SIT191 Problem solving task 2
Due: Friday 15th December 2023

Total marks: 60

Weighting: 30%

2.1 People have one of four different blood types.

Blood types	Percentage (%)
O	30
A	35
B	15
AB	Rest of them

a) What is the probability that a person involved in an accident
i. does not have type A blood?
ii. has type O or type A blood?
iii. is neither type AB or type B?

b) Among three accident victims, what is the probability that
i. all have type A?
ii. none of them are type O?
iii. at least one person is type B?
iv. the first accident victim only is type A?

[(1+1+1)+(1+1+1+2) = 8 marks]

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2.2 Assume that 45% of households have at least one dog, 25% of households have at least one cat and that 10% of households have at least one of each animal.

a) What is the probability that a randomly selected household has a dog but not a cat?
b) What is the probability that a randomly selected household does not have either animal?
c) What is the probability that a randomly selected household has a cat or a dog?
d) If a household has a dog, what is the probability that they also have a cat?
e) Are owning dogs and cats mutually exclusive? Explain.
f) Are owning dogs and cats independent events? Explain.

[1+1+1+2+2+2 = 9 marks]

2.3 Measurement of the thumb nail length in a sample of healthy middle-aged men native to a particular region found a mean length of 12.4 mm and a standard deviation of 2.7 mm. Suppose that the sample provides an accurate representation of the whole population of native men in this said region and that a Normal model applies.

a) Draw the model for the thumb nail length of a native man of this region.
b) About what percent of native men would have a thumbnail below 9.7 mm in length.
c) About what percent of native men would have a thumbnail between 7 mm and 15.1 mm in length?

[2+1+1 = 4 marks]

2.4 Based on the Normal model *N*(12.4, 2.7) describing the average length, What proportion of native men would have a thumbnail

a) at least 17 mm?
b) between 8 mm and 13 mm?

[2+2 = 4 marks]

2.5 Based on the Normal model *N*(12.4, 2.7) describing the average length, What is the approximate thumbnail length above which, 16% of the native men with the longest thumbnail lengths are expected?

[2 marks]

2.6 In a particular rural area of Victoria, 80% of learner drivers pass their driving test on the first attempt.

a) Amongst If 8 learner drivers are selected, what is the probability of a successful driving test outcome for
   i) all 8 drivers?
   ii) 0 or 1 drivers?
   iii) at least two drivers?

b) For 120 learner drivers are selected,
   i) how many on average would you expect to pass the test? Compute the standard deviation.
   ii) what is the probability that more than 100 drivers pass?
   [Hint: use your answers from (b) (i)]

[(1+2+2) +(2+3) = 10 marks]

2.7 A random sample of 262 people tried a special exercise routine that lasted 2 months to lose weight. The weights of these people were measured both at the beginning and the end of the routine. Out of them, 226 individuals experienced a weight loss of at least 5%. Is there evidence that this exercise results in an average weight loss of at least 5% in more than 81% of individuals?

a) Write appropriate hypotheses.
b) Check the assumptions and conditions.
c) Perform the hypothesis test (state $\hat{p}$, $\hat{q}$, z-score and P-value).
d) State your conclusion. Use a significance level of 5%.
e) Give a 90% confidence interval for the true proportion of people who may have an average weight loss of at least 5% by this exercise and interpret your interval (state also Critical value z*, formula and value for Standard Error, and formula for the confidence interval).

[1+2+3+1+5 = 12 marks]

2.8 A survey was conducted to compare customer satisfaction on the service in two branches of a particular bank. Various customers visiting the two branches were selected randomly, and asked if they were satisfied with the services provided by the branch. The results of this survey are shown in the following table. Is a significant difference between the two bank branches regarding the responses from the customers about their satisfaction on the service of the two branches?
Use indices "1" (like n_1, p_1, q_1) for Branch A and "2" (like n_2, p_2,q_2) for Branch B.

	Number satisfied	Not satisfied
Branch A	558	53
Branch B	496	77

a) Write appropriate hypotheses.
b) Perform the hypothesis test: find $\hat{p_1}$, $\hat{q_1}$, $\hat{p_2}$, $\hat{q_2}$, $\hat{p}_{pooled}$, $\hat{q}_{pooled}$, z-score, the P-value and state your conclusion in plain English. Use $\alpha = 0.05$.
c) Create a 95% confidence interval for the difference in the proportions between the two bank branches regarding the responses from the customers satisfied on the services, and interpret your interval (also state the critical value z*, formula and value for Standard Error, and formula for the confidence interval).
d) Explain how your interval is consistent with your conclusion from b).

[1+4+4+2 = 11 marks]

Marking Scheme:

Questions		Maximum marks
Question 1	part(a-i), (a-ii) and (a-iii) – Calculation of probability to two decimal places (1 mark each)	3
	part (b-i), (b-ii) and (b-iii) – Calculation of probability to two decimal places (1 mark each)	3
	part(b-iv) – Calculation of probability to two decimal places (2 marks)	2
Question 2	part(a)– Calculation of probability to two decimal places (1 mark)	1
	part(b)– Calculation of probability to two decimal places (1 mark)	1
	part(c) – Calculation of probability to two decimal places (1 mark)	1
	part(d) – Calculation of conditional probability to two decimal places (2 marks)	2
	part(e) – Reasonable explanation of mutually exclusive events (2 marks)	2
	part(f) – Reasonable explanation of independent events (2 marks)	2
Question 3	part(a) – Bell shaped curve with correct mean (0.5 marks) and correct label of 68-95-99.7% Rule (0.5 marks each)	2
	part(b) – Calculation of percentage (1 mark)	1
	part(c) - Calculation of percentage (1 mark)	1

Question 4	part(a) – Correct percentage (2 marks)	2
	part(b) – Correct percentage (2 marks)	2
Question 5	Correct z-score (1 mark) and correct calculation (1 mark)	2
Question 6	part(a-i)– Calculation of probability to two decimal places (1 mark)	1
	part(a-ii), (a-iii)– Calculation of probability to two decimal places (2 marks each)	4
	part(b-i) – Correct formula (1 mark) and correct mean calculation (1 mark)	2
	part (b-ii) – correct standard deviation calculation (1 mark) and calculation of probability to two decimal places (2 marks)	3
Question 7	part(a) – correct null and alternative hypotheses (0.5 marks each)	1
	part(b) – one relevant assumption (1 mark) and one relevant condition (1 mark) with respect to one proportion	2
	part(c) – Correct formula for z-score (1 mark); correct calculation to 4 decimal places: z-score (1 mark) and P-value (1 mark)	3
	part(d) – reasonable comparison of P-value and significance level (0.5 marks) and correct conclusion (0.5 marks)	1
	part(e) - Correct formula for Standard Error (1 mark) and for Confidence interval (1 mark), correct calculation to 4 decimal places: z* (0.5 marks), Standard Error (1 mark), Confidence interval (1 mark); and correct interpretation (0.5 marks)	5
Question 8	part(a) – Correct null and alternative hypotheses (0.5 marks each)	1
	part(b) – Correct formula for z-score (0.5 marks), correct calculation to 4 decimal places: z-score (0.5 marks), P-value (2 marks), correct comparison of P-value and significance level and correct conclusion (0.5 marks); correct interpretation (0.5 marks)	4
	part(c) - Correct formula for Standard Error (0.5 marks) and for Confidence interval (0.5 marks), Correct calculation to 4 decimal places: z* (0.5 marks), Standard Error (1 mark), Confidence interval (1 mark); and correct interpretation (0.5 marks)	4
	part(d) – Reasonable comparison of part(b) and (c) conclusion with respect to null hypotheses (2 marks)	2