Task 1: Quadratics

1.1) State the number of real solutions + explain using discriminant

Discriminant equation

$b^2 - 4ac$

a) $-x^2 - 3x - 10 = 0$

Discriminant

-> $-3^2 - 4(-1)(-10)$

-> $9 - 4 \times 10$

-> $= -31$

There are no real solutions as the discriminant is negative, $\sqrt{-31}$ being a complex number.

b) $5x^2 - 150 = 0$

Discriminant

-> $0^2 - 4(5)(-150)$

-> -20×-150

-> $-2\cancel{0} \times -15\cancel{0}$

-> 30

There are two real solutions as the discriminant is a positive, non-zero value.

c) $25x^2 - 20x + 4 = 0$

Discriminant

-> $-20^2 - 4(25)(4)$

-> $400 - 100 \times 4$

-> $400 - 400 = 0$

The discriminant is zero, meaning there is only one unique solution.

1.2) Complete the square and identify stationary points

a) $y = x^2 - 6x + 17$

Find $\frac{b^2}{4}$

-> $\frac{-6^2}{4} = \frac{36}{4} = 9$

Add $\frac{b^2}{4}$ to the equation

-> $y = x^2 - 6x + 9 + 17 - 9$

-> $y = x^2 - 6x + 9 + 8$

Form a perfect square
-> $y = (x^2 - 6x + 9) + 8$
-> $-3 \times -3 = 9$ and $-3 + -3 = -6$
-> $y = (x-3)^2 + 8$

Extract the vertex
$y = (x-3)^2 + 8$
-> x is translated by -3, so $x = 3$
-> $y = 8$

Vertex = $(3, 8)$

Verification
$x = \frac{-b}{2a} \rightarrow \frac{--6}{2(1)} = \frac{6}{2} = 3$
$y = 3^2 - 6(3) + 17$
-> $9 - 18 + 17$
-> $y = -9 + 17 = 8$
$(3, 8)$

b) $y = 3x^2 - 12x + 6$

Divide by a
$y = 3x^2 - 12x + 6$
-> $y = \frac{3x^2}{2} - \frac{12x}{2} + \frac{6}{2}$
-> $3(x^2 - 4x + 2)$

Find $\frac{b^2}{4}$
$3(x^2 - 4x + 2)$
-> $\frac{-4^2}{4} = \frac{16}{4} = 4$

Add $\frac{b^2}{4}$ **to the equation**
$3(x^2 - 4x + 2)$
-> $3(x^2 - 4x + 4 + 2 - 4)$
-> $3(x^2 - 4x + 4 - 2)$

Form a perfect square
$3(x^2 - 4x + 4 - 2)$
-> $3((x^2 - 4x + 4) - 2)$
-> $-2 \times -2 = 4$, added they $= -4$. -2 is therefore the factor.
-> $3((x-2)^2 - 2)$

Extract the vertex
$y = 3((x-2)^2 - 2)$
-> x is translated by -2, so $x = 2$
-> $y = 3(-2) = -6$

Vertex = $(2, -6)$

Verification
$x = \frac{-b}{2a} = \frac{--12}{2(3)} = \frac{12}{6} = 2$
-> $x = 2$

$y = 3x^2 - 12x + 6$
-> $3(2^2) - 12(2) + 6$
-> $3(4) - 24 + 6$
-> $12 - 24 + 6$
-> $-12 + 6$
-> $y = -6$

$= (2, -6) = \text{vertex}$

1.3) Find the solutions for $4x^2 - 3x = 10$

$4x^2 - 3x = 10$
-> $4x^2 - 3x - 10 = 0$
-> $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-10)}}{2(4)}$
-> $x = \frac{3 \pm \sqrt{9 - -160}}{8}$
-> $x = \frac{3 \pm \sqrt{169}}{8}$
-> $x = \frac{3 \pm 13}{8}$

$x^+ = \frac{3+13}{8} = \frac{16}{8} = 2$
$x^- = \frac{3-13}{8} = \frac{-10}{8} = -\frac{5}{4}$

Task 2: Logarithms and Exponentials

2.1) Solve for x using log and exponent rules

a) $2y = 10 + e^{3x-4}$

-> $2y - 10 = e^{3x-4}$
-> $ln(2y - 10) = ln(e^{3x-4})$
-> $ln(2y - 10) = 3x - 4$
-> $ln(2y - 10) + 4 = 3x$
-> $\frac{ln(2y-10)+4}{3} = x$
$x = \frac{ln(2y-10)+4}{3}$

b) $\ln(2x - 3) + \ln(x + 2) = \ln(9x - 24)$

Apply $log_a(a) = 1$ to both sides of the equation

-> $e^{ln(2x-3)} + e^{ln(x+2)} = e^{ln(9x-24)}$
-> $2x - 3 + x + 2 = 9x - 24$
-> $3x - 1 = 9x - 24$
-> $-1 + 24 = 9x - 3x$

-> $23 = 6x$
$x = \frac{23}{6}$

2.2) Expand and simplify $(2e^{2x} - e^{-2x})^2$

$a^2 - 2ab + b^2$
-> $(2e^{2x})^2 - 2(2e^{2x})(-e^{-2x}) + (-e^{-2x})^2$

- $2^2e^{4x} = 4e^{4x}$
- $-2(2e^{2x}) = -4e^{2x}$
- $-4e^{2x} \times e^{-2x} = -4e^{2x+-2x} = 4e^0 = -4$
  - $(-e^{-2x})(-e^{-2x}) = e^{-2x+-2x} = e^{-4x}$
  $4e^{4x} + e^{-4x} - 4$

Verification
$4e^{4x} + e^{-4x} - 4$
-> $4e^{4(0)} + e^{-4(0)} - 4$
-> $4 * 1 + 1 - 4$
-> $4 + 1 - 4 = 1$

$(2e^{2x} - e^{-2x})^2$
-> $2e^{2(0)} - e^{-2(0)}$
-> $(2 - 1)^2 = (1)^2$
-> 1

2.3) Simply $2ln(xy) + ln(x) - ln(x^3y^2)$

$2ln(xy) + ln(x) - ln(x^3y^2)$

Prepare $2ln(xy)$ for addition using the expansion log law

$log_a(m^n) = (n)log_a(m)$
-> $2ln(xy) \rightarrow ln((xy)^2)$

Add them together using the addition log law

$log_a(m) + log_a(n) = log_a(mn)$
-> $ln((xy)^2 \times x)$
-> $x^2y^2 \times x$
-> x^3y^2
$ln(x^3y^2)$

Subtraction
$log_a(m) - log_a(n) = log_a(\frac{m}{n})$
-> $ln(x^3y^2) - ln(x^3y^2) = ln(\frac{x^3y^2}{x^3y^2})$
-> $ln(\frac{x^3y^2}{x^3y^2})$

-> $ln(\frac{x^0y^0}{x^0y^0})$
-> $ln(\frac{1\times1}{1\times1})$

$ln(1) = 0$

Task 3: Trigonometry

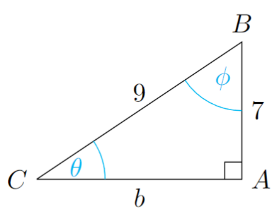

FIGURE 1. Triangle

3.1) Find b's length, θ, and ϕ. Angles must be in radians and to 4 decimal places.

$b = 4\sqrt{2}$
$\theta = 0.8911$ radians
$\phi = 0.6797$ radians

$b = \sqrt{c^2 - a^2}$
-> $b = \sqrt{9^2 - 7^2}$
-> $b = \sqrt{81 - 49}$
-> $b = \sqrt{32}$
-> $32 = 2 * 16$
-> $b = \sqrt{16}\sqrt{2}$
$b = 4\sqrt{2}$

$\theta = \arctan\left(\frac{7}{4\sqrt{2}}\right) = 0.8911$ radians

- $\theta = \arccos\left(\frac{4\sqrt{2}}{9}\right) = 0.8911$ radians
- $\theta = \arcsin(\frac{7}{9}) = 0.8911$ radians

$360^\circ = 2\pi$ radians, so $180^\circ = \pi$ radians and $90^\circ = \frac{\pi}{2}$ radians
According to the triangle law, a triangle's sum of angles is 180°.
-> $180^\circ - 51.06^\circ - 90^\circ = 38.94^\circ$
$\pi - 0.8911 - \frac{\pi}{2} = 0.6797$ radians

3.2) If $\cos(\theta) = \frac{-\sqrt{3}}{2}$ and $\sin(\theta) = \frac{-1}{2}$, which quadrant does angle θ belong to?

A negative value divided by a positive still leads to a negative value, so $\cos = -x$ and $\sin = -y$. Following ASTC, Angle θ must therefore belong to the third quadrant as $\tan$ would be positive when both $\sin$ and $\cos$ are negative.

3.3) Sketch $y = 2\cos(x)$ for $x \in [0, 2\pi]$. What is the range and x/y-intercepts?

The domain is $0 \leq x \leq 2\pi$.
The range of $\cos$ is $-1 \leq x \leq 1$, but as it is multiplied by $2\times$, the range of this function is $-2 \leq x \leq 2$

y-intercept
$y = 2\cos(x)$
-> $y = 2\cos(0)$
-> $y = 2 \times 1$
$y = 2$

x-intercept
$y = 2\cos(x)$
-> $0 = 2\cos(x)$
-> $\frac{0}{2} = \cos(x)$
-> $\arccos(0) = x$
$x = 1.5708 = \frac{\pi}{2}$
It took $x = \frac{\pi}{2}$ to do half of a full dip, reaching $y = 0$. $\cos$ waves rise back up, so it must take $\frac{\pi}{2}$ for the first half of the down dip to reach $y = 0$, $\frac{\pi}{2}$ for the second half of the dip ($y = -1$), and on the upwards rise, another $\frac{\pi}{2}$, totalling $3\frac{\pi}{2} = \frac{3\pi}{2}$ where $y = 0$ again.
At this point, it will have reached another x intercept. So, the x intercepts are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.
A general formula found with the help of the internet is $\frac{\pi}{2} + 2\pi n$.

Graph