3

Exponentials and logarithms

Introduction

Two of the most widely applicable functions in mathematics are the exponential function and the logarithmic function. Exponential growth is common in biology, and exponential decay is prevalent in both physics and chemistry. Logarithmic functions occur in chemistry and geology, and are used to solve exponential equations.

This topic covers the index laws, the use of the base *e* for both exponentials and logarithms, the rules governing exponentials and logarithms, the inverse relationship of the two functions, and solutions of both exponential and logarithmic equations. After studying this topic, you should be able to:

- understand and apply the index laws;
- understand and use the base *e*;
- use the rules for exponentials to simplify expressions;
- use the rules for logarithms to simplify expressions;
- solve exponential equations;
- solve logarithmic equations.

3.1 Index laws

1. An **exponential** is formed when a number (the base) is raised to a power (also called the exponent or **index**). For instance, in 4^2, the base is 4 and the index is 2. Similarly, in x^3, the base is x and the index is 3.
2. The rules governing exponentials are called the **index laws**.There are 7 index laws, as follows.

I $a^m \times a^n = a^{m+n}$ **(to multiply, add indices)**

e.g. $4^2 \times 4^3 = 4^5$, and $x^2 \times x^4 = x^6$.

II $\dfrac{a^m}{a^n} = a^{m-n}$ **(to divide, subtract indices)**

e.g. $\dfrac{2^5}{2^2} = 2^3$, and $\dfrac{x^8}{x^3} = x^5$.

III $(ab)^n = a^n b^n$ **(all terms in brackets raised to the power *n*)**

e.g. $(2 \times 5)^2 = 2^2 \times 5^2$, and $(xy)^4 = x^4 y^4$.

IV $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ **(all terms in brackets raised to the power *n*)**

e.g. $\left(\dfrac{3}{5}\right)^2 = \dfrac{3^2}{5^2}$, and $\left(\dfrac{x}{y}\right)^3 = \dfrac{x^3}{y^3}$.

V $(a^m)^n = a^{mn}$ **(to find a power of a power, multiply powers)**

e.g. $(2^3)^4 = 2^{12}$, and $(x^2)^3 = x^6$.

VI $a^0 = 1$ and $a^1 = a$ **(by definition)**

e.g. $x^0 = 1$, $3x^0 = 3 \times 1 = 3$, and $4x^1 = 4x$.

VII $a^{-n} = \dfrac{1}{a^n}$ **(switching a power from top to bottom)**

e.g. $4^{-2} = \dfrac{1}{4^2}$, and $x^{-3} = \dfrac{1}{x^3}$

NOTE: The index laws apply for both $a \neq 0$ and $b \neq 0$.

3. Law VII is used to convert negative powers of a variable to positive powers of that variable. In general, a power in the top line changes sign when switched to the bottom line. Similarly, a power in the bottom line changes sign when switched to the top line. For instance,

   $\dfrac{1}{x^2} = x^{-2}$, and $\dfrac{1}{x^{-2}} = x^2$.

4. When **sums** of variables appear in an expression, law VII cannot be used, e.g. $\dfrac{x^{-1}}{4+x^{-3}} \neq \dfrac{4+x^3}{x^1}$, i.e. the sum $4+x^{-3}$ cannot be switched to the top line. Such expressions can be simplified (and written in terms of non-negative indices) as follows:

   first locate the most negative power of x, say x^{-n};

   then multiply by $\dfrac{x^n}{x^n}$.

   For instance, in $\dfrac{x^{-1}}{4+x^{-3}}$, the most negative power of x is x^{-3}.

   Multiplying by $\dfrac{x^3}{x^3}$ gives $\dfrac{x^{-1}}{4+x^{-3}} = \dfrac{x^3x^{-1}}{x^3(4+x^{-3})} = \dfrac{x^2}{4x^3+x^3x^{-3}}$

   $$= \frac{x^2}{4x^3+x^0} \qquad = \frac{x^2}{4x^3+1}.$$

Examples

1. Simplify, and express all answers with non-negative indices

(i)	$\dfrac{x^4x^3}{x^2x^8}$	(ii)	$\dfrac{5x^3y^{-1}}{x^{-2}y^4}$	(iii)	$\left(\dfrac{3x^{-2}}{y}\right)(2x^3y^{-4})$
(iv)	$\left(\dfrac{x^2}{xy^{-2}}\right)^3$	(v)	$\dfrac{x^{-2}+2x^{-4}}{x-3x^{-3}}$	(vi)	$\dfrac{7x^2}{x^3+4x^{-1}}$.

1. (i) $\dfrac{x^4x^3}{x^2x^8} = \dfrac{x^7}{x^{10}} = x^{7-10} = x^{-3} = \dfrac{1}{x^3}$

(ii) $\dfrac{5x^3y^{-1}}{x^{-2}y^4} = \dfrac{5x^3x^2}{y^1y^4}$ (switching lines for y^{-1} and x^{-2})

$= \dfrac{5x^5}{y^5}$

(iii) $\left(\dfrac{3x^{-2}}{y}\right)(2x^3y^{-4}) = \dfrac{3 \times 2x^{-2}x^3y^{-4}}{y} = \dfrac{6x^1y^{-4}}{y} = \dfrac{6x}{yy^4}$

$= \dfrac{6x}{y^5}$

(iv) $\left(\dfrac{x^2}{xy^{-2}}\right)^3 = \dfrac{x^{2\times 3}}{x^3y^{-2\times 3}} = \dfrac{x^6}{x^3y^{-6}} = \dfrac{x^6y^6}{x^3} = x^{6-3}y^6$

$= x^3y^6$

(v) For $\dfrac{x^{-2}+2x^{-4}}{x-3x^{-3}}$, a sum is involved. The most negative power of x is x^{-4}. Multiplying by $\dfrac{x^4}{x^4}$ gives

$$\frac{x^{-2}+2x^{-4}}{x-3x^{-3}} = \frac{x^4(x^{-2}+2x^{-4})}{x^4(x-3x^{-3})} = \frac{x^4x^{-2}+2x^4x^{-4}}{x^4x+-3x^4x^{-3}}$$

$$= \frac{x^2+2x^0}{x^5-3x^1} \qquad = \frac{x^2+2}{x^5-3x}$$

(vi) For $\dfrac{7x^2}{x^3+4x^{-1}}$, a sum is involved. The most negative power of x is x^{-1}. Multiplying by $\dfrac{x^1}{x^1}$, i.e. $\dfrac{x}{x}$ gives

$$\frac{7x^2}{x^3+4x^{-1}} = \frac{x \times 7x^2}{x(x^3+4x^{-1})} = \frac{7x^3}{xx^3+4xx^{-1}} = \frac{7x^3}{x^4+4x^0}$$

$$= \frac{7x^3}{x^4+4}.$$

Problems

1. Simplify, and express all answers with non-negative indices

(i) $\dfrac{x^6x^3}{x^2x^9}$ (ii) $\dfrac{6x^{-4}y}{x^{-2}y^{-3}}$ (iii) $\left(\dfrac{x^{-2}}{4y^2}\right)(2x^5y^{-2})$

(iv) $\left(\dfrac{3x^{-1}}{xy^{-3}}\right)^2$ (v) $\dfrac{x^2+4x^{-3}}{x^3-3}$ (vi) $\dfrac{7x^{-2}}{5x^{-1}+x^{-3}}$.

Answers

1. (i) $\dfrac{1}{x^2}$ (ii) $\dfrac{6y^4}{x^2}$ (iii) $\dfrac{x^3}{2y^4}$

(iv) $\dfrac{9y^6}{x^4}$ (v) $\dfrac{x^5+4}{x^6-3x^3}$ (vi) $\dfrac{7x}{5x^2+1}$.

3.2 Fractional indices

1. Fractional indices are equivalent to n$^{\text{th}}$ roots:

   e.g. $a^{1/2} = \sqrt{a}$ (square root of a);

   $a^{1/3} = \sqrt[3]{a}$ (cube root of a);

   $a^{1/4} = \sqrt[4]{a}$ (fourth root of a), etc.

   In general $a^{\frac{1}{n}} = \sqrt[n]{a}$ (n$^{\text{th}}$ root of a).

   Two very useful consequences of this are:

   $\sqrt{x} = x^{1/2}$ and $\dfrac{1}{\sqrt{x}} = \dfrac{1}{x^{1/2}} = x^{-1/2}$.

2. The 7 index laws apply to fractional indices, and can be used to simplify expressions containing square roots.

   To achieve such simplifications the following rules are required.

   For both $a \geq 0$ and $b \geq 0$,

   $\sqrt{a^2} = a$ (as $\sqrt{a^2} = (a^2)^{1/2} = a^{2\times 1/2} = a^1 = a$)

   $\sqrt{ab} = \sqrt{a}\sqrt{b}$ (as $\sqrt{ab} = (ab)^{1/2} = a^{1/2}b^{1/2} = \sqrt{a}\sqrt{b}$)

   $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$ (as $\sqrt{\dfrac{a}{b}} = \left(\dfrac{a}{b}\right)^{1/2} = \dfrac{a^{1/2}}{b^{1/2}} = \dfrac{\sqrt{a}}{\sqrt{b}}$) (here $b > 0$).

Examples

1. Simplify, and express all answers with non-negative indices

(i) $\dfrac{x^{4/3}x}{x^{-2/3}}$ (ii) $\dfrac{(x^{2/3})^{3/4}}{x^{-5/2}}$.

2. Simplify

(i) $(5+2\sqrt{6})(5-2\sqrt{6})$ (ii) $\sqrt{x}(3\sqrt{x}-\sqrt{4x})$.

1. (i) $\dfrac{x^{4/3}x}{x^{-2/3}} = x^{2/3}x^{4/3}x$ (switching $x^{-2/3}$)

$= x^{6/3}x \qquad = x^2x = x^3$

(ii) $\dfrac{(x^{2/3})^{3/4}}{x^{-5/2}} = \dfrac{x^{2/3\times3/4}}{x^{-5/2}} = \dfrac{x^{1/2}}{x^{-5/2}}$ (as $\dfrac{2}{3}\times\dfrac{3}{4} = \dfrac{2}{4} = \dfrac{1}{2}$)

$= x^{1/2}x^{5/2}$ (switching $x^{-5/2}$)

$= x^3$ (as $\dfrac{1}{2}+\dfrac{5}{2} = \dfrac{6}{2} = 3$).

2. (i) $(5+2\sqrt{6})(5-2\sqrt{6}) = 5(5-2\sqrt{6})+2\sqrt{6}(5-2\sqrt{6})$

$= 25-10\sqrt{6}+10\sqrt{6}-4\sqrt{6}\sqrt{6}$

$= 25-4\times6 \qquad = 25-24 = 1$

(ii) $\sqrt{x}(3\sqrt{x}-\sqrt{4x}) = 3\sqrt{x}\sqrt{x}-\sqrt{x}\sqrt{4x}$

$= 3x-2\sqrt{x}\sqrt{x}$ (as $\sqrt{4x} = \sqrt{4}\sqrt{x} = 2\sqrt{x}$)

$= 3x-2x = x$.

Problems

1. Simplify, and express all answers with non-negative indices

(i) $\dfrac{x^{7/4}x^{-1}}{x^{-1/4}}$ (ii) $\dfrac{(x^{5/3})^{3/4}}{x^{-3/4}}$.

2. Simplify

(i) $(8+5\sqrt{11})(8-5\sqrt{11})$ (ii) $3\sqrt{x}(6\sqrt{x}-\sqrt{9x})$.

Answers

1. (i) $\dfrac{x^{7/4}x^{-1}}{x^{-1/4}} = x^{7/4}x^{1/4}x^{-1}$ (switching $x^{-1/4}$)

$$= x^2 x^{-1} \qquad (\text{as } \frac{7}{4} + \frac{1}{4} = \frac{8}{4} = 2)$$

$$= x^1 = x$$

(ii) $$\frac{(x^{5/3})^{3/4}}{x^{-3/4}} = \frac{x^{5/3 \times 3/4}}{x^{-3/4}} = \frac{x^{5/4}}{x^{-3/4}} \qquad (\text{as } \frac{5}{3} \times \frac{3}{4} = \frac{5}{4})$$

$$= x^{5/4} x^{3/4} \qquad (\text{switching } x^{-3/4})$$

$$= x^2 \qquad (\text{as } \frac{5}{4} + \frac{3}{4} = \frac{8}{4} = 2).$$

2. (i) $$(8 + 5\sqrt{11})(8 - 5\sqrt{11}) = 8(8 - 5\sqrt{11}) + 5\sqrt{11}(8 - 5\sqrt{11})$$

$$= 64 - 40\sqrt{11} + 40\sqrt{11} - 25\sqrt{11}\sqrt{11}$$

$$= 64 - 25 \times 11 \qquad = 64 - 275 = -211$$

(ii) $$3\sqrt{x}(6\sqrt{x} - \sqrt{9x}) = 18\sqrt{x}\sqrt{x} - 3\sqrt{x}\sqrt{9x}$$

$$= 18x - (3\sqrt{x})(3\sqrt{x}) \qquad (\text{as } \sqrt{9x} = \sqrt{9}\sqrt{x} = 3\sqrt{x})$$

$$= 18x - 9x = 9x.$$

3.3 Exponential functions

1. A simple exponential function has the form $y = a^x$, where the base a is greater than zero, and the variable x is the power or exponent. Common bases are $a = 2$ (in computing) and $a = 10$ (in the physical sciences).
2. The number $\boldsymbol{e}$ is used as the base for exponentials in almost all mathematics. To 3 decimal places, $e \approx 2.718$. This choice of base is important in the calculus of exponentials.
3. The standard exponential function can be written in one of the two forms:

   $y = ke^x$ (exponential growth); or

   $y = ke^{-x}$ (exponential decay), where k is a constant.

   The fundamentals of exponential growth and decay can be observed when the graphs of the functions $y = e^x$ and $y = e^{-x}$ are sketched on the same set of axes.

   Both graphs have:

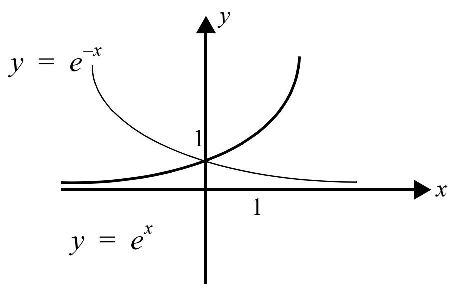
   I **no x-intercept;**

   II **a y-intercept of 1.**

For $y = e^x$, y increases as x increases (growth),

whereas for $y = e^{-x}$, y decreases as x increases (decay).

More generally, the same properties are observed for the graphs of both $y = ke^x$ and $y = ke^{-x}$, except that the y-intercept is then k. This follows from the fact that, when $x = 0$, $y = ke^x = ke^0 = k \times 1 = k$. Similarly, when $x = 0$, $y = ke^{-x} = ke^0 = k \times 1 = k$.



4. Expressions involving exponentials can be simplified using the following three rules, all of which follow from the index laws:

$e^a \times e^b = e^{a+b}$ (Product of powers)

$\dfrac{e^a}{e^b} = e^{a-b}$ (Quotient of powers)

$(e^a)^b = e^{ab}$ (Power of a power)

$\dfrac{1}{e^a} = e^{-a}$ (Negative powers)

Examples

1. Simplify (by writing as a single exponential)

   (i) $\dfrac{e^x}{e^{x-2}}$ (ii) $\dfrac{(e^{x+3})^4}{e^{x+2}}$ (iii) $e^{2x-5}(e^{3-x})^2$.

2. Expand and simplify

   (i) $(e^x + e^{-x})^2$ (ii) $(e^{2x} + e^{-2x})(e^{2x} - e^{-2x})$.

1. (i) $\dfrac{e^x}{e^{x-2}} = e^{x-(x-2)} = e^{x-x+2} = e^2$

(ii) $\dfrac{(e^{x+3})^4}{e^{x+2}} = \dfrac{e^{4x+12}}{e^{x+2}} = e^{4x+12-x-2} = e^{3x+10}$

(iii) $e^{2x-5}(e^{3-x})^2 = e^{2x-5} \times e^{6-2x}$

$= e^{2x-5+6-2x} = e^{-5+6} = e\,.$

2. (i) $(e^x + e^{-x})^2 = (e^x)^2 + 2e^x e^{-x} + (e^{-x})^2$

$= e^{2x} + 2e^0 + e^{-2x} = e^{2x} + 2 + e^{-2x}$

(ii) $(e^{2x} + e^{-2x})(e^{2x} - e^{-2x}) = (e^{2x})^2 - (e^{-2x})^2$

$= e^{4x} - e^{-4x}\,.$

Problems

1. Simplify (by writing as a single exponential)

(i) $\dfrac{e^{3x}}{e^{2x-3}}$ (ii) $\dfrac{(e^{x+1})^3}{e^{5-3x}}$ (iii) $e^{6x-5}(e^{3-2x})^3\,.$

2. Expand and simplify

(i) $(e^x - e^{-x})^2$ (ii) $e^{3x}(e^{2x} + e^{-2x})\,.$

Answers

1. (i) e^{x+3} (ii) e^{6x-2} (iii) $e^4\,.$

2. (i) $e^{2x} - 2 + e^{-2x}$ (ii) $e^{5x} + e^x\,.$

3.4 Logarithmic functions

1. If $y = a^x$, then ***x* is called the logarithm of *y* to base *a***, and denoted

$x = \log_a y\,.$

The expression $\log_a y$ is thus the power of *a* needed to obtain *y*.

For instance, $\log_2 8$ is equal to 3, because $2^3 = 8\,.$

Similarly, $\log_2\left(\dfrac{1}{4}\right) = -2$, because $2^{-2} = \dfrac{1}{4}\,.$

2. As is the case with exponentials, the base *e* is the usual base for logarithms in mathematics.Using base *e*, the natural logarithm of *x* is always written as $\ln x$.

3. If $y = \ln x$, then *y* is the power to which *e* must be raised to give *x*.

So, $e^y = x$ and $y = \ln x$ mean the same thing.

More importantly, $e^{\ln x} = x$, and $\ln(e^x) = x$.

Thus, the operations e and ln are inverse operations, in the same way as multiplying and dividing by 3 are inverse operations.

4. The graph of the function $y = \ln x$ can be sketched by analogy with the exponential function. Since $e^y = x$ and $y = \ln x$ mean the same thing, the required sketch is the sketch of the equation $e^y = x$, i.e. $x = e^y$.

   Now, when $x = 0$, $e^y = 0$, which has no solution.

   So, the graph has no y-intercept.

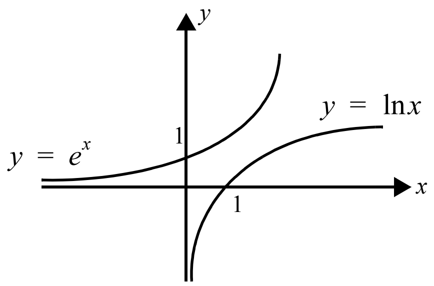
   When $y = 0$, $x = e^0 = 1$.

   So, the graph has an x-intercept at $x = 1$.

   Also, as x increases, y increases. This can be checked by plotting selected points on the curve $y = \ln x$,

   e.g. when $x = -1$, $y = e^{-1} \approx 0.3$, and when $x = 1$, $y = e^1 \approx 2.718$.

   The graphs of both $y = e^x$ and $y = \ln x$ are sketched on the same set of axes in the diagram below.



5. The properties of logarithms are very important. Stated in base e they are as follows;

   Law 1: $\ln x + \ln y = \ln(xy)$ (Sum of logs)

   Law 2: $\ln x - \ln y = \ln\left(\frac{x}{y}\right)$ (Difference of logs)

   Law 3: $r\ln x = \ln(x^r)$ (Multiple of a log)

Examples

1 Simplify (by expressing as the natural logarithm of a single number)

(i) $4\ln 2 + 2\ln 5 - \ln 10$ (ii) $\ln 12 - 2\ln 3 + \ln 6$.

(i) $4\ln 2 + 2\ln 5 - \ln 10 = \ln(2^4) + \ln(5^2) - \ln 10$ (Law 3)

$= \ln 16 + \ln 25 - \ln 10 = \ln(16 \times 25) - \ln 10$ (Law 1)

$= \ln\left[\frac{16 \times 25}{10}\right]$ (Law 2)

$= \ln\left[\frac{16 \times 5}{2}\right] \quad = \ln 40$

ii) $\ln 12 - 2\ln 3 + \ln 6 \quad = \ln 12 - \ln(3^2) + \ln 6$ (Law 3)

$= \ln 12 - \ln 9 + \ln 6 \quad = \ln\left[\frac{12}{9}\right] + \ln 6$ (Law 2)

$= \ln\left[\frac{12 \times 6}{9}\right]$ (Law 1)

$= \ln\left[\frac{72}{9}\right] \quad = \ln 8\,.$

2. Simplify (by expressing as a single natural logarithm)

(i) $\ln(5x) - 4\ln(2x) + 2\ln(4x^2)$

(ii) $2\ln(x^2y) + 3\ln x - \ln(x^3y)$.

(i) $\ln(5x) - 4\ln(2x) + 2\ln(4x^2)$

$= \ln(5x) - \ln[(2x)^4] + \ln[(4x^2)^2]$ (Law 3)

$= \ln(5x) - \ln[16x^4] + \ln[16x^4]$

$= \ln(5x)$

(ii) $2\ln(x^2y) + 3\ln x - \ln(x^3y)$

$= \ln[(x^2y)^2] + \ln(x^3) - \ln(x^3y)$ (Law 3)

$= \ln[x^4y^2] + \ln(x^3) - \ln(x^3y)$

$= \ln(x^4y^2x^3) - \ln(x^3y)$ (Law 1)

$= \ln(x^7y^2) - \ln(x^3y)$

$= \ln\left[\frac{x^7y^2}{x^3y}\right]$ (Law 2)

$= \ln(x^4y)\,.$

Problems

1. Simplify (by expressing as the natural logarithm of a single number)

(i) $2\ln 3 + 2\ln 5 - \ln 15$ (ii) $5\ln 2 - 2\ln 4 + \ln 7$.

2. Simplify (by expressing as a single natural logarithm)

   (i) $\ln(8x) - 2\ln(2x) + 3\ln(2x^2)$

   (ii) $3\ln(xy) + 2\ln x - \ln(x^3y^2)$.

Answers

1. (i) $\ln 15$ (ii) $\ln 14$.

2. (i) $\ln(16x^5)$ (ii) $\ln(x^2y)$.

3.5 Logarithmic and exponential equations

1. When solving any equation for x, the operations performed on x and the order of those operations should be noted. These operations should then be 'undone' in reverse order to find x (see Chapter 1 of this Study Guide).'Undoing' an operation means performing the corresponding inverse operation (on both sides of the equation). Example 1.(i) below gives an elementary illustration of this, and indicates that the techniques of solving equations covered in Chapter 1 still apply.

2. As exponentials and logarithms are inverse operations, an exponential can be 'undone' by taking logarithms of both sides. Similarly, a logarithm can be 'undone' by equating exponentials of both sides (see Examples below).

Examples

1. Solve the following equations for x:

   (i) $\dfrac{3x-2}{5} + 3 = -1$ (ii) $3 + 2\ln(x-5) = 7$

   (iii) $\dfrac{e^{3x-4}+2}{6} = 1$.

   (i) $\dfrac{3x-2}{5} + 3 = -1$

   [Note that the order of operations performed on x to form the left side of the equation is as follows: multiply by 3; subtract 2; divide by 5; add 3. So, solving for x requires the following operations to be performed to both sides of the equation **in this order**: subtract 3; multiply by 5; add 2; divide by 3]. So,

$$\frac{3x-2}{5} = -4 \qquad \therefore 3x - 2 = -20$$

$$\therefore 3x = -18 \qquad \therefore x = \frac{-18}{3}$$

Hence, $x = -6$.

(ii) $3 + 2\ln(x-5) = 7$

(subtract 3) $\therefore 2\ln(x-5) = 4$

(divide by 2) $\therefore \ln(x-5) = 2$

(exponentials of both sides) $\therefore e^{\ln(x-5)} = e^2$

So, $x - 5 = e^2$

(add 5) Hence, $x = e^2 + 5$.

(iii) $\dfrac{e^{3x-4}+2}{6} = 1$

(multiply by 6) $\therefore e^{3x-4} + 2 = 6$

(subtract 2) $\therefore e^{3x-4} = 4$

(ln of both sides) $\therefore \ln(e^{3x-4}) = \ln 4$.

So, $3x - 4 = \ln 4$

(add 4) $\therefore 3x = 4 + \ln 4$

(divide by 3) Hence, $x = \dfrac{4+\ln 4}{3}$.

2. Solve the following equations for x

(i) $7 + \frac{1}{2}\ln(3x-2) = y$ (ii) $y\ln(5\sqrt{x}) = \dfrac{y+3}{2}$

(iii) $\dfrac{3e^{5x}+2}{6} = y - 1$.

(i) $7 + \frac{1}{2}\ln(3x-2) = y$

$$\therefore \frac{1}{2}\ln(3x-2) = y - 7$$

$$\therefore \ln(3x-2) = 2(y-7)$$

$$\therefore 3x - 2 = e^{2(y-7)}$$

$$\therefore 3x = e^{2(y-7)} + 2$$

$$x = \frac{e^{2(y-7)}+2}{3}$$

(ii) $y\ln(5\sqrt{x}) = \dfrac{y+3}{2}$

$$\therefore \ln(5\sqrt{x}) = \frac{y+3}{2y}$$

$$\therefore 5\sqrt{x} = e^{(y+3)/2y}$$

$$\therefore \sqrt{x} = \frac{e^{(y+3)/2y}}{5}$$

$$\therefore x = \left[\frac{e^{(y+3)/2y}}{5}\right]^2$$

$$\therefore x = \frac{e^{(y+3)/y}}{25}$$

(iii) $$\frac{3e^{5x}+2}{6} = y-1$$

$$\therefore 3e^{5x}+2 = 6(y-1)$$

$$\therefore 3e^{5x}+2 = 6y-6$$

$$\therefore 3e^{5x} = 6y-8$$

$$\therefore e^{5x} = \frac{6y-8}{3}$$

$$\therefore 5x = \ln\left[\frac{6y-8}{3}\right]$$

$$\therefore x = \frac{1}{5}\ln\left[\frac{6y-8}{3}\right].$$

Problems

1. Solve the following equations for x

   (i) $4 - 2\ln(2x-1) = 0$ (ii) $3 + 6\ln(x-1) = 0$

   (iii) $\dfrac{e^{3x-2}-7}{4} = -1$.

2. Solve the following equations for x

   (i) $7 - 3\ln(9x-1) = y$ (ii) $\dfrac{e^{3x-1}-11}{2} = 3y-7$

   (iii) $5y\ln(1+\sqrt{x}) = y-4$.

Answers

1. (i) $x = \dfrac{1+e^2}{2}$ (ii) $x = 1+e^{-1/2}$ (iii) $x = \dfrac{2+\ln 3}{3}$.

2. (i) $x = \dfrac{1+e^{(7-y)/3}}{9}$ (ii) $x = \dfrac{1+\ln(6y-3)}{3}$.

   (iii) $x = [e^{(y-4)/5y} - 1]^2$