1 Power Rule

1.1 The derivative

1. (i) $y = 3\sqrt{x} = 3x^{1/2}$, $\frac{dy}{dx} = 3 \times \frac{1}{2}x^{\frac{1}{2}-1} = \frac{3}{2}x^{-\frac{1}{2}}$.

   (ii) $y = \sqrt{3x} = \sqrt{3}x^{1/2}$, $\frac{dy}{dx} = \sqrt{3}/2x^{\frac{1}{2}-1} = \frac{\sqrt{3}}{2}x^{-\frac{1}{2}}$.

   (iii) $y = \frac{3}{\sqrt{x}} = 3x^{-\frac{1}{2}}$, $\frac{dy}{dx} = 3 \times (-\frac{1}{2})x^{-\frac{1}{2}-1} = -\frac{3}{2}x^{-\frac{3}{2}}$.

   (iv) $y = \frac{1}{3\sqrt{x}} = \frac{1}{3}x^{-\frac{1}{2}}$, $\frac{dy}{dx} = \frac{1}{3} \times (-\frac{1}{2})x^{-\frac{1}{2}-1} = -\frac{1}{6}x^{-\frac{3}{2}}$

   (v) $y = x^4 - \frac{1}{x^4} = x^4 - x^{-4}$, $\frac{dy}{dx} = 4x^3 - (-4)x^{-5} = 4x^3 + 4x^{-5}$.

   (vi) $y = 4x^{1/6} + \pi^3$, $\frac{dy}{dx} = 4 \times \frac{1}{6}x^{\frac{1}{6}-1} = \frac{2}{3}x^{-\frac{5}{6}}$.

   (vii) $y = 4x^2 - 5x + 6$, $\frac{dy}{dx} = 4 \times 2x - 5 = 8x - 5$.

   (viii) $y = 2x^3 + 7x^2 + 9x - 1$, $\frac{dy}{dx} = 2 \times 3x^2 + 2 \times 7x + 9 = 6x^2 + 14x + 9$.

2. (i) $(-1, 1)$
   $\frac{dy}{dx} = 5 \times 2x + 1 = 10x + 1$ when $x = -1$ we have
   $\frac{dy}{dx} = 10 \times (-1) + 1 = -9$.
   (ii) $(2, 19)$, when $x = 2$ we have
   $\frac{dy}{dx} = 10 \times 2 + 1 = 21$.

   We can see that the points $(-1, 1)$ and $(2, 19)$ both belongs to the curve $y = 5x^2 + x - 3$.

3. (i) $y = 2\sqrt{x}$, $\frac{dy}{dx} = 2 \times \frac{1}{2}x^{1/2-1} = x^{-1/2}$.

   (ii) $y = \sqrt{2x}$, $\frac{dy}{dx} = \sqrt{2} \times \frac{1}{2}x^{-\frac{1}{2}} = \frac{\sqrt{2}}{2}x^{-\frac{1}{2}}$.

   (iii) $y = \frac{2}{\sqrt{x}} = 2x^{-\frac{1}{2}}$, $\frac{dy}{dx} = 2 \times (-\frac{1}{2})x^{-1/2-1} = -x^{-\frac{3}{2}}$.

   (iv) $y = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-\frac{1}{2}}$, $\frac{dy}{dx} = \frac{1}{2} \times (-\frac{1}{2})x^{-\frac{3}{2}} = -\frac{1}{4}x^{-\frac{3}{2}}$.

   (v) $y = x^3 - \frac{1}{x^2} = x^3 - x^{-2}$, $\frac{dy}{dx} = 3x^2 - (-2)x^{-3} = 3x^2 + 2x^{-3}$.

   vi) $y = 6x^{2/3} + \sqrt{3}$, $\frac{dy}{dx} = 6 \times \frac{2}{3}x^{2/3-1} = 4x^{-1/3}$.

   (vii) $y = 7x^2 - 9x + 8$, $\frac{dy}{dx} = 7 \times 2x - 9 = 14x + 9$.

   (viii) $y = 3x^3 - 7x^2 + 10x - 1$, $\frac{dy}{dx} = 3 \times 3x^2 - 7 \times 2x + 10 = 9x^2 - 14x + 10$.

4. We have for all x that $\frac{dy}{dx} = 14x - 4$.

   (i) Then for for the point $(-1, 5)$, we have $x = -1$ substituting we have $\frac{dy}{dx} = 14 \times (-1) - 4 = -18$. The slope is -18.

   (ii) For the point $(2, 14)$, $x = 2$ we have $\frac{dy}{dx} = 14 \times 2 - 4 = 24$. The slope is 24.