1) A particle has an initial displacement of $s = 0$ when $t = 0$ and the velocity is $v = 7t^2 - 53t - 402 \text{ m/s}^2$ when $t \geq 0$

a) Find the displacement of the particle at time t

$v = 7t^2 - 53t - 402 \text{ m/s}^2$

$\int(7t^2 - 53t - 402) + C$
-> $\frac{7}{3}t^3 - \frac{53}{2}t^2 - \frac{402}{1}t + C$
$s = \frac{7}{3}t^3 - \frac{53}{2}t^2 - 402t + C$

b) Find the acceleration of the particle at time t

$v = 7t^2 - 53t - 402 \text{ m/s}^2$
-> $7 \times 2t^1 - 53$
$a = 14t - 53$

c) What is the acceleration and displacement when $v = 0$?

$v = 7t^2 - 53t - 402 \text{ m/s}^2$
-> $0 = 7t^2 - 53t - 402 \text{ m/s}^2$
-> $t = \frac{-(-53) \pm \sqrt{(-53)^2 - 4(7)(-402)}}{2(7)}$
-> $t = \frac{53 \pm \sqrt{2809 + 11256}}{14}$
-> $t = \frac{53 \pm \sqrt{14{,}065}}{14}$
$t^+ = 12.2569$ seconds
$t^- = -4.6854$ seconds. $t \geq 0$ is not fulfilled so we ignore this.

Acceleration when $v = 0$
$a = 14t - 53$
-> $14(12.2569) - 53$
$a = 118.5959 \text{ m/s}^2$ when $v = 0$

Displacement when $v = 0$
$s = \frac{7}{3}t^3 - \frac{53}{2}t^2 - 402t + C$
-> $s = \frac{7}{3}(12.2569)^3 - \frac{53}{2}(12.2569)^2 - 402(12.2569) + C$
$s = -4611.8726 + C$ metres

2) Evaluate this integral

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\sin(3x) - 27\cos(9x))dx$
-> $\frac{1}{3}\cos(3x) - 27\cos(9x)$
-> $\frac{1}{3}\cos(3x) - \frac{27}{9}\sin(9x)$

$\frac{1}{3}\cos(3x) - 3\sin(9x)$
$[\frac{1}{3}\cos(3x) - 3\sin(9x)]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$

a: $\frac{1}{3}\cos\left(3\frac{\pi}{2}\right) - 3\sin(9 \times \frac{\pi}{2})$
-> $\frac{1}{3}\cos\left(\frac{3\pi}{2}\right) - 3\sin(\frac{9\pi}{2})$
-> $\frac{1}{3} \times 0 - 3 \times 1$
$= -3$

b: $\frac{1}{3}\cos\left(3\frac{3\pi}{2}\right) - 3\sin(9 \times \frac{3\pi}{2})$
-> $\frac{1}{3}\cos\left(\frac{9\pi}{2}\right) - 3\sin(\frac{27\pi}{2})$
-> $\frac{1}{3} \times 0 - 3 \times -1$
$= 3$

Following the integral interval equation, $b - a$
$3 - -3 = 6$

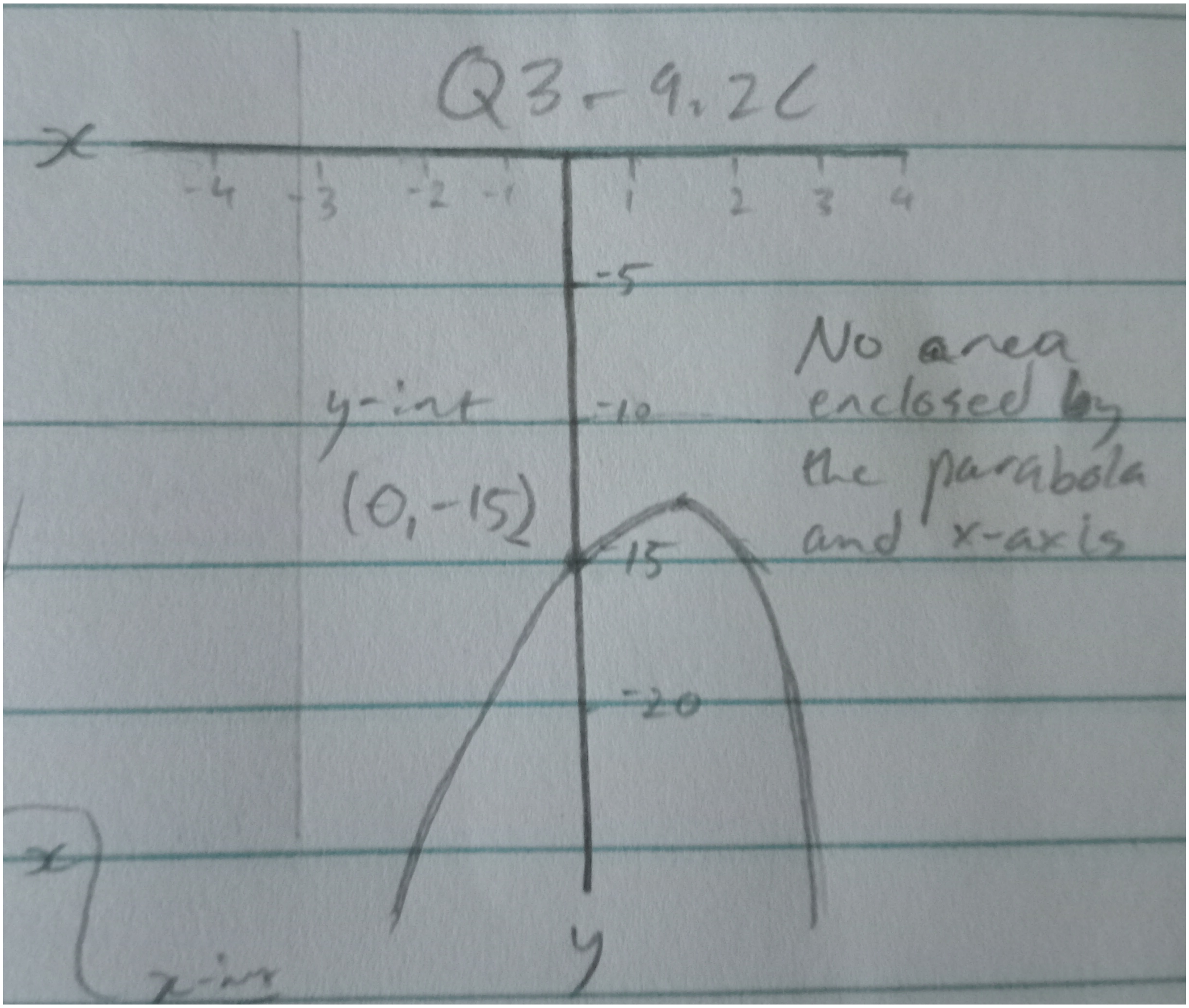
This definite integral's value is 6

3) Sketch the parabola $y = -15 + 3x - x^2$ and shade the area enclosed by the parabola and the x-axis. Find the area enclosed by this parabola and the x-axis using integration.

$y = -15 + 3x - x^2$

After sketching, the graph does not have any enclosed area between the parabola and the x-axis. The integration will not give useful information for us.

Q3 graph



4) Sketch the graphs of the line $y = 2 + x$ and the parabola $y = 2x^2 - 3x - 31$ on the same set of axis. Show all working including finding the x- and y-intercepts, and the co-ordinates where the two graphs intersect. Shade the area enclosed by the parabola and the line.

Note that any approximations are purely for graphing purposes

- Find the intercepts of both graphs
- Find the point of intersection

$y = 2 + x$

- y-intercept is $+2$
  -> $0 = 2 + x$
- x-intercept is -2

$y = 2x^2 - 3x - 31$

- the y-intercept is $(0, -31)$
- the x-intercepts are $(\frac{3-\sqrt{257}}{4}, 0)$ and $(\frac{3+\sqrt{257}}{4}, 0)$
- the vertex is $\left(\frac{3}{4}, -\frac{257}{8}\right)$

x-intercepts

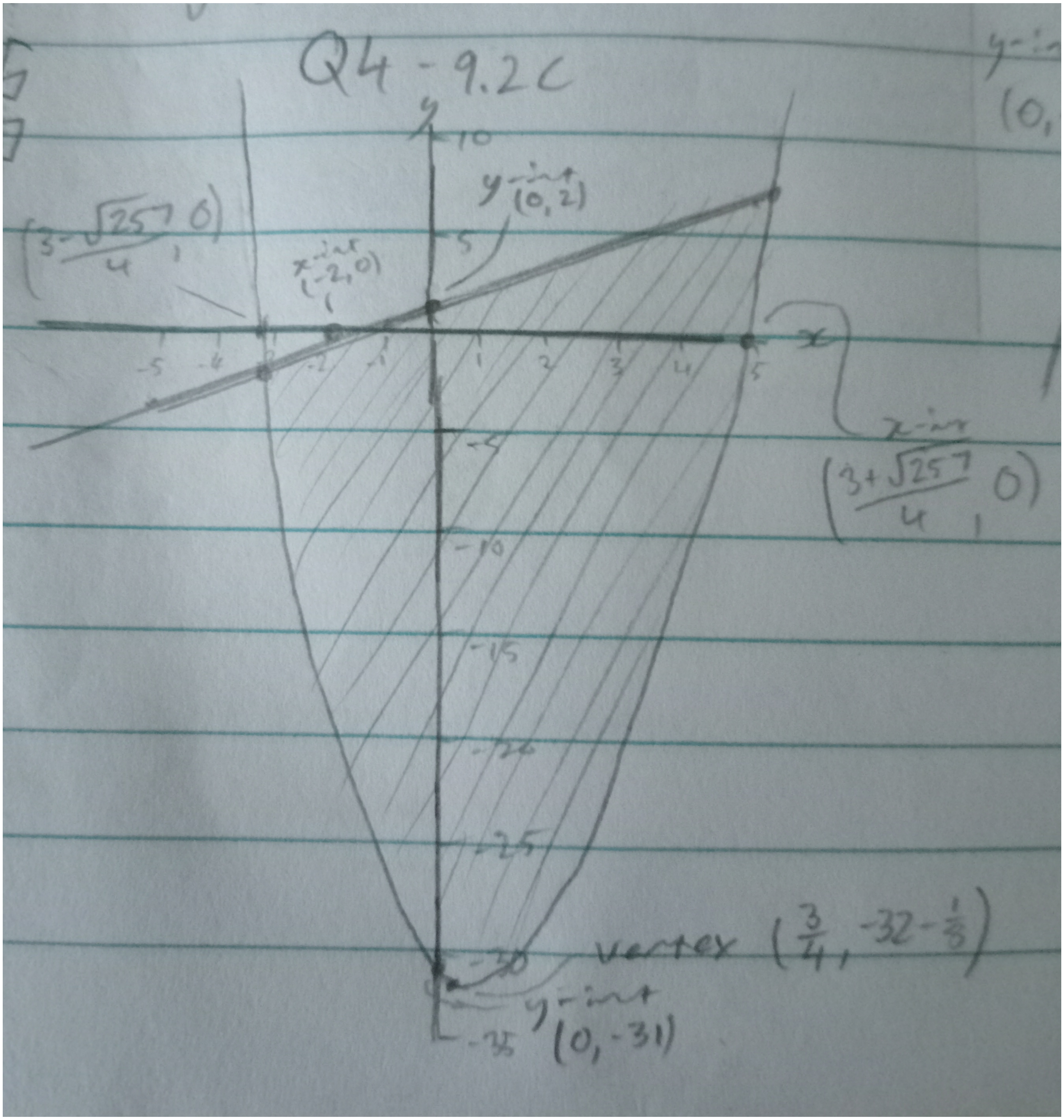
-> $0 = 2x^2 - 3x - 31$
-> $x = \frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-31)}}{2(2)}$
-> $x = \frac{3\pm\sqrt{9+248}}{4}$
-> $x = \frac{3\pm\sqrt{9+248}}{4}$
-> $x = \frac{3\pm\sqrt{257}}{4}$
$x^+ = \frac{3+\sqrt{257}}{4} \approx 4.758$
$x^- = \frac{3-\sqrt{257}}{4} \approx -3.257$

Vertex

$x = \frac{-b}{2a}$
->$\frac{--3}{2(2)}$
-> $\frac{3}{4}$

$y = 2(\frac{3}{4})^2 - 3(\frac{3}{4}) - 31$
-> $2\left(\frac{9}{16}\right) - \frac{9}{4} - 31$
-> $\frac{18}{16} - \frac{9\times4}{4\times4} - \frac{31\times16}{16}$
-> $\frac{18}{16} - \frac{36}{16} - \frac{496}{16}$
-> $-\frac{18}{16} - \frac{496}{16}$
-> $-\frac{514}{16}$
$= -\frac{257}{8} = -32 - \frac{1}{8}$

Q4 graph



5) Find the area between the line $y = 2 + x$ and the parabola $y = 2x^2 - 3x - 31$

$2 + x = 2x^2 - 3x - 31$
-> $0 = 2x^2 - 4x - 33$

First, we find the integrals. We need to find the intersection x values so we know the correct upper and lower bounds to use.

Intersection coordinates

Intersect coords 1: $(\frac{4+\sqrt{280}}{4}, 2+\frac{4+\sqrt{280}}{4})$
Intersect coords 2: $(\frac{4-\sqrt{280}}{4}, 2+\frac{4-\sqrt{280}}{4})$

$$\begin{cases} y = 2 + x \\ y = 2x^2 - 3x - 31 \end{cases}$$

$2 + x = 2x^2 - 3x - 31$
-> $0 = 2x^2 - 3x - 31 - 2 - x$
-> $0 = 2x^2 - 4x - 33$
-> $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-33)}}{2(2)}$
-> $x = \frac{4 \pm \sqrt{16+264}}{4}$
-> $x = \frac{4 \pm \sqrt{280}}{4}$
$x^+ = \frac{4+\sqrt{280}}{4} \approx 5.183$
$x^- = \frac{4-\sqrt{280}}{4} \approx -3.183$

Substituting into $y = 2 + x$
$y^+ = 2 + \frac{4+\sqrt{280}}{4} \approx 7.183$
$y^- = 2 + \frac{4-\sqrt{280}}{4} \approx -1.183$

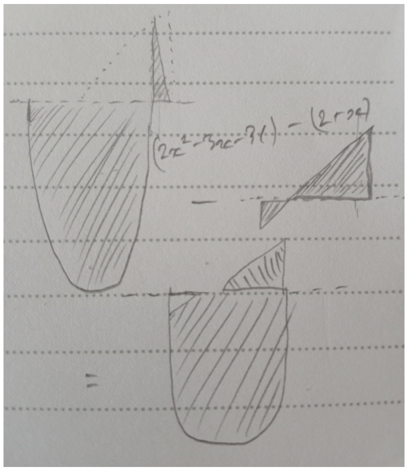
Setup of the definite integral

From these, we can setup a definite integral with the x range, and since the graph has no complicated intersections, we just need the one correct integral.
We are strictly trying to find the area below the line so using $f(x) - g(x)$, we'll calculate the area of parabola minus area of line

$(2x^2 - 3x - 31) - (2 + x)$
-> $2x^2 - 3x - 31 - 2 - x$
-> $2x^2 - 4x - 33$

$\int_{\frac{4-\sqrt{280}}{4}}^{\frac{4+\sqrt{280}}{4}} (2x^2 - 4x - 33)dx$
-> $\int_{\frac{4-\sqrt{280}}{4}}^{\frac{4+\sqrt{280}}{4}} (2x^2)dx - \int_{\frac{4-\sqrt{280}}{4}}^{\frac{4+\sqrt{280}}{4}} (4x)dx - \int_{\frac{4-\sqrt{280}}{4}}^{\frac{4+\sqrt{280}}{4}} (33)dx$
-> $(\frac{41\sqrt{70}}{3}) - (4\sqrt{70}) - (33\sqrt{70})$
-> $\frac{41\sqrt{70}}{3} - 4\sqrt{70} - 33\sqrt{70}$
-> $\frac{41\sqrt{70}}{3} - 37\sqrt{70}$
$= -\frac{70\sqrt{70}}{3} \approx -195.2206728 \rightarrow 195.2206728$ units^2

I drew this loosely-accurate graph as a companion to understand what I was actually doing by subtracting the line from the parabola equation. The graph assumes the $\frac{4\pm\sqrt{280}}{4}$ integral is applied to both equations.


(2x² − 3x − 3) − (2 + x)
−
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