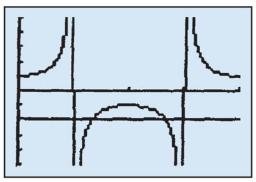
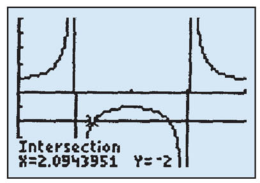
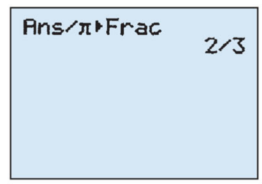
Using a graphics calculator

Check that the calculator is in **Radian** mode.

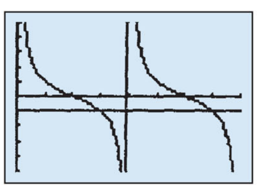
a Enter the functions $y = 1/\cos(x)$ and $y = -2$ into the **Y**= window. Press $\boxed{\text{WINDOW}}$ and enter **Xmin** $= 0$, **Xmax** $= 2\pi$, **Xscl** $= \frac{\pi}{2}$ (to split the period of 2π into four equal intervals), **Ymin**$= -5$ and **Ymax** $= 5$. Press $\boxed{\text{GRAPH}}$ to display the graphs. Use **5:intersect** from the **CALCULATE** menu to find the points of intersection. Return to the home screen and press $\boxed{X, T, \theta, n}$ $\boxed{\div}$ $\boxed{\text{2ND}}$ $\boxed{\wedge}$ $\boxed{\text{MATH}}$ and choose **1:Frac** from the **MATH** menu to express each solution of x as an exact fraction of π. For example, $2/3$ on the screen represents the solution $x = \frac{2\pi}{3}$.

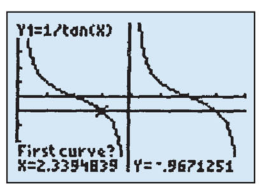


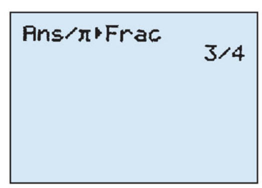




b Enter the functions $y = 1/\tan(x)$ and $y = -1$ into the **Y**= window. Press $\boxed{\text{WINDOW}}$ and enter **Xmin** $= 0$, **Xmax** $= 2\pi$, **Xscl** $= \frac{\pi}{2}$ (to split the period of π into four equal intervals), **Ymin** $= -5$ and **Ymax** $= 5$. Press $\boxed{\text{GRAPH}}$ to display the graphs. Use **5:intersect** from the **CALCULATE** menu to find the points of intersection. Return to the home screen and press $\boxed{X, T, \theta, n}$ $\boxed{\div}$ $\boxed{\text{2ND}}$ $\boxed{\wedge}$ $\boxed{\text{MATH}}$ and choose **1:Frac** from the **MATH** menu to express each solution of x as an exact fraction of π. For example, $3/4$ on the screen represents the solution $x = \frac{3\pi}{4}$.







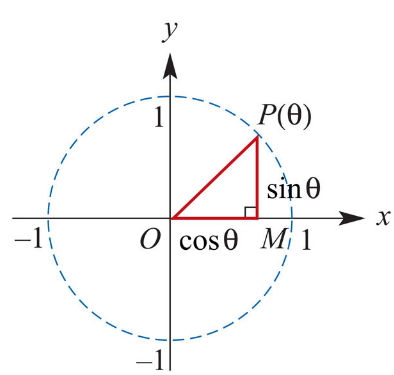
The Pythagorean identity

Consider a point, $P(\theta)$, on the unit circle.

By Pythagoras' theorem:

$$OP^2 = OM^2 + MP^2$$

$$\therefore \quad 1 = (\cos\theta)^2 + (\sin\theta)^2$$



Now $(\cos\theta)^2$ and $(\sin\theta)^2$ may be written as $\cos^2\theta$ and $\sin^2\theta$.
Since this is true for all values of θ it is called an identity.
In particular this is called the **Pythagorean identity**.

$$\cos^2\theta + \sin^2\theta = 1$$

Other forms of the identity can be derived.
Dividing both sides by $\cos^2\theta$ gives:

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{1 + \tan^2\theta = \sec^2\theta}$$

Dividing both sides by $\sin^2\theta$ gives:

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{\cot^2\theta + 1 = \operatorname{cosec}^2\theta}$$

Example 7

a If $\operatorname{cosec} x = \frac{7}{4}$, find $\cos x$.

b If $\sec x = -\frac{3}{2}$, find $\sin x$ where $\frac{\pi}{2} \le x \le \pi$.

Solution

a Since $\operatorname{cosec} x = \frac{7}{4}$, $\sin x = \frac{4}{7}$

Now $\cos^2 x + \sin^2 x = 1$

so $\cos^2 x + \frac{16}{49} = 1$

$\therefore \quad \cos^2 x = \frac{33}{49}$

$\therefore \quad \cos x = \pm\frac{\sqrt{33}}{7}$

b Since $\sec x = -\frac{3}{2}$, $\cos x = -\frac{2}{3}$

$\cos^2 x + \sin^2 x = 1$

$\therefore \quad \frac{4}{9} + \sin^2 x = 1$

$\therefore \quad \sin x = \pm\frac{\sqrt{5}}{3}$

For $P(x)$ in the 2nd quadrant, $\sin x$ is positive

$\therefore \quad \sin x = \frac{\sqrt{5}}{3}$

Example 8

If $\sin\theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$, find the value of $\cos\theta$ and $\tan\theta$.

Solution

Since $\cos^2\theta + \sin^2\theta = 1$

then $\cos^2\theta + \frac{3^2}{5^2} = 1$

$\therefore \quad \cos^2\theta = 1 - \frac{9}{25}$

$= \frac{16}{25}$

$\therefore \quad \cos\theta = -\frac{4}{5}$ since $\frac{\pi}{2} < \theta < \pi$

$\therefore \quad \tan\theta = -\frac{3}{4}$ as $\tan\theta = \frac{\sin\theta}{\cos\theta}$