Task 1: Give it a go

Click on a question number to see how your answers were marked and, where available, full solutions.

Question Number	Score	
Question 1	2 / 2	Review
Question 2	1 / 1	Review
Question 3	1 / 1	Review
Question 4	1 / 1	Review
Question 5	2 / 2	Review
Question 6	2 / 2	Review
Question 7	10 / 10	Review
Total	**19 / 19 (100%)**	

Performance Summary

Exam Name:	SIT190 - Week 7 - Quiz - Short
Session ID:	13874488195
Student's Name:	COWLISHAW, Ethan Del (edcowlishaw)
Exam Start:	Thu Jan 01 1970 10:00:00
Exam Stop:	Tue Apr 30 2024 15:25:16
Time Spent:	0:14:44

Click on a question number to see how your answers were marked and, where available, full solutions.

Question Number	Score	
Question 1	2 / 2	Review
Question 2	1 / 1	Review
Question 3	1 / 1	Review
Question 4	1 / 1	Review
Question 5	2 / 2	Review
Question 6	2 / 2	Review
Question 7	10 / 10	Review
Total	**19 / 19 (100%)**	

Performance Summary

Exam Name:	SIT190 - Week 7 - Quiz - Short
Session ID:	12394967142
Student's Name:	COWLISHAW, Ethan Del (edcowlishaw)
Exam Start:	Fri May 03 2024 15:10:29
Exam Stop:	Fri May 03 2024 15:23:27
Time Spent:	0:12:58

I achieved full marks on both attempts of the quizzes. Though I did get full marks, I did continuously make silly mistakes like forgetting the order in which you subtract values to find the gradient (doing $y_1 - y_2$ instead of $y_2 - y_1$), so there is definitely room for improvement on both attempts.
For the last question where you needed to find the y-values of the stationary points, it completely slipped my mind on how to do so. I did a bunch of trial-and-error to remember how.

I understand how to find the answers most of the time, I simply have not consolidated the information properly. I will focus on doing so, and these quizzes have already helped me identify gaps and force me to slow down.

I did improve by about 12% time-wise so I am happy with that performance. As aforementioned, I made a few silly mistakes that would easily be fixed by slowing down.

(also a fun thing I noticed - my time on the first attempt thinks we're at the Unix epoch back in 1970)

Task 2: Derivatives

1) If $f(x) = 4x^2 + 1$, find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x+h) = 4(x+h)^2 + 1$
-> $4(x^2 + h^2 + 2xh) + 1$
-> $4x^2 + h^2 + 8xh + 1$

$\lim_{h \to 0} \frac{(4x^2+h^2+8xh+1)-(4x^2+1)}{h}$
-> $\frac{4x^2+h^2+8xh+1-4x^2-1}{h}$
-> $\frac{\cancel{4x^2}+h^2+8xh+\cancel{1}\cancel{-4x^2}\cancel{-1}}{h}$
-> $\frac{h^2+8xh}{h}$
-> $\frac{h^2}{h} + \frac{8xh}{h}$
-> $h + 8x$

-> $0 + 8x$
$= 8x$

Verification
$4x^2 + 1$
-> $4 \times 2x^{2-1} \cancel{+1}$
-> $8x^1$
$= 8x$

2) The below table gives the displacement of a vehicle from a starting position. Give the average rate of change in displacement between 4-8 hours.

Time (h)	0	2	4	6	8	10
Displacement (km)	0	135	210	298	329	428

$\frac{329-210}{8-4} = \frac{119}{4} = 29.75$km

3) Find the derivative of the following functions

a) $f(x) = 7x^3 - \frac{5}{x^3} - \frac{4}{x^{-1}} - \frac{5}{\sqrt{x+1}}$

Answer: $21x^2 + 15x^{-4} - 4 - \frac{5}{2(x+1)^{\frac{3}{2}}}$
Alternate answer: $21x^2 + \frac{15}{x^4} - 4 - \frac{5}{2(x+1)^{\frac{3}{2}}}$

Working out

$1 : 7x^3$
$2 : -\frac{5}{x^3}$
$3 : -\frac{4}{x^{-1}}$
$4 : -\frac{5}{\sqrt{x+1}}$

1: $7x^3$

-> $7 * 3x^{3-1}$
$= 21x^2$

2: $-\frac{5}{x^3}$

-> $-5x^{-3}$
-> $-3 * -5x^{-3-1}$
$= 15x^{-4}$ or $\frac{1}{15x^4}$

3: $-\frac{4}{x^{-1}}$

-> $-4 \div \frac{1}{x}$
-> $-4 \times \frac{x}{1}$

-> $\frac{-4x}{1} = -4x$

Or: -> $\frac{-4}{\frac{1}{x}}$

-> $\frac{1}{x} = 1x^1 = x$

-> $\frac{-4}{x}$

-> $-4x^1 = -4x$

-> $-4 * 1x^{1-1}$

$= -4$

4:

$-\frac{5}{\sqrt{x+1}}$

-> $\sqrt{x+1} = (x+1)^{\frac{1}{2}}$

-> $-\frac{5}{(x+1)^{\frac{1}{2}}}$

-> $-5(x+1)^{\frac{1}{2}}$

-> $-5 * \frac{1}{2} * (x+1)^{\frac{1}{2}-1}$

-> $-\frac{5}{2}(x+1)^{-\frac{3}{2}}$

-> $-\frac{5}{2} \times \frac{1}{\sqrt{(x+1)^3}}$

$-\frac{5}{2\sqrt{(x+1)^3}}$ or $-\frac{5}{2(x+1)^{\frac{3}{2}}}$

b) $y = 3\cos(4x^2) - 2\sin\left(\frac{x}{3}\right) + \tan(x)$

$f'(x) = -k * p * \sin(kx^p)$

-> $3\cos(4x^2)$

-> $3 \times -4 * 2\sin(4x^2)$

-> $-24\sin(4x^2)$

$f'(x) = k\cos(kx)$

-> $-2\sin(\frac{x}{3})$

-> $k = 1,\ x \to \frac{x}{3}$

-> $-2 \times \frac{1}{3}\cos(\frac{x}{3})$

-> $-\frac{2}{3}\cos(\frac{x}{3})$

$\tan(kx) \to f'(x) = k * p * \sec(kx^p)^2$

-> $\tan(x)$

-> $k = 1,\ x \to x$

-> $1 \times \sec(x)^2$

-> $\sec(x)^2$

$-24\sin(4x^2) - \frac{2}{3}\cos\left(\frac{x}{3}\right) + \sec(x)^2$

c) $y = 3e^{7x^2}$

$y = 3e^{7x^2}$

$u = 7x^2$

$u' = 14x^1$

$v = 3e^u$
$v' = 3e^u$

$f'(x) = v' \times u$
$3e^{7x^2} \times 7x^2$
-> $3e^{7x^2} \times 7 \times 2x^{2-1}$
-> $3e^{7x^2} \times 14x$
-> $14x \times 3e^{7x^2}$
$42e^{7x^2}x$

d) $y = \ln(5x^3)$

Derivative of $f(x) = \ln(f_2(x))$ is $f'(x) = \frac{f_2'(x)}{f_2(x)}$

$u = 5x^3$
$u' = 15x^2$

$v = \ln(u)$
$v' = \frac{1}{u}$

$u' \times v'$
-> $15x^2 \times \frac{1}{5x^3}$
-> $\frac{15x^2}{5x^3}$
-> $\frac{3x^{2-2}}{x^{3-2}}$
-> $\frac{3x^0}{x^1}$
$= \frac{3}{x}$

4) Sketch the graph $y = x^2 + 4x - 13$

Requirements:

- Find the derivative of this function and use this to find the stationary point.
- Give all working for finding the x and y-intercepts.
- Use a sign diagram or the second derivative test to identify if the stationary point of $y = x^2 + 4x - 13$ is a local maximum or a local minimum

Derivative + classification

$f(x) = x^2 + 4x - 13$
-> $f'(x) = 1 \times 2x^{2-1} + 4 \times 1x^{1-1} \cancel{-13}$
-> $f'(x) = 2x^1 + 4x^0$
$f'(x) = 2x + 4$
$f''(x) = 2$, which is > 0, meaning the stationary point is a local minimum, and as it is a parabola, it is a global minimum too.

Stationary point

$(-2, -17)$

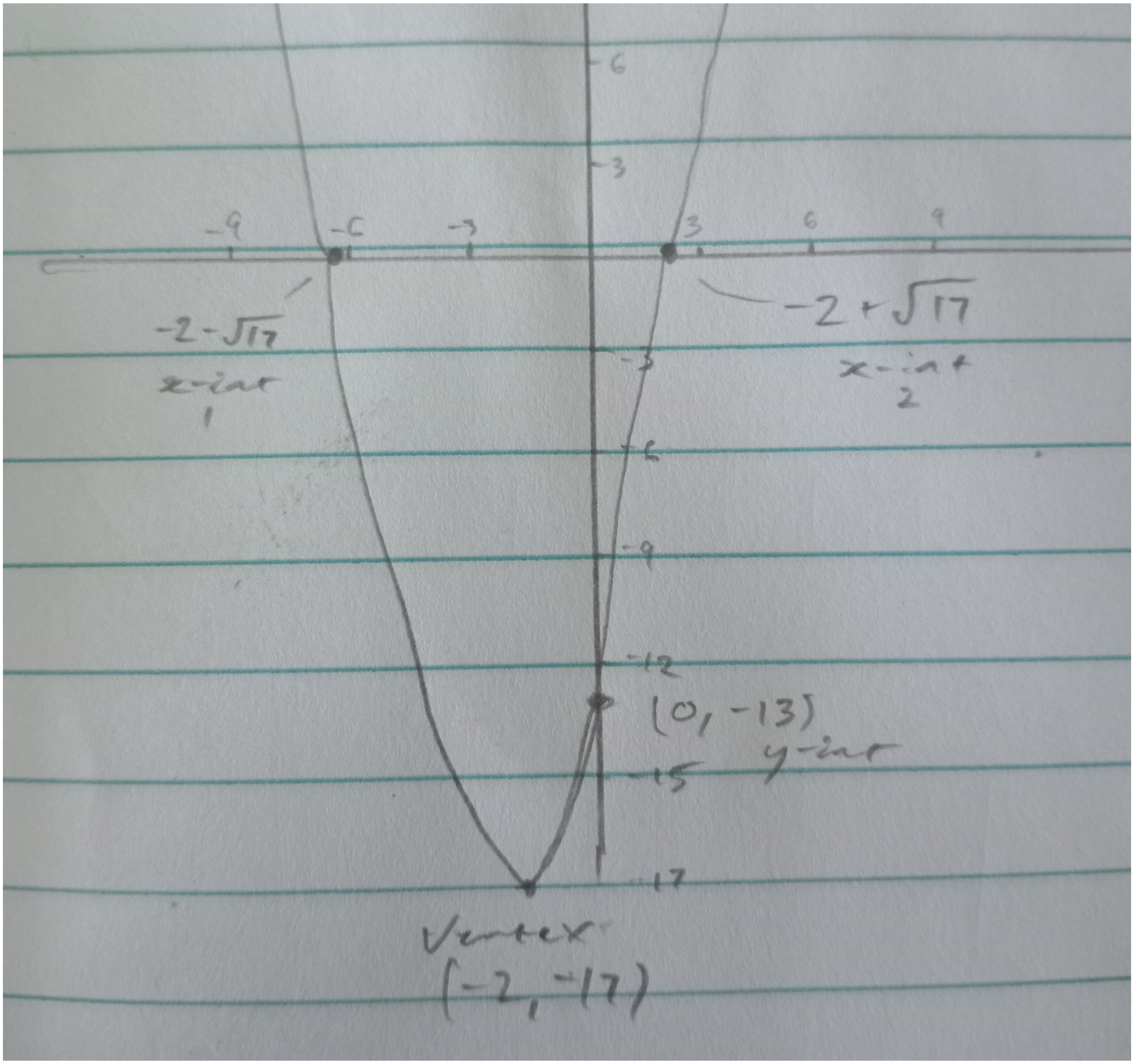
$0 = 2x + 4$
-> $2x = -4$
-> $x = -2$

$(-2)^2 + 4(-2) - 13$
-> $4 - 8 - 13$
$y = -17$

y and x intercepts

The y-intercept is the constant of the $f(x)$ function, -13
For the x in $x^2 + 4x - 13$, we can use the quadratic equation
$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-13)}}{2(1)}$
-> $x = \frac{-4 \pm \sqrt{16+52}}{2}$
-> $x = \frac{-4 \pm \sqrt{68}}{2}$
-> $x = \frac{-4 \pm \sqrt{68}}{2}$
-> $\sqrt{68} = \sqrt{4}\sqrt{17} = 2\sqrt{17}$
-> $x = -2 \pm \frac{2\sqrt{17}}{2}$
-> $x = -2 \pm \sqrt{17}$
The x-intercepts are $x_1^+ = -2 - \sqrt{17}$ and $x_2^- = -2 + \sqrt{17}$

Graph



-2 - √17
x-int 1
-2 + √17
x-int 2
(0, -13)
y-int
Vertex
(-2, -17)