1 Logarithmic laws

1. (i) $4\ln 2 + 2\ln 5 - \ln 10 = \ln(2^4) + \ln(5^2) - \ln 10 = \ln 16 + \ln 25 - \ln 10 = \ln(16 \times 25) - \ln 10 = \ln 400 - \ln 10 = \ln(\frac{400}{10}) = \ln 40.$

   (ii) $\ln 12 - 2\ln 3 + \ln 6 = \ln 12 + \ln 6 - \ln 3^2 = \ln(12 \times 6) - \ln 9 = \ln(\frac{72}{9}) = \ln 8.$

2. (i) $\ln(5x) - 4\ln(2x) + 2\ln(4x^2) = \ln(5x) - \ln(2x)^4 + \ln(4x^2)^2 = \ln\left(\frac{5x \times (4x^2)^2}{(2x)^4}\right) =$
   $= \ln\left(\frac{5 \times 16x x^4}{16x^4}\right) = \ln(5x).$

   (ii) $2\ln(x^2y) + 3\ln x - \ln(x^3y) = \ln(x^2y)^2 + \ln x^3 - \ln(x^3y) = \ln\left(\frac{x^4y^2 \times x^3}{x^3y}\right) = \ln(x^4y).$

3. (i) $2\ln 3 + 2\ln 5 - \ln 15 = \ln(3^2) + \ln(5^2) - \ln 15 = \ln(\frac{9 \times 25}{15}) = \ln 15.$

   (ii) $5\ln 2 - 2\ln 4 + \ln 7 = \ln(2^5) - \ln(4^2) + \ln 7 = \ln(\frac{32 \times 7}{16}) = \ln 14.$

4. (i) $\ln(8x) - 2\ln(2x) + 3\ln(2x^2) = \ln(8x) - \ln(2x)^2 + \ln(2x^2)^3 = \ln\left(\frac{8x \times 8x^6}{4x^2}\right) = \ln(16x^5).$

   (ii) $3\ln(xy) + 2\ln x - \ln(x^3y^2) = \ln(xy)^3 + \ln x^2 - \ln(x^3y^2) = \ln\left(\frac{x^3y^3 \times x^2}{x^3y^2}\right) = \ln(x^2y).$