I apologise for the somewhat inconsistent formatting, I'm trying out some new things to see what I like best. The blocks with the blue bar are my favourite so far but I do like my $\rightarrow$ arrows.

(1) Find the stationary points of $y = (x^4 - 2x^3)e^{2x}$. Note: you must give exact values - do not approximate the stationary point values

Final answers:

	1	2	3
x	0	$+\sqrt{3}$	$-\sqrt{3}$
y	0	$9e^{2\sqrt{3}} - 6\sqrt{3}e^{2\sqrt{3}}$	$\frac{9+6\sqrt{3}}{e^{2\sqrt{3}}}$
$\approx (x, y)$*	$(0, 0)$	$(1.732, -44.481)$	$(-1.732)(0.607)$

*approximates for curiosity's sake

Working out for (1)

Finding y'

$y = (x^4 - 2x^3)e^{2x}$
$u = x^4 - 2x^3$
$u' = 4x^3 - 6x^2$

$v = e^{2x}$
$v' = 2e^{2x}$

$u \times v' + v \times u'$
-> $(x^4 - 2x^3) \times 2e^{2x} + e^{2x}(4x^3 - 6x^2)$
-> $e^{2x}(2x^4 - 4x^3) + e^{2x}(4x^3 - 6x^2)$
-> $2e^{2x}x^4 - 4e^{2x}x^3 + 4e^{2x}x^3 - 6e^{2x}x^2$
-> $2e^{2x}x^4 \cancel{- 4e^{2x}x^3 + 4e^{2x}x^3} - 6e^{2x}x^2$
-> $2e^{2x}x^4 - 6e^{2x}x^2$
$y' = 2e^{2x}x^4 - 6e^{2x}x^2$

Stationary points

$x = 0, \; x = \sqrt[3]{3}$

$y' = 2e^{2x}x^4 - 6e^{2x}x^2$
-> $0 = 2e^{2x}x^4 - 6e^{2x}x^2$
-> $2e^{2x}x(x^3 - 3x)$
-> $2e^{2x}x^2(x^2 - 3)$

Setup a difference of two squares formula, where $a^2 - b^2 = (a+b)(a-b)$
-> $-3 = (\sqrt{3})^2$
-> $(x^2 - (\sqrt{3})^2)$
-> $x^2 - (\sqrt{3})^2$
-> $(x+\sqrt{3})(a-\sqrt{3})$

Apply the zero-product principle
-> $0 = 2e^{2x}x^2(x^2-3)(x+\sqrt{3})(x-\sqrt{3})$
$e^{2x} = 0$ ($2 \neq 0$ so it's unneeded) or
$x = 0$ (for the x^2 term) or
$x - \sqrt{3} = 0$ or
$x + \sqrt{3} = 0$

Sifting for valid values
$e^{2x} = 0$
-> $e = \sqrt[2x]{0}$
-> $\sqrt[2x]{0} = 0$
$e \neq 0$, so it is not a stationary point

$x_1 = 0$
-> Can be true, so it's valid

$x - \sqrt{3} = 0$
-> $x_2 = +\sqrt{3}$

$x + \sqrt{3} = 0$
-> $x_3 = -\sqrt{3}$

So, the points on the x-axis are 0, $\sqrt{3}$, and $-\sqrt{3}$

Finding the stationary point y-values

From the x-values $0, -\sqrt{3}$, and $\sqrt{3}$

Finding y_1 from $x_1 = 0$
$y_1 = ((0)^4 - 2(0)^3)e^{2(0)}$
-> $(0-0) \times 1$
$y_1 = 0$

Finding y_1 from $x_2 = \sqrt{3}$
$y_2 = ((\sqrt{3})^4 - 2(\sqrt{3})^3)e^{2(\sqrt{3})}$

> $(\sqrt{3})^4 = 9$

Applying the root rule $(\sqrt[n]{a})^n = a$
-> $-2(\sqrt{3})^3$

-> $-2(\sqrt[2]{3})^2\sqrt{3}$
-> $-2(3)\sqrt{3}$
$= -6\sqrt{3}$

-> $e^{2\sqrt{3}}(9-6\sqrt{3})$
$y_2 = 9e^{2\sqrt{3}} - 6\sqrt{3}e^{2\sqrt{3}}$

Finding y_1 from $x_3 = -\sqrt{3}$
$y_2 = ((-\sqrt{3})^4 - 2(-\sqrt{3})^3)e^{2(-\sqrt{3})}$

> $(-\sqrt{3})^4 = 9$
> $-2(-\sqrt{3})^3 = -2(-\sqrt{3})^2 \times -\sqrt{3}$
> $-\sqrt{3}^2 = -3$
> $-2 \times -3 = 6$

$(9 - 6 \times -\sqrt{3})e^{2(-\sqrt{3})}$
-> $(9+6\sqrt{3})e^{2(-\sqrt{3})}$
-> $9e^{2-\sqrt{3}} + 6\sqrt{3}e^{2(-\sqrt{3})}$

Apply the reciprocal index law $x^{-y} = \frac{1}{x^y}$
$9e^{2(-\sqrt{3})} + 6\sqrt{3}e^{2(-\sqrt{3})}$
-> $9e^{-2\sqrt{3}} + 6\sqrt{3}e^{-2\sqrt{3}}$
-> $\frac{9}{e^{2\sqrt{3}}} + \frac{6\sqrt{3}}{e^{2\sqrt{3}}}$
$y_3 = \frac{9+6\sqrt{3}}{e^{2\sqrt{3}}}$

(2) Draw a single sign table and an accompanying sign diagram.

For each interval in the sign table, select a value of x and show all working to obtain the sign of the gradient in that interval.
Note: there should be no approximations in working to obtain gradient for the selected values of x. Please leave the expressions with square roots and powers of e.

Finding y''

$y' = 2e^{2x}x^4 - 6e^{2x}x^2$

Left side
$u_{LL} = 2e^{2x}$ and $u'_{LL} = 4e^{2x}$
$v_{LR} = x^4$ and $v'_{LR} = 4x^3$

$uv' + vu' \rightarrow u_{LL}v'_{LR} + v_{LR}u'_{LL}$
$2e^{2x} \times 4x^3 + x^4 \times 4e^{2x}$
-> $u'_L = 8e^{2x}x^3 + 4e^{2x}x^4$

Right side
$u_{RL} = 6e^{2x}$ and $u'_{RL} = 12e^{2x}$
$v_{RR} = x^2$ and $v'_{RR} = 2x$

$6e^{2x} \times 2x + x^2 \times 12e^{2x}$
-> $v'_R = 12e^{2x}x + 12e^{2x}x^2$

Combining left and right
$y'' = (u'_L) - (u'_R)$
-> $(8e^{2x}x^3 + 4e^{2x}x^4) - (12e^{2x}x + 12e^{2x}x^2)$
-> $8e^{2x}x^3 + 4e^{2x}x^4 - 12e^{2x}x - 12e^{2x}x^2$
-> $y'' = 4e^{2x}x^4 + 8e^{2x}x^3 - 12e^{2x}x^2 - 12e^{2x}x$

Filled out sign table

Working out is below

	1	**2**	**3**
x	0	$+\sqrt{3}$	$-\sqrt{3}$
y	0	$9e^{2\sqrt{3}} - 6\sqrt{3}e^{2\sqrt{3}}$	$\frac{9+6\sqrt{3}}{e^{2\sqrt{3}}}$
$\approx (x, y)$*	$(0, 0)$	$(1.732, -44.481)$	$(-1.732)(0.607)$
y''	0	$12\sqrt{3}e^{2\sqrt{3}}$	$-\frac{12\sqrt{3}}{e^{2\sqrt{3}}}$
$\approx y''$	0, need sign test	644.02	-0.65
	$+, 0, -$, local max	> 0, local min	< 0, local max

Calculating 0

$y'' = 4e^{2x}x^4 + 8e^{2x}x^3 - 12e^{2x}x^2 - 12e^{2x}x$
-> $4e^{2(0)}(0)^4 + 8e^{2(0)}(0)^3 - 12e^{2(0)}(0)^2 - 12e^{2(0)}(0)$
-> $4 \times 0 + 8 \times 0 - 12 \times 0 - 12 \times 0$
$= 0$
As $0 = 0$, the sign test has to be used.

Sign test calculations

	$x < 0$ **at** $x = -0.1$	$x = 0$	$x > 0$ **at** $x = 0.1$
Exact $(=)$	$-4e^{-2} = -\frac{4}{e^2}$	0	$-12e^2$
Approx $(\approx)$	0.8780	0	-1.6020
Results	$+$	0	$-$

The results are $+, 0, -$, that means the point must be a local maximum.

Calculating $+0.1$
$y'' = 4e^{2x}x^4 + 8e^{2x}x^3 - 12e^{2x}x^2 - 12e^{2x}x$
-> $4e^{2(0.1)}(0.1)^4 + 8e^{2(0.1)}(0.1)^3 - 12e^{2(0.1)}(0.1)^2 - 12e^{2(0.1)}(0.1)$
-> $4e^{0.2}(0.0001) + 8e^{0.2}(0.001) - 12e^{0.2}(0.01) - 12e^{0.2}(0.1)$

-> $\frac{4}{10000}e^{\frac{1}{5}} + \frac{8}{1000}e^{\frac{1}{5}} - \frac{12}{100}e^{\frac{1}{5}} - \frac{12}{10}e^{\frac{1}{5}}$
-> $\frac{4}{10000}e^{\frac{1}{5}} + \frac{80}{10000}e^{\frac{1}{5}} - \frac{1200}{10000}e^{\frac{1}{5}} - \frac{12000}{10000}e^{\frac{1}{5}}$
$4 + 80 - 1200 - 12,000 = -13,116$
-> $-\frac{13116}{10000}e^{\frac{1}{5}}$
$= \frac{-3279e^{\frac{1}{5}}}{2500}$
≈ -1.6020

Calculating -0.1
$y'' = 4e^{2x}x^4 + 8e^{2x}x^3 - 12e^{2x}x^2 - 12e^{2x}x$
-> $4e^{2(-0.1)}(-0.1)^4 + 8e^{2(-0.1)}(-0.1)^3 - 12e^{2(-0.1)}(-0.1)^2 - 12e^{2(-0.1)}(-0.1)$
-> $4e^{-0.2}(0.0001) + 8e^{-0.2}(-0.001) - 12e^{-0.2}(0.01) - 12e^{-0.2}(-0.1)$
-> $\frac{4}{10000}e^{-\frac{1}{5}} - \frac{8}{1000}e^{-\frac{1}{5}} - \frac{12}{100}e^{-\frac{1}{5}} + \frac{12}{10}e^{-\frac{1}{5}}$
-> $\frac{4}{10000}e^{-\frac{1}{5}} - \frac{80}{1000}e^{-\frac{1}{5}} - \frac{1200}{100}e^{-\frac{1}{5}} + \frac{12000}{10}e^{-\frac{1}{5}}$
$4 - 80 - 1200 + 12000 = 10724$
-> $\frac{10724}{10000}e^{-\frac{1}{5}}$
-> $\frac{2681}{2500}e^{-\frac{1}{5}}$
≈ 0.8780

Calculating $\sqrt{3}$

$y'' = 4e^{2x}x^4 + 8e^{2x}x^3 - 12e^{2x}x^2 - 12e^{2x}x$
-> $4e^{2\sqrt{3}}(\sqrt{3})^4 + 8e^{2(\sqrt{3})}(\sqrt{3})^3 - 12e^{2\sqrt{3}}\sqrt{3}^2 - 12e^{2\sqrt{3}}\sqrt{3}$
-> $(9 \times 4e^{2\sqrt{3}}) + (3 \times 8e^{2(\sqrt{3})} \times \sqrt{3}) - (3 \times 12e^{2\sqrt{3}}) - (12e^{2\sqrt{3}}\sqrt{3})$
-> $(36e^{2\sqrt{3}}) + (24\sqrt{3}e^{2(\sqrt{3})}) - (36e^{2\sqrt{3}}) - (12\sqrt{3}e^{2\sqrt{3}})$
-> $\cancel{(36e^{2\sqrt{3}})} + (24\sqrt{3}e^{2(\sqrt{3})}) - \cancel{(36e^{2\sqrt{3}})} - (12\sqrt{3}e^{2\sqrt{3}})$
-> $24\sqrt{3}e^{2\sqrt{3}} - 12\sqrt{3}e^{2\sqrt{3}}$
$= 12\sqrt{3}e^{2\sqrt{3}}$
≈ 644.02
$644.02 > 0$ so it is a local minimum

Calculating $-\sqrt{3}$

$y'' = 4e^{2x}x^4 + 8e^{2x}x^3 - 12e^{2x}x^2 - 12e^{2x}x$
-> $(4e^{2(-\sqrt{3})}(-\sqrt{3})^4) + (8e^{2(-\sqrt{3})}(-\sqrt{3})^3) - (12e^{2(-\sqrt{3})}(-\sqrt{3})^2) - (12e^{2(-\sqrt{3})}(-\sqrt{3}))$
$(4e^{2(-\sqrt{3})}(-\sqrt{3})^4)$
-> $4e^{2(-\sqrt{3})} \times 9$
-> $36e^{2(-\sqrt{3})}$

$(8e^{2(-\sqrt{3})}(-\sqrt{3})^3)$
-> $(8e^{2(-\sqrt{3})}(-\sqrt{3})^2 \times (-\sqrt{3}))$
-> $8 \times 3 = 24$
-> $-24\sqrt{3}e^{2(-\sqrt{3})}$

$(12e^{2(-\sqrt{3})}(-\sqrt{3})^2)$
-> $(12e^{2(-\sqrt{3})} \times 3)$
-> $(36e^{2(-\sqrt{3})})$

-> $\times -1$
-> $-36e^{2(-\sqrt{3})}$

$(12e^{2(-\sqrt{3})}(-\sqrt{3}))$
-> $-12\sqrt{3}e^{2(-\sqrt{3})}$
-> $\times -1$
-> $12\sqrt{3}e^{2(-\sqrt{3})}$

$36e^{2(-\sqrt{3})} - 36e^{2(-\sqrt{3})} - 24\sqrt{3}e^{2(-\sqrt{3})} + 12\sqrt{3}e^{2(-\sqrt{3})}$
-> $\cancel{36e^{2(-\sqrt{3})} - 36e^{2(-\sqrt{3})}} - 24\sqrt{3}e^{2(-\sqrt{3})} + 12\sqrt{3}e^{2(-\sqrt{3})}$
-> $-24\sqrt{3}e^{2(-\sqrt{3})} + 12\sqrt{3}e^{2(-\sqrt{3})}$
-> $-12\sqrt{3}e^{2(-\sqrt{3})}$
-> $-12\sqrt{3}e^{-2\sqrt{3}}$
$= -\frac{12\sqrt{3}}{e^{2\sqrt{3}}}$
≈ -0.65

As $-0.65 < 0$, it is a local maximum