not** part of the High Distinction On-Track task.