c the time it takes to reach its maximum height
d the maximum height reached
e the time taken to return to the point of projection.

6 In a tall building the lift passes the 50th floor with a velocity of −8 m/s and an acceleration of $\frac{1}{9}(t-5)$ m/s^2. If each floor spans a distance of 6 m, find at which floor the lift will stop.

19.3 Constant acceleration

When considering motion of a particle due to a constant force, e.g. gravity, the acceleration is constant. There are a number of rules that we may establish by considering the case where acceleration remains constant or uniform.

Given that $\dfrac{dv}{dt} = a$

by antidifferentiating we have

$v = at + c$ where c is the initial velocity.

Using the symbol u for initial velocity we have

$$\boxed{v = u + at} \quad \boxed{1}$$

Now given that $\dfrac{dx}{dt} = v$

by antidifferentiating a second time we have

$x = ut + \frac{1}{2}at^2 + d$, where d is the initial position.

If we consider $s = x - d$ as the change in position of the particle from its starting point, i.e. the particle's displacement from its initial position, we have

$$\boxed{v = ut + \frac{1}{2}at^2} \quad \boxed{2}$$

If we transform the formula $v = u + at$ so that t is the subject we have

$$t = \frac{v-u}{a}$$

By substitution in

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ s &= \frac{u(v-u)}{a} + \frac{a(v-u)^2}{2a^2} \\ 2as &= 2u(v-u) + (v-u)^2 \\ &= 2uv - 2u^2 + v^2 - 2uv + u^2 \\ &= v^2 - u^2 \end{aligned}$$

i.e.

$$\boxed{v^2 = u^2 + 2as} \quad \boxed{3}$$

Also we know that distance travelled = average velocity × time.

i.e. $s = \frac{1}{2}(u + v)t$ $\boxed{4}$

These four formulas are very useful but it must be remembered that they only apply when dealing with constant acceleration.

When approaching problems involving constant acceleration it is a good idea to list the quantities you are given, establish which quantity or quantities you require and then use the appropriate formula. Ensure that all quantities are converted to compatible units.

Constant acceleration summary

If acceleration is constant, the following formulas may be applied, where u is the initial velocity, v is the final velocity, a is the acceleration, t is the time and s is the displacement.

- $v = u + at$
- $s = ut + \frac{1}{2}at^2$
- $v^2 = u^2 + 2as$
- $s = \frac{1}{2}(u + v)t$

Example 7

A body is moving in a straight line with uniform acceleration at an initial velocity of 12 m/s. After 5 s its velocity is 20 m/s. Find

a the acceleration **b** the distance travelled in this time

c the time taken to travel a distance of 200 m.

Solution

Given $u = 12$

$v = 20$

$t = 5$

a Find a using $v = u + at$

$$20 = 12 + 5a$$
$$a = 1.6$$

The acceleration is 1.6 m/s^2.

b Find s using $s = ut + \frac{1}{2}at^2$

$$= 12(5) + \frac{1}{2}(1.6)5^2$$
$$= 80$$

The distance travelled is 80 m.

c Using the formula $s = ut + \frac{1}{2}at^2$ gives

$$200 = 12t + \frac{1}{2} \times (1.6) \times t^2$$
$$200 = 12t + \frac{4}{5}t^2$$
$$1000 = 60t + 4t^2$$
$$250 = 15t + t^2$$

i.e. $t^2 + 15t - 250 = 0$

$(t - 10)(t + 25) = 0$

$\therefore$ $t = 10$ or $t = -25$

As $t \geq 0$, $t = 10$ is the acceptable solution.

The body takes 10 s to travel a distance of 200 m.