Task 1: Give it a go

SIT190 - week 9 - quiz -short

Click on a question number to see how your answers were marked and, where available, full solutions.

Question Number	Score	
Question 1	4 / 4	Review
Question 2	4 / 4	Review
Total	**8 / 8 (100%)**	

Performance Summary

Exam Name:	SIT190 - week 9 - quiz -short
Session ID:	11577204828
Student's Name:	COWLISHAW, Ethan Del (edcowlishaw)
Exam Start:	Tue May 14 2024 18:38:13
Exam Stop:	Tue May 14 2024 20:29:15
Time Spent:	1:51:02

SIT190 - week 9 - quiz -short

Click on a question number to see how your answers were marked and, where available, full solutions.

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Question 1	4 / 4	Review
Question 2	4 / 4	Review
Total	**8 / 8 (100%)**	

Performance Summary

Exam Name:	SIT190 - week 9 - quiz -short
Session ID:	1548162105
Student's Name:	COWLISHAW, Ethan Del (edcowlishaw)
Exam Start:	Tue May 14 2024 20:31:27
Exam Stop:	Tue May 14 2024 20:50:23
Time Spent:	0:18:55

For nearly every question, there were indefinite integrals that required a C constant to be placed at the end. I forgot every single time. I need to work on remembering this important step. I understand how to solve for C which is excellent.
In the first attempt, I struggled to understand why fractions like $\frac{1}{x^3}$ becomes $-\frac{1}{2x^2}$ instead of $-\frac{2}{x^2}$, and I still do struggle to understand to some degree but it is slowly sinking in.
Another bonus is that I am beginning to be able to calculate this stuff all mentally with no writing down. With this new power though, I am repeating silly mistakes all over again but that seems to be the case with any sort of new skill.

I will focus on mental calculations more to hone the skill further as it is extremely gratifying and also speeds up calculations significantly.

Task 2: Anti-differentiation

1) Integrate the following functions

a) Integrate $\int(5x^4 - 27x^3 + 38x^2)$

$\int(5x^4 - 27x^3 + 38x^2) + C$
-> $\frac{5x^{4+1}}{4+1} - \frac{27x^{3+1}}{3+1} + \frac{38x^{2+1}}{2+1} + C$
-> $\frac{5x^5}{5} - \frac{27x^4}{4} + \frac{38x^3}{3} + C$
-> $\frac{5}{5}x^5 - \frac{27}{4}x^4 + \frac{38}{3}x^3 + C$
$= x^5 - \frac{27}{4}x^4 + \frac{38}{3}x^3 + C$

b) Integrate $\int(\frac{13}{x^3} - \frac{26}{x} + 10x^{\frac{17}{23}})$

$\int\left(\frac{13}{x^3} - \frac{26}{x} + 10x^{\frac{17}{23}}\right) + C$
When written in power notation, $\frac{26}{x}$ becomes $26x^{-1}$. If we then find the integral of this, it becomes $\frac{26x^0}{0}$. A division by 0 is not allowed, so we instead convert it to the log version where $\ln(u)' = \frac{1}{u}$ and then reverse it.
We apply the $|x|$ absolute function to the x as in log functions, it can not be ≤ 0.
-> $13x^{-3} - 26\ln(|x|) + 10x^{\frac{17}{23}} + C$

-> $\frac{13x^{-3+1}}{-3+1} - 26\ln(|x|) + \frac{10x^{\frac{17}{23}+\frac{23}{23}}}{\frac{17}{23}+\frac{23}{23}} + C$
-> $\frac{13x^{-2}}{-2} - 26\ln(|x|) + \frac{10x^{\frac{40}{23}}}{\frac{40}{23}} + C$
-> $-\frac{13}{2}x^{-2} - 26\ln(|x|) + \frac{23}{4}x^{\frac{40}{23}} + C$
$= -\frac{13}{2x^2} - 26\ln(|x|) + \frac{23}{4}x^{\frac{40}{23}} + C$

c) Integrate $\int(10\sin(3x) + 8\tan(\frac{x}{2}))$

$\int(10\sin(3x) + 8\tan(\frac{x}{2}))$
-> $-\frac{1}{3} \times 10\cos(3x) + 8 \times -\frac{1}{\frac{1}{2}}\ln\left(\cos\left(\frac{x}{2}\right)\right) + C$
-> $-\frac{10}{3}\cos(3x) + 8 \times -2\ln\left(\cos\left(\frac{x}{2}\right)\right) + C$
$= -\frac{10}{3}\cos(3x) - 16\ln\left(\cos\left(\frac{x}{2}\right)\right) + C$

d) Integrate $\int(6e^{4x} - 27e^{-9x})$

$\int(6e^{4x} - 27e^{-9x}) + C$
-> $\frac{6}{4}e^{4x} - \frac{27}{-9}e^{-9x} + C$
$= \frac{3}{2}e^{4x} + 3e^{-9x} + C$

2) Find the original function $f(x)$ given $f'(x) = 8x^3 - 38x^2 + 56$ and $f(-2) = 1$

Finding the non-definite integral
$\int(8x^3 - 38x^2 + 56) + C$
-> $\frac{8}{4}x^4 - \frac{38}{3}x^3 + 56x + C$
-> $2x^4 - \frac{38}{3}x^3 + 56x + C$

Calculating using $f(-2) = 1$
$y = 2x^4 - \frac{38}{3}x^3 + 56x + C$
-> $1 = 2(-2)^4 - \frac{38}{3}(-2)^3 + 56(-2) + C$
-> $1 = 2(16) - \frac{38}{3}(-8) - 112 + C$
-> $1 = 32 + \frac{304}{3} - 112 + C$
-> $C = 1 - 32 - \frac{304}{3} + 112$
-> $C = 81 - \frac{304}{3}$
$C = -\frac{61}{3}$

Answer
$f(x) = 2x^4 - \frac{38}{3}x^3 + 56x - \frac{61}{3}$ given $y = 1$ and $x = -2$

3) Find the original function $f(x)$ given $f'(x) = 8\sin(3x) + 12\cos(13x)$ and $f(-\pi) = 2$

Finding the non-definite integral
$\int(8\sin(3x) + 12\cos(13x)) + C$
-> $-\frac{1}{3} \times 8\cos(3x) + \frac{1}{13} \times 12\sin(13x) + C$
-> $-\frac{8}{3}\cos(3x) + \frac{12}{13}\sin(13x) + C$

Calculating using $f(-\pi) = 2$

$y = -\frac{8}{3}\cos(3x) + \frac{12}{13}\sin(13x) + C$
-> $2 = -\frac{8}{3}\cos(3(-\pi)) + \frac{12}{13}\sin(13(-\pi)) + C$
-> $2 = -\frac{8}{3}\cos(-3\pi)) + \frac{12}{13}\sin(-13\pi)) + C$
-> $2 = -\frac{8}{3} \times -1 + \frac{12}{13} \times 0 + C$
-> $2 = \frac{8}{3} + C$
-> $C = 2 - \frac{8}{3}$
-> $C = \frac{6}{3} - \frac{8}{3}$
$C = -\frac{2}{3}$

Answer

$f(x) - \frac{8}{3}\cos(3x) + \frac{12}{13}\sin(13x) - \frac{2}{3}$ given $x = -\pi$ and $y = 2$

4) Find the original function $f(x)$ given $f'(x) = \frac{23}{x}$ and $f(e) = 3$

Finding the non-definite integral

$\int \left(\frac{23}{x}\right) + C$
-> $23\ln(|x|) + C$

Calculating using $f(e) = 3$

$y = 23\ln(|x|) + C$
-> $3 = 23\ln(|e|) + C$
-> We apply the $\log_e(e) = 1$ rule
-> $3 = 23 \times 1 + C$
-> $3 - 23 = C$
$C = -20$

Answer

$f(x) = 23\ln(|x|) - 20$ given $x = e$ and $y = 3$