1 Discovering the derivative

1.1 Rates of change

1. (i) over the first 5 seconds is $\frac{12-0}{5-0} = \frac{12}{5} = 2.4(m/s)$.

   (ii) between 10 and 20 seconds is $\frac{42-23}{20-10} = \frac{19}{10} = 1.9(m/s)$.

   (iii) over the last 5 seconds is $\frac{60-51}{30-25} = \frac{9}{5} = 1.8(m/s)$.

   (iv) over the entire 30 seconds is $\frac{60-0}{30-0} = 2(m/s)$.

   Note: The units of the rate of change here are the units for distance (m.) divided by the units for time (sec.).

2. (i) between 7 a.m. and 3 p.m. is $\frac{29-9}{15-7} = \frac{20}{8} = 2.5$ ($°C$ per hour).

   (ii) between 11 a.m. and 11 p.m. is $\frac{21-27}{23-11} = \frac{-6}{12} = -0.5$ ($°C$ per hour).

   (iii) Since the average rate of change of temperature between 7 p.m. and 11 p.m. is -1.5 $°C$ per hour,

   $\frac{21-x}{11-7} = -1.5°C \implies 21 - x = 4 \times (-1.5) = -6 \implies x = 21 + 6 = 27.$
   Hence, the temperature at 7 p.m. is $27°C$.

3. (i) over the first hour is $\frac{3.9-0}{1-0} = 3.9(km/h)$.

   (ii) between 1.5 and 3 hours is $\frac{8.4-4.8}{3-1.5} = \frac{3.6}{1.5} = 2.4(km/h)$.

   (iii) over the last half hour is $\frac{8.4-7.1}{3-2.5} = \frac{1.3}{0.5} = 2.6(km/h)$.

   (iv) over the entire 3 hours is $\frac{8.4-0}{3-0} = 2.8(km/h)$.

4. (i) Between 11 a.m. and 3 p.m. is $\frac{14.8-11.6}{15-11} = 0.8$°C/h

   (ii) Between 7 a.m. and 3 p.m. is $\frac{14.8-2.8}{15-7} = \frac{12}{8} = 1.5$°C/h

   (iii) Since the average rate of change of temperature between 7 p.m. and 11 p.m. is -1.1 °C/h, $\frac{8.4-x}{11-7} = -1.1 \implies 8.4 - x = -1.1 \times 4 = -4.4$, then $x = 8.4 + 4.4 = 12.8$. Hence, the temperature at 7 p.m. is $12.8°C$.

1.2 The derivative

1. (i) $y = f(x) = 4x - 7$
   $f(x+h) = 4(x+h) - 7 = 4x + 4h - 7$ and $f(x+h) - f(x) = 4x + 4h - 7 - (4x - 7) = 4h$ then,
   $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{4h}{h} = 4.$
   Then $f'(x) = 4$.

Note: this agrees with the fact that the slope of the straight line r $y = f(x) = 4x - 7$ is 4 (constant slope for all values of x).