SIT190 - WEEK 7

- Average rate of change
- Instantaneous rate of change
- First principles
- Derivative
    - Power rule
    - Exponentials and logarithmic functions
    - Trigonometric functions
- Gradient of a function
    - Finding stationary points
        - Sign test
        - Second derivative
    - Sketching polynomials

The Island of Antikythera

Your band has arrived safely at the coast of the Island of Antikythera. You plan to use the boat to travel off-shore in your search for the missing Antikythera fragments. In order to work out the time it will take you to reach the location in the boat, you want to calculate the average speed of your boat on the journey from the mainland. The journey was 130km and took you 3 hours.

Find the average speed of the boat on this journey. The first band to find the average speed (km/h) gains 10 points.

The Island of Antikythera

A member of your band points out that although you have the **average** speed for the entire trip this is not indicative of your average speed when you were closer to the coast. This was due to dangerous rocks and currents that needed to be avoided.

Table: Distance d at time t hours.

Time t (hours)	0	5	30	70	105	125	130
Distance d (km)	0	0.5	1	1.5	2	2.5	3

What was the average speed in the last half hour of your trip? Your band decides it is better to use this estimate for your search.

Reflection

Average speed $= \frac{\text{Total distance travelled}}{\text{Time taken}}$.

- Can you find the average speed between $t = 0.5$ and $t = 2$?
- Can you find the average speed between $t = 1$ and $t = 2$?
- Why does the average speed differ when we take it across these different intervals of time?

Distance d (km)	0	5	30	70	105	125	130
Time t (hours)	0	0.5	1	1.5	2	2.5	3

Average rate of change

Before you boat to the location, you decide you need to head to the village for supplies. A local gives you the following table that contains the time it takes him to get to and from the village. Here s is the distance from your current location in metres, and t is the time in minutes. The village is 1.8 km (1800 metres) from your current location.

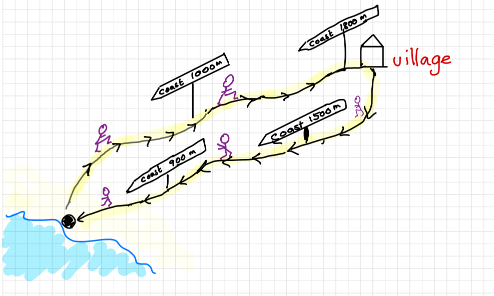
Time t (minutes)	0	13	26	38	48	70
Distance from coast s (m)	0	1000	1800	1500	900	0

What was the average speed that the local takes to get to the village from the coast? Did he walk at the same rate for the entire trip?

Displacement

- Displacement is the distance from a given point.
- Average velocity $= \frac{\text{change in displacement}}{\text{Change in time}} = \frac{s_2 - s_1}{t_2 - t_1}$.

> Compare the average velocity and the average speed over the entire trip, that is, from $t = 0$ to $t = 70$.

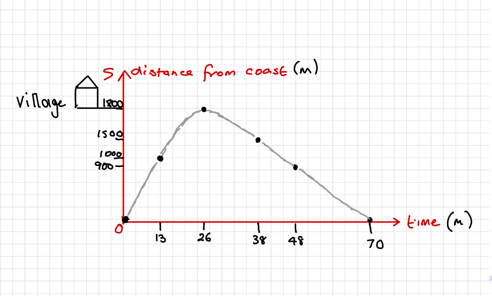


Time (minutes)	0	13	26	38	48	70
Distance from coast (m)	0	1000	1800	1500	900	0

Trip to the village

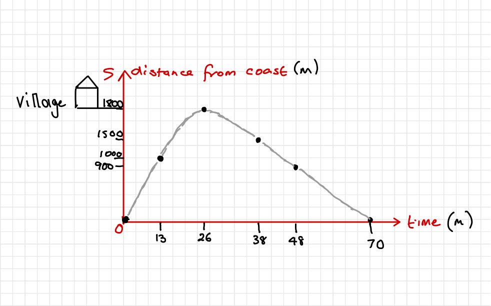
We can represent the trip to the village by a graph.

- We measure the displacement from the coast by the s-axis.
- We measure the elapsed time by the t-axis.
- At time $t = 0$, the local was at the coast (ie displacement was 0) represented by the point $(0, 0)$
- At time $t = 13$, the local was 1000m from the coast, represented by the point $(13, 1000)$.
- At time $t = 70$, the local is back at the coast (ie displacement is 0) represented by the point $(70, 0)$.



Trip to the village

- At time $t = 13$, the local was 1000m from the coast, represented by the point $(13, 1000)$.
- At time $t = 70$, the local is back at the coast (ie displacement is 0) represented by the point $(70, 0)$.
- To find the average change in displacement from $t = 13$ and $t = 70$
$$\frac{0 - 1000}{70 - 13} \approx -17.54 \text{ metre/min.}$$



Average rate of change

- Next week we will look at how to solve problems involving displacement, velocity and acceleration.
- This week we will look at the rate of change of a function $y = f(x)$ where $f(x)$ is a mathematical expression.
  For example:
  - $y = 3x^2 - 4x + 6$
  - $y = \ln(x)$
  - $y = x^3 - 10x^2 + 31x + 10$

Average rate of change

The average rate of change of f is

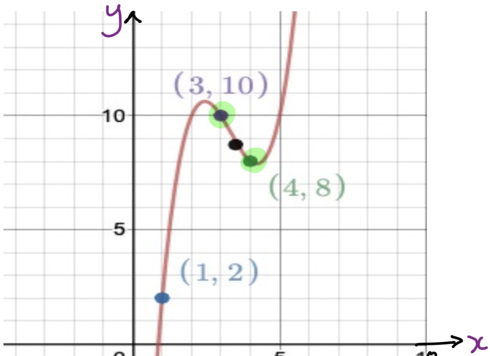
$$\frac{\text{change in } f(x)}{\text{change in } x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

This is the gradient of the line connecting the points

$$(x_1, f(x_1)) \text{ and } (x_2, f(x_2)).$$

For example, find the average rate of change of
$f(x) = x^3 - 10x^2 + 31x - 20$ between $x = 3$ and $x = 4$.

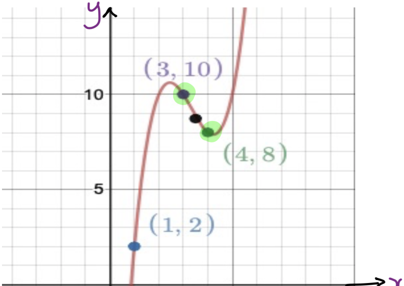
1. Find the values for y or $f(x)$ for these values of x.
   - When $x = 3$ the value of $f(x) = 10$ and when $x = 4$ the value of $f(x) = 8$.
2. Find the change in the $f(x)$ values
   - $f(x_2) - f(x_1) = 8 - 10 = -2$.
3. Find the change in the x values
   - $x_2 - x_1 = 4 - 3 = 1$.
4. Average rate of change $= \dfrac{\text{Change in } f(x)}{\text{Change in } x} = \dfrac{-2}{1} = -2$.



Investigate

$$f(x) = x^3 - 10x^2 + 31x - 20$$

- The average rate of change of $f(x)$ in this interval was -2.
- What is the average rate of change between $x = 1$ and $x = 4$?
- What is the average rate of change between $x = 1$ and $x = 3$?
- What is the average rate of change between $x = 1$ and $x = 2$?
- Compare the line connecting the pairs of points to the section of the curve in each interval.



Reflection

- How well does the average rate of change describe how the behaviour of the function?
- Do large intervals or small intervals work better?
- We want the exact rate of change at **a** point $(x_1, f(x_1))$ on the curve.
- The average rate of change was the gradient of the line connecting the two points, $(x_1, f(x_1))$ and $(x_2, f(x_2))$.
- The *Instantaneous Rate of Change* is the gradient of the tangent line to the point $(x_1, f(x_1))$.
- We now need to work out how to find this gradient.

The Plan

An idea of the plan to find the instantaneous rate of change at the point $(x, f(x))$:

- Find a point that is very close to this point. We use $(x+h, f(x+h))$ where h is a small number.
- Use the gradient of the line connecting this point to $(x, f(x))$ to estimate the instantaneous rate of change.
  We can do this by making h smaller and smaller.
- Move this point closer and closer to $(x, f(x))$ to get a better estimate.

Let us find the instantaneous rate of change of the function $f(x) = x^3 - 10x^2 + 31x - 20$ at the point $(1, 2)$.

What is happening as h gets closer to 0?

h	$(1+h, f(1+h))$	Average Rate of Change
1	$(2, 10)$	$\frac{10-2}{2-1} = 8$
$\frac{1}{2}$	$(\frac{3}{2}, \frac{59}{8})$	$\frac{\frac{59}{8}-2}{\frac{3}{2}-1} = \frac{43}{8} \div \frac{1}{2} = 10\frac{3}{4} = 10.75$
$\frac{1}{4}$	$(\frac{5}{4}, \frac{325}{64})$	$\frac{\frac{325}{64}-2}{\frac{5}{4}-1} = \frac{197}{64} \div \frac{1}{4} = 12\frac{5}{16} = 12.3215$
0.01	$\approx (1.01, 2.1393)$	$\approx \frac{0.1393}{0.01} = 13.93$
0.001	$\approx (1.001, 2.0139)$	$\approx \frac{0.0139}{0.001} = 13.9$

The instantaneous rate of change at a point $(x, f(x))$ is

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

When $f(x) = x^3 - 10x^2 + 31x - 20$,

$$\begin{aligned}
&\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{(x+h)^3 - 10(x+h)^2 + 31(x+h) - 20 - (x^3 - 10x^2 + 31x - 20)}{h} \\
&= \lim_{h \to 0} \frac{3hx^2 + 3h^2x + h^3 - 20xh - h^2 + 31h}{h} \\
&= \lim_{h \to 0} (3x^2 + 3hx + h^2 - 20x - h + 31) \\
&= \lim_{h \to 0} (3x^2 - 20x + 31 + 3hx + h^2 - h) \\
&= 3x^2 - 20x + 31.
\end{aligned}$$

At the point $(1, 2)$, we evaluate this at $x = 1$ and find the instantaneous rate of change is

$$3(1)^2 - 20(1) + 31 = 3 - 20 + 31 = 14.$$

Derivative

- We use the notation $f'(x)$ to denote the derivative of $f(x) =$ some expression in x.
  For example, if $f(x) = x^3 - 10x^2 + 31x - 20$ then $f'(x) = 3x^2 - 20x + 31$.
- We use the notation $\frac{dy}{dx}$ to denote the derivative of $y =$ some expression in x.
  For example, if $y = x^3 - 10x^2 + 31x - 20$ then $\frac{dy}{dx} = 3x^2 - 20x + 31$.
- To find the instantaneous rate of change at a specific point $(x_1, f(x_1))$, we evaluate the derivative at $x = x_1$.
  For example, the instantaneous rate of change of the function $f(x) = x^3 - 10x^2 + 31x - 20$ at the point $(3, 10)$ is $f'(x) = 3(3)^2 - 20(3) + 31 = -2$.

Derivative

The **derivative** gives the instantaneous rate of change of f at x and can be found using first principles using:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Can your band find the derivative of the following functions using first principles, and then find the instantaneous rate of change when $x = 1$.

$$y = 3x + 2$$

$$y = 2 - 3x$$

$$y = x^2 - 4x + 6$$

$$y = 25 - x^2$$

$$y = 3$$

Reflection

- Finding the derivative using first principles.
- Gives the gradient of tangent line
- Horizontal line has gradient 0 so the derivative is . . ..
- The line $y = mx + c$ has constant gradient so the derivative is . . .
- Polynomials of degree ≥ 2 do not have constant gradient, so we need to evaluate at a specific point.

Power Rule

Finding the derivative using first principles may be a little laborious, so this week and next week we will look at some rules that can be used.

Power Rule: If $y = x^n$ then the derivative is $\frac{dy}{dx} = nx^{n-1}$.

Some examples:

- Positive power: $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$
- Constant: $y = 3 \Rightarrow y = 3x^0 \Rightarrow \frac{dy}{dx} = 0x^{-1} = 0$.
- Negative power: $y = \frac{1}{x} \Rightarrow y = x^{-1} \Rightarrow \frac{dy}{dx} = -x^{-2} = \frac{-1}{x^2}$.
- Fractional power: $y = x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{\frac{-1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$.

Power Rule

Power Rule: If $y = x^n$ then the derivative is $\frac{dy}{dx} = nx^{n-1}$.

- Constant multiple k:
  If $y = kx^n$ then the derivative is $\frac{dy}{dx} = knx^{n-1}$.
  - For example: $y = 3x^2 \Rightarrow \frac{dy}{dx} = 3 \times 2x = 6x$
- Addition Rule: If y is expressed as a sum of terms, we can find the derivative of each term.
  - For example:
    $y = 3x^2 + 4x^3 - 2x^4$
    $\Rightarrow \frac{dy}{dx} = 3 \times 2x + 4 \times 3x^2 - 2 \times 4x^3 = 6x + 12x^2 - 8x^3$
    and
    $f(x) = 2x - 2x^{\frac{5}{2}} \Rightarrow f'(x) = 2 - \frac{5}{2}2x^{\frac{3}{2}} = 2 - 5x^{\frac{3}{2}}$.

Visiting the wreck

Greek sponge divers found the wreck of an ancient ship near the coast of Antikythera in 1901 and recovered a vast treasure of jewellery, bronze and marble statutes, ceramics and other precious items. They also found the mangled parts of the device known now as the Antikythera Mechanism.

Image:Lyre Discovery Site: Antikythera shipwreck. Photographer: Timothy J. Moore. Lyre. 07/17/2016, ca. 300BC-101BC. Artstor, `library.artstor.org/asset/SS37414_37414_42501979`

Visiting the wreck

The plan is for your band to head to the dive site. You have a good idea how to locate the wreck and how long it will take you to get there. But you have little experience diving and must work out the rate you must descend and the rate you must ascend.

If you descend too fast, pressure changes can damage your eardrums. If you ascend too fast you can suffer decompression sickness. Note: In real life, there are standards for these rates and you should follow these.

But on this quest, we are on a mythical journey and so we find the rate of descent by finding the derivative of

$$f(x) = x^4 - 3x^2 + 12x - \frac{8}{x} + 400 \text{ when } x = 1.$$

10 points for the first band to find the rate and reach the ocean floor.

Visiting the wreck

You find the wreck and dig in the swirling sand until you uncover what you believe is the missing fragment of the Antikythera mechanism. Unfortunately, as you were digging, a giant sea-dragon has crept up ready to pounce. You must ascend to the surface as quickly as you can in order to avoid being attacked.

Image: Unknown. Marine thiasos from the so-called Altar of Domitius Ahenobarbus, Image View Description:from front, Image View Type:overall. ca. 1890-1900, late 2nd to early 1st c. BCE. Artstor

Escape

You must find the rate of ascent by finding the derivative of

$$f(x) = 2x^{\frac{3}{2}} - x^{\frac{1}{2}} + 3x - \frac{4}{x} + 40 \text{ when } x = 4.$$

10 points for the first band to find the rate and reach the surface.

Image: Unknown. Marine thiasos from the so-called Altar of Domitius Ahenobarbus, Image View Description:from front, Image View Type:overall. ca. 1890-1900, late 2nd to early 1st c. BCE. Artstor

Another Artefact Aquired

- Your team successfully makes it back to the boat with the precious fragment in hand.
- You will soon be departing to the next location, India.
- At this point, we have seen how to find the derivative using first principles, and how to use the power rule to find some derivatives.
- We will briefly look at some further rules for finding derivatives of other functions
- Then for the remainder of this workshop we will look at an application of the derivative - finding and identifying types of stationary points.

Rules for finding the derivative

Apply the rule to find the derivative:

Function and Derivative	Function	Derivative
$y = e^{kx} \Rightarrow \frac{dy}{dx} = ke^{kx}$	$y = e^{2x}$	
$f(x) = e^{kx} \Rightarrow f'(x) = ke^{kx}$	$f(x) = \frac{1}{4}e^{4x}$	
$f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$	$f(x) = \ln(4x)$	
$y = \ln(x) \Rightarrow \frac{dy}{dx} = \frac{1}{x}$	$y = \ln(x^2)$	

Rules for finding the derivative

Apply the rule to find the derivative:

Function and Derivative	Function	Derivative
$y = \sin(kx) \Rightarrow \frac{dy}{dx} = k\cos(kx)$	$y = \sin(3x)$	
$f(x) = \sin(kx) \Rightarrow f'(x) = k\cos(kx)$	$y = 14\sin(\frac{x}{2})$	
$f(x) = \cos(kx) \Rightarrow f'(x) = -k\sin(kx)$	$f(x) = \cos(\frac{x}{2})$	
$y = \cos(kx) \Rightarrow \frac{dy}{dx} = -k\sin(kx)$	$y = \frac{3}{4}\cos(2x)$	

Some more examples:

Find the derivative of the following functions:

$$y = 4\cos(3x) - 2\sin(\frac{3x}{2}) - e^{-3x} + \ln(4x^2)$$

$$f(x) = e^{2x} - e^{3x} + \cos(4x) - \sin(4x) + \ln(x)$$

Stationary Points

- The stationary points occur when the gradient is 0.
- The derivative gives us the gradient at a point.
- So to find stationary points:
  1. Find the derivative $\frac{dy}{dx}$.
  2. Find the values of x when the derivative $\frac{dy}{dx} = 0$.
  3. For each value of x, find the corresponding values of y.
- For example:
  1. $y = 3x^2 - 4x + 6 \Rightarrow \frac{dy}{dx} = 6x - 4$.
  2. $\frac{dy}{dx} = 6x - 4 = 0 \Rightarrow 6x = 4 \Rightarrow x = \frac{4}{6} = \frac{2}{3}$.
  3. When $x = \frac{2}{3}$, we find that
     $y = 3(\frac{2}{3})^2 - 4(\frac{2}{3}) + 6 = \frac{4}{3} - \frac{8}{3} + 6 = \frac{14}{3} = 4\frac{2}{3}$.
  4. Stationary point is $(\frac{2}{3}, 4\frac{2}{3})$.

Stationary Points

Another example: find the stationary points in
$f(x) = x^3 - 6x^2 - 15x + 20$

1. $f(x) = x^3 - 6x^2 - 15x + 20 \Rightarrow f'(x) = 3x^2 - 12x - 15.$
2. $f'(x) = 3x^2 - 12x - 15 = 0 \Rightarrow x^2 - 4x - 5 = 0$
   $\Rightarrow (x-5)(x+1) = 0$
   $\Rightarrow x - 5 = 0$ or $x + 1 = 0 \Rightarrow x = 5,$ or $x = -1$
3. When $x = 5$, we find that $f(x) = 125 - 150 - 75 + 20 = -80$ and when $x = -1$, we find that $f(x) = -1 - 6 + 15 + 20 = 28.$
4. Statrionry points are $(5, -80)$ and $(-1, 28)$.

Stationary Points

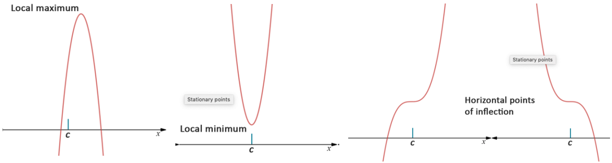
Can your band find the stationary points of these functions:

1. $y = -2x^2 + 4x + 3$
2. $f(x) = x^3 - 3x^2 - 144x + 432$
3. $y = x^3 + 3x^2 + 3x + 9$
4. $f(x) = x^4 - 16$

Stationary Points

There are three types of stationary point:

- a local maximum: gradient goes up and then becomes zero at the stationary point and then goes down.
- a local minimum : gradient goes down and then becomes zero at the stationary point and then goes up.
- a point of inflexion: (a) : gradient goes up and then becomes zero at the stationary point and then goes up, or
  (b) : gradient goes down and then becomes zero at the stationary point and then goes down.



How to classify your stationary points

There are two methods:

- Sign method
  - Derivative only changes sign when the curve passes through a stationary point.
  - Look at the sign changes
    1. positive, zero, negative = a local maximum
    2. negative, zero, positive = a local minimum
    3. postive, zero, positive OR negative, zero, negative = horizontal point of inflection.
- Second derivative method
  - Find the second derivative $f''(x)$ or $\frac{d^2y}{dx^2}$ (the derivative of the derivative)
  - Evaluate the $f''(x)$ at the x value of the stationary point.
    1. if $f''(x) < 0$ then it is a local maximum
    2. if $f''(x) > 0$ then it is a local minimum
    3. if $f''(x) = 0$ then we have to use the sign test.

Find the stationary point - Example

$$y = x^3 + 3x^2 + 3x + 9$$

Step 1: Find the derivative:

$$\frac{dy}{dx} = 3x^2 + 6x + 3$$

Step 2: Find the values of x where the derivative is 0.

$$3x^2 + 6x + 3 = 0$$

$$\Rightarrow x = \frac{-6 \pm \sqrt{36 - 36}}{6} = -1.$$

Step 3: Find the corresponding y values:

When $x = -1$, $y = -1 + 3 - 3 + 9 = 8$.

Stationary Point is $(-1, 8)$.

Classify the stationary point

Method 1:

Region	$x < -1$	$x = -1$	$x > -1$
Sign of gradient	$+$	0	$+$

$(-1, 8)$ is a horizontal point of inflection.

Method 2:

$$\frac{dy}{dx} = 3x^2 + 6x + 3 \Rightarrow \frac{d^2y}{dx^2} = 6x + 6$$

When $x = -1$, $\frac{d^2y}{dx^2} = -6 + 6 = 0$ so we cannot use this method.

Find the stationary point - Example

$$y = 3x^2 + 6x + 9$$

Step 1: Find the derivative:

$$\frac{dy}{dx} = 6x + 6$$

Step 2: Find the values of x where the derivative is 0.

$$6x + 6 = 0 \Rightarrow 6x = -6 \Rightarrow x = -1.$$

Step 3: Find the corresponding y values:

When $x = -1$, $y = 3 - 6 + 9 = 6$.
Stationary Point is $(-1, 6)$.

Classify the stationary point

Method 1:

Region	$x < -1$	$x = -1$	$x > -1$
Sign of gradient	$-$	0	$+$

$(-1, 6)$ is a local minimum stationary point.

Method 2:

$$\frac{dy}{dx} = 6x + 6 \Rightarrow \frac{d^2y}{dx^2} = 6$$

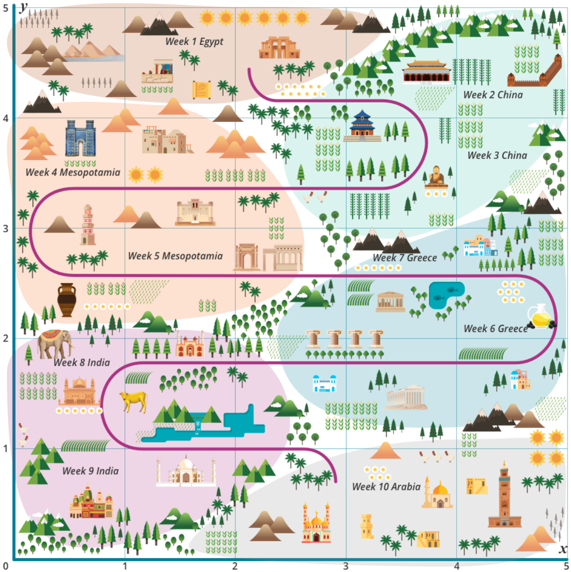
When $x = -1$, $\frac{d^2y}{dx^2} = 6 > 0$ so we have a local minimum stationary point.

Classifying stationary points

Given the polynomial $y = x^2 - 5x + \frac{31}{4}$,

- Find the y intercept.
- Can you find an x-intercept? Explain.
- Find the the stationary point.
- Use both methods to find the type of the stationary point - which was easier to use?
- Sketch the graph. raph.

The stationary point will give you the location on the map that your band needs to head to in order to travel for the next leg of the journey.


y
x
0
1
2
3
4
5
Week 1 Egypt
Week 2 China
Week 3 China
Week 4 Mesopotamia
Week 5 Mesopotamia
Week 6 Greece
Week 7 Greece
Week 8 India
Week 9 India
Week 10 Arabia

Reflection

Although, we have reached the end of this leg of the journey, we will take time to reflect on how we can combine our skills to sketch graphs of polynomials.

- Apply algebra to find the x and y intercepts.
  - Substitute $y = 0$ and solve for x (for quadratics we used the quadratic formula)
  - Substitute $x = 0$ and solve for y
- Apply differentiation and algebra to find stationary points.
  - Differentiate using the power rule to find $\frac{dy}{dx}$.
  - Apply algebra to solve $\frac{dy}{dx} = 0$
  - Apply algebra to find the corresponding y values for each x.
- Classify stationary points.
  - Apply algebra to find the signs in regions using Method 1, or
  - Differentiate using power rule to find $\frac{d^2y}{dx^2}$ and use Method 2.

> Review the following example, and use the information to sketch the graph.

Graph sketching - Example

Sketch the cubic
$y = (x-1)\left(x^2-2x-26\right) = x^3-3x^2-24x+26.$

Find the y intercept:
When $x=0$, $y=26$, so $(0,26)$ is the y-intercept.

Find the x intercept: When $y=0$, we have

$$\begin{aligned}
&(x-1)\left(x^2-2x-26\right)=0\\
&\Rightarrow x-1=0 \text{ or } x^2-2x-26=0\\
&\Rightarrow x=1 \text{ or } x=\frac{2\pm\sqrt{108}}{2}=\frac{2\pm 6\sqrt{3}}{2}\\
&\Rightarrow x=1, \text{ or } x=1\pm 3\sqrt{3}
\end{aligned}$$

The x-intercepts are $(1,0)$, $(1-3\sqrt{3},0)$ and $(1+3\sqrt{3},0)$.

Graph sketching - Example

Sketch the cubic
$y = (x-1)\left(x^2-2x-26\right) = x^3-3x^2-24x+26.$

To find the stationary point find the values of x where $\frac{dy}{dx} = 0$

$$\begin{aligned}\frac{dy}{dx} =& 3x^2-6x-24 = 0\\ \Rightarrow& x^2-2x-8=0\\ \Rightarrow&(x-4)(x+2)=0\\ \Rightarrow& x-4=0 \text{ or } x+2=0\\ \Rightarrow& x=4 \text{ or } x=-2\end{aligned}$$

For each x, we find the corresponding y by substituting this value of x into the original cubic.
When $x=4$, $y=64-48-96+26=-54$ and when $x=-2$, $y=-8-12+48+26=54$, so the stationary points are $(4,-54)$ and $(-2,54)$.

Classify the stationary point

Method 1: $\frac{dy}{dx} = 3x^2 - 6x - 24$

Region	$x < -2$	$x = -2$	$-2 < x < 4$	$x = 4$	$x > 4$
Sign of gradient	$+$	0	$-$	0	$+$

$(-2, 54)$ is a local maximum stationary point.

$(4, -54)$ is a local minimum stationary point.

Classify the stationary point

Method 2: $\frac{dy}{dx} = 3x^2 - 6x - 24$

$$\frac{d^2y}{dx^2} = 6x - 6$$

When $x = -2$, $\frac{d^2y}{dx^2} = -12 - 6 = -18 < 0$ so we have a local maximum stationary point.
When $x = 4$, $\frac{d^2y}{dx^2} = 24 - 6 = 18 > 0$ so we have a local minimum stationary point.

Sketching the graph

$$y = x^3 - 3x^2 - 24x + 26$$

The x-intercepts are $(1, 0)$, $(1 - 3\sqrt{3}, 0)$ and $(1 + 3\sqrt{3}, 0)$.
The y-intercept is $(0, 26)$.
Stationary points: local min: $(4, -54)$; local max: $(-2, 54)$

Give it a go

Can you sketch the following graph $y = x^2 - 14x + 48$ showing the intercepts and stationary point?