# Task 1: Differentiation - Rates of change

A tank is leaking. The amount of water in the tank after $t$ days is $V = 4(49 - 3t)^3$ litres where $t \in [0, 28]$.

## (1) What was the original amount of water in the tank, that is, the amount at t = 0?

$V = 4(49 - 3t)^3$
-> $4(49 - 3(0))^3$
-> $4(49)^3$
$470,596L$ at $t = 0$

## (2) When does the amount of water reach an eighth of the original amount?

$\frac{1}{8} \times 470,596L = 58,824.5L$

$58,824.5L = 4(49 - 3t)^3$
-> $\frac{58,824.5L}{4} = (49 - 3t)^3$
-> $\sqrt[3]{14,706.125L} = 49 - 3t$
-> $\frac{49}{2}L - 49 = -3t$
-> $\frac{49}{2}L - \frac{98}{2} = -3t$
-> $-\frac{49}{2} = -3t$
-> $-\frac{49}{2} \div -\frac{3}{1} = t$
-> $-\frac{49}{2} \times -\frac{1}{3} = t$
$t = \frac{49}{6} = 8.1\dot{6}$ days to reach $\frac{1}{8}$ of it's original volume

## (3) What is the rate of change of V with respect to time t?

$V = 4(49 - 3t)^3$

**Finding** $V'$
$V = 4(49 - 3t)^3$
$u = 49 - 3t$
-> $u' = -3$

$v = 4(u)^3$
-> $v' = 12u^2$

$u' \times v'$
$V' = -3 \times 12(49 - 3t)^2$
$V' = -36(49 - 3t)^2$

### (a) Evaluate this function when t = 0

$V' = -36(49 - 3t)^2$
-> $-36(49 - 3(0))^2$
-> $-36(2401)$
$= -86,436$

**(b) Evaluate this function when $t$ is your answer for question (2).**

$V' = -36(49 - 3t)^2$
$Q2$ answer: $8.1\dot{6}$

-> $-36(49 - 3(8.1\dot{6}))^2$
-> $-36(49 - 24.5)^2$
-> $-36(24.5)^2$
-> $-36(600.25)$
$= -21,609$

**c) Compare your answers for question (3)(a) and question (3)(b). At which of these two times was the tank emptying at a greater rate? Explain why.**

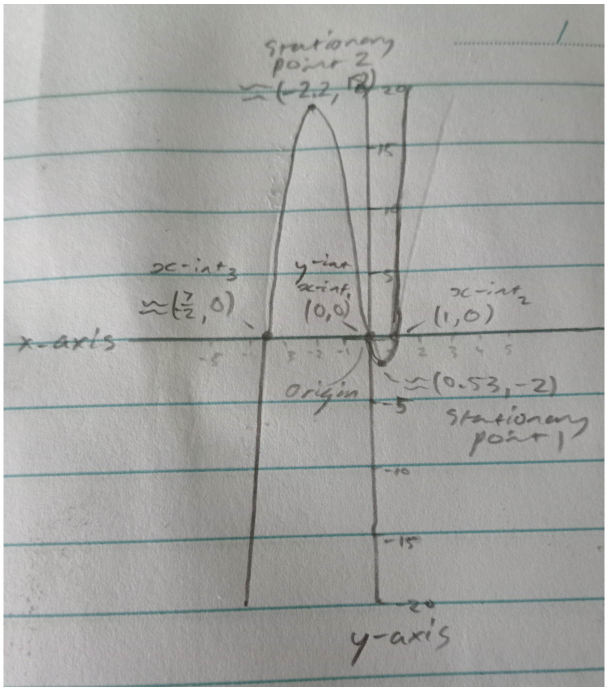
The number given out represents the magnitude of the rate of change at a given point. As we want to know which rate emptied faster, we want a more negative number.
Therefore, as (a) is more negative, it at zero days ($t = 0$) was emptying faster than at $t = 8.1\dot{6}$

# Task 2: Differentiation - Sketching graphs

## 1) Sketch the graph $y = 2x^3 + 5x^2 - 7x$. Provide all working for finding the intercepts and stationary points. You must use either a 2nd derivative test or a sign diagram to classify each stationary point.

$y = 2x^3 + 5x^2 - 7x$

**Graph**




## Intercepts

### $x$-intercepts

To find these without a calculator, we'll employ the zero-product principle

$0 = 2x^3 + 5x^2 - 7x$

$x(2x^2 + 5x - 7)$

> $x = 0$ or
> $x = 2x^2 + 5x - 7$

$0 = 2x^2 + 5x - 7$

$x = \frac{-(5)\pm\sqrt{(5)^2-4(2)(-7)}}{2(2)}$

-> $\frac{-5\pm\sqrt{25+56}}{4}$

-> $\frac{-5\pm\sqrt{81}}{4}$

-> $\frac{-5\pm\sqrt{81}}{4}$

-> $x^{\pm} = \frac{-5\pm 9}{4}$

$x^{+} = 1$

$x^{-} = -3.5 = -\frac{7}{2}$

> $x_1 = 0$, $x_2 = 1$, and $x_3 = -\frac{7}{2}$
> $(0, 0)$, $(1, 0)$, and $(-\frac{7}{2}, 0)$

### $y$-intercept

$y = 2(0)^3 + 5(0)^2 - 7(0)$

> $y = 0$
> $(0, 0)$

## Derivatives

Finding $y'$
$y = 2x^3 + 5x^2 - 7x$
-> $3 \times 2x^{3-1} + 2 \times 5x^{2-1} - 1 \times 7x^{1-1}$
-> $6x^2 + 10x^1 - 7x^0$
$y' = 6x^2 + 10x - 7$

Finding $y''$
$y' = 6x^2 + 10x - 7$
-> $2 \times 6x^{2-1} + 1 \times 10x^{1-1} \cancel{-7}$
-> $12x^1 + 10x^0$
$y'' = 12x + 10$

## Stationary points

### Solutions

$(0.5308, -2.0078)$ and $(-2.1976, 18.3040)$

### Finding the stationary points

$y' = 6x^2 + 10x - 7$
-> $0 = 6x^2 + 10x - 7$
-> $x = \frac{-(10) \pm \sqrt{(10)^2 - 4(6)(-7)}}{2(6)}$
-> $x = \frac{-10 \pm \sqrt{100 + 168}}{12}$
-> $x = \frac{-10 \pm \sqrt{268}}{12}$
$x_1^+ = \frac{-10 + \sqrt{268}}{12} = 0.5308$
$x_2^- = \frac{-10 - \sqrt{268}}{12} = -2.1976$

$y = 2x^3 + 5x^2 - 7x$
-> $f(x) = 2x^3 + 5x^2 - 7x$
-> $f(\frac{-10 \pm \sqrt{268}}{12}) = 2x^3 + 5x^2 - 7x$
$y_1^+ = -2.0078$
$y_2^- = 18.3040$

### Classifying the stationary points

$y'' = 12x + 10$

$y_1'' = 12(0.5308) + 10$
-> $6.3696 + 10$
$y_1'' = 16.3696$
This is $> 0$, so $(x_1, y_1)$ is a local minimum

$y_2'' = 12(-2.1976) + 10$
-> $-26.3712 + 10$
$y_2'' = -16.3712$
This is $< 0$, so $(x_2, y_2)$ is a local maximum