Week 4

Monday, 13 November 2023 9:07 AM

Random Phenomena :

Example 1: TRIAL → Tossing a coin one time

OUTCOME → Heads or Tails

SAMPLE SPACE → All possible outcomes

Probability = 1/2 = 50% $\quad S = \{H\} \ \{T\}$

TRIAL → Toss two coins simultaneously

SAMPLE SPACE → $S = \{H,H\} \ \{T,T\} \ \{H,T\} \ \{T,H\}$

Probability $= \frac{1}{4} = 0.25 = 25\%$

Example 2: TRIAL → Rolling a die

SAMPLE SPACE → $S = \{1\} \ \{2\} \ \{3\} \ \{4\} \ \{5\} \ \{6\}$

Probability $= \frac{1}{6} = 0.167 = 16.7\%$

TRIAL → Rolling two dices together

SAMPLE SPACE → $S = \{1,1\} \ \{1,2\} \ \{1,3\} \ldots\ldots \{2,2\},$
$\{2,1\} \ \{2,3\} \ldots\ldots\ldots\ldots \{6,6\}$

- The probability of an event is the number of outcomes in the event divided by the total number of possible outcomes. For a particular event, A:

$$P(\mathbf{A}) = \frac{\text{number of outcomes in } \mathbf{A}}{\text{Total number of outcomes}}$$

Example: $P(\text{Green}) = \frac{3}{10} = 0.3 = 30\%$

WARDROBE

3 Blue shirts
4 Black shirts
3 Green shirts

$P(\text{Black}) = \frac{4}{10} = 0.4 = 40\%$

$P(\text{not Black}) = \frac{6}{10} = 0.6 = 60\%$

$P(\text{Black}) + P(\text{not Black}) = 100\%$

Total probability = 1 or 100%

Question 1 : Probability distribution activity $\frac{45}{100} = 0.45$

A person's blood type is A, B, AB or O. Suppose 45% of the population has type O blood, 40% type A, 11% type B and the rest type AB.

Display this information as a probability distribution.

$$\begin{array}{r} 0.40 \\ 0.11 \\ +\ 0.45 \\ \hline 0.96 \end{array}$$

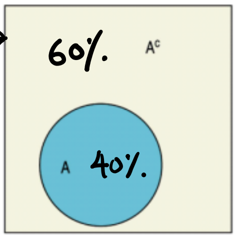
Outcome	A	B	AB	O
Probability	0.40	0.11	0.04	0.45

= 1

Complement Rule:

- The set of outcomes that are *not* in the event **A** is called the complement of **A**, denoted $\mathbf{A}^C$.
- The probability of an event occurring is 1 minus the probability that it doesn't occur: $P(\mathbf{A}) = 1 - P(\mathbf{A}^C)$

Universal set →



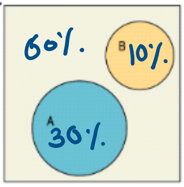
The set **A** and its complement.

Previous example :

P(Black) = 40%

$P(\text{Black})^C$ = P(not Black) = 60%

Events that have no outcomes in common (and, thus, cannot occur together) are called disjoint (or mutually exclusive).



P(Blue) = 30%

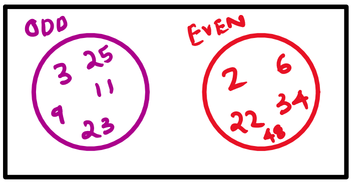
P(Yellow) = 10%

P(Blue or Yellow) = 30% + 10% = 40%

+

Addition Rule for disjoint sets A and B.

$$P(A \text{ OR } B) = P(A) + P(B)$$



P(Even) = {2, 6, 22, 48, 34}

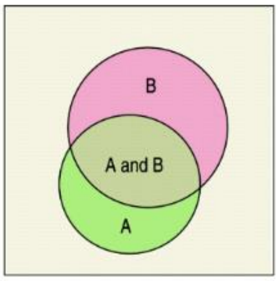
P(odd) = {3, 9, 11, 23, 25}

P(Even OR ODD) = P(Even ∪ Odd)

= {2, 3, 6, 9, 11, 22, 23, 25, 34, 48}

OR → ∪ → Union of sets

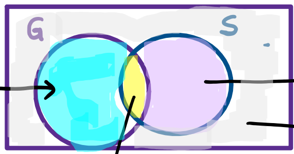
Addition Rule for sets A and B which are NOT DISJOINT



P(A or B) = P(A) + P(B) − P(A and B)

+

P(only G)



P(only S)

no G
no S

G → Garages

S → Swimming pool

P(G and S)

P(**A** *and* **B**) = *P*(**A**) x *P*(**B**), <u>provided that **A** and **B** are independent</u>.

Question 2 : Blood type

A person's blood type is A, B, AB or O. Suppose 45% of the population has type O blood, 40% type A, 11% type B and the rest type AB.

1. What is the probability that a random blood donor
   a) has blood type AB? 0.04 = 4%
   b) has type A or type B blood? = P(A) + P(B) = 0.40 + 0.11 = 0.51 = 51%.
   c) is not type O? 1 − P(O) = 1 − 0.45 = 0.55

Outcome	A	B	AB	O
Probability	0.40	0.11	0.04	0.45

0.40 + 0.11 + 0.04 = 0.55

2. Among four random donors, what is the probability that
   a) all are type O ?
   b) no-one is type AB?
   c) they are not all type A?
   d) at least one person is type B?
   e) the second person only is type O?

2) (a) P(all are type O) = P(O) AND P(O) AND P(O) AND P(O)
= P(O) X P(O) X P(O) X P(O)
= 0.45 X 0.45 X 0.45 X 0.45 = 0.041 = 4.1%

(b) P(no one AB) = 0.96 X 0.96 X 0.96 X 0.96 = $(0.96)^4$
= 0.849 = 84.9%

(c) P(not all type A) = 1 − P(all type A) = 1 − 0.0256
= 0.9744 = 97.44%

AAAB ABABO AABB

P(All type A) = AAAA → 0.40 X 0.40 X 0.40 X 0.40 = 0.0256

(d) P(at least 1 person is type B) = 1 − P(all not B) = 1 − 0.627
= 0.373 = 37.3%

P(not B x not B x not B x not B) = $(0.89)^4$ = 0.627

(e) P(second person only is type O)

= P(not O) x P(O) x P(not O) x P(not O)

$= 0.55 \times 0.45 \times 0.55 \times 0.55 = 0.0748 = 7.48\%$

Question 3 : Probability notation activity

The table below provides figures on the distribution for marital status for a group of adults by gender.

	Single	Married	Widowed	Divorced	TOTALS
Male	187	510	19	45	761
Female	149	520	106	66	841
TOTALS	336	1030	125	111	1602

$P(A) = P(\text{Male})$

$P(B) = P(\text{Divorced})$

For a randomly selected person from this group, let A be the event that the person is male and let B be the event that the person is divorced.

1. Find:
   a) P(A)
   b) P(B)
   c) P(A and B)
   d) P(B^c) $\longrightarrow$ P(not divorced)
   e) P(A or B)

2. Are events A and B mutually exclusive? Explain.
   No, $P(A \text{ and } B) \neq 0$

(a) $= P(A) = \frac{761}{1602} = 0.475$ ✓

(b) $= P(B) = \frac{111}{1602} = 0.069$ ✓

(c) $= P(A \text{ and } B) = \frac{45}{1602} = 0.028$ ✓

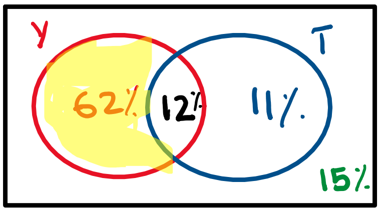
(d) $= P(B^c) = 1 - P(B)$
$= 1 - 0.069$
$= 0.931$

(e) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$= 0.475 + 0.069 - 0.028 = 0.516$

Question 4 : Venn diagram exercise

$P(Y) = 0.74$ $\quad P(T) = 0.23$ $\quad P(Y \text{ and } T) = 0.12$

For a particular demographic 74% of the group use YouTube, 23% use Twitter and 12% use YouTube and Twitter.



62 + 12 + 11 = 85 ✓

1. Construct a Venn diagram to display the information.
2. Use your diagram to answer the following.
   a) What percentage of people only use YouTube? 62%
   b) What is the probability that someone uses neither social media? 15% or 0.15
   c) What percentage of people use YouTube or Twitter?
   d) Are using YouTube and using Twitter mutually exclusive events? Explain.
   e) Are using YouTube and using Twitter independent events? Explain.

(c) $P(Y \text{ or } T) = P(Y) + P(T) - P(Y \text{ and } T)$
$= 0.74 + 0.23 - 0.12$
$= 0.85$

(d) No, $P(Y \text{ and } T) \neq 0$

(e) For independent events, $P(A \text{ and } B) = P(A) \times P(B)$

$P(Y \text{ and } T) = P(Y) \times P(T)$

$0.12 \neq 0.74 \times 0.23 = 0.17$

$\therefore$ The events are not independent

Conditional probability (cont.)

- To find the probability of the event **B** *given* the event **A**, we restrict our attention to the outcomes in **A**. We then find in what fraction of *those* outcomes **B** also occurred.

$$P(\mathbf{B}|\mathbf{A}) = \frac{P(\mathrm{A\ and\ B})}{P(\mathbf{A})}$$

Question 5 : Conditional probability

Suppose 74% of the population use YouTube, 23% use Twitter and 12% use YouTube and Twitter.

$P(Y) = 0.74$ ✓

$P(T) = 0.23$ ✓

$P(Y \text{ and } T) = 0.12$ ✓

1. Given that someone uses Twitter, what is the probability that they also use YouTube?

Condition

2. What is the probability that someone uses Twitter if it is known that they use YouTube?

Condition

1) $P(Y/T) = \frac{P(Y \text{ and } T)}{P(T)} = \frac{0.12}{0.23} = 0.521$

2) $P(T/Y) = \frac{P(Y \text{ and } T)}{P(Y)} = \frac{0.12}{0.74} = 0.162$

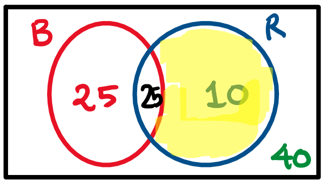
QUIZ PRACTICE

$P(\text{Bike}) = P(B) = 0.50$

$P(\text{Run}) = P(R) = 0.35$

$P(B \text{ and } R) = 0.25$

A recent survey of adults found that 50% of them regularly bike ride for exercise, 35% are runners and 25% ride and run.



$25 + 25 + 10 = 60$

a) Using a Venn diagram or otherwise, find the percentage of these adults that:

(i) run but don't ride (1 mark)

$35\% - 25\% = 10\%$

○ 40%

○ 60%

○ 32%

○ 57%

✓ 10%

(ii) run or ride (1 mark)

$P(\text{Bike OR RUN}) = P(B) + P(R) - P(B \text{ and } R)$

$= 0.50 + 0.35 - 0.25$

$= 0.60$

✓ 60%

○ 28%

○ 97%

○ 25%

○ 47%

(iii) ride if it is known that they run (2 marks)

Condition

- 43.9%
- 71.4% ✓
- 166.7%
- 25%
- 32%

$P(B/R) = \frac{P(B \text{ and } R)}{P(R)} = \frac{0.25}{0.35} = 0.714$

(iv) neither run nor ride (1 mark)

- 47%
- 25%
- 3%
- 40% ✓
- 53%

$1 - P(B \text{ OR } R)$

$1 - 0.6 = 0.4$

b) Are running and riding disjoint? (1 mark)

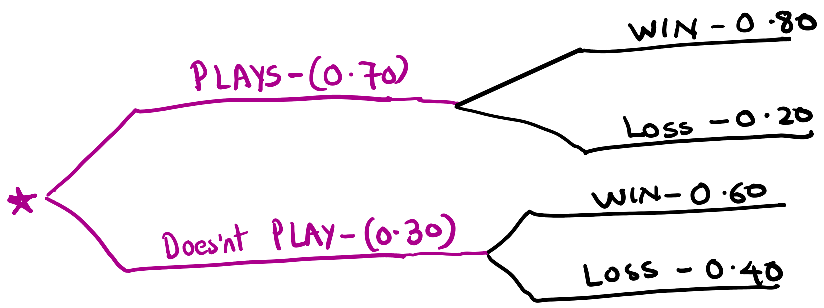
- Yes, because adults only ride or run
- No, because an adult can ride and run for exercise. ✓
- Yes, because riding a bike doesn't affect the likelihood of being a runner.
- Yes, because an adult can ride and run for exercise.
- No, because riding a bike and running are independent events.

$P(B \text{ and } R) = 0.25$

Question 6 : Tree diagram

A star player for a team is injured and has been given a 70% chance of playing in the next game. If the star plays, the team has an 80% chance of winning their next game. Without the star, the team has a 60% chance of winning.

Use a tree diagram to find the probability of the team winning the next game.



P (team winning the next game)

= P (Plays and Win) OR P (Doesn't play and Win)

= $0.70 \times 0.80 + 0.30 \times 0.60$

= $0.56 + 0.18 = 0.74$

The expected value of a (discrete) random variable can be found by summing the products of each possible value and the probability that it occurs:

$$\mu = E(X) = \sum x \cdot P(x)$$

The variance for a random variable is:

$$\sigma^2 = Var(X) = \sum (x-\mu)^2 \cdot P(x)$$

$\sigma = \sqrt{\sum (x-\mu)^2 \cdot P(x)}$

The standard deviation for a random variable is:

$$\sigma = SD(X) = \sqrt{Var(X)}$$

Question 7 : Discrete random variable

Follow example 3.58 on p. 118 of the text then find the mean and standard deviation for the following random variable.

x	1	2	3	4	5
$P(x)$	0.6	0.1	0.1	0.1	0.1

x	$P(x)$	$x \cdot P(x)$	$x-\mu$	$(x-\mu)^2$	$(x-\mu)^2 \times P(x)$
1	0.6	0.6	1-2 = -1	1	1 × 0.6 = 0.6
2	0.1	0.2	0	0	0 × 0.1 = 0.0
3	0.1	0.3	1	1	1 × 0.1 = 0.1
4	0.1	0.4	2	4	4 × 0.1 = 0.4
5	0.1	0.5	3	9	9 × 0.1 = 0.9
		2.0			2.0

Mean, $\mu = E(X) = \sum x \cdot P(x) = 2$

OR

$$\mu = 1 \times 0.6 + 2 \times 0.1 + 3 \times 0.1 + 4 \times 0.1 + 5 \times 0.1 = 2$$

Standard deviation, S.D $\sigma = \sqrt{\sum (x-\mu)^2 \cdot P(x)} = \sqrt{2} = 1.414$

OR

$$\begin{aligned}\sigma^2 &= (1-2)^2 \times 0.6 + (2-2)^2 \times 0.1 + (3-2)^2 \times 0.1 + (4-2)^2 \times 0.1 + (5-2)^2 \times 0.1 \\ &= 2 \\ \sigma &= \sqrt{2} = 1.42\end{aligned}$$

Online calculator:

https://www.mathportal.org/calculators/statistics-calculator/probability-distributions-calculator.php

Enter Values for X (separate by , : ; or blank space)

```
1,2,3,4,5
```

Enter Values for P(X) (separate by , : ; or blank space)

```
0.6,0.10,0.10,0.10,0.10
```

All values of P(X) must sum to one.
Example: 1/3, 1/6, 0.5

The mean (expectation) of the given distribution is:

$$\mu = 2$$

explanation

In order to find the mean (expectation) of the given distribution, we will use the following formula:

$$\mu = \sum x \cdot p(x)$$

So, first we need to multiply each value of X by each probability P(X), then add these results together. In this example we have:

$$\mu = 1 \cdot 0.6 + 2 \cdot 0.1 + 3 \cdot 0.1 + 4 \cdot 0.1 + 5 \cdot 0.1$$
$$\mu = 2$$

The standard deviation of the given distribution is:

$$\sigma = 1.4142$$