SLE123 Physics for the Life Sciences

Week 11

Optical Instruments

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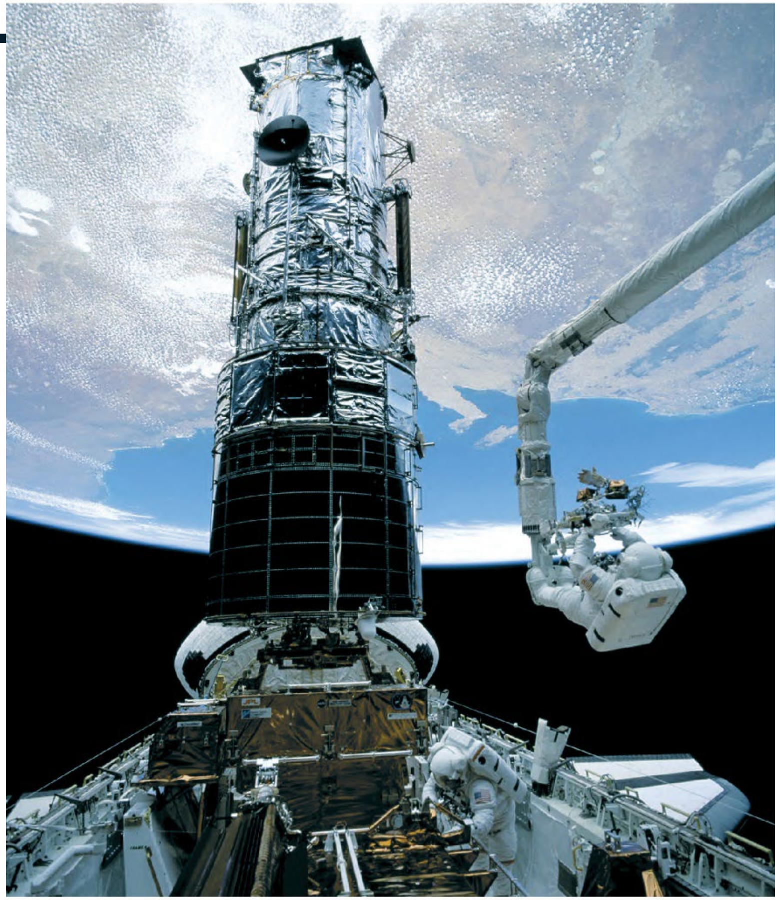
Optical Instruments



Source: NASA

Topics:

- Transverse magnification
- Thin Lens equation
- Camera
- The Eye
- Angular magnification
- Simple magnifier
- Compound microscope

Sample question:

The Hubble Space Telescope, orbiting earth at an altitude of about 600 km, was launched in 1990 by the crew of the Space Shuttle discovery. What is the advantage of having a telescope in space when there are telescopes on Earth with larger light-gathering capabilities?

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Transverse Magnification

- The image formed by a mirror or a lens is, in general, not the same size as the object. It may also be inverted (upside down).
- The **transverse magnification** *m* (also called the *lateral* or *linear* magnification) is a ratio that gives both the relative size of the image—in any direction perpendicular to the principal axis—and Its orientation.
  - The magnitude of *m* is the ratio of the image size to the object size:

$$|m| = \frac{\text{image size}}{\text{object size}}$$

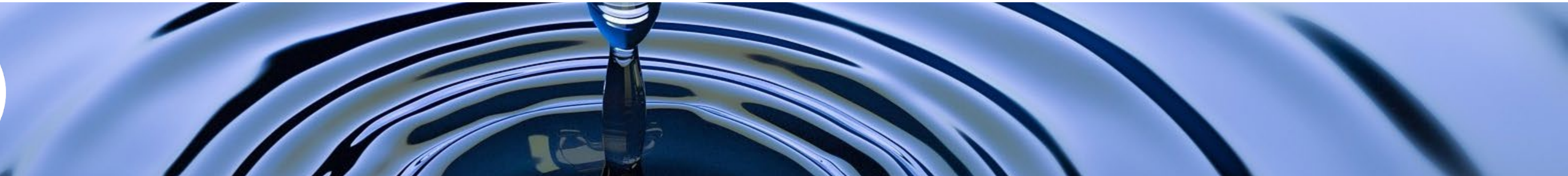
  - If $m < 1$, the image is smaller than the object.
  - The sign of *m* is determined by the orientation of the image. For an inverted (upside down) image, $m < 0$; for an upright (right side up) image, $m > 0$.

Transverse Magnification (continued)

- Let *h* be the height of the object (really the *displacement* of the top of the object from the axis) and *h*′ be the height of the image. If the image is inverted, *h*′ and *h* have opposite signs.

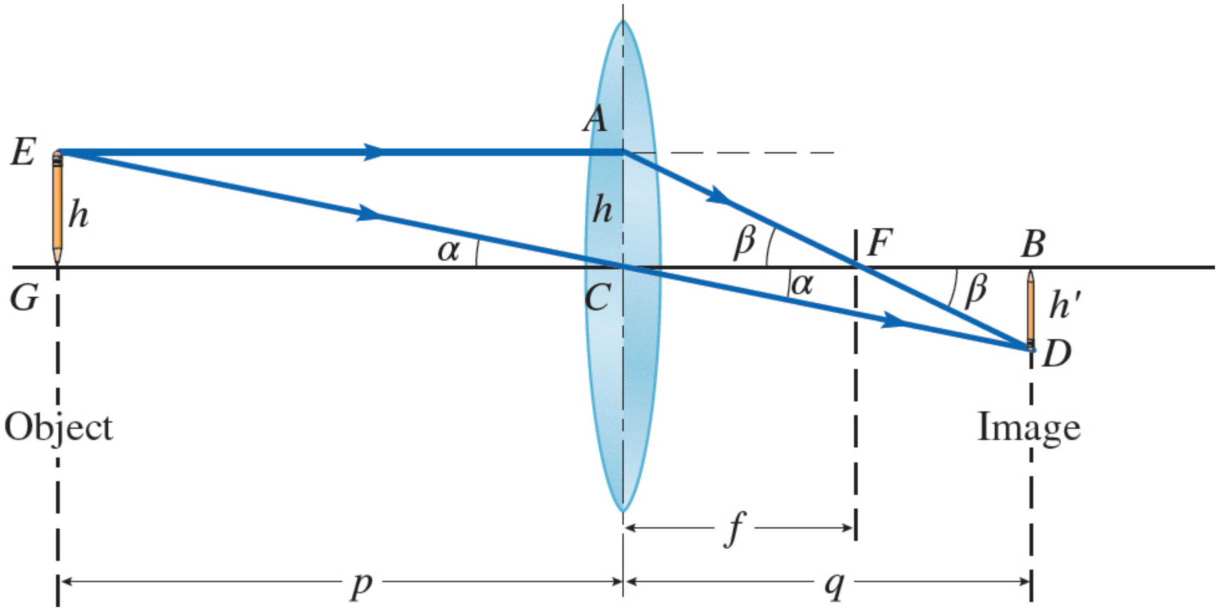
- Then the definition of the transverse magnification is

$$m = \frac{h'}{h}$$



Magnification for Thin Lenses

- Triangles ΔEGC and ΔDBC are similar right triangles.

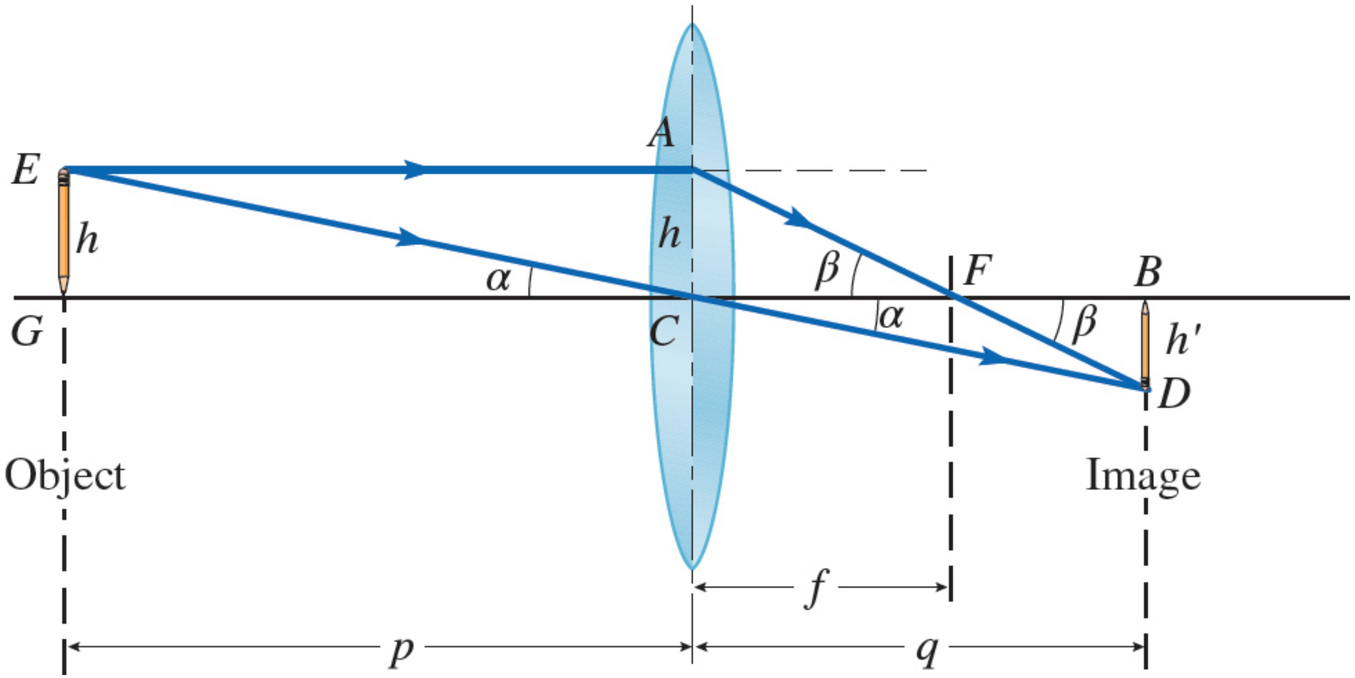


$$\tan\alpha = \frac{h}{p} = \frac{-h'}{q}$$

Magnification equation: $$m = \frac{h'}{h} = -\frac{q}{p}$$

The Thin Lens Equation

- From two other similar right triangles ΔACF and ΔDBF,



$$\tan \beta = \frac{h}{f} = \frac{-h'}{q - f}$$

The Thin Lens Equation (continued)

- After dividing through by q and rearranging, we obtain the
- **Thin lens equation**:

$$\tan\beta = \frac{h}{f} = \frac{-h'}{q-f}$$

$$\frac{q-f}{f} = \frac{-h'}{h} = \frac{q}{p}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Example 23.9

- A wild daisy 1.2 cm in diameter is 90.0 cm from a camera’s zoom lens. The focal length of the lens has magnitude 150.0 mm.

(a) Find the distance between the lens and the image sensor.

(b) How large is the image of the daisy?

Example 23.9 Strategy

- The problem can be solved using the lens and magnification equations.
- The lens must be *converging* to form a real image on the sensor, so f = +150.0 mm. The image must be formed on the sensor, so the distance from the lens to the sensor is q.
- After finishing the algebraic solution, we sketch a ray diagram as a check.

- Given: h = 1.2 cm; p = 90.0 cm; f = +15.00 cm
  Find: q, h'

Example 23.9 Solution 1

- The sensor is 18.0 cm from the lens.
  - (a)

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$q=\left(\frac{1}{f}-\frac{1}{p}\right)^{-1}$$

$$q=\left(\frac{1}{15.00\text{ cm}}-\frac{1}{90.0\text{ cm}}\right)^{-1}=+18.0\text{ cm}$$

Example 23.9 Solution 2

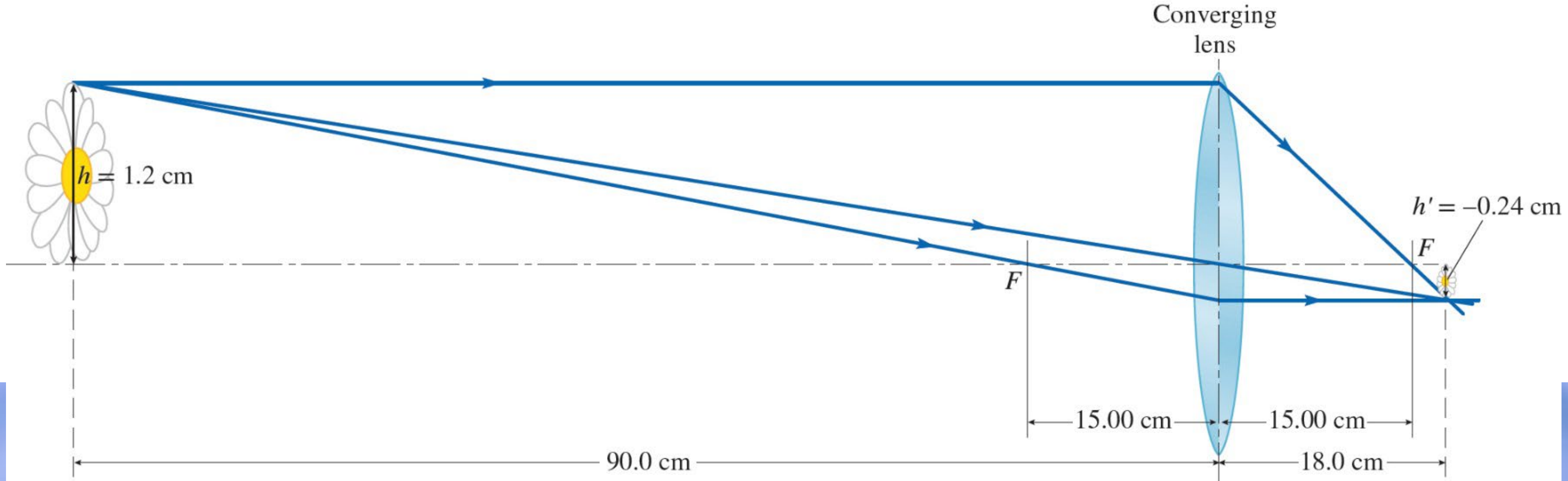
- (b)

$$m = \frac{h'}{h} = -\frac{q}{p} = -\frac{18.0\ \text{cm}}{90.0\ \text{cm}} = -0.200$$

$$h' = mh = -0.200 \times 1.2\ \text{cm}$$

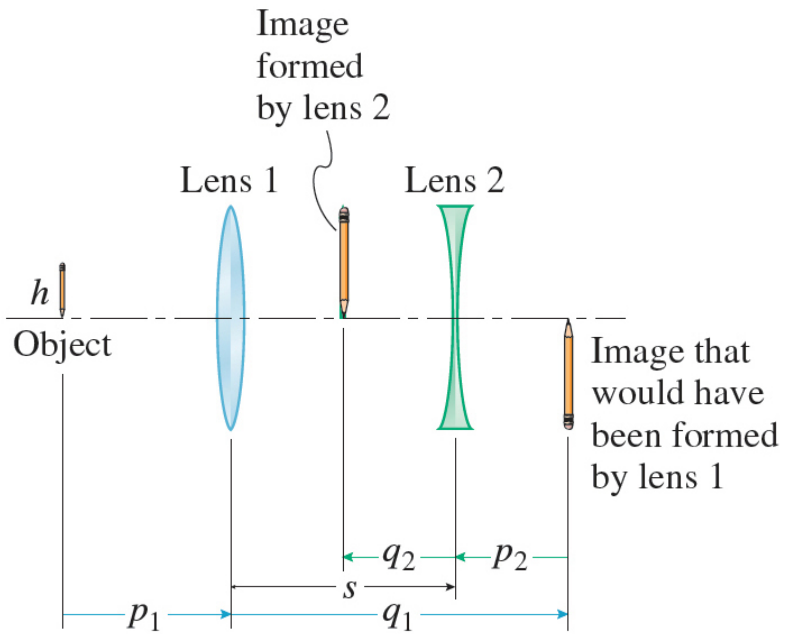
$$= -0.24\ \text{cm}$$

  - The image of the daisy is 0.24 cm in diameter.



Lenses in Combination

- Optical instruments generally involve two or more lenses in combination.
- What happens when light rays emerging from a lens pass through another lens?
- We will find that the image formed by the first lens serves as the object for the second lens.



Lenses in Combination (continued)

- For lenses in combination, we apply the lens equation to each lens in turn, where the object for a given lens is the image formed by the previous lens.

  This same procedure holds true for combinations of lenses and mirrors.

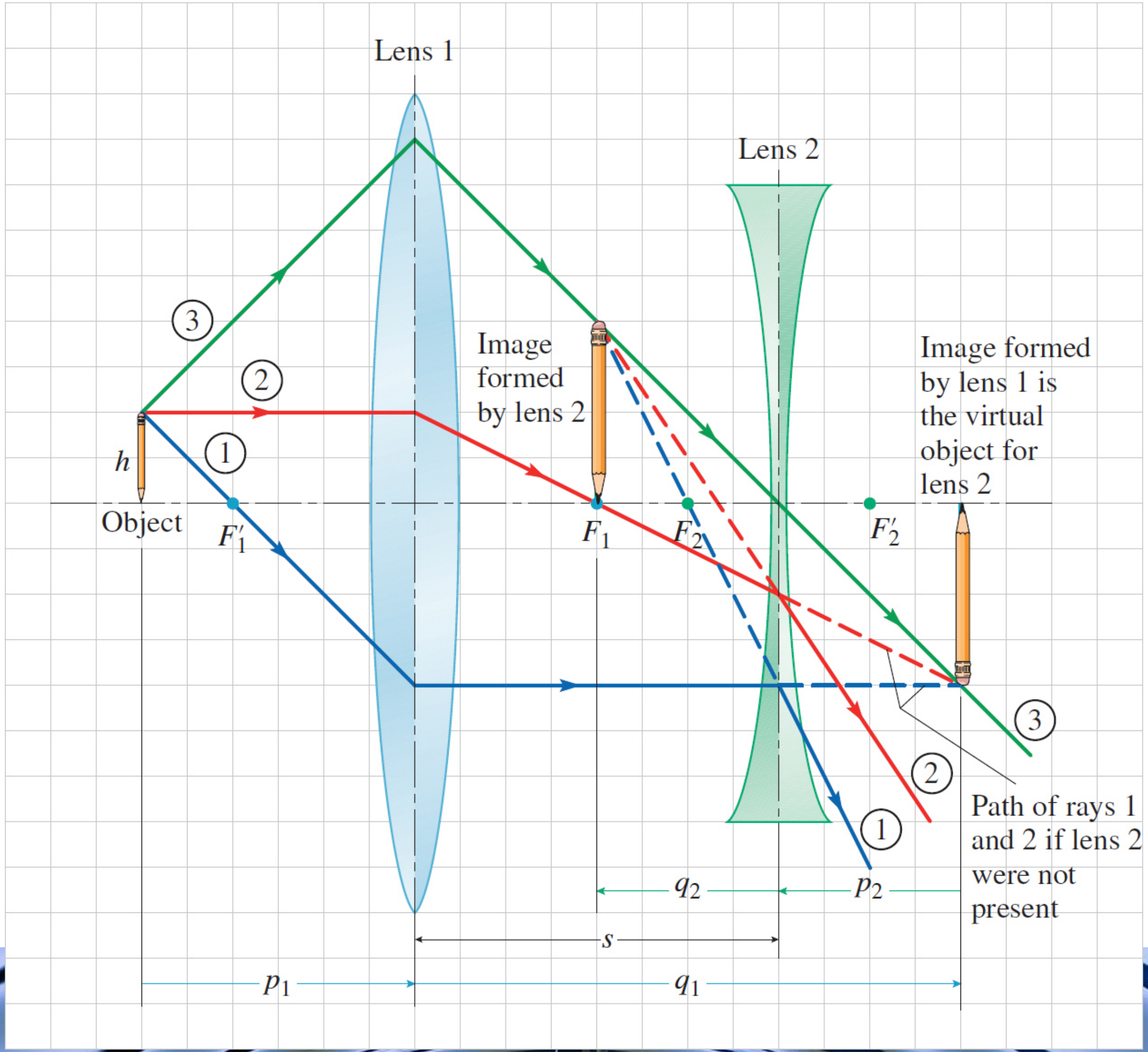
For a system of two thin lenses separated by a distance *s*, we can apply the thin lens equation separately to each lens:

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}$$

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

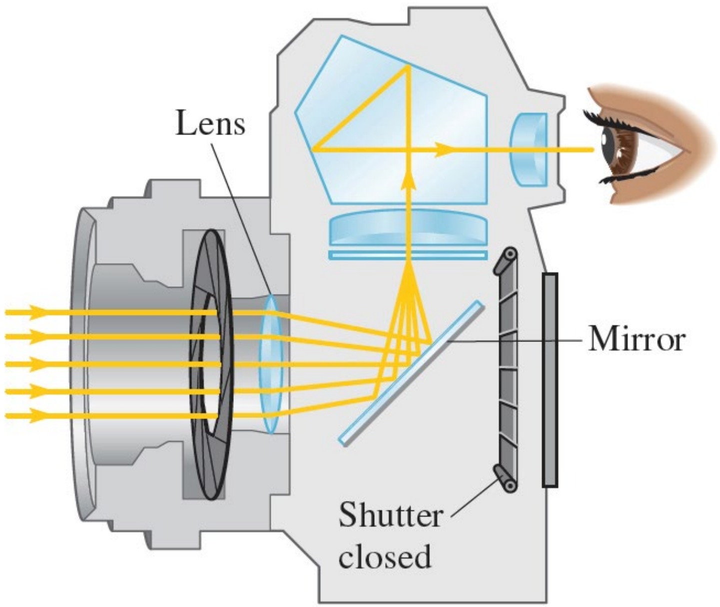
The object distance p_2 for the second lens is, $p_2 = s - q_1$

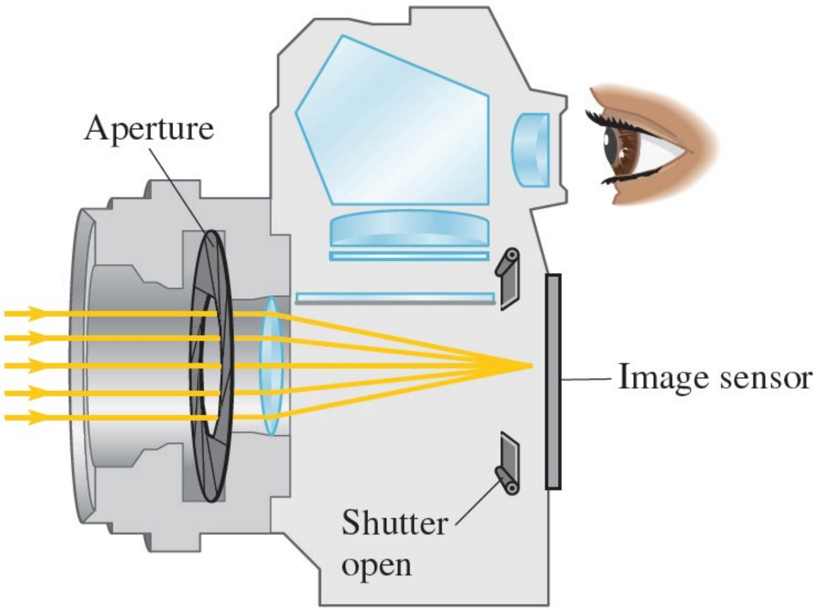
Ray Diagrams for Two Lenses



Cameras

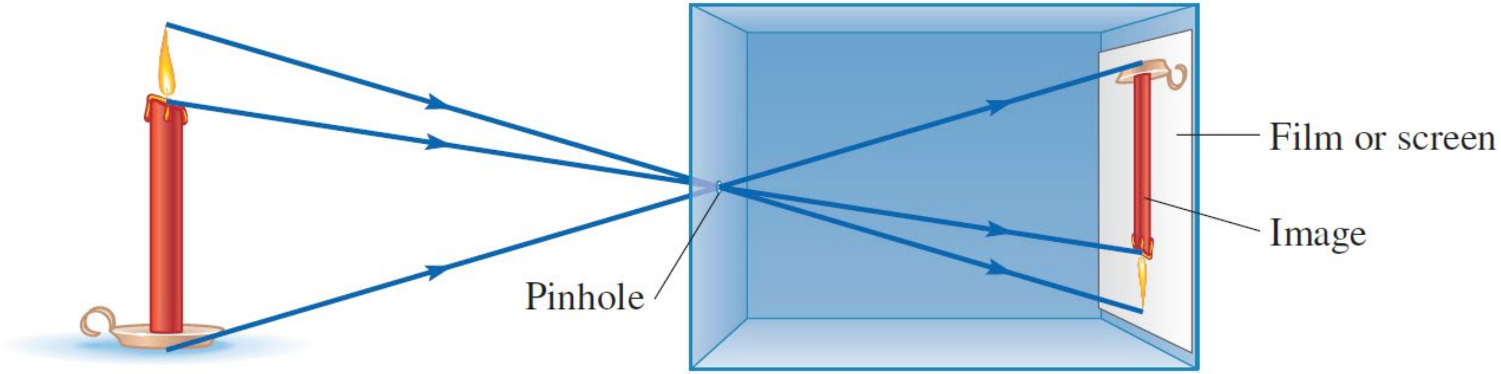
- One of the simplest optical instruments is the camera, which often has only one lens to produce an image, or even—in a pinhole camera—no lens.





Pinhole Camera

- Even simpler than a camera with one lens is a **pinhole camera**, or *camera obscura* (Latin “dark room”).
- To make a pinhole camera, a tiny pinhole is made in one side of a box.



An inverted, real “image” is formed on the opposite side of the box.
A photographic plate (a glass plate coated with a photosensitive emulsion) or film placed on the back wall can record the image.

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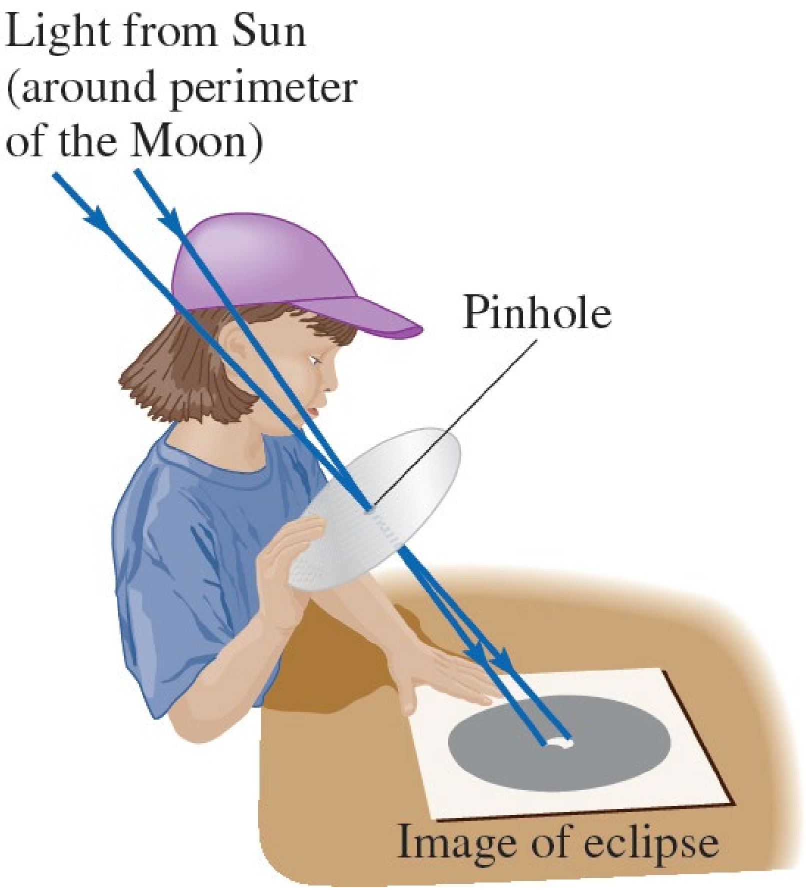
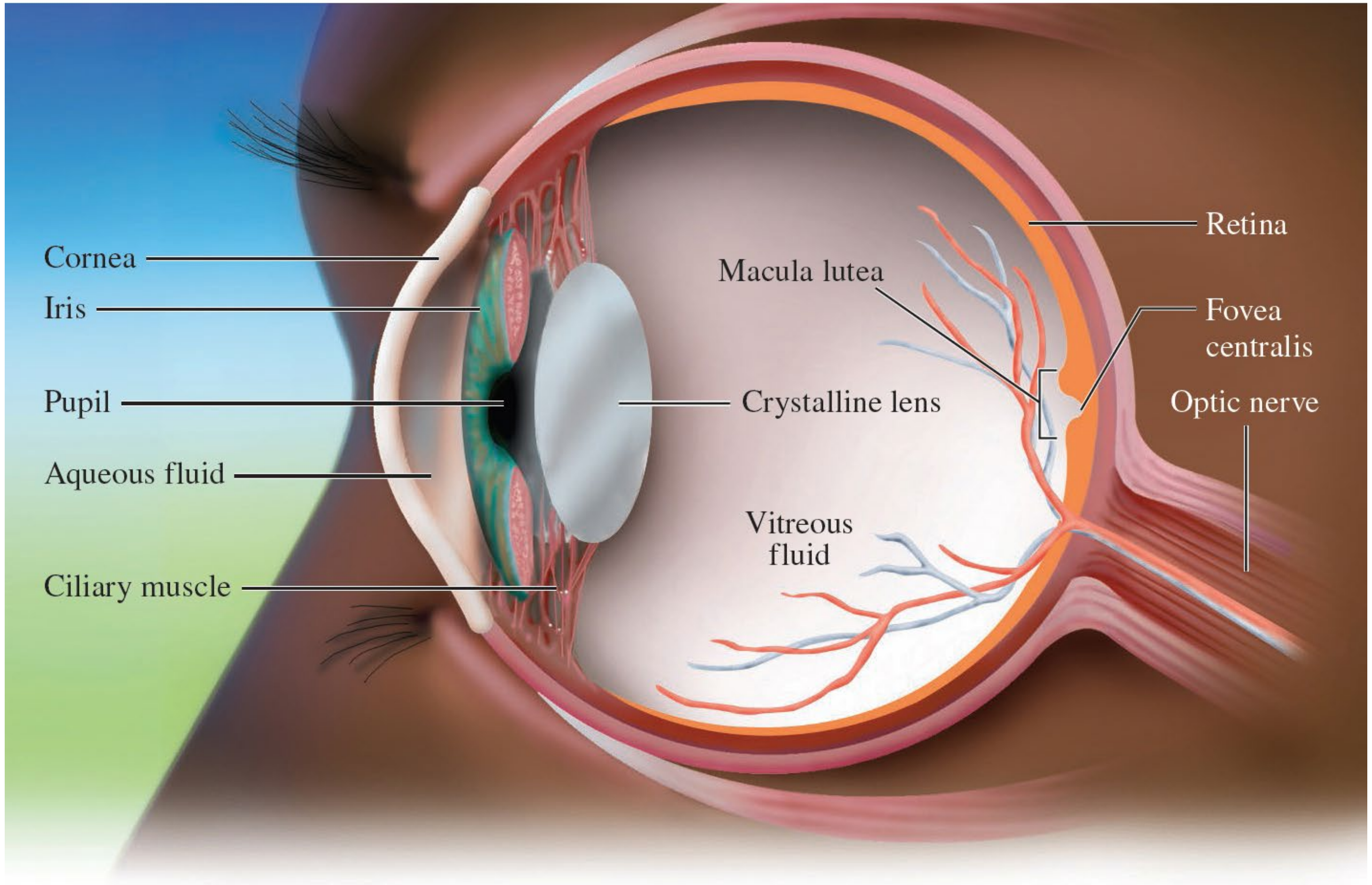
Pinhole Camera (continued)



The Eye

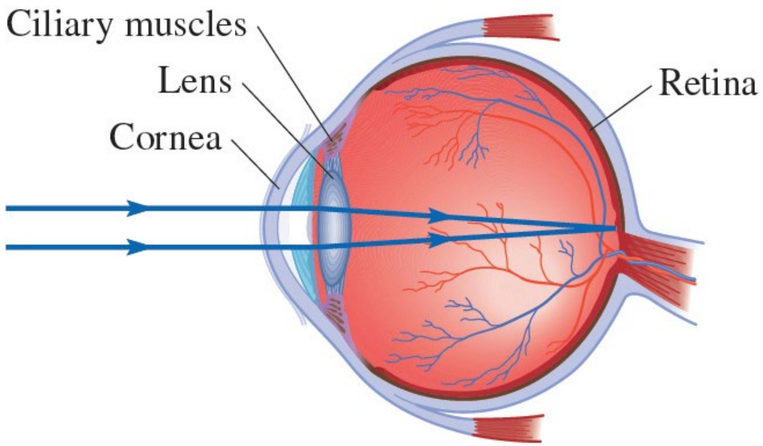
- The human eye is similar to a digital camera.
- The camera forms a real image on a CCD array; the eye forms a real image on the *retina*, a membrane with approximately 125 million photoreceptor cells (the *rods* and *cones*).
- The focusing mechanism is different, though. In the camera, the lens moves toward or away from the image sensor to form an image on the sensor as the object distance p changes. In the eye, the lens is at a fixed distance from the retina, but it has a variable focal length; the focal length is adjusted to keep the image distance constant as the object distance varies.

The Anatomy of the Human Eye



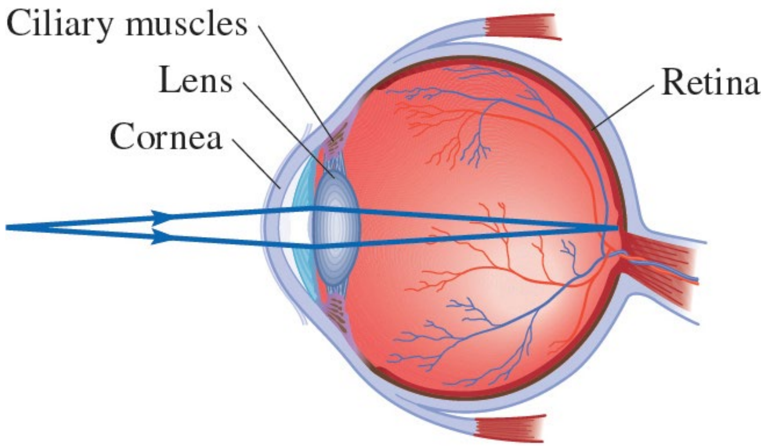
Accommodation

- Variation in the focal length of the flexible lens is called **accommodation**; it is the result of an actual change in the shape of the lens of the eye through the action of the *ciliary muscles*.
- The adjustable shape of the lens allows for accommodation for various object distances, while still forming an image at the fixed image distance determined by the separation of lens and retina.



Viewing distant object, longer focal length

(a)



Viewing nearby object, shorter focal length

(b)

Near Point and Far Point

- Accommodation enables an eye to form a sharp image on the retina of objects at a range of distances from the **near point** to the **far point**.
- A young adult with good vision has a near point at 25 cm or less and a far point at infinity.
- A child can have a near point as small as 10 cm. Corrective lenses (eyeglasses or contact lenses) or surgery can compensate for an eye with a near point greater than 25 cm or a far point less than infinity.

Refractive Power

- Optometrists write prescriptions in terms of the **refractive power** (P) of a lens rather than the focal length.

- (Refractive power is different from "magnifying power," which is a synonym for the angular magnification of an optical instrument.)

- The refractive power is simply the reciprocal of the focal length:

$$P = \frac{1}{f}$$

Why use refractive power instead of focal length?
When two or more thin lenses with refractive powers P_1, P_2, . . . are sufficiently close together, they act as a single thin lens with refractive power

$$P = P_1 + P_2 + \cdots$$

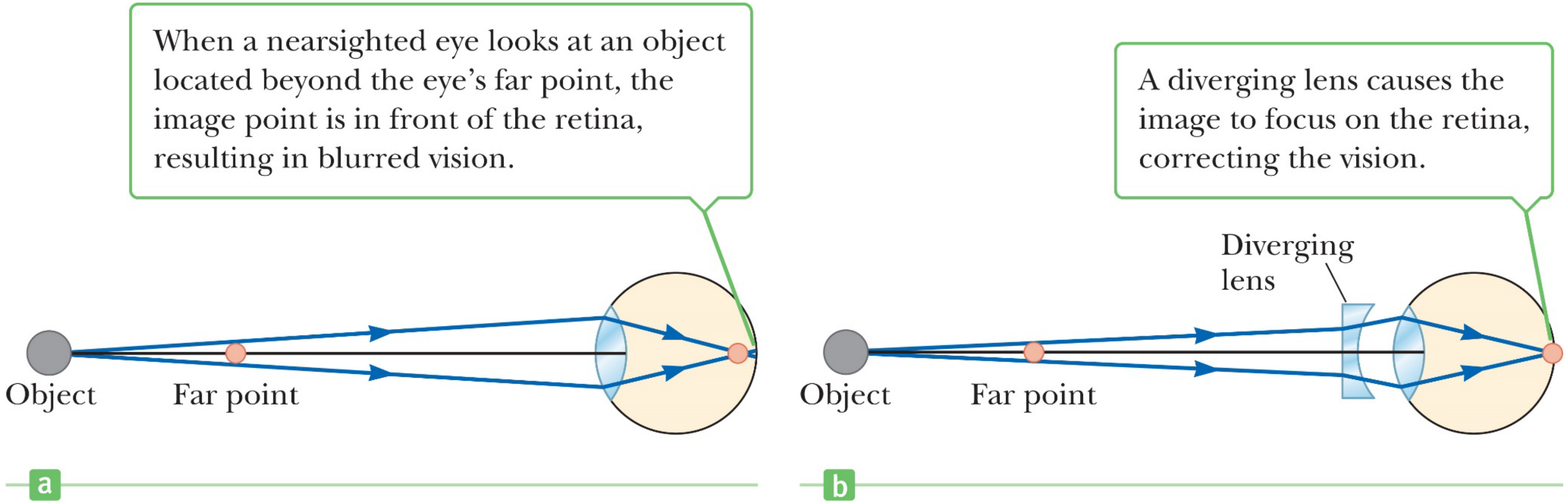
Diopters

- Refractive power is usually measured in **diopters** (symbol D).
- One diopter is the refractive power of a lens with focal length f = 1 m (1 D = 1 m^{-1}).

- The shorter the focal length, the more "powerful" the lens because the rays are bent more.

- Converging lenses have positive refractive powers, and diverging lenses have negative refractive powers.

Myopia



- The refractive power of the lens is too large; the eye makes the rays converge too soon.
- A diverging corrective lens (with negative refractive power) can compensate for nearsightedness by bending the rays outward.

Example 24.4

- Without her contact lenses, Dana cannot see clearly an object more than 40.0 cm away.
- What refractive power should her contact lenses have to give her normal vision?

Example 24.4 Strategy

- The far point for Dana's eyes is 40.0 cm.
- For an object at infinity, the corrective lens must form a virtual image 40.0 cm from the eye.
- We use the lens equation with $p = \infty$ and $q = -40.0$ cm to find the focal length or refractive power of the corrective lens.
- The image distance is negative because the image is virtual—it is formed on the same side of the lens as the object.

Example 24.4 Solution

- Since $p = \infty$, $1/p = 0$. Then

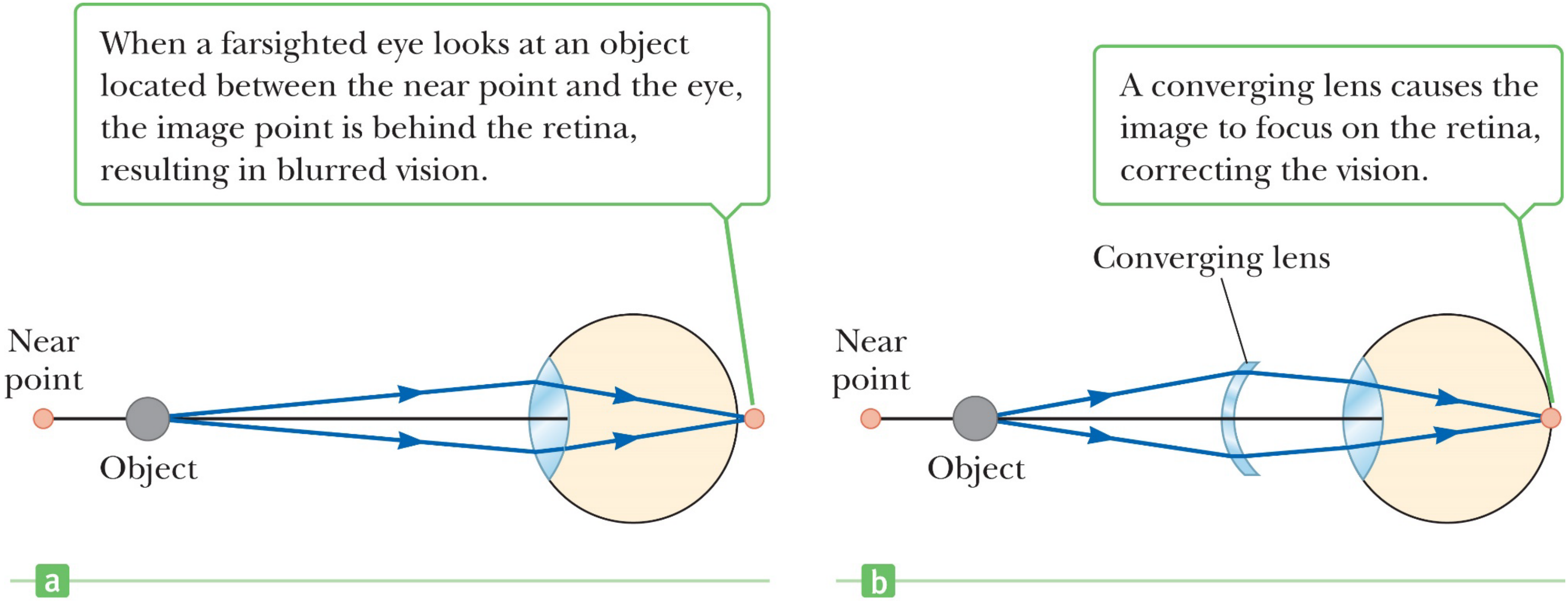
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = P$$

$$0 + \frac{1}{-40.0\ \text{cm}} = \frac{1}{f}$$

- Solving for the focal length,
  - f = -40.0 cm

$$P = \frac{1}{f} = \frac{1}{-40.0 \times 10^{-2}\ \text{m}} = -2.50\ \text{D}$$

Hyperopia



- A hyperopic (farsighted) eye can see distant objects clearly but not nearby objects; the near point is too large.
- The refractive power of the eye is too small; the cornea and lens do not refract the rays enough to make them converge on the retina.

Example 24.5

- Winifred is unable to focus on objects closer than 2.50 m from her eyes.
- What refractive power should her corrective lenses have?

Example 24.5 Strategy

- For an object 25 cm from Winifred's eye, the corrective lens must form a virtual image at the near point of Winifred's eye (2.50 m from the eye).
- We use the thin lens equation with p = 25 cm and q = −2.50 m to find the focal length.
- As in the last example, the image distance is negative because it is a virtual image formed on the same side of the lens as the object.

Example 24.5 Solution

- Substituting p = 0.25 m and q = -2.50 m,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{0.25\ \mathrm{m}}+\frac{1}{-2.50\ \mathrm{m}}=\frac{1}{f}$$

  - Solving for the focal length,
    - f = 0.28 m

$$P=\frac{1}{f}=+3.6\ \mathrm{D}$$

Presbyopia

- As a person ages, the lens of the eye becomes less flexible and the eye's ability to accommodate decreases, a phenomenon known as presbyopia.
- Older people have difficulty focusing on objects held close to the eyes; from the age of about 40 years many people need eyeglasses for reading. At age 60, a near point of 50 cm is typical; in some people it may be 1 m or even more.
- Reading glasses for a person suffering from presbyopia are similar to those used by a farsighted person.

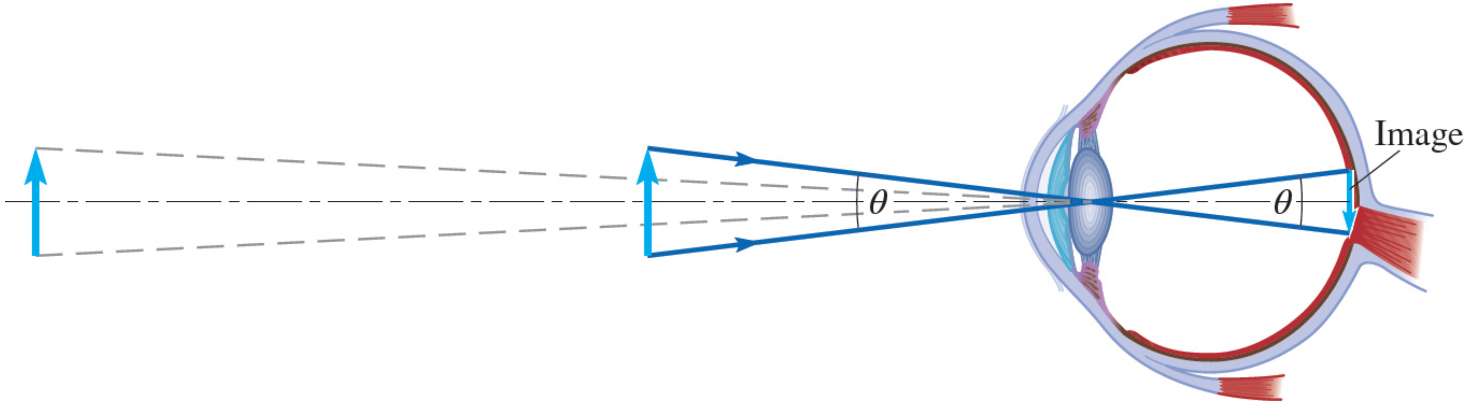
Angular Magnification

- Imagine rays from the top and bottom of each object that are incident on the center of the lens of the eye. The angle θ is called the **angular size** of the object.
- The image on the retina subtends the same angle θ; the angular size of the image is the same as that of the object.
- Rays from the object at a greater distance subtend a smaller angle; the angular size depends on distance from the eye.

Angular Magnification (continued)

- For the unaided eye, the retinal image size is proportional to the angle subtended by the object. The figure shows two identical objects being viewed from different distances.

Image Size is Proportional to Angle Subtended



Angular Magnification (continued)

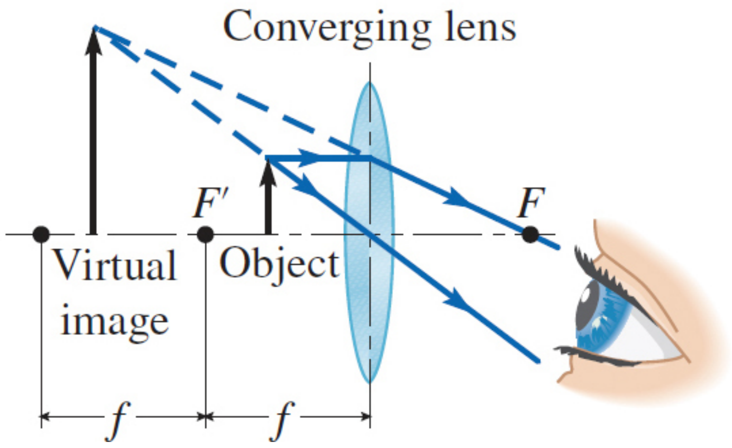
- A magnifying glass, microscope, or telescope serves to make the image on the retina larger *than it would be if viewed with the unaided eye*.
- Since the size of the image on the retina is proportional to the angular size, we measure the usefulness of an optical instrument by its **angular magnification**—the ratio of the angular size using the instrument to the angular size with the unaided eye.
- **Definition of angular magnification:**

$$M = \frac{\theta_{\text{aided}}}{\theta_{\text{unaided}}}$$

Simple Magnifier

- A **simple magnifier** is a converging lens placed so that the object distance is less than the focal length (i.e., a magnifying glass).

- The virtual image formed is enlarged, upright, and farther away from the lens than the object.

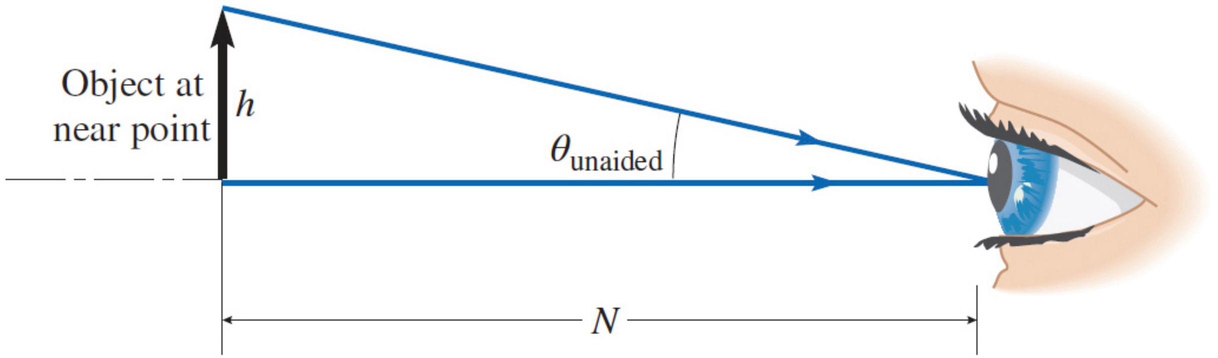


Simple Magnifier Continued 1

- If a small object of height *h* is viewed with the unaided eye, the angular size when it is placed at the near point (a distance *N* from the eye) is

$$\theta_{\text{unaided}} \approx \frac{h}{N} \qquad \text{(in radians)}$$

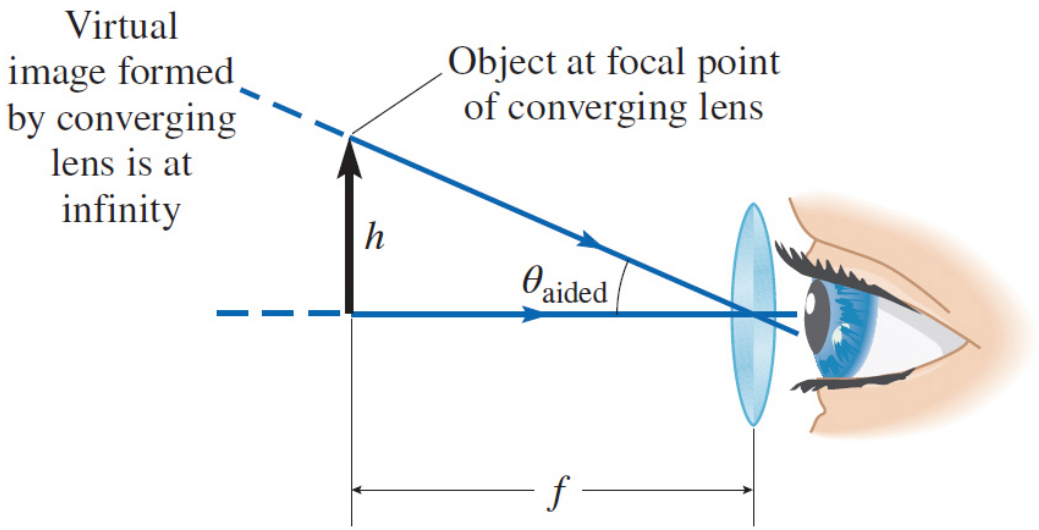
  - where we assume $h << N$ and, thus, θ_{unaided} is small enough that $\tan \theta_{\text{unaided}} \approx \theta_{\text{unaided}}$.



Simple Magnifier Continued 2

- If the object is now placed at the focal point of a converging lens, the image is formed at infinity and can be viewed with a relaxed eye. The angular size of the image is

$$\theta_{\text{aided}} \approx \frac{h}{f} \qquad \text{(in radians)}$$



Simple Magnifier: Angular Magnification

- Then the angular magnification M is

$$M = \frac{\theta_{\text{aided}}}{\theta_{\text{unaided}}} = \frac{h/f}{h/N} = \frac{N}{f}$$

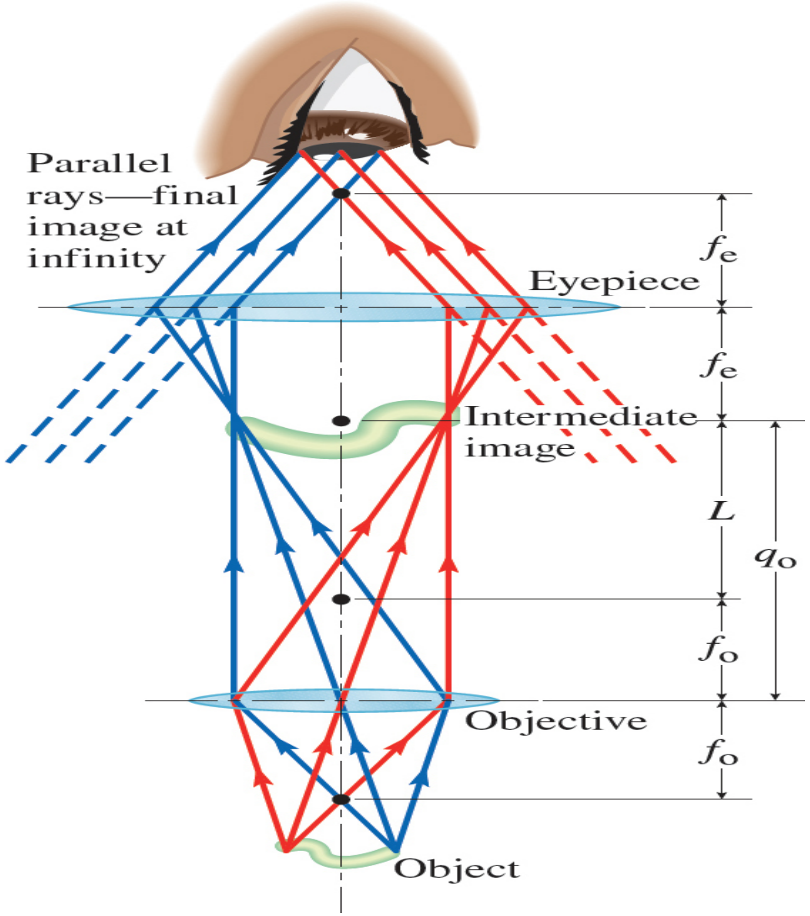
  - If the object is placed closer to the magnifier ($p < f$), the angular magnification is somewhat larger. The angular size of the image would then be $\theta_{\text{aided}} = h/p$, and the angular magnification would be

$$M = \frac{\theta_{\text{aided}}}{\theta_{\text{unaided}}} = \frac{h/p}{h/N} = \frac{N}{p}$$

Compound Microscopes

- The simple magnifier is limited to angular magnifications of 15-20 at most.
- By contrast, the **compound microscope**, which uses two converging lenses, enables angular magnifications of 2000 or more.
- A small object to be viewed under the microscope is placed *just beyond* the focal point of a converging lens called the **objective**.
- The function of the objective is to form an enlarged real image. A second converging lens, called the **ocular** or **eyepiece**, is used to view the real image formed by the objective lens.

Ray Diagram for Compound Microscope



Magnification due to the Eyepiece

- If we used just the eyepiece as a simple magnifier to view the object, the angular magnification would be

$$M_e = \frac{N}{f_e} \qquad \text{(due to eyepiece)}$$

  - where f_e is the focal length of the eyepiece and the virtual image is at infinity for ease of viewing.
  - Customarily we assume N = 25 cm. The objective forms an image that is larger than the object.

Magnification due to the Objective

- The transverse magnification due to the objective is

$$m_o = -\frac{L}{f_o} \qquad \text{(due to objective)}$$

- where *L* (the **tube length**) is the distance between the *focal points* of the two lenses, not the distance between the lenses.
- Since the image of the objective is placed at the focal point of the eyepiece the tube length is

$$L = q_o - f_o$$

Angular Magnification of a Microscope

$$M_{\text{total}} = m_{\text{o}} M_{\text{e}} = -\frac{L}{f_{\text{o}}} \times \frac{N}{f_{\text{e}}}$$

Example 24.7

- A compound microscope has an objective lens of focal length 1.40 cm and an eyepiece with a focal length of 2.20 cm. The objective and the eyepiece are separated by 19.6 cm. The final image is at infinity.

(a) What is the angular magnification?

(b) How far from the objective should the object be placed?

Strategy

- Given: f_o = 1.40 cm, f_e = 2.20 cm, lens separation = 19.6 cm
- To find: (a) total angular magnification M; (b) object distance p_o

Example 24.7 Solution 1

- (a)

$$L = \text{distance between lenses} - f_o - f_e$$
$$= 19.6\text{ cm} - 1.40\text{ cm} - 2.20\text{ cm} = 16.0\text{ cm}$$

$$M = -\frac{L}{f_o} \times \frac{N}{f_e}$$
$$= -\frac{16.0\text{ cm}}{1.40\text{ cm}} \times \frac{25\text{ cm}}{2.20\text{ cm}} = -130$$

Example 24.7 Solution 2

- (b)

$$q_o = L + f_o = 16.0\text{ cm} + 1.40\text{ cm} = 17.4\text{ cm}$$

$$\frac{1}{p_o} + \frac{1}{q_o} = \frac{1}{f_o}$$

$$p_o = \frac{f_o q_o}{q_o - f_o}$$

$$= \frac{1.40\text{ cm} \times 17.4\text{ cm}}{17.4\text{ cm} - 1.40\text{ cm}}$$

$$= 1.52\text{ cm}$$