Problem Solving task 3

Q1. 8 colours Green Yellow Red Orange Pink Purple Blue Brown

120 smarties 12 14 17 15 16 17 11 18

a) If equal, how many of each colour? Chi-square Goodness of fit

$\frac{120 \text{ smarties}}{8 \text{ colours}}$ = 15 of each colour in the bag.

b) A goodness-of-fit test.

Observed counts = Expected counts

c) H_0: The observed smarty counts match the expected counts of each (15)

H_A: The observed smarty counts do not match the expected count of each. → observed counts ≠ Expected counts

d) There are (8-1=7) 7 degrees of freedom.

e) The chi-square² value is 2.933 and the p-value is 0.8911

Calculated on Statskingdom.com/310GoodnessChi.html

Test: Goodness of fit, the rest set to default.

Sample data:

Observed	Expected
12	15
14	15
17	15
15	15
16	15
17	15
11	15
18	15

f) Conclusion given an alpha of 0.05

The null hypothesis is accepted as the p-value (0.8911) is greater than the alpha/significance level (0.05). The observed smarty counts strongly fit the expected model values.

Chi-square-independence

Q2. Are security level & position independent?

	Very Secure	Secure	Insecure
Non-manager	22	55	15
Manager	45	35	15

a) Hypotheses

H_0: The position held and the level of securing clearance of staff members is independent $A \perp B$

H_A: The position held and the securing level of staff members is dependent. $A \not\perp B$

b) Degrees of freedom: $\underset{2}{\text{Rows}} - 1 \underset{=1}{} \times \underset{3}{\text{Columns}} - 1 \underset{=2}{} = 2$ degrees of freedom

$$\chi^2 = \sum \frac{(Obs - exp)^2}{Exp}$$

c) Find χ^2 & p-value $\chi^2 = 12.295$ p-value $= 0.002$

From Statskingdom.com/310GoodnessChi.html

- Test: Independence association
- Other parameters at default
- Data entered in to 'Sample Data' fields

d) There is a significant correlation between the a staff member's held position and security level. The null hypothesis is rejected and the alternative hypothesis supported. This conclusion was reached as $\underset{0.002}{\text{p-value}} < \underset{0.05}{\text{alpha}}$.

Page 3 — One mean

Q3. Catheter expected diameter = 2.00 mm

$n = 36$ Average = 2.03mm   SD = 0.05mm

a) Confidence interval $DF = \underbrace{\overset{n-1}{35}}$ alpha = 0.05

$\bar{y} \pm t^*_{n-1} \times \frac{s}{\sqrt{n}}$ on T-table, $t^* \approx 2.030$

$2.03 + 2.030 \times \frac{0.05}{\sqrt{36}} = 2.047$

$2.03 - 2.030 \times \frac{0.05}{\sqrt{36}} = 2.013$

[2.013 mm, 2.047 mm]

b) We are 95% confident that the confidence interval, [2.013, 2.047] contains the true mean diameter of catheters.

c) Hypothesis test

H_0: There is no significant difference between the average catheter diameter and the expected. $\mu_1 = \mu_2$

H_A: There is significant difference between the average catheter diameter and the expected. $\mu_1 \neq \mu_2$

$t = \frac{\bar{y} - \mu_0}{s / \sqrt{n}}$

Calculated from statskingdom.com/130meanT1.html

"One sample T test calculator"

Settings

Tails: Two   Sig. level: 0.05   Info   Sample average: 2.03

Exp. mean: 2.00, rest as default   n: 36   SD: 0.05

p-value: 0.00098 $\underset{\text{p-value}}{0.00098} < \underset{\text{alpha}}{0.05}$

The p-value is significantly less than the alpha, rejecting the null hypothesis and supporting that there is a significant difference in the selected catheter diameter to the expected diameter.

 Paired means.

Q4. Gas mileage - premium vs. regular miles/gallon

Using statskingdom.com/paired-t-test-calculator.html

Tails: Two, other settings default. Alpha: 0.05

Input data

Regular 16, 20, 21, 22, 23, 22, 24, 28, 27, 28

Premium 18, 21, 23, 24, 25, 23, 26, 26, 28, 29

a)

p-value: 0.0129 $t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} \rightarrow$ 3.087 Test statistic.

The p-value is less than the alpha (0.0129 < 0.05), rejecting the null hypothesis. This shows that there is a significant difference between regular and premium gasoline mileage.

$d_1 = d_2$ H_0: There is no significant difference in gas mileage between petrol and premium fuels

$d_1 \neq d_2$ H_A: There is a significant difference in gas mileage between petrol and premium fuels.

b) Used statskingdom, paired T-test. Alpha = 0.10.

Confidence interval: 90% [0.4874, 1.9126]

$\bar{d} \pm t^*_{n-1} \times \frac{s_d}{\sqrt{n}}$

We are 90% confident that the difference in mileage is between [0.4874, 1.9126]. miles/gallon

*

ΔΔ

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Q5. Blood $\overset{\text{systolic}}{\text{pressure}}$ in women v. men.

Women	105	12	$n_2 = 35$	Two means.
Men	115	15	$n_1 = 23$	
	$\bar{y}$	σ		

a)

H_0: There is no significant increase in blood pressure in men compared to women $\mu_2 = \mu_1$

H_A: There is a significant increase in blood pressure in men compared to women $\mu_1 > \mu_2$

$$t = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

DF = 35 + 23 − 2 = 56 Alpha = 0.05

Statskigdom.com/150MeanT2uneq.html Tails: Right

"Two sample T-test calculator (Welch's T-test)

p-value = 0.0053 (p-value < 0.05) T-statistic = 2.683

Mens systolic blood pressure has a significant increase is supported and the H_0 rejected compared to women

b. $\left(\overset{(115)}{\bar{y}_1} - \overset{(105)}{\bar{y}_2}\right) \pm t_{56_{0.10}} \times \sqrt{\frac{15^2}{23} + \frac{12^2}{35}}$ 90% confidence

approx 1.297 (t-value)

+: 14.84

−: 5.16

[5.16, 14.84]. We are 90% confident that the average increase in systolic blood pressure in men is between [5.16, 14.84] mm Hg.

c) Yes, it supports the (p-value < 0.05) null hypothesis rejection, as it shows an increase compared to women systolic blood pressure. If it didn't show an increase, the values would be negative.

Q6 - Phosphorus content in 3 trees' leaves

a) H_A: The phosphorus content does differ in the varieties. $H_A \mu_1 \neq \mu_2 \neq \mu_3$ | $H_0: \mu_1 = \mu_2 = \mu_3$ The phosphorus content does not differ in the varieties.

b) F-stat: 16.972 → p-value: 0.000... alpha = 0.05
The null hypothesis is rejected as the obtained p-value < 0.05. This supports the H_A with a high degree of certainty.

c) Yes. It is important to do a test like Bonferroni to find what specific groups are statistically different. The H_0 being rejected only informs that there is a statistically significant difference but not on what group. This importance is supported by the precalculated Bonferroni table - showing statistically significant differences in all comparisons

Like 2,3 showing a +1 mean difference, 1,3 showing a ~2.6 difference, and 1,2 showing a ~3 difference. The 1,2 being the most different and 2,3 the least. This would be unknown without a Bonferroni test.

Page 7 - Bear chest circumference to weight

a) Response: Weight
Explanatory - Chest Circumference

b) y= (Constant + CHEST x
→ y= -251.948 + 12.380x

c) The y-intercept in this equation is simply the starting point of where circumference is a theoretical 0 inches. A bear cannot have a 0 inch diameter or a negative weight, meaning the y-intercept is only a point for later, reasonable data.

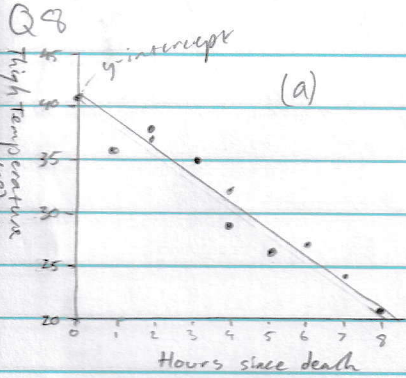
d) For each inch of circumference, a given bear is modelled as gaining an additional 12.380 pounds
Rise: 12.380 pounds Run: Inches (1)

e) H_0: There is no linear relationship between a bear's chest circumference and its weight y=0
H_A: There is a linear relationship between a bear's chest circumference and its weight. y≠0

~~y-intercept~~ T-statistic=-7.451 Slope T-statistic=15.277

The p-value (0.000...) < alpha 0.05, rejecting the null hypothesis. There is a very strong relationship between a bear's chest circumference and weight, supporting the alternate hypothesis.

Q8



(a) There is a strong negative linear relationship between the hours since death and the thigh temperature of deer carcasses

Statskingdom.com/linear-regression-calculator.html
- Hours since death in the X column
- Thigh temperatures in the Y column

b) $y = 40.912 - 2.479x$

c) It is meaningful. The y-intercept, a temperature, is not an extreme value both in terms of being an outlier and plausability. $\approx 41°C$ is higher than a regular body temperature but this could be explained by a particularly hot day or perhaps illness, like a fever.

d) $25°C = 40.912 - 2.479x$ $\quad -40.912 \quad -2.479$ $\qquad 25 - 40.912 = \frac{-15.912}{-2.479} = x = 6.419$

Time since death of a 25°C deer carcass is modelled to be 6.42 hours. This is a mostly reliable prediction. The prediction is well within the data range so it is not unreliable extrapolation but instead interpolation. Since it is a model and not real, it does not have a guaranteed reliability.