1 Kinematics

1. The instantaneous rate of change at the time t is given by
$\frac{dV}{dt} = 24 - 2t$.

   (i) When $t = 10$, $\frac{dV}{dt} = 24 - 2 \times 10 = 4$ (litres per minute).

   (ii) The question is to find V when $\frac{dV}{dt} = 23$ litres/minute, when $24 - 2t = 23$ then $2t = 24 - 23 \implies t = 1/2$. Then $V = 24 \times 1/2 - (1/2)^2 = 12 - 1/4 = \frac{47}{4}$
$V = 11.75$(litres per minute).

   (iii) The question is to find $\frac{dV}{dt}$ when $V = 44$ then
$24t - t^2 = 44$, then $t^2 - 24t + 44 = 0 \implies (t - 22)(t - 2) = 0$ when $t = 22$ and $t = 2$, since $0 \leq t \leq 12$ we get $t = 2$ as the only permissible solution. Now, for $t = 2$, $\frac{dV}{dt} = 24 - 4 = 20$ (litres per minute).

2. (i) The velocity is given by $v = \frac{ds}{dx} = -3t^2 + 6t + 6 (m/s)$.
   The acceleration by $a = \frac{dv}{dt} = -6t + 6 (m/s^2)$.

   (ii) For $t = 2$, $v = -3 \times 2^2 + 6 \times 2 + 6 = 6 m/s$

   (iii) For $t = 7$, $a = -6 \times 7 + 6 = -36 m/s^2$.

   (iv) The accelerations is 0 when $-6t + 6 = 0 \implies t = 1$. When $t = 1$ we have $v = -3 \times 1^2 + 6 \times 1 + 6 = 9 m/s$.

   (v) The velocity is -3 when $v = -3t^2 + 6t + 6 = -3$ then dividing by 3 we obtain $t^2 - 2t - 3 = 0 \implies (t - 3)(t + 1) = 0$ we have $t = 3$ and $t = -1$, as we cannot have a negative time, we only have the time $t = 3$. When $t = 3$ we have $a = -6 \times 3 + 6 = -12 m/s^2$. The acceleration when the velocity is $-3 m/s$ is $a = -12 m/s^2$.

3. (i) The instantaneous rate of change of volume when $t = 9$ is $\frac{dV}{dt} = 20 - 2t$ computed when $t = 9$ is $\frac{dV}{dt} = 20 - 2 \times 9 = 2$(litre per minute).

   (ii) When $\frac{dV}{dx} = 14$litres/minute, we have $20 - 2t = 14 \implies t = 3$, now the volume at this time is $V = 20 - 3^2 = 11$ litres.

   (iii) When $v = 84$ litres, we have $20 - t^2 = 84 \implies t^2 - 64 = 0$ which give us the values for the time $t = 8$ and $t = -8$, but we cannot have negative time, then the only permissible time is $t = 8$. Now for $t = 8$ we have $\frac{dV}{dt} = 20 - 2 \times 8 = 4$ litres per minute.

4. The displacement s (metres) of an object at time t (seconds) is given by $s = -2t^3 + 6t^2 + 18t$, for $t \geq 0$. Find

   (i)$v = \frac{ds}{dt} = -6t^2 + 12t + 18$ and $a = \frac{dv}{dt} = -12t + 12$.

   (ii) the velocity when$t = 2$ is $v = -6 \times 2^2 + 12 \times 2 + 18 = 18 m/s$

(iii) the acceleration when $t = 3$ is $a = -12 \times 3 + 12 = -24m/s^2$.

(iv) When acceleration is 0, we have $a = -12t + 12 = 0 \implies t = 1$, then the velocity for $t = 1$ is $v = -6 \times 1^2 + 12 \times 1 + 18 = 24m/s^2$.

(v) When the velocity is -30 we have $-6t^2 + 12t + 18 = -30$ then $-6t^2 + 12t + 48 = 0$ dividing by -6 we have $t^2 - 2t - 8 = 0$ and $(t-4)(t+2) = 0$ then $t = 4$ and $t = -2$ but the time cannot be negative. Now when $t = 4$ we have $a = -12 \times 4 + 12 = -36m/s^2$.