Week 5

Monday, 20 November 2023 9:15 AM

Problem solving task 2 ⟶ Due 15th December 2023

2.1 People have one of four different blood types.

Blood types	Percentage (%)
O	30 %
A	35 %
B	15 %
AB	Rest of them 20 %

a) What is the probability that a person involved in an accident
i. does not have type A blood?
ii. has type O or type A blood? $P(O) + P(A)$
iii. is neither type AB or type B? $1 - [P(AB) + P(B)]$

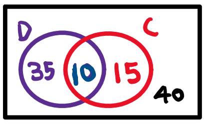
b) Among three accident victims, what is the probability that
i. all have type A? $P(A) \times P(A) \times P(A)$
ii. none of them are type O? $P(\text{not } O) \times P(\text{not } O) \times P(\text{not } O)$
iii. at least one person is type B?
iv. the first accident victim only is type A?

[(1+1+1)+(1+1+1+2) = 8 marks]

2.2 Assume that 45% of households have at least one dog, 25% of households have at least one cat and that 10% of households have at least one of each animal.

$P(D) = 0.45$
$P(C) = 0.25$
$P(D \text{ and } C) = 0.10$

a) What is the probability that a randomly selected household has a dog but not a cat?
b) What is the probability that a randomly selected household does not have either animal?
→ c) What is the probability that a randomly selected household has a cat or a dog?
d) If a household has a dog, what is the probability that they also have a cat? $P(C/D) = \frac{P(D \text{ and } C)}{P(C)}$
e) Are owning dogs and cats mutually exclusive? Explain.
f) Are owning dogs and cats independent events? Explain.



$P(D \text{ and } C) = P(D) \times P(C)$

[1+1+1+2+2+2 = 9 marks]

$P(\text{Cat or Dog}) = P(D) + P(C) - P(D \text{ and } C)$

The Standard Deviation as a Ruler

- Suppose the average male height for a particular age is 1.78m with standard deviation 8cm and for females, the average is 1.62m with standard deviation 5cm. Which height is more extreme? A male who is 1.91m or a 1.73m female?

MALES

Average $= 1.78$ m

S.D $= 8$cm $= \frac{8}{100} = 0.08$ m

FEMALES

Average $= 1.62$m

S.D $= 5$cm $= 0.05$ m

Find Z-Score [standardising the scores]

$$\boxed{Z = \frac{X - \mu}{\sigma}}$$

μ → Average

σ → S.D

$Z_{MALE} = \frac{1.91 - 1.78}{0.08}$

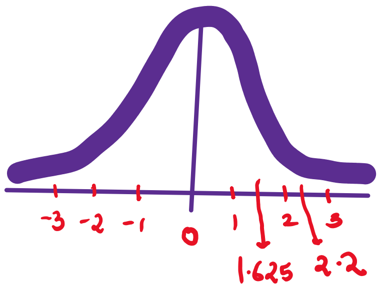
$Z_{FEMALE} = \frac{1.73 - 1.62}{0.05}$

z_{MALE} = $\frac{\quad}{0.08}$

$= 1.625$

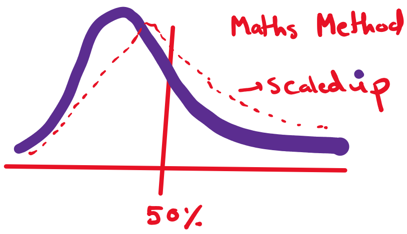
z_{FEMALE} = $\frac{\quad}{0.05}$

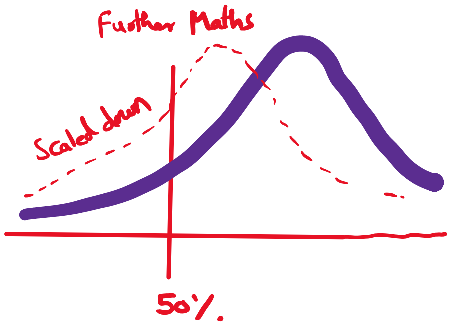
$= 2.2$



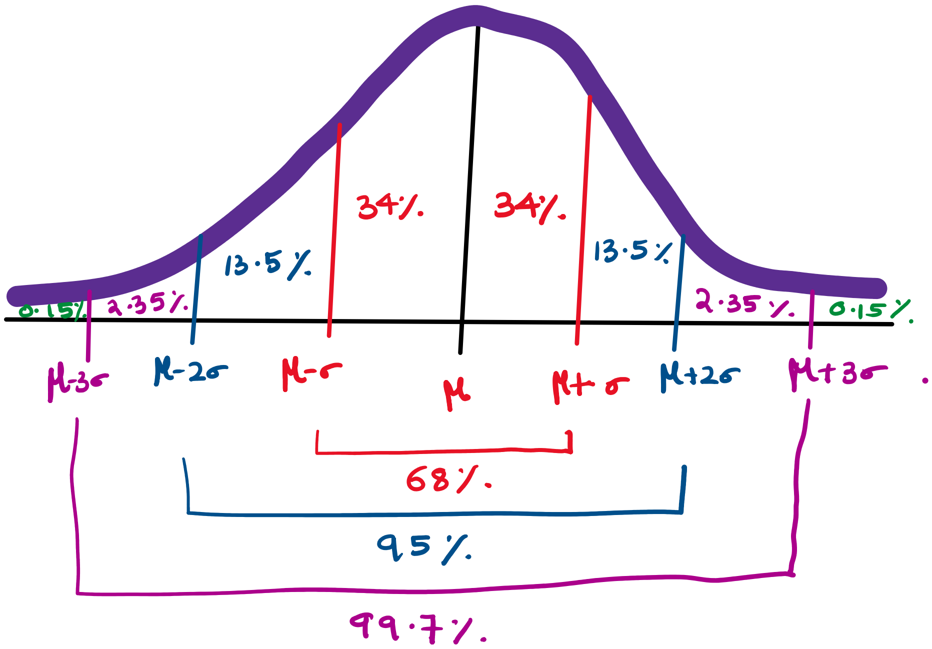
Example of Normal Model :

VCE Units





The following diagram displays the 68-95-99.7 rule:



Question 2 : Normal distribution example – Worms

Suppose a particular species of worm is 6.1mm long on average, with a standard deviation of 1.3 mm, and that worm lengths are normally distributed.

$\mu = 6.1$ mm

$\sigma = 1.3$mm

a) Draw a model representing worm lengths

b) Within what interval are the central 68% of worms? 4.8 and 7.4

c) Approximately what percentage of worms are longer than 8.7mm? 2.5%

d) What percentage of worms are smaller than 4mm?

e) Below what length are the shortest 25% of worms?

f) What is the cut-off length for the longest 10% of worms?

68%

$\mu + \sigma = 7.4$

$\mu - \sigma = 4.8$

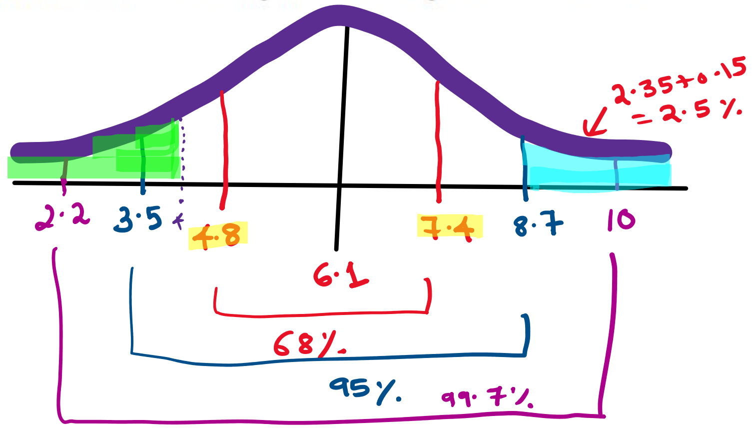
95%

(a)

e) Below what length are the shortest 25% of worms?

f) What is the cut-off length for the longest 10% of worms?

(a)



95%

$\mu + 2\sigma = 8.7$

$\mu - 2\sigma = 3.5$

99.7%

$\mu + 3\sigma = 10$

$\mu - 3\sigma = 2.2$

(b) $50 - (34 + 13.5) = 2.5\%$

(c) Smaller than 4mm

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{4 - 6.1}{1.3}$$

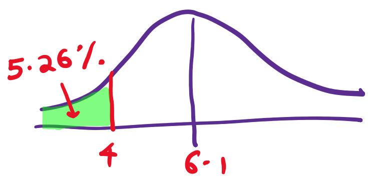
$$\boxed{Z = -1.62}$$

Look up Normal Table

→ Use the formula and find Z-score

$$Z = \frac{x - \mu}{\sigma}$$

→ Use the Normal table to find the Z-score



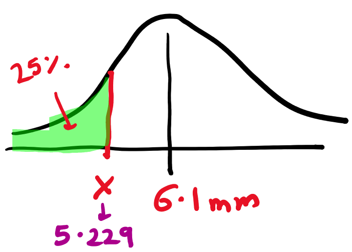
9	8	7	6	5	4	3	$\boxed{2}$	1	0	← 2nd decimal place
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	−1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	−1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	−1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	$\boxed{0.0526}$	0.0537	0.0548	$\boxed{-1.6}$ ←

$0.0526 \times 100 = 5.26\%$

(d) shortest 25%

→ Lookup Normal table for Z-score

→ Use the formula for calculation



9	8	$\boxed{7}$	6	5	4	3	2	1	0	
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	−0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	−0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2451	0.2483	$\boxed{0.2514}$	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	$\boxed{-0.6}$ ←
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5

Z-score for shortest 25% → −0.67

$$\boxed{Z = -0.67}$$

$\boxed{Z = \frac{x-\mu}{\sigma}}$ $1.3 \times -0.67 = \frac{X - 6.1}{\cancel{1.3}} \times \cancel{1.3}$

$+6.1 \quad -0.871 = X - 6.\cancel{1} + 6.\cancel{1}$

$\boxed{5.229 = X}$

Suppose body temperature for healthy adults follows a normal distribution with mean 37.3°C and standard deviation 0.4°C.

$\mu = 37.3$
$\sigma = 0.4$

a) In which temperature interval would you expect the central 68% of adults? (1 mark)

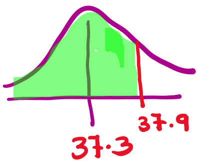
- ○32.8 to 40.8°C
- ○36.4 to 37.6°C
- ○36.8 to 37.2°C
- ✓36.9 to 37.7°C
- ○36.0 to 37.6°C

$\mu - \sigma$ and $\mu + \sigma$

36.9 and 37.7

b) What percentage of temperatures are lower than 37.9°C? (2 marks)

Using the Z table or technology, calculate your answer then enter the percentage, correct to 1 decimal place.



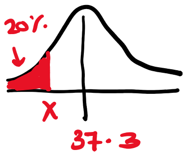
93.3

$Z = \frac{x-\mu}{\sigma} = \frac{37.9 - 37.3}{0.4}$

$\boxed{Z = 1.5}$ Normal table → 0.9332 ×100

c) Below what body temperature are the coolest 20% of adults? (2 marks)

Use the Z table or technology and enter the temperature correct to 1 decimal place (without the °C symbol).



37

Look up Normal Table, $Z = -0.84$

$Z = \frac{x-\mu}{\sigma}$ $\quad -0.84 = \frac{X - 37.3}{0.4}$

$X = 36.964$

	0	1	2	3	4	5	6	7	8	9
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Using statsKingdom.com

https://www.statskingdom.com/distribution-calculator.html

Distribution

Normal distribution

Mean (μ): 37.3

Standard deviation (σ): 0.4

Probability (p) or Score (x): x_1

x_1 - score: 37.9

Rounding: 6

Chart Rounding: 2

$Z_1 = 1.5$

$P(X \le 37.9) = \mathbf{0.933193}$. → 93.3%

$P(X > 37.9) = \mathbf{0.0668072}$.

(c)

9	8	7	6	5	4	3	2	1	0	
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	−0.9
0.1867	0.1894	0.1922	0.1949	0.1977	$\boxed{0.2005}$	0.2033	0.2061	0.2090	0.2119	−0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	−0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5

Distribution

Normal distribution

Mean (μ): 37.3

Standard deviation (σ): 0.4

Probability (p) or Score (x): $P(X \le x_1)$

p_1: 0.20

Rounding: 6

Chart Rounding: 2

$x_1 = \mathbf{36.963352}$, $Z_1 = -0.841621$.

PST2 Questions on Normal distribution :

2.3 Measurement of the thumb nail length in a sample of healthy middle-aged men native to a particular region found a mean length of 12.4 mm and a standard deviation of 2.7 mm. Suppose that the sample provides an accurate representation of the whole population of native men in this said region and that a Normal model applies.

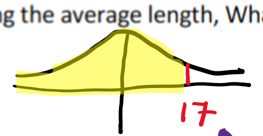
$\mu = 12.4$
$\sigma = 2.7$

a) Draw the model for the thumb nail length of a native man of this region. Bell shape curve → 68-95-99.7% Rule
b) About what percent of native men would have a thumbnail below 9.7 mm in length.
c) About what percent of native men would have a thumbnail between 7 mm and 15.1 mm in length?

[2+1+1 = 4 marks]

2.4 Based on the Normal model $N(\overset{\mu}{12.4}, \overset{\sigma}{2.7})$ describing the average length, What proportion of native men would have a thumbnail

a) at least 17 mm?
b) between 8 mm and 13 mm?



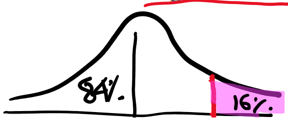
$Z = \dfrac{X - \mu}{\sigma}$ → Look up Normal table for Area

[2+2 = 4 marks]

2.5 Based on the Normal model $N(\overset{\mu}{12.4}, \overset{\sigma}{2.7})$ describing the average length, What is the approximate thumbnail length above which, 16% of the native men with the longest thumbnail lengths are expected?

[2 marks]

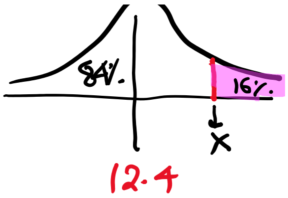
$Z = \dfrac{x - \mu}{\sigma}$ $\quad Z = \dfrac{x - \mu}{\ }$



$Z = 0.99$ OR 1.00

$Z = \frac{x-\mu}{\sigma}$ $\quad Z = \frac{x-\mu}{\sigma}$

$Z = \frac{x-\mu}{\sigma}$



[2 marks]

$Z = 0.99$ OR 1.00

BINOMIAL DISTRIBUTIONS

- The basis for the Binomial model is a Bernoulli trial.
  We have Bernoulli trials if:
  - there are only two possible outcomes (success and failure)
  - the probability of success, *p*, is constant
  - the trials are independent

For example: Flipping a coin → HEAD, TAIL

Rolling a Die → EVEN, ODD

- Two parameters define the Binomial model: *n*, the number of trials; and, *p*, the probability of success. We denote this **Binom(*n, p*)**.
- The mean is $\boxed{\mu = np}$
  and standard deviation $\boxed{\sigma = \sqrt{npq}}$

Normal(μ, σ) → Mean, S.D

$$\boxed{Z = \frac{x-\mu}{\sigma}}$$

p → probability of Success
q → probability of Failure

$p + q = 1$

n = number of trials
p = probability of success
q = 1 – *p* = probability of failure
X = number of successes in *n* trials

$$P(X = x) = \binom{n}{x} p^x q^{(n-x)}$$

$$P(X = x) = {}^nC_x \, p^x q^{n-x}$$

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$$

$${}^8C_3 = \frac{8!}{3!\ 5!} = 56$$

$$= \frac{8 \times 7 \times \cancel{6} \times \cancel{5 \times 4 \times 3 \times 2 \times 1}}{\underbrace{\cancel{3 \times 2 \times 1}}_{6} \ \cancel{5 \times 4 \times 3 \times 2 \times 1}}$$

$= 56$

8 Shift + ÷ 3

${}^nC_r$

8 C 3 = 56

Question 6 : Binomial distribution example – plant cuttings

A horticulturalist finds that the probability of a particular plant species growing successfully from a cutting is 0.4.
If 5 cuttings are planted, find the probability that:

$p = 0.4$
$q = 0.6$
$n = 5$

a) exactly 3 cuttings will grow. x

b) at least 1 cutting will grow.

$$P(X = x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

(a) $P(X=3) = {}^{5}C_{3}(0.4)^{3}(0.6)^{5-3}$

$= 10 \times 0.064 \times 0.36$

$= 0.2304$ or 23.04%

(b) $P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

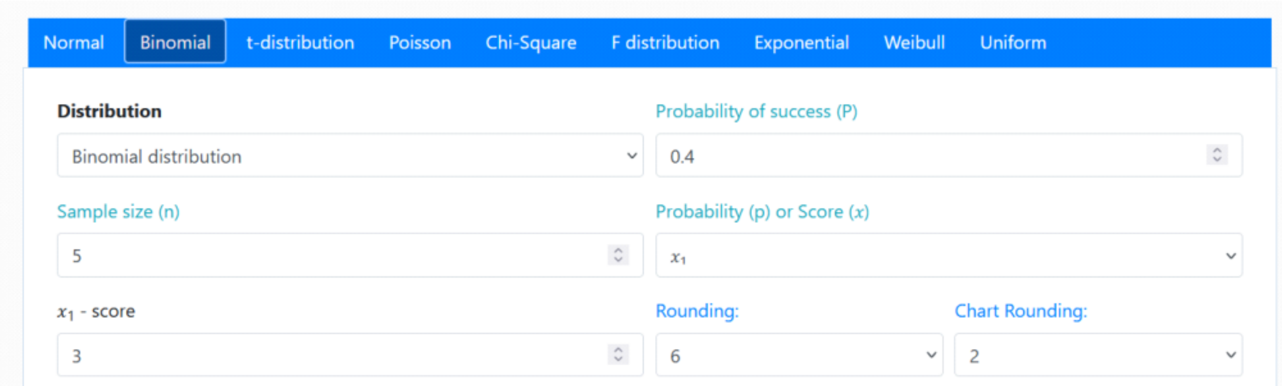
$1 - P(X=0) = 1 - 0.07776 = 0.9222$

$P(X=0) = {}^{5}C_{0}(0.4)^{0}(0.6)^{5-0}$

$= 1 \times 1 \times 0.6^{5} = 0.07776$

Using Statkingdom.com

(a)



$P(X \le 3) =$ **0.91296**.
$P(X < 3) =$ **0.68256**.
$P(X > 3) =$ **0.08704**.
$P(X \ge 3) =$ **0.31744**.
$P(X = 3) =$ **0.2304**.

(b)

Normal	Binomial	t-distribution	Poisson	Chi-Square	F distribution	Exponential	Weibull	Uniform

Distribution

Binomial distribution

Probability of success (P)

0.4

Sample size (n)

5

Probability (p) or Score (x)

x_1

x_1 - score

1

Rounding:

6

Chart Rounding:

2

P(X ≤ 1) = **0.33696**.
P(X < 1) = **0.07776**.
P(X > 1) = **0.66304**.
P(X ≥ 1) = **0.92224**.
P(X = 1) = **0.2592**.

Question 8 : Normal approximation calculation – plant cuttings

A horticulturalist finds that the probability of a particular plant species growing successfully from a cutting is 0.4.
If 100 cuttings are planted, find:

$p = 0.4$
$q = 0.6$
$n = 100$

✓a) the expected number of cuttings that will grow successfully.
b) the probability that at least 50 of the cuttings will grow. 2.07%

$P(X=50)+P(X=51)+P(X=52)+ \dots P(X=100)$

$\text{Binomial}(n, p) \longrightarrow \text{Normal}(\mu, \sigma)$, $\mu \to 40$, $\sigma \to 4.9$

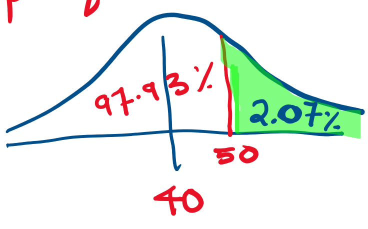
(a) Mean, $\mu = np = 100 \times 0.4 = 40$

S.D, $\sigma = \sqrt{npq} = \sqrt{100 \times 0.4 \times 0.6} = 4.898 \simeq 4.9$

(b) $P(X=x) = {}^nC_x\, p^x q^{n-x} \longrightarrow Z = \dfrac{X-\mu}{\sigma}$

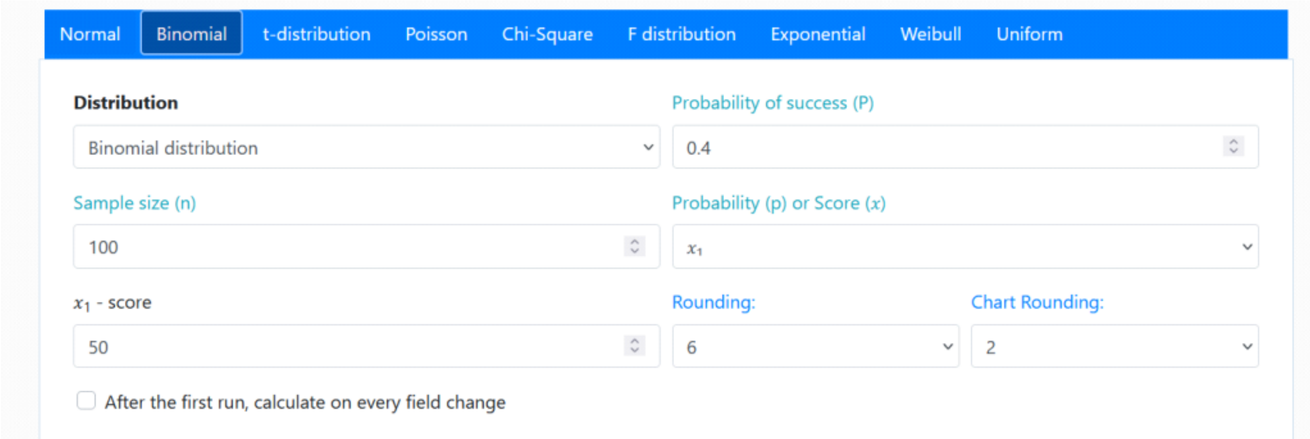
$= \dfrac{50-40}{4.9}$

$\boxed{Z = 2.04}$



$100 - 97.93 = 2.07\%$

	.0	1	2	3	$\boxed{4}$	5	6	7	8	9
$\boxed{2.0}$	0.9772	0.9778	0.9783	0.9788	$\boxed{0.9793}$	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936



P(X ≤ 50) = **0.983238**.
P(X < 50) = **0.972901**.
P(X > 50) = **0.0167617**.
P(X ≥ 50) = **0.0270992**.
P(X = 50) = **0.0103375**.

PST2 Question on Binomial Distribution:

2.6 In a particular rural area of Victoria, 80% of learner drivers pass their driving test on the first attempt.

$p = 0.80$
$q = 0.20$
$n = 8$

a) Amongst If 8 learner drivers are selected, what is the probability of a successful driving test outcome for
i) all 8 drivers? $P(X=8)$
ii) 0 or 1 drivers? $P(X=0) + P(X=1)$
iii) at least two drivers? $1 - [P(X=0) + P(X=1)]$

$P(X=x) = {}^nC_x p^x q^{n-x}$

b) For 120 learner drivers are selected, Now, $n = 120$
i) how many on average would you expect to pass the test? Compute the standard deviation.
ii) what is the probability that more than 100 drivers pass?
[Hint: use your answers from (b) (i)]

$\mu = np$
$\sigma = \sqrt{npq}$

[(1+2+2) +(2+3) = 10 marks]

$P(X > 100) = P(X = 101) + P(102) + \ldots\ldots P(120)$