

1 Discovering the derivative

1.1 Rates of change

1. The following table shows the distance covered by a swimmer at various times during a 30 second swim

Distance (m)	0	12	23	32	42	51	60
Time (s)	0	5	10	15	20	25	30

Find the average speed

- (i) over the first 5 seconds
 - (ii) between 10 and 20 seconds
 - (iii) over the last 5 seconds
 - (iv) over the entire 30 seconds
2. The temperatures at various times on a certain day are shown in the following table.

Temperature	9°C	27°C	29°C	x	21°C
Time	7 a.m.	11 a.m.	3 p.m.	7 p.m.	11 p.m.

- (i) Find the average rate of change of temperature between 7 a.m. and 3 p.m.
 - (ii) Find the average rate of change of temperature between 11 a.m. and 11 p.m.
 - (iii) Given that the average rate of change of temperature between 7 p.m. and 11 p.m. is -1.5°C per hour, find the temperature, x , at 7 p.m.
3. The following table shows the total distance covered by a walker at various times during a hike

Distance (m)	0	2.4	3.9	4.8	6.0	7.1	8.4
Time (s)	0	0.5	1.0	1.5	2.0	2.5	3.0

Find the average speed

- (i) over the first hour
- (ii) between 1.5 and 3 hours

- (iii) over the last half hour
- (iv) over the entire 3 hours.
4. The temperatures at various times on a certain day are shown in the following table.

Temperature	2.8°C	11.6°C	14.8°C	x	8.4°C
Time	7 a.m.	11 a.m.	3 p.m.	7 p.m.	11 p.m.

- (i) Find the average rate of change of temperature between 11 a.m. and 3 p.m.
- (ii) Find the average rate of change of temperature between 7 a.m. and 3 p.m.
- (iii) Given that the average rate of change of temperature between 7 p.m. and 11 p.m. is -1.1°C per hour, find the temperature, x , at 7 p.m.

1.2 The derivative

1. Use the first-principles formula $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of the following functions

(i) $y = f(x) = 4x - 7$

(ii) $y = f(x) = x^2$.