Finding the derivative using first principles.

This step-by-step guide to finding the derivatives of a simple polynomial function using first principles, that is,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Example: $f(x) = 3x^2 - 4x + 3$

1. Calculate $f(x+h)$ by replacing x with $x+h$ in $f(x)$	$f(x) = 3x^2 - 4x + 3$ so $f(x+h) = 3(x+h)^2 - 4(x+h) + 3$ $= 3x^2 + 6xh + 3h^2 - 4x - 4h + 3$
2. Substitute $f(x)$ and $f(x+h)$ into $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	$f'(x) = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 4x - 4h + 3 - 3x^2 + 4x - 3}{h}$
3. Simplify the expression $f'(x) = \frac{f(x+h) - f(x)}{h}$	$f'(x) = \lim_{h \to 0} \frac{6xh + 3h^2 - 4h}{h}$ $= \lim_{h \to 0} (6x + 3h - 4)$
4. The derivative is the limit of this expression as $h \to 0$	$f'(x) = \lim_{h \to 0} (6x + 3h - 4) = 6x - 4.$ (Note: as $h \to 0$, $3h \to 0$.)