SLE123

Week 9

Electrical Potential

DEAKIN COLLEGE

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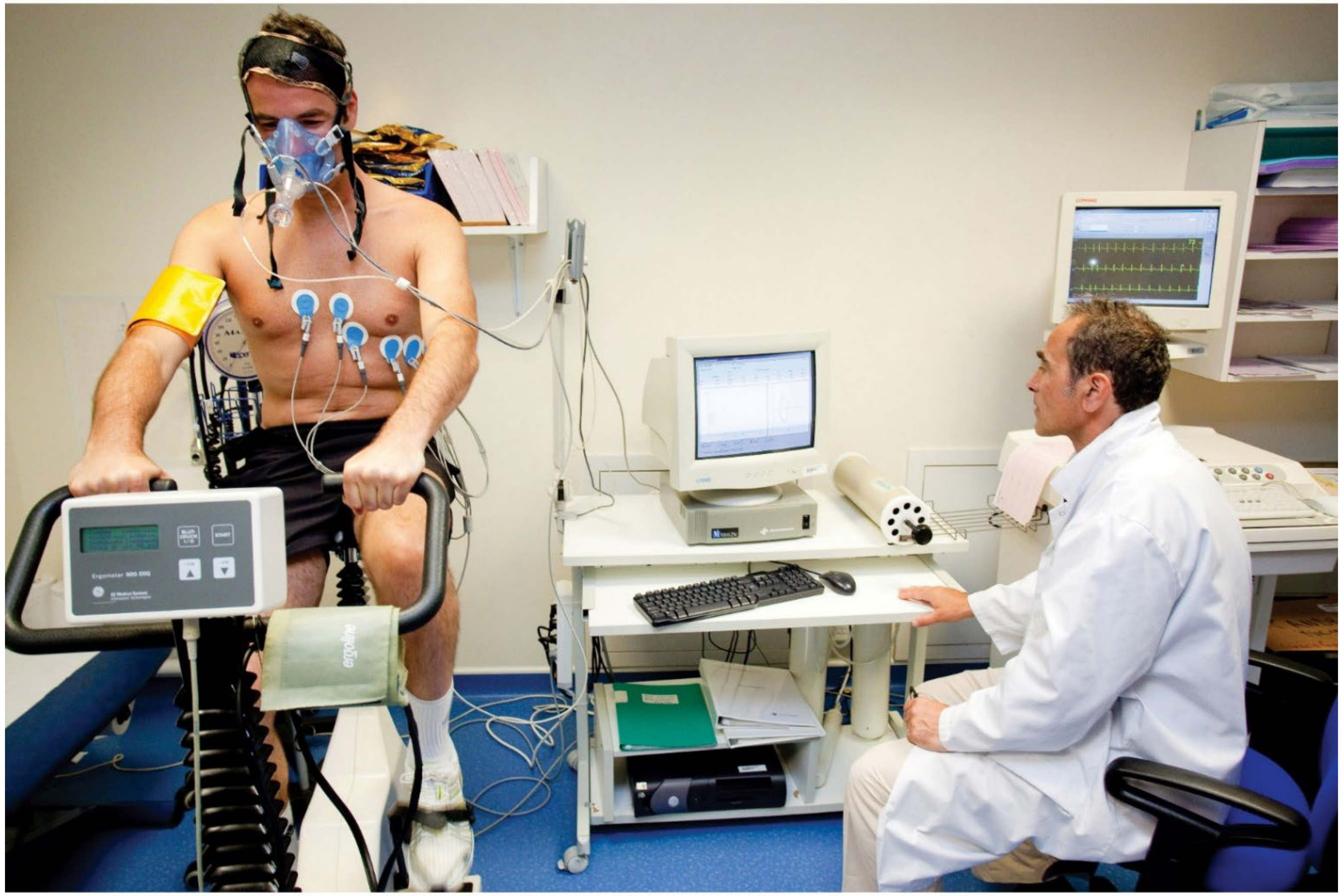
Electrical Potential



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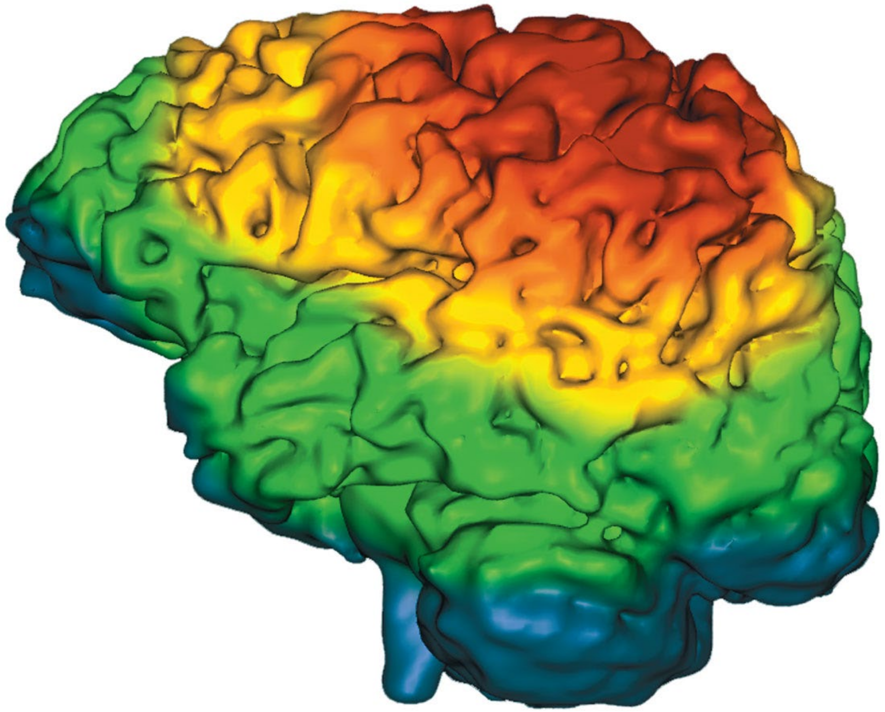
A tool widely used in medicine to diagnose the condition of the heart is the electrocardiograph (ECG).

Topics

- Electric potential energy
- Electric potential
- Conservation of energy
- Potential and field
- Equipotential lines
- Heart & ECG



Electrical Potential

- **In this chapter we study the concept of electric potential, remembering that**

  - **Electromagnetic forces are the most significant forces in governing the behaviour of particles**

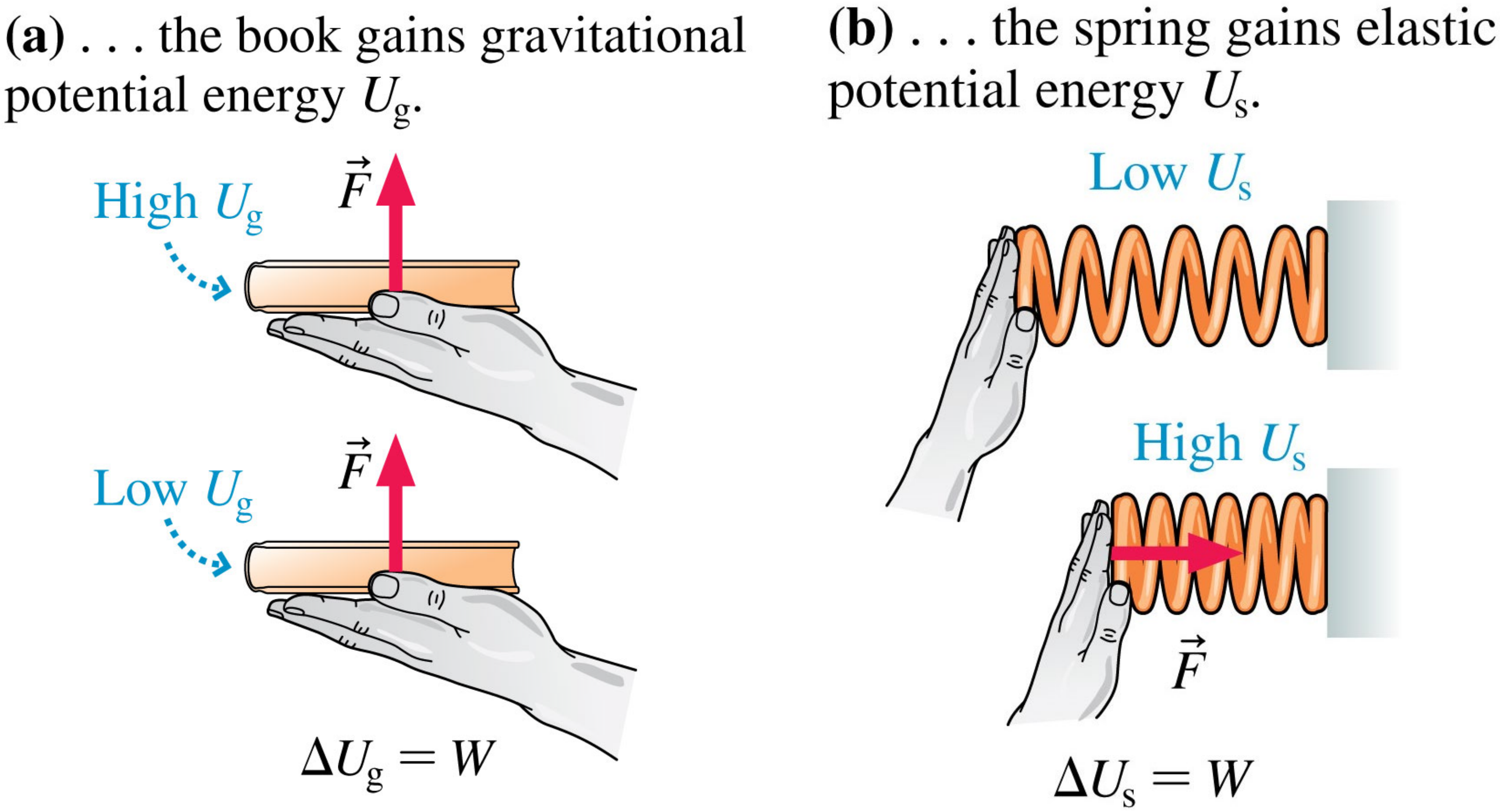
  - **Gravitational forces are significant but weaker than electromagnetic forces**

  - **Nuclear forces are effective essential only in the nucleus and are thought not play a role in the behaviour of particles outside the nucleus, unless the nucleus in broken down**

- **Hence this concept is fundamental to an understanding of the principles of how matter behaves, including biological systems.**

- **Force is directly related to potential.**

Force is directly related to potential



Electrical Potential

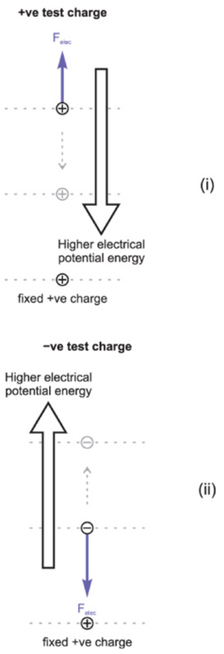


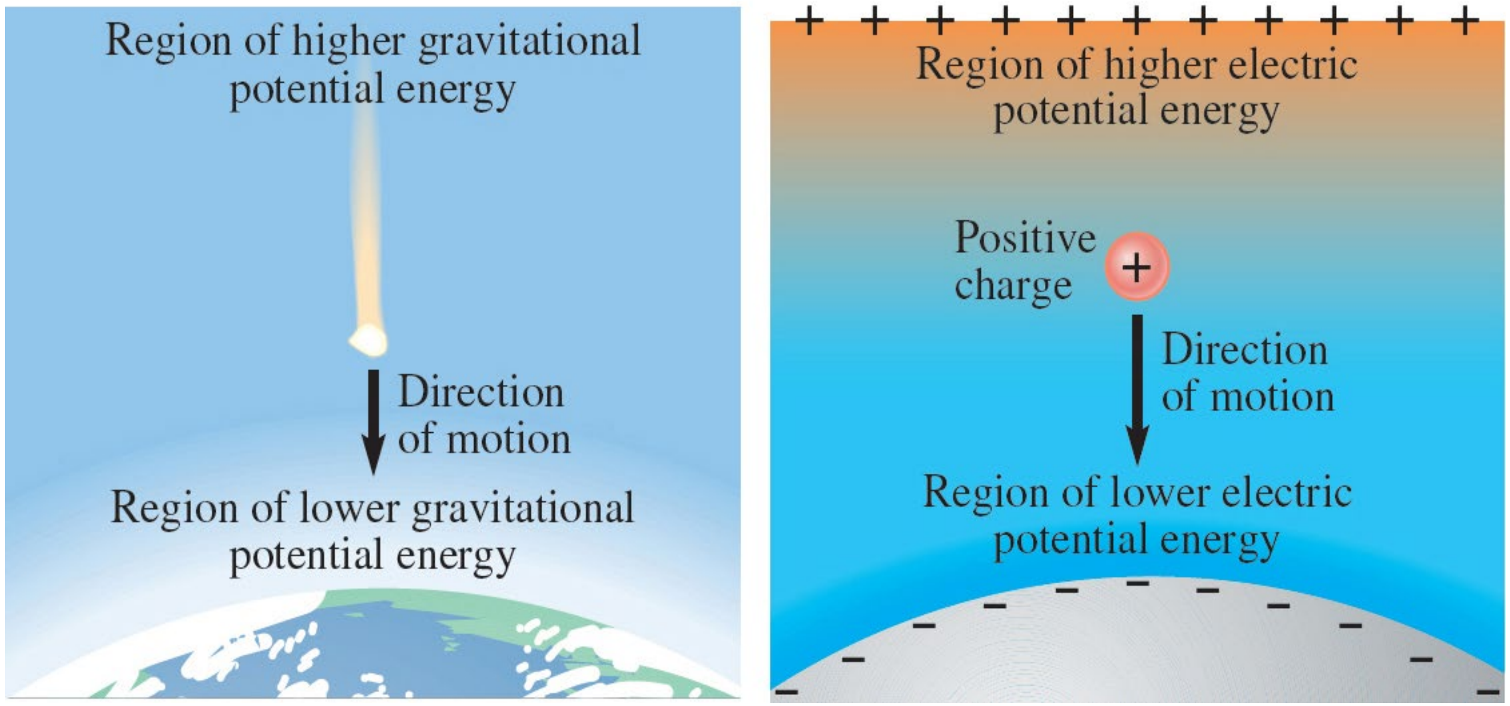
Figure 25.1

(i) Moving two 'like' charges closer together increases the electrical potential energy of each charge. (ii) Moving two 'unlike' charges further apart increases the electrical potential energy of each charge

Introduction to Biological Physics for the Health and Life Sciences Franklin, Muir, Scott, Wilcocks and Yates

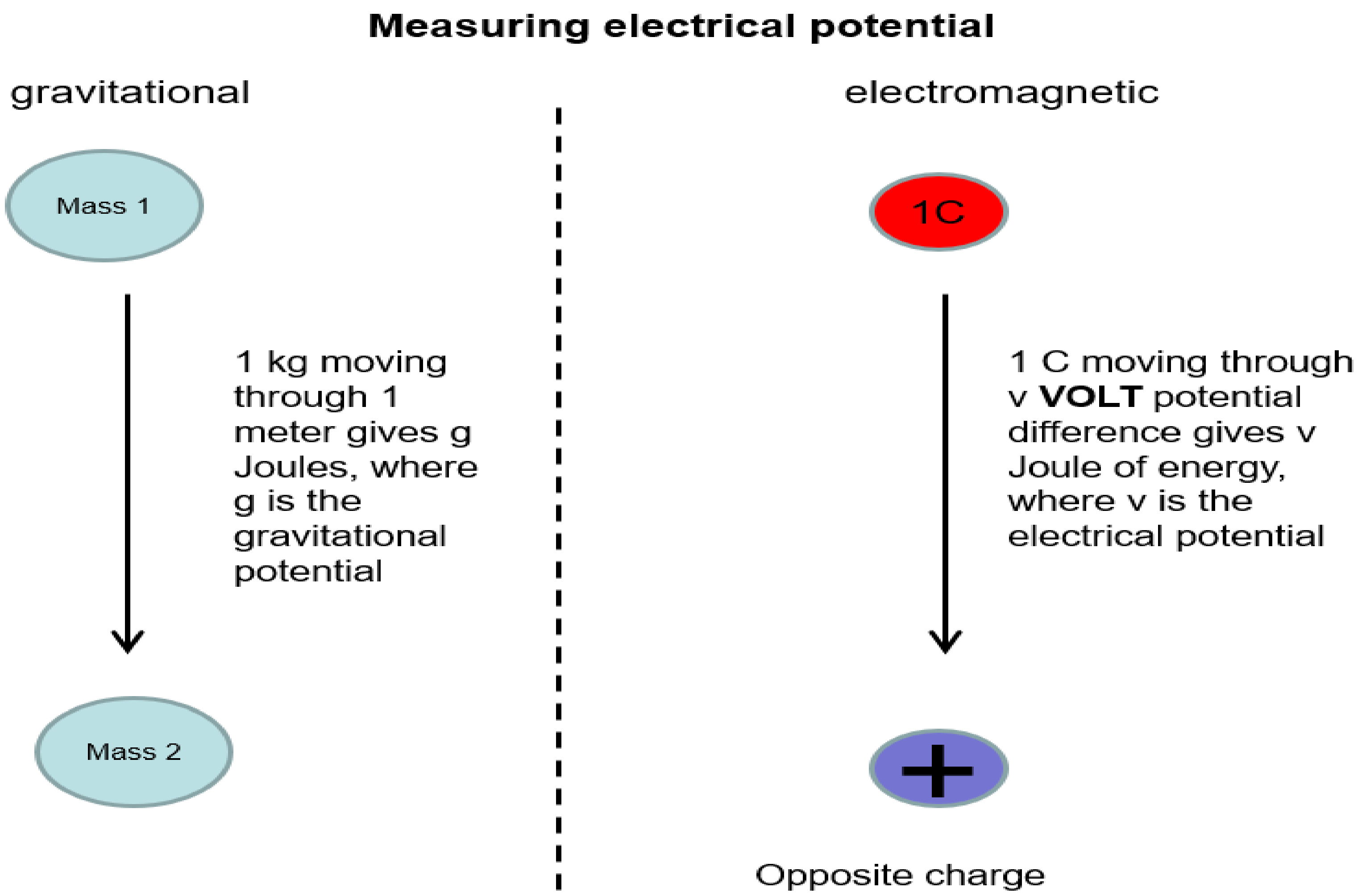
What is electrical potential? It results from charges moving with respect to each other – (through an electric field) – resulting in a change electric potential energy of the charges

Electrical potential



A useful analogy can be drawn between the concept of gravitational potential energy and electrical potential energy. Like most analogies it is imperfect, however. When considering electrical potential energy, we have both positive and negative charges which means both attractive and repulsive forces are possible, unlike gravitational potential energy, when all forces are attractive.

Electrical Potential



Electrical Potential measurement

Electrical potential – the volt – a measure of energy per unit charge

$$\mathbf{V = U_{elec}/Q}$$

V = voltage (electrical potential) (V)
U_{elec} = electrical potential energy (J)
Q (or q)* = charge (C)

Note : V and ΔV (potential difference) are often used interchangeably; but if you use the latter, equation really should be $\mathbf{\Delta V = \Delta U_{elec}/Q}$.

Hint: think of electrical potential (voltage) as a sort of electrical "height" that indicates how much energy was gained by a 1 C unit charge being brought from infinity to that point in space where there is the electric field concerned.

Example calculation

$V = U_{elec}/Q$ (or $\Delta V = \Delta U_{elec}/Q$)

1. If a 3C charge passes through a potential energy difference of 12 J, what is the voltage

Ans: $\Delta V = \Delta U_{elec}/Q = 12J/3C = 4$ volts (= 4V)

2. If a 32C charge passes through a 600 V electrical potential difference, what is the electrical energy given to the charge

Ans: $\Delta U_{elec} = Q.\ \Delta V = 32\ C \times 600\ V = 19200\ J$

Electrical Potential

- Potential difference or voltage, ΔV, is the change in electrical potential energy per unit of charge.

$$\Delta V = \frac{\Delta \text{EPE}}{q} = \frac{-\text{work done}}{q}$$

- The units: J/C = volts

$$\Delta \text{potential energy} = -(\text{work done by the field})$$

Hint: think of electrical potential (voltage) as a sort of electrical "height" that indicates how much energy can be given to charge due to the field concerned.

Some Typical Voltages

Source	Voltage (approx.)
Thundercloud to ground	10^8 V
High-voltage power line	10^6 V
Power supply for TV tube	10^4 V
Automobile ignition	10^4 V
Household outlet	10^2 V
Automobile battery	12 V
Flashlight battery	1.5 V
Resting potential across nerve membrane	10^{-1} V
Potential changes on skin (EKG and EEG)	10^{-4} V

Electrical Potential

Typical electric potentials

Source of potential	Approximate potential
Cells in human body	100 mV
Battery	1–10 V
Household electricity	100 V
Static electricity	10 kV
Transmission lines	1 MV

m = milli = 10^{-3}
k = kilo = 10^{+3}
M = mega = 10^{+6}

TABLE 21.1 Distinguishing electric potential and potential energy

The *electric potential* is a property of the source charges. The electric potential is present whether or not a charged particle is there to experience it. Potential is measured in J/C, or V.

The *electric potential energy* is the interaction energy of a charged particle with the source charges. Potential energy is measured in J.

(KJF 2e, p. 678)

Electric Field Strength and Voltage

We can express the electric field strength in terms of the number of volts per meter (V/m).

- $V = E.d$

- V= electrical potential (volts)
- E=electric field strength (volts/meter)
- d=distance or length (m)

Eg

- If E= 5 V/m and d= 20 cm then V=5 x 20/100= 1V

Electrical Potential

Electrical charges moving through a constant electric field

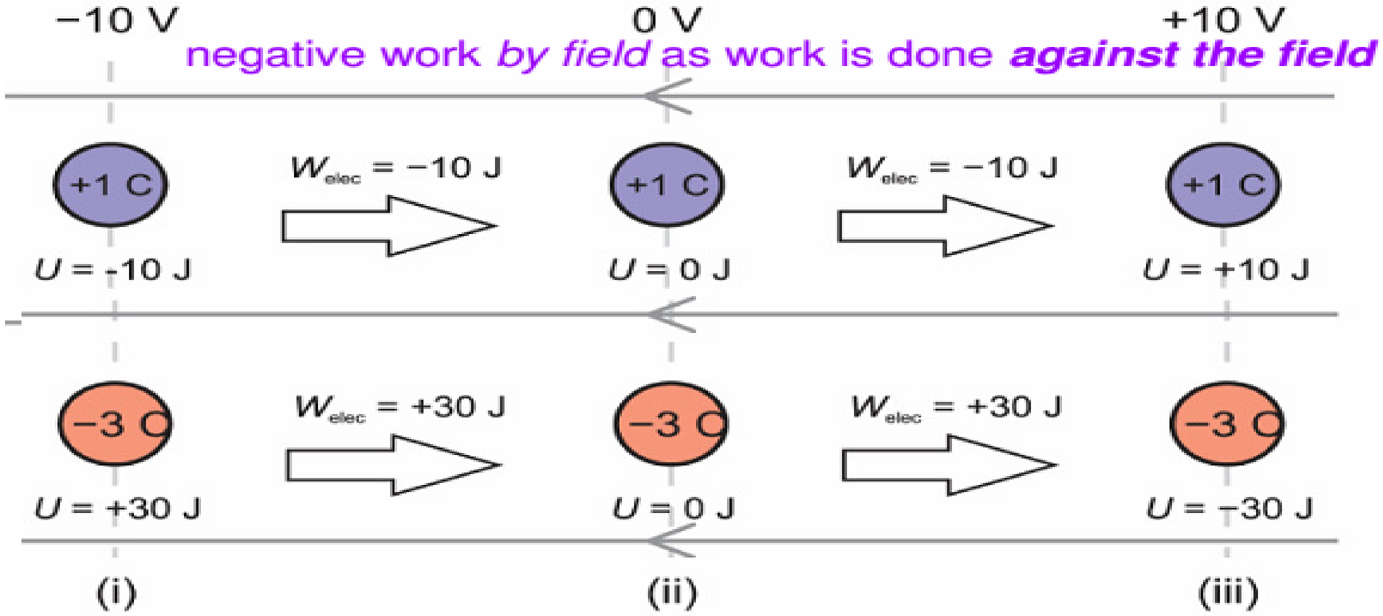


Figure 25.4

positive work *by field* as work is done ***by the field***

Two charges, with magnitudes of +1 C and −3 C, are moved through a uniform electric field. Shown is the amount of work done by the *electric field* when moving the charges, and the electrical potential energy *at those points in space* where the electrical potential is:
(i) −10 V, (ii) 0 V and (iii) +10 V.

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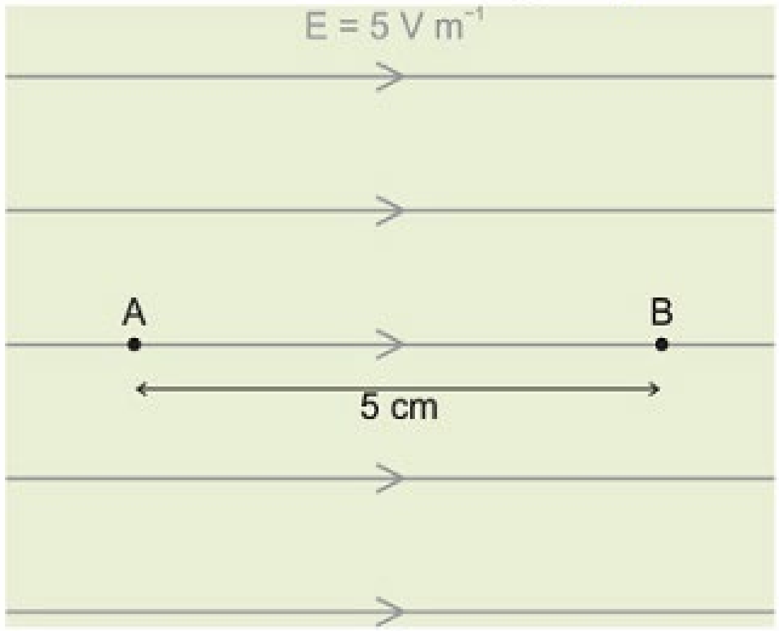
Electric Field Strength and Voltage

- For example, in the diagram below (E=electric field strength): what is the potential difference between points A and B?

$$E = \frac{F}{q} = -\frac{\Delta V}{d}$$

$\Delta V = -E\ d =$
$-5V/m \times (5 \times 10^{-2}m) = -\ 0.25V$
Note that negative sign indicates that a positive charge will have lower electrical potential at point B than at point A)



Two points in a region of uniform electric field.

Equipotential Lines

- **Equipotential lines & Field lines**

- Equipotential lines are lines that connect points of equal potential.

- A useful analogy is to think of walking around a hill always at the same height. In this case you are walking at the same gravitational potential.

- In the same way we can traverse electrical "hills" at constant potential.

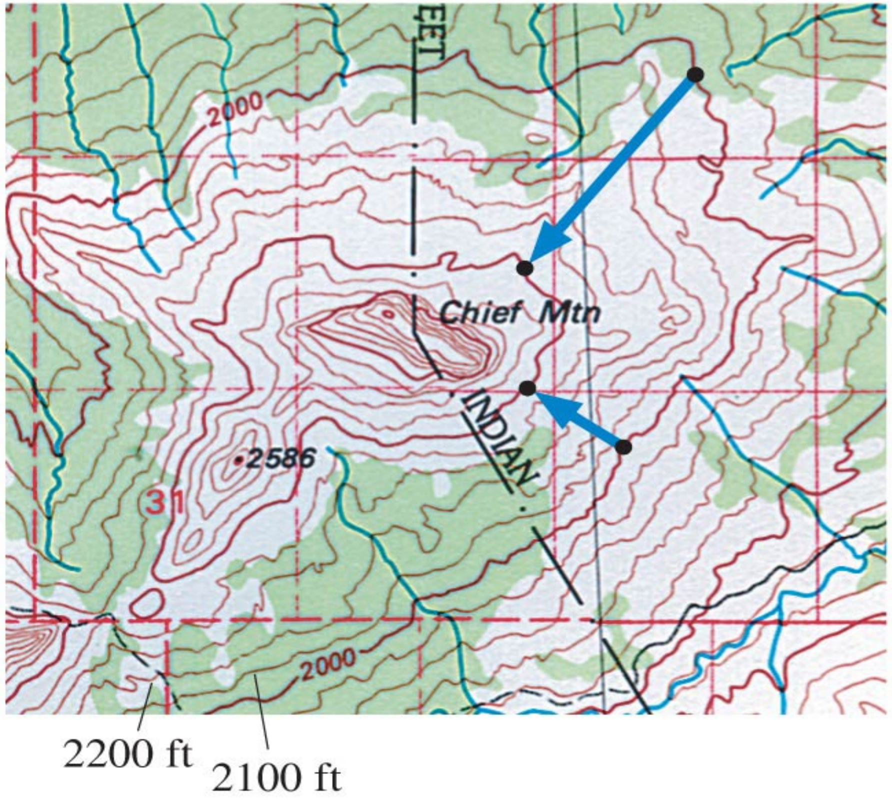
- There is also a direct relationship between electric field lines and electrical potential.

A Topographic Map

(a)

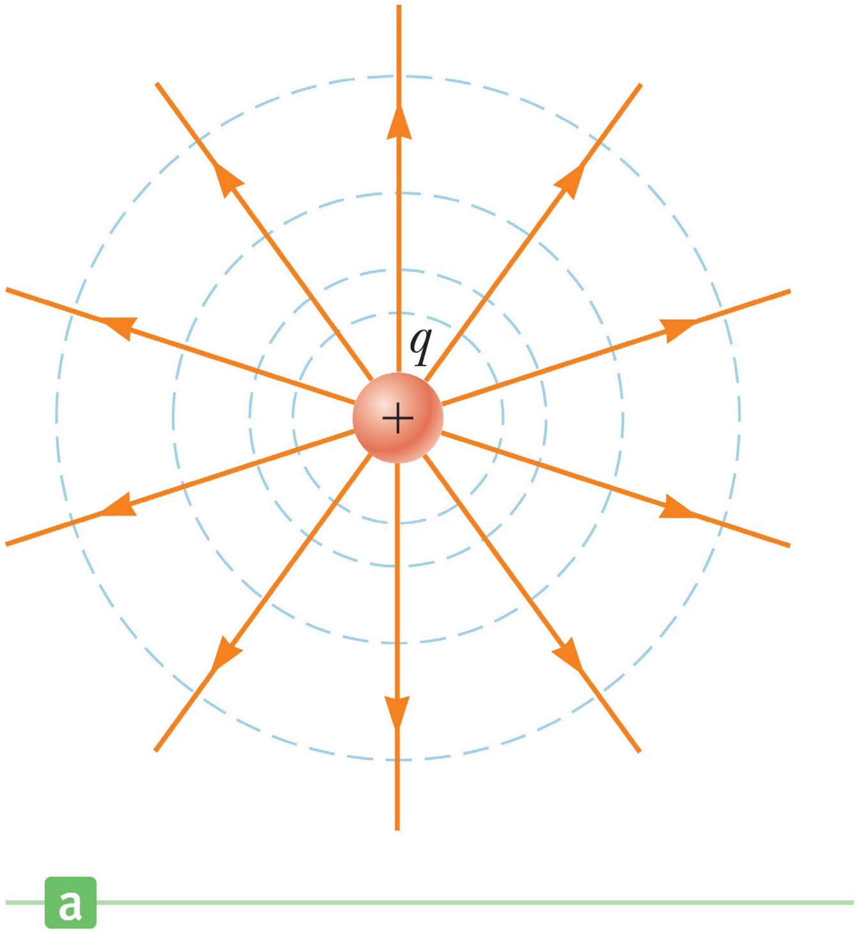
Chief Mountain in Glacier National Park, Montana

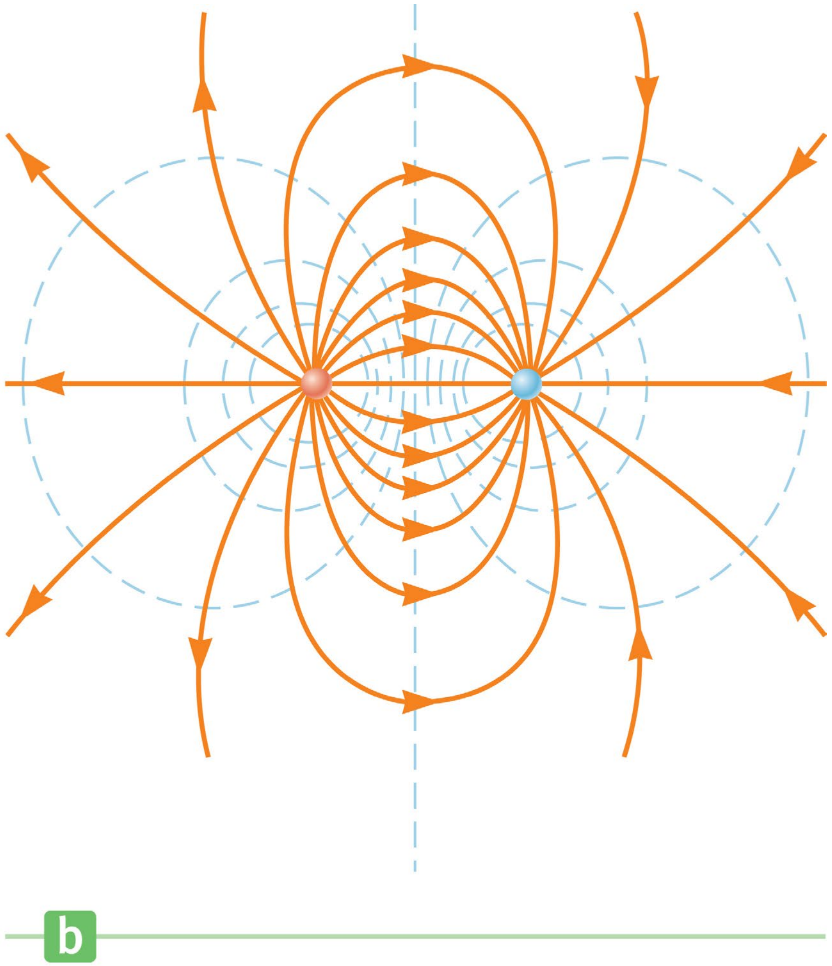
(b)



Equipotential lines show the electrical topography of a collection of charges.

Equipotential Surfaces





Equipotential lines and Field Lines

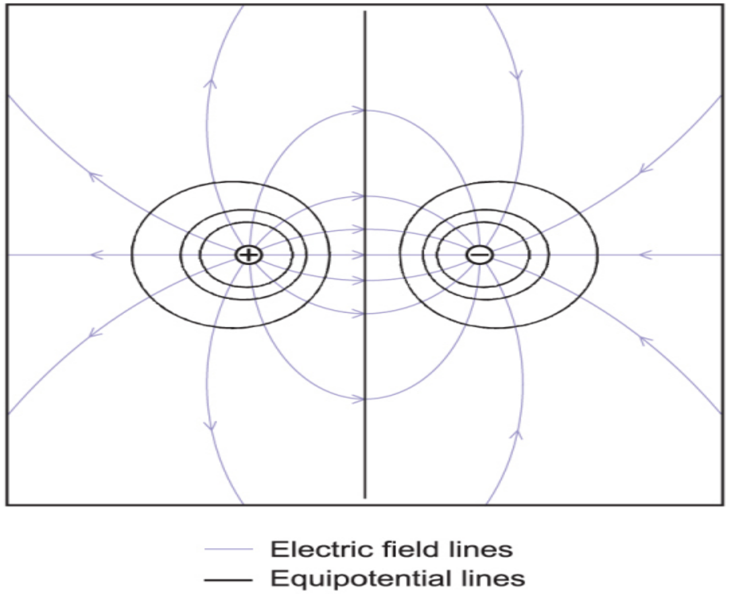


Figure 25.6

A selection of electric field lines and equipotential lines for two unlike charges forming an electric dipole.

Introduction to Biological Physics for the Health and Life Sciences Franklin, Muir, Scott, Wilcocks and Yates

Equipotential Lines and Field Lines

Electric field vectors are *tangential* to electric field lines because they show the direction - as well as magnitude - of the field

BUT

Because any change in position that has a component in the direction of the field will result in a change in potential; then equipotential lines are *always perpendicular* to field lines.

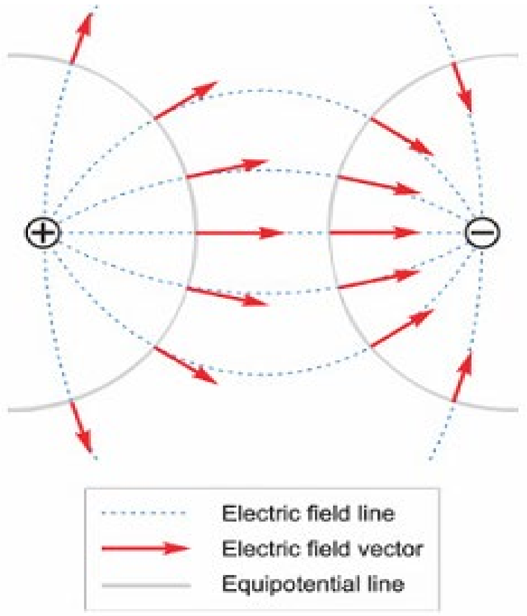
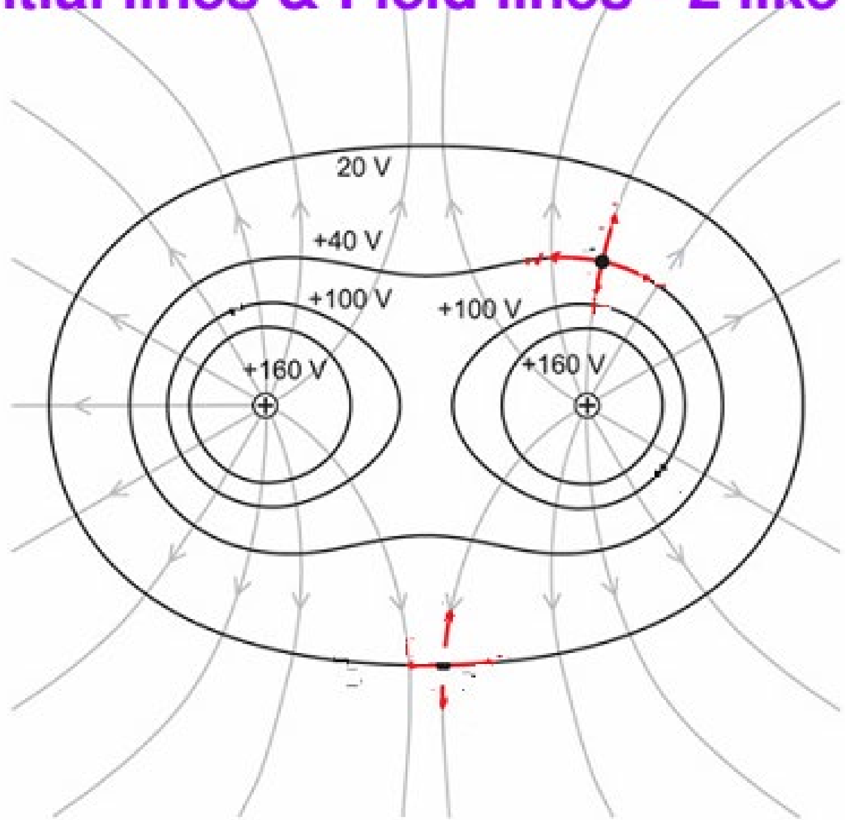


Figure 25.7

Electric field lines, electric field vectors, and equipotentials around an electric dipole. The electric field vector is perpendicular to the equipotential line that it lies upon. Note that the electric field strength is not the same at all points on a single equipotential line.

Equipotential lines and Field lines

Equipotential lines & Field lines - 2 like charges



The electric field and some equipotentials in the region around two positive charges.

Problem

1. Positive charge, *no external force* - Problem

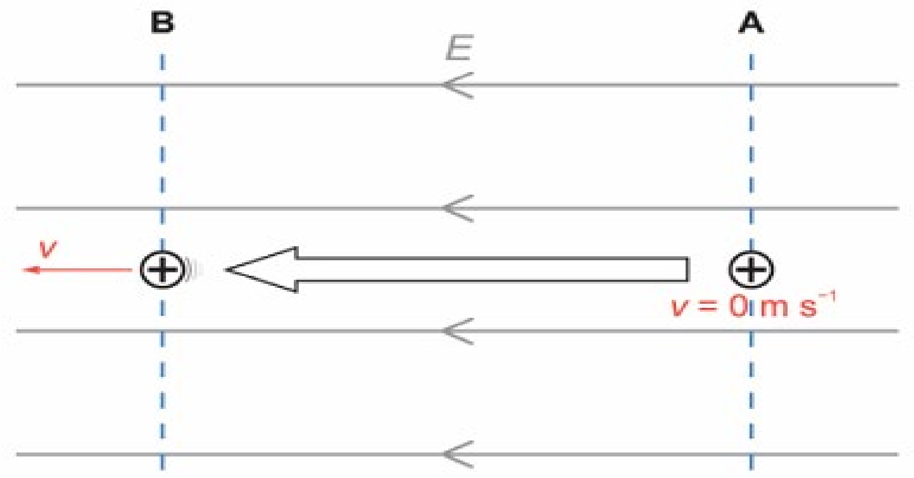


Figure 25.9

Qn: If the charge in Fig 25.9 was a proton, what would be the velocity if
E=5 V/m and
A and B are 5 cm apart?

Hint: Find the total energy transferred,

Ans: $|\Delta U| = |q.E.d| = |\Delta \text{ Kinetic energy}| = |\frac{1}{2}mv^2|$ as the proton starts from rest

From Data Sheet: $q = 1.6\times 10^{-19}$ C; $m = 1.67 \times 10^{-27}$ kg
Also, $d = 5\text{cm} = 0.05\text{m}$

So $|q.E.d| = (1.6\times 10^{-19}) \times 5 \times 0.05 \text{ J} = 0.4 \times 10^{-19} \text{ J}$
$= |\frac{1}{2}mv^2| = \frac{1}{2}(1.67 \times 10^{-27}\text{kg})v^2$
$\Rightarrow v^2 = 0.4_{79} \times 10^8 \text{ m}^2/\text{s}^2$; and $v = 7000\text{m/s}$

Problem

2. Positive charge, *external force* opposing the electrical force - problem

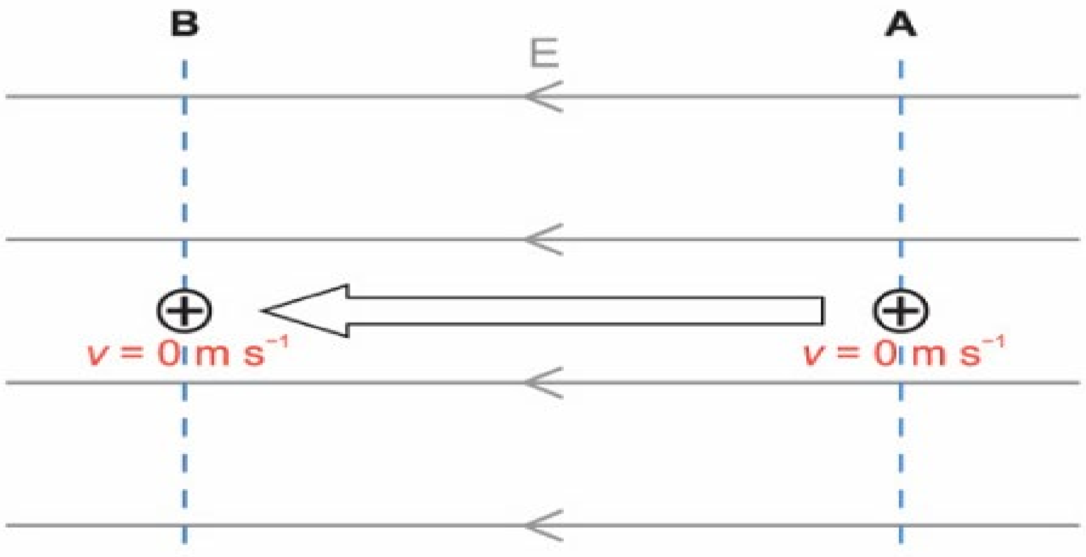


Figure 25.10

Qn: If the charge in Fig 25.9 was a proton, and if E=5 V/m, what force would be necessary to *keep the proton stationary*?

Hint: Once the proton is stationary, the external force needs to be opposite to the electrical force.

Ans: $|F_{ext}| = |F_{elec}| = |q.E.| = q = 1.6\times 10^{-19}\ C \times 5\ V/m = 8 \times 10^{-19}\ N$

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Problem

Charge moving perpendicular to the field direction

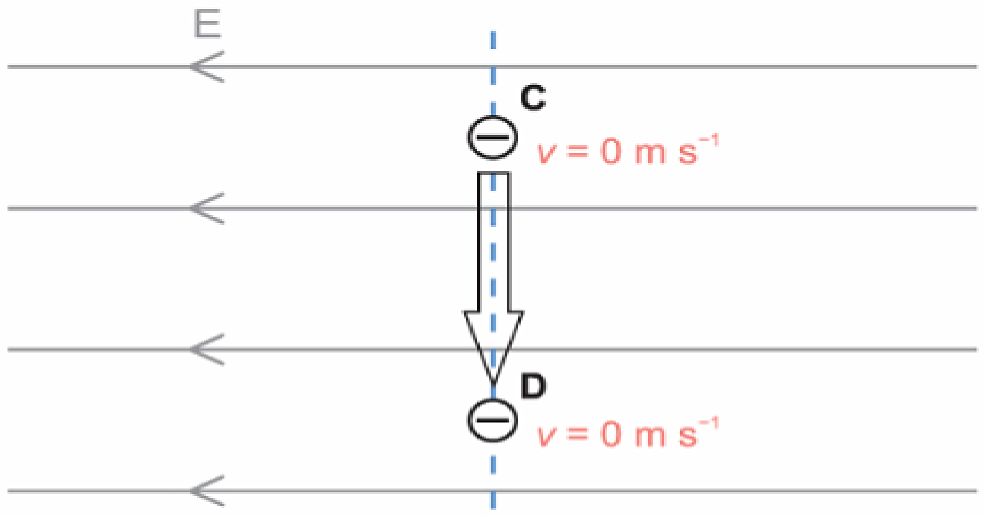


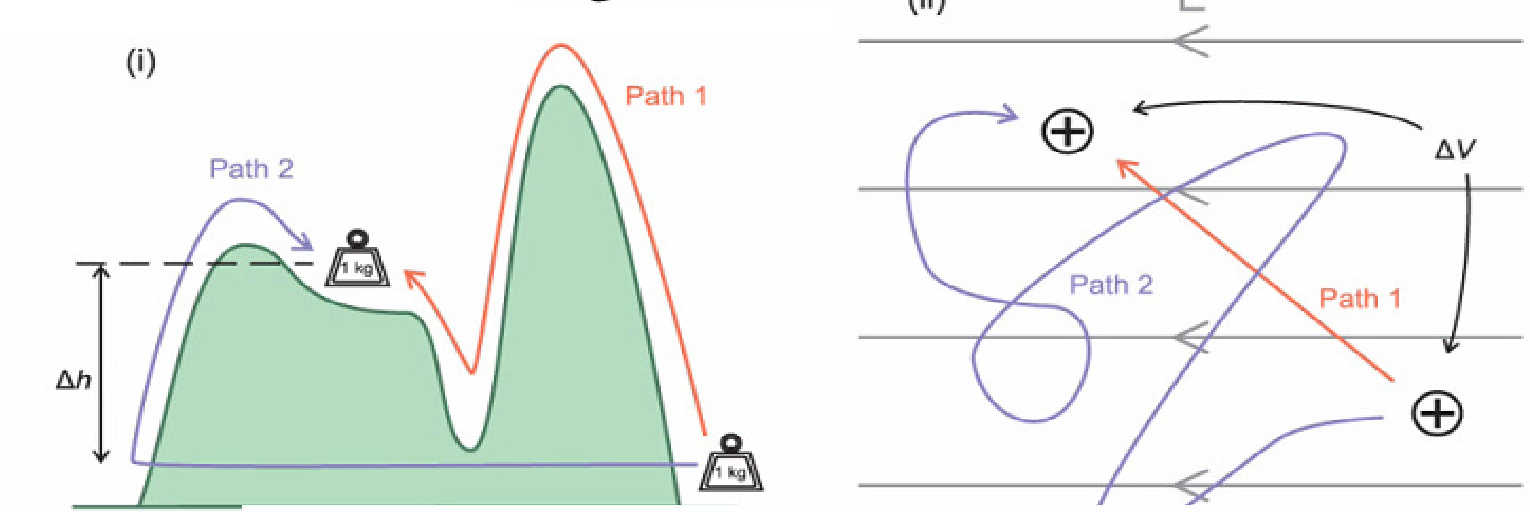
Figure 25.12

or negative

A positive charge is moved from C to D along an equipotential line. There is no work done by the electric field.

Work Done

Figure 25.13



The work done by an external force to move
(i) a mass up a hill and
(ii) a charge in a uniform electric field depends only upon the start and end points.

In the case of the mass, the work done on the weight must be $W = mg\Delta h$ for either path,
while for the charge it must be $W = -q\Delta V$.

Problem

25.1 A 20000NC^{-1} uniform electric field does +5000 J of work on a +0.20 C charged object

(a) **Did the charged object move in the direction of the electric field or against it?**

Ans: Moves with the field - because the direction of an electric field is *defined* as the same as the direction of a force on a positive charge

(b) **How far did the object move?**

Ans: $W_{elec} = F \times d$ (force x distance) $= (q \times E) \times d$,

so $d = W_{elec}/qE$

$= 5000J/(0.20C)(20000N/C) = 5000J/4000N$

$= 1.25m \approx 1.3m$

Problem

25.1 A 20000NC^{-1} uniform electric field does +5000 J of work on a +0.20 C charged object

(c) **What was the change in electrical potential through which the object moved?**

Ans: $W_{elec} = -q\,\Delta V$, so $\Delta V = -W_{elec}/q$ =

$-$ 5000J/ 0.20C = $-$ 25000 V

(d) **If the object was initially at a point with an electrical potential of –2000 V, *what was the electrical potential its end point?***

Ans: $-$ 25000 V = $\Delta V = V_{final} - V_{initial}$

so $V_{final} = \Delta V + V_{initial}$ =

$-$ 25000 V + $-$ 2000V = –27000 *V*

Problem

25.2 A proton is moved at a constant velocity from a position at which the electrical potential is 100 V to one at which the electrical potential is –50 V

(a) How much work was done on the proton by the electric field?

Ans: Remember that charge on a proton $q = 1.6 \times 10^{-19}$ C

Now $W_{elec} = -\mathbf{q}\,\mathbf{\Delta V}$,

But $\Delta V = V_{final} - V_{initial}$, so $\Delta V = (-50V) - (100V) = -150V$

so $W_{elec} = -q\,\Delta V = -(1.6 \times 10^{-19}\,C) \times (-150V) = +\,2.4\times10^{-17}$ J

(Positive because the proton moved in the direction of the field.)

(b) **How much work was done on the proton by the external force?**

Ans: $W_{external} = -\,W_{elec} = -\,2.4\times10^{-17}$ J

(Negative because the proton is moving against the direction of the external force - which is just stopping it accelerate.) 34

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Problem

Which position changes result in a change of potential?

A→B
A→C
B→C
Calculations in Qn 25.3

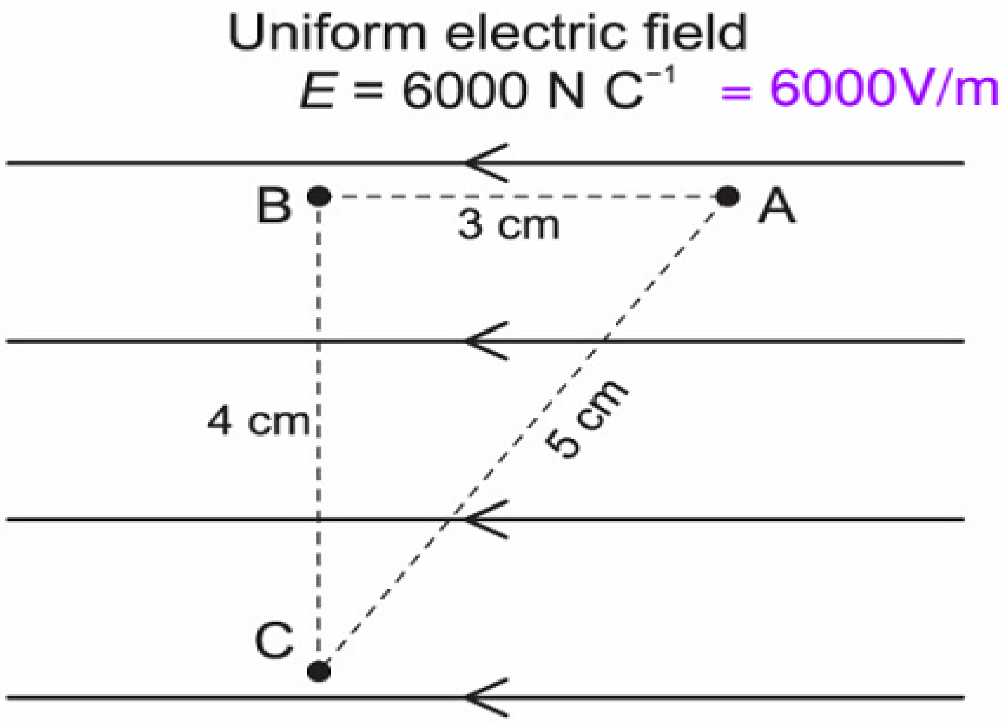

Figure 25.21

Three points in a region of uniform electric field.

Problem

25.3 In a region of space there is a uniform 6000NC^{-1} electric field like that shown in Figure 25.21.

(a) What is the potential difference between points A and B? Which point is at the lower electrical potential?

Ans: $V_{AB} = \Delta V = -E\,d = -(6000 V/m)(3 \times 10^{-2}\ m) = -180\ V$, B is at the lower electrical potential

(b) **What is the potential difference between points A and C? Which point is at the lower electrical potential?**

Ans: $V_{AC} = \Delta V = -E\,d$ - where d is in the direction of the field $= -(6000 V/m)(3 \times 10^{-2}\ m) = -180\ V$, C is at the lower electrical potential

(c) **What is the potential difference between points B and C? Which point is at the lower electrical potential?**

Ans: $V_{BC} = \Delta V = -E\,d$ - where d is in the direction of the field $= -(6000 V/m)(0 \times 10^{-2}\ m) = 0\ V$, both points are at the same electrical potential

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Problem

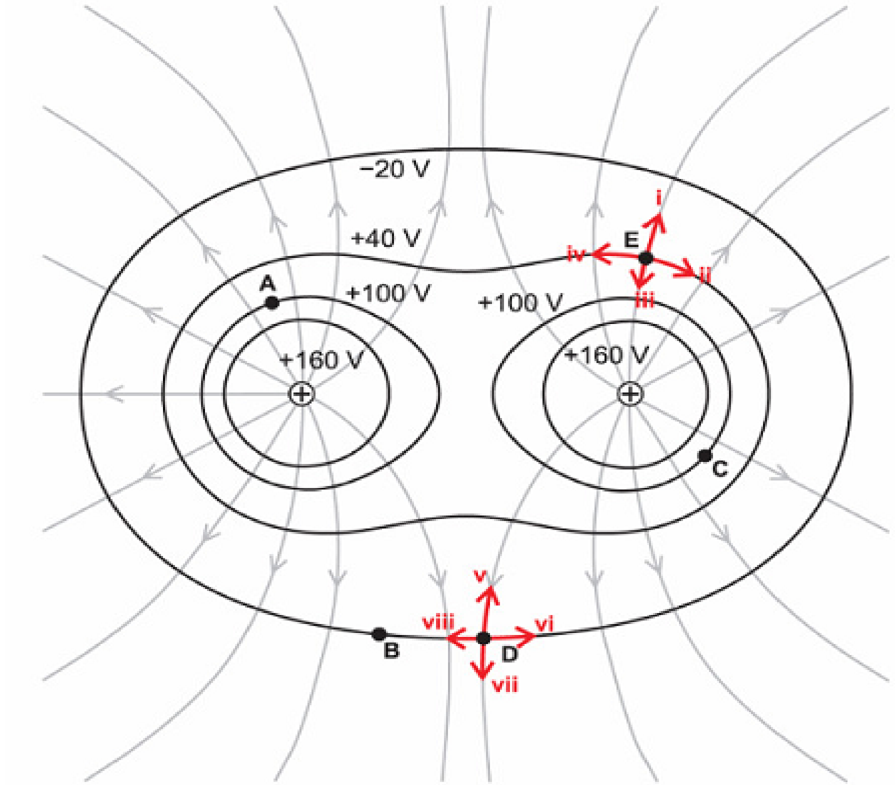


Figure 25.22 The electric field and some equipotentials in the region around two positive charges.

Diagram for Question 26 – next Slide

Problem

Qn 25.6 (a) What is the potential difference between points A and B?

Ans: V_{AB} = **ΔV =** V_B - V_A= - 20V - (100V) = -120 V, B is at the lower electrical potential

(b) How much work does the electric field do on a –0.5 C charge that is moved from A to C?

Ans: W_{elec} **= – q ΔV , but ΔV = 0V, so** W_{elec} = 0V

(c) If a –5 C charge is released from point D which path would it take?

Ans: The electric field lines point *in the direction that a positive charge would move,* so a negative charge will move against the field line - in direction **v** towards the positive charge.

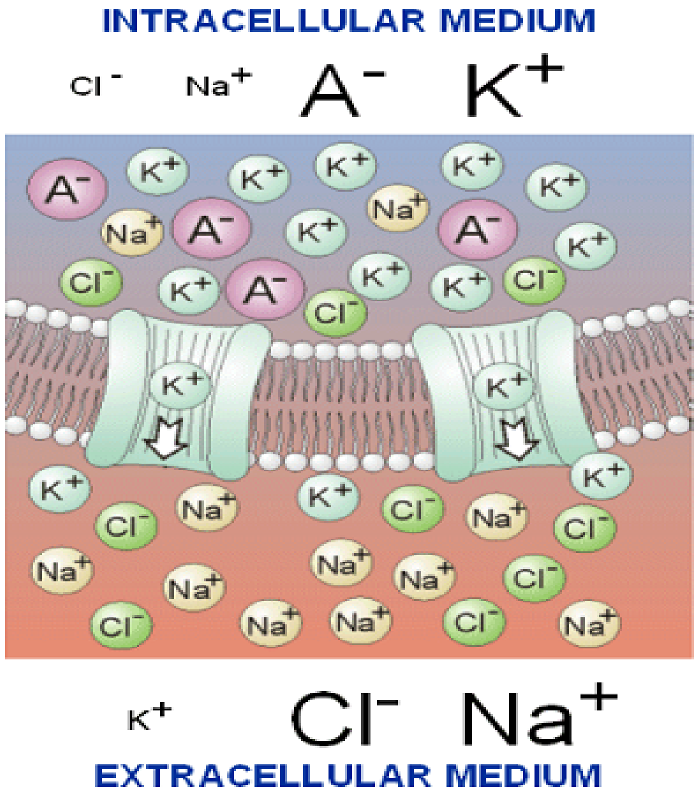
(d) If a +6 C charge is released from point E which path would it take?

Ans: The electric field lines point *in the direction that a positive charge would move,* so it would move along the field line - in direction **i** away from the positive charge.

Electrical Potential in Biology

Cell Membrane Potentials

Cell membrane potentials provide the source of electrical force and thus stored energy in muscle and nerve cells (neurons).



Cell membranes are established by ion pumps - that move ions across a plasma membrane against their concentration gradient, in contrast to ion channels, where ions go through by passive transport

For example, most muscle & nerve cells have ***Na^+/K^+* pumps** that maintain the concentration of Na^+ high outside the cell, and K^+ high within the cell.

(http://highered.mheducation.com/sites/0072495855/student_view0/chapter2/animation__how_the_sodium_potassium_pump_works.html)

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Electrical potential in Biology

25.7 The Heart and ECG

Electrical impulses from the heart muscle cause your heart to beat (contract). This electrical signal begins in the sinoatrial (SA) node, located at the top of the heart's upper-right chamber (the right atrium) and travels through the heart muscle (myocardium).

The "pacemaking" signal occurs at a rate of around 60-100 beats per minute.

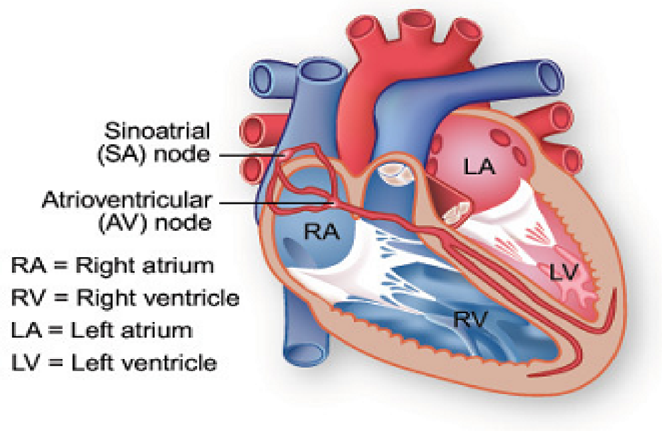

Diagram from the Texas Heart Institute*

For efficient pumping of the blood without backflow, the muscles in the upper chambers of the heart (the atria) contract first.
The signal then passes through the atrioventricular (AV) node - which checks the signal and sends it through the muscle fibres of the lower chambers (the ventricles), causing them to contract.
The SA node sends electrical impulses at a certain rate, but your heart rate may still change depending on physical demands, stress, or hormonal factors.

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Electrical Potential in Biology

Heart potential

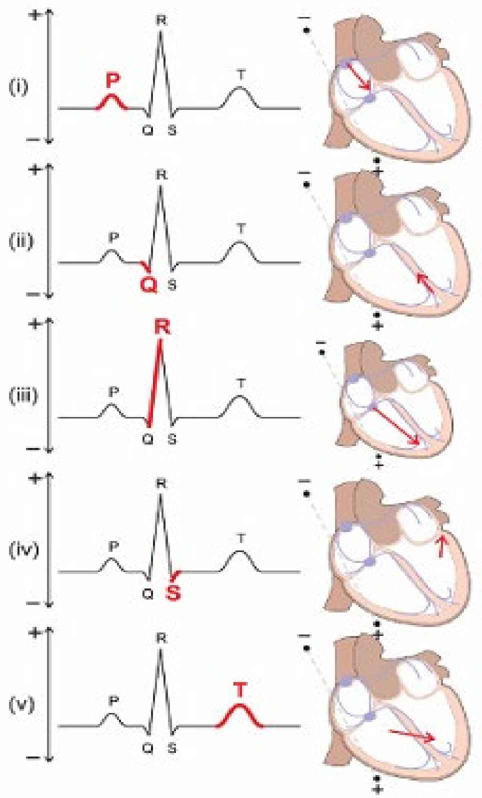


Figure 25.15

The heart vector at various times during the cardiac cycle.

P wave cause by depolarisation of the atria

QRS complex shows the depolarisation of the ventricles (and masks the repolarisation of the atria)

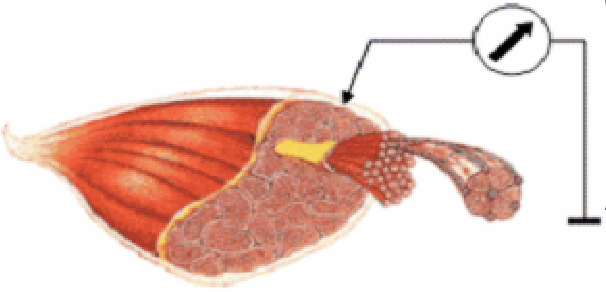
T wave is cause by repolarisation of the ventricles

Electrical Potential in Biology

The Heart and ECG

A typical heart muscle is around 100 μm long, and 15 μm wide. The outer membrane is around the cell is around 8-10 nm thick, and, in its resting state a potential difference of around 90mV exists between the inside of the cell and the outside of the cell membrane with the outside being positive.

Example 25.2 (p247) In order to function, the heart cell must "pump" ions from one side of the membrane to another



Picture *

(a) How much energy is required to pump a single Na^+ ion from the inside of the cell membrane to the outside, if the outside of the cell is positively charged with respect to the inside?

Ans: *Energy needs to be provided to move a cation to the more positively-charged cell exterior, with $\Delta U = q\,\Delta V = (1.6 \times 10^{-19}C) \times (90 \times 10^{-3}V)$ $= 1.4_4 \times 10^{-20}J$*

(b) A typical chocolate bar releases about 1000kJ of energy into the body once metabolised. How many Na^+ ions can be transported across a heart cell membrane with this amount of energy?

Ans: *Number of Na^+ cations $= 1.00 \times 10^6 J / 1.44 \times 10^{-20} J$ per Na^+*
$= 6.9 \times 10^{25}$ Na^+ cations

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