Week 7

Monday, 4 December 2023 9:18 AM

Common alpha levels are 0.10, 0.05, and 0.01.

The alpha level is also called the significance level.

$H_0: p = 0.20$

Alternative hypothesis: H_A: $p \neq 0.20$ } Two-sided test

$p > 0.20$ } One-sided test

$p < 0.20$

Because confidence intervals are two-sided, they correspond to two-sided tests.

- In general, a confidence interval with a confidence level of *C*% corresponds to a two-sided hypothesis test with an α-level of 100 – *C*%.
- For example – 90% confidence level corresponds to a two-sided hypothesis test with an α-level of 100 – *90*% = 10%.

The relationship between confidence intervals an one-sided hypothesis tests is a little more complicated.

- A confidence interval with a confidence level of *C*% corresponds to a one-sided hypothesis test with an α-level of ½(100 – *C*)%.
- For example – 90% confidence level corresponds to a one-sided hypothesis test with an α-level of ½(100 – *90*)% = 5%.

Confidence intervals	Two-sided test	One-sided test
90%	$\alpha = 100 - 90 = 10\%$	$\alpha = \frac{100-90}{2} = \frac{10}{2} = 5\%$
95%	$\alpha = 100 - 95 = 5\%$	$\alpha = \frac{100-95}{2} = \frac{5}{2} = 2.5\%$
98%	$\alpha = 100 - 98 = 2\%$	$\alpha = \frac{100-98}{2} = \frac{2}{2} = 1\%$

Question 1 : Two proportions

Which of the following situations involve two independent proportions?

1. Comparing the proportion of heads from 100 coin tosses with 0.5. One proportion
2. Analysing the percentages of fails between 2 campuses. Two proportions
3. Investigating the proportion of people who believe in ghosts. One proportion
4. Comparing the proportions of males and females who sleep for longer than 8 hours a night. Two proportion

Confidence Interval

Two proportions $(\hat{p}_1 - \hat{p}_2) \pm z^* \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$

Hypothesis Test

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_1} + \frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_2}}}$$

Assumptions and Conditions

- Independence Assumptions:
  - Randomisation Condition: The data in each group should be drawn independently and at random from the population or obtained from randomised comparative experiment.
  - The 10% Condition: If the data are sampled without replacement, the sample should not exceed 10% of the population.
  - Independent Groups Assumption: The two groups we're comparing must be independent *of each other*.
  - Sample Size Condition:
    - *Each* of the groups must be big enough…
    - Success/Failure Condition: Both groups are big enough that at least 10 successes and at least 10 failures have been observed in each. i.e. n_1p_1>10, n_1q_1>10 and n_2p_2>10, n_2q_2>10

Question 2 : Two proportion confidence intervals example

To explore the benefits of a Mediterranean diet for heart patients, a hospital set up two nutritional programs for patients who had suffered a heart attack. A traditional low fat diet was given to 358 randomly selected patients and a Mediterranean diet (including fruit, olive oil and bread) was given to 397 randomly selected patients. When the programs ended, it was found that 17 patients on the traditional diet had suffered a second heart attack and 4 patients receiving the Mediterranean diet has suffered a second heart attack. Calculate a 98% confidence interval for the difference in second heart attack proportions between the diets.

GROUP 1 → Traditional diet

Sample size, $n_1 = 358$ ✓

$\hat{p}_1 = \frac{17}{358} = 0.047$ ✓

$\hat{q}_1 = 0.953$ ✓

GROUP 2 – Mediterranean diet

$n_2 = 397$ ✓

$\hat{p}_2 = \frac{4}{397} = 0.01$ ✓

$\hat{q}_2 = 0.99$ ✓

$$\hat{q}_1 = 0.953 \quad (358)$$

$$\hat{q}_2 = 0.99$$

$$z^*(98\%) = 2.33$$

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$= (0.047 - 0.01) \pm 2.33 \sqrt{\frac{0.047 \times 0.953}{358} + \frac{0.01 \times 0.99}{397}}$$

$$= 0.037 \pm 2.33 \times 0.0122$$

$$= 0.037 \pm 0.0285$$

0.0085 and 0.0655

0.85% 6.6%.

We are 98% sure that the traditional diet results in between 0.9% and 6.6% more second heart attacks compared with the Mediterranean diet.

Confidence Interval Calculator For Two Proportions

Confidence interval calculator for the difference between two proportions with calculation steps, using the *normal distribution approximation*.

Confidence Level (CL): 0.98

Bounds: Two-sided: L < p_1-p_2 < U

Sample proportion ($\hat{p}_1$) or #successes (x_1): 0.047

Sample proportion ($\hat{p}_2$) or #successes: (x_2): 0.01

Sample size (n_1): 358

Sample size (n_2): 397

Continuity correction: Don't use

Rounding: 4

Confidence interval: **[0.008503 , 0.0655]**.
Alternatively: **0.037 ± 0.0285**

Two proportions confidence interval formula

$$CI = \hat{p}_1 - \hat{p}_2 \pm Z_{1-\alpha/2} \sqrt{\left(\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}\right)}$$

$$CI = 0.047 - 0.01 \pm 2.3263 \sqrt{\left(\frac{0.047(1 - 0.047)}{358} + \frac{0.01(1 - 0.01)}{397}\right)}$$

$$CI = 0.037 \pm 0.0285$$

Hypotheses test:

Step 1: Write Null and Alternative Hypotheses

$H_0: \hat{p}_1 = \hat{p}_2$ OR $H_0: \hat{p}_1 - \hat{p}_2 = 0$

$H_A: \hat{p}_1 \neq \hat{p}_2$

Two sided test

$\hat{p}_1 > \hat{p}_2$ } one-sided test
$\hat{p}_1 < \hat{p}_2$

Step2 : Test statistic – Two proportions

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_1} + \frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_2}}}$$

$$p_{pooled} = \frac{p_1 + p_2}{n_1 + n_2} \qquad q_{pooled} = \frac{q_1 + q_2}{n_1 + n_2}$$

Question 4 : Two proportion hypothesis tests example

To explore the benefits of a Mediterranean diet for heart patients, a hospital set up two nutritional programs for patients who had suffered a heart attack. A traditional low fat diet was given to 358 randomly selected patients and a Mediterranean diet (including fruit, olive oil and bread) was given to 397 randomly selected patients. When the programs ended, it was found that 17 patients on the traditional diet had suffered a second heart attack and 4 patients receiving the Mediterranean diet has suffered a second heart attack.
Is there evidence that a Mediterranean diet significantly reduces the proportion of people suffering a second heart attack compared with a traditional diet? Perform a hypothesis test using α=0.01.

GROUP 1 → Traditional diet

Sample size, $n_1 = 358$ ✓

$\hat{p}_1 = \frac{17}{358} = 0.047$ ✓

$\hat{q}_1 = 0.953$ ✓

GROUP 2 – Mediterranean diet

$n_2 = 397$ ✓

$\hat{p}_2 = \frac{4}{397} = 0.01$ ✓

$\hat{q}_2 = 0.99$ ✓

Null and Alternative Hypotheses

$H_0 : \hat{p}_1 = \hat{p}_2$

$H_A : \hat{p}_1 > \hat{p}_2$

Test-statistic :

$$p_{pooled} = \frac{p_1 + p_2}{n_1 + n_2} = \frac{17 + 4}{358 + 397} = \frac{21}{755}$$

$p_{pooled} = 0.0278$

$q_{pooled} = 1 - 0.0278 = 0.9722$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_1} + \frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_2}}} = \frac{0.047 - 0.01}{\sqrt{\frac{0.0278 \times 0.9722}{358} + \frac{0.0278 \times 0.9722}{397}}}$$

$$z = \frac{0.037}{0.01198} = 3.088$$

$$\boxed{z = 3.09}$$

Look up normal table

	0	1	2	3	4	5	6	7	8	9
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993

p-value $= 1 - 0.9990 = 0.001$

P-value $= 0.001$ Alpha level $= 0.01$

P-value $< \alpha = 0.01$

P-value is Low, Reject Null Hypothesis

Since the P-value is < α=0.01, reject H_0.

There appears to be evidence that a Mediterranean diet significantly reduces the proportion of people suffering a second heart attack compared with a traditional diet.

Tails: Right (H_1: $p_1 > p_2$)

Digits: 6

Significance level (α): 0.01

Continuity correction: Don't use

h effect size: 0.5

Calculate the expected h effect size: Calculate h

Group-1: Traditional diet

Group-2: Mediterranean diet

Proportion ($\hat{p}_1$) or successes (x_1): 0.047

Proportion ($\hat{p}_2$) or successes (x_2): 0.01

Sample size$_1$ (n_1): 358

Sample size$_2$ (n_2): 397

Step by step

The test statistic **Z** equals **3.101799**,

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_1} + \frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_2}}}$$

The p-value equals **0.000961742**,

Since the p-value < α, H_0 is rejected.
The **Traditional diet** population's proportion is considered to be greater than the **Mediterranean diet** population's proportion.
In other words, the sample proportion of Traditional diet is greater than Mediterranean diet, and the difference is big enough to be statistically significant.

2.8 A survey was conducted to compare customer satisfaction on the service in two branches of a particular bank. Various customers visiting the two branches were selected randomly, and asked if they were satisfied with the services provided by the branch. The results of this survey are shown in the following table. Is a significant difference between the two bank branches regarding the responses from the customers about their satisfaction on the service of the two branches?
Use indices "1" (like n_1, p_1, q_1) for Branch A and "2" (like n_2, p_2, q_2) for Branch B.

	Number satisfied	Not satisfied
Branch A	558 ✓	53 ✓
Branch B	496 ✓	77 ✓

$H_0: \hat{p}_1 = \hat{p}_2$
$H_A: \hat{p}_1 \neq \hat{p}_2$

a) Write appropriate hypotheses.
b) Perform the hypothesis test: find $\hat{p_1}$, $\hat{q_1}$, $\hat{p_2}$, $\hat{q_2}$, $\hat{p}_{pooled}$, $\hat{q}_{pooled}$, z-score, the P-value and state your conclusion in plain English. Use α = 0.05.
c) Create a 95% confidence interval for the difference in the proportions between the two bank branches regarding the responses from the customers satisfied on the services, and interpret your interval (also state the critical value z*, formula and value for Standard Error, and formula for the confidence interval).
d) Explain how your interval is consistent with your conclusion from b).

GROUP 1 – BRANCH A

$n_1 = 558 + 53 = 611$

$\hat{p}_1 = \frac{558}{611} = 0.913$

$\hat{q}_1 = 0.087$

GROUP 2 – BRANCH B

$n_2 = 496 + 77 = 573$

$\hat{p}_2 = \frac{496}{573} = 0.866$

$\hat{q}_2 = 0.134$

Two proportions $(\hat{p}_1 - \hat{p}_2) \pm \overset{1.96}{z^*} \times \underbrace{\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}}_{\text{Standard Error}}$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_1} + \frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_2}}}$$

$p_1 = 558$
$p_2 = 496$
$n_1 = 611$
$n_2 = 573$

$\hat{p}_{pooled} = \frac{p_1 + p_2}{n_1 + n_2}$

$q_{pooled} = 1 - p_{pooled}$

CHI-SQUARE TESTS → Qualitative Variables

One proportion → One Categorical variable

Example : Left handed people

Example: Left handed people

p q

Two proportions → One categorical variable

Example: Left handed people

Males: p_1 q_1

Females: p_2 q_2

More than one categorical variable → CHI-SQUARE TEST

Goodness-of-Fit

- A test of whether the distribution of counts in one categorical variable matches the distribution predicted by a model is called a goodness-of-fit test.
- Eg. Consider a genetic experiment where we are interested in investigating characteristics of the offspring resulting from mating white chickens with small combs. The offspring are categorised according to their feather colour and comb size, with data for 190 offspring shown in the following table. Are the data consistent with the ratio 9:3:3:1 specified by a particular genetic model for characteristics A:B:C:D?

Observed Counts

$n=4$

Type	Number of offspring (obsv.)	Expected Counts	$\frac{(\text{obsv} - \text{Exp})^2}{\text{Exp}}$	
A: White feathers, small comb	111	106.9	$(111-106.9)^2/106.9 =$	0.157
B: White feathers, large comb	37	35.6	$(37-35.6)^2/35.6 =$	0.055
C: Dark feathers, small comb	34	35.6	$(34-35.6)^2/35.6 =$	0.072
D: Dark feathers, large comb	8	11.9	$(8-11.9)^2/11.9 =$	1.278
TOTAL	190		$X^2 =$	1.562

genetic model : A : B : C : D

9 : 3 : 3 : 1

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp}$$

9+3+3+1 = 16

$A \rightarrow \frac{9}{16} \times 190 = 106.9$

$B \text{ and } C \rightarrow \frac{3}{16} \times 190 = 35.6$

$D \rightarrow \frac{1}{16} \times 190 = 11.9$

$$\chi^2 = \sum \left(\frac{(\textbf{observed} - \textbf{expected})^2}{\textbf{expected}} \right) \text{ for each cell}$$

$$\chi^2 = \frac{(111 - 106.9)^2}{106.9} + \frac{(37 - 35.6)^2}{35.6} + \frac{(34 - 35.6)^2}{35.6} + \frac{(8 - 11.9)^2}{11.9}$$

$$= 1.57$$

Assumptions and Conditions

- ✓ **Counted Data Condition:** Check that the data are *counts* for the categories of a categorical variable.
- ✓ **Independence Assumption**: The counts in the cells should be independent of each other.
  - Randomisation Condition
- ✓ **Sample Size Assumption**: We must have enough data for the methods to work.
  - Expected Cell Frequency Condition: We should expect to see at least 5 individuals in each cell.

9:3:3:1

✓ Hypotheses
✓ Test statistic
✓ P-value
✓ Comparison: P-value & Alpha level
✓ Conclusion

H_0: Model is demonstrated → observed counts = Expected counts

H_A: Model is not demonstrated → observed counts ≠ Expected counts

Test-statistic $\chi^2 = 1.562$

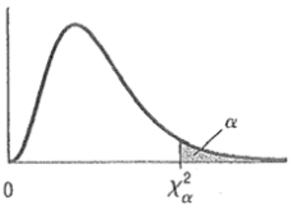
degrees of freedom, $df = n - 1 = 4 - 1 = 3$

P-value → Look up chi-square table

P-value is HIGH

P-value is LOW

Right tail probability	df	0.10 (10%)	0.05 (5%)	0.025 (2.5%)	0.01 (1%)	0.005 (0.5%)
Table X Values of χ^2_α	1	2.706	3.841	5.024	6.635	7.879
	2	4.605	5.991	7.378	9.210	10.597
1.562	$\boxed{3}$	6.251	7.815	9.348	11.345	12.838
	4	7.779	9.488	11.143	13.277	14.860
	5	9.236	11.070	12.833	15.086	16.750
	$\boxed{6}$	10.645	12.592	14.449	16.812	18.548
	7	12.017	14.067	16.013	18.475	20.278
	8	13.362	15.507	17.535	20.090	21.955
	9	14.684	16.919	19.023	21.666	23.589
	10	15.987	18.307	20.483	23.209	25.188



P-value > 0.10

P-value > 0.10

P-value is HIGH, FAIL to Reject Ho

∴ 9:3:3:1 model is demonstrated

Chi-Square test calculator

Goodness of fit test, Test of independence, McNemar test

Video Information Chi-squared test for variance Chi-Square Calculator

Test calculation

Right-tailed - for the goodness of fit test, the test of independence / the test for association, or the McNemar test, you can use only the right tail test.
The calculator includes results from the Fisher calculator, binomial test, McNemar Mid-p, simulation.

Test:	Goodness of fit	Name:	Population
Significance level (α):	0.05	m:	0
Effect:	Medium	Effect size (w):	0.3
Continuity correction:	False	Simulation repeats:	100000
Digits:	4		

Enter sample data

Categories	Observed Frequency	*Expected Value
A	111	106.9
B	37	35.6
C	34	35.6
D	8	11.9

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp}$$

$$\chi^2 = \frac{(111-106.9)^2}{106.9} + \frac{(37-35.6)^2}{35.6} + \frac{(34-35.6)^2}{35.6} + \frac{(8-11.9)^2}{11.9} = 1.562$$

DF	3

The test statistic χ^2 equals **1.5624**,

The p-value equals **0.668**,

Since p-value > α, H_0 is accepted.
The statistical model fits the observations

Question 6 : Chi-square Goodness of fit test activity

During a two month period at a particular hospital, 70 babies were born.
The table below shows how many babies were born on each day of the week.

Day	Births (obsv.)	Exp.
Monday	12	10
Tuesday	19	10
Wednesday	8	10
Thursday	11	10
Friday	9	10
Saturday	7	10
Sunday	4	10
	70	

Perform a hypothesis test to determine if the sampled results are consistent with births being uniformly distributed throughout the week. Use a significance level of 5%.

H_0: Births are uniformly distributed / observed counts = Expected counts being uniformly distributed throughout the week. Use a significance level of 5%.

H_0: Births are uniformly distributed / observed counts = Expected counts

H_A: Births are not uniformly distributed / observed counts ≠ Expected counts

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp}$$

$$\chi^2 = \frac{(12-10)^2}{10} + \frac{(19-10)^2}{10} + \frac{(8-10)^2}{10} + \frac{(11-10)^2}{10} + \frac{(9-10)^2}{10} + \frac{(7-10)^2}{10} + \frac{(4-10)^2}{10} = 13.6$$

$df = n - 1 = 7 - 1 = 6$

Looking at chi-square table, P_value is between 5% and 2.5%

The p-value equals **0.03444**,

Since p-value < α, H_0 is rejected.
The statistical model does not fit the observations

∴ Births are not uniformly distributed

Test for Independence

A test of whether the two categorical variables are independent examines the distribution of counts for one group of individuals classified according to both variables in a contingency table.

Question 7 : Chi-square Independence tests activity

In a study with 307 patients to evaluate the effectiveness of the drug Timolol in preventing angina attacks, patients were randomly allocated to receive either Timolol or a placebo daily for 28 weeks. The results are shown below:

	Obsv.1 Timolol	Obsv.2 Placebo	TOTAL	Exp1	Exp2	$\frac{(obsv_1 - Exp_1)^2}{Exp_1}$	$\frac{(obsv_2 - Exp_2)^2}{Exp_2}$
Angina free	44	19	63	32.83	30.17	3.80	4.14
Not angina free	116	128	244	127.17	116.83	0.90	1.07
TOTAL	160	147	307				

Perform the hypothesis test for the Timolol study described above, using α=0.05.

$\boxed{\chi^2 = 9.91}$

H_0: Angina attacks are independent of the drug taken

H_A: Angina attacks are not independent of the drug taken

$df = 1$

$$\textbf{Expected Count} = \frac{\textbf{row total} \times \textbf{column total}}{\textbf{total observations, } n}$$

The chi-square independence test has $df = (R-1)(C-1)$

$= (2-1) \times (2-1) = 1$

Exp_1

Exp_2

$$\frac{63 \times 160}{307} = 32.83 \qquad \frac{63 \times 147}{307} = 30.17$$

$$\frac{244 \times 160}{307} = 127.17 \qquad \frac{244 \times 147}{307} = 116.83$$

χ^2

$$\frac{(44 - 32.83)^2}{32.83} = 3.80$$

$$\frac{(116 - 127.17)^2}{127.17} = 0.98$$

χ^2

$$(19 - 30.17)^2 / 30.17 = 4.14$$

$$(128 - 116.83)^2 / 116.83 = 1.07$$

P-value < 0.05

P-value is Low, Reject Null Hypothesis

Angina attacks are not independent of the drug taken

Test:	Independence (Association)	Name:	Population
Significance level (α):	0.05	m:	0
Effect:	Medium	Effect size (w):	0.3
Continuity correction:	False	Simulation repeats:	100000
Digits:	4		

Var A / Var B	Timolol	Placebo
Angina free	44	19
Not Angina free	116	128

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp}$$

$$\chi^2 = \frac{(44-32.83)^2}{32.83} + \frac{(19-30.17)^2}{30.17} + \frac{(116-127.17)^2}{127.17} + \frac{(128-116.83)^2}{116.83} = 9.978$$

DF	1	(Rows-1)*(Columns-1) = (2-1)*(2-1) = 1

The p-value equals **0.001584**

Since p-value < α, H_0 is rejected.
The statistical model does not fit the observations
A significant association was found between variable A and variable B

Angina Attacks

Drug taken