Problem solving task 21

1. Blood types

O	30%
A	35%
B	15%
AB	20%

a i) Not have type A?

$p(\text{NOT } A)$

$100 - 35 = 65\%$

$p = 0.65$

ii) Has type O or A?

$p(O) + p(A) = 65\%$

$30 + 35\% \quad p = 0.65$

iii) Neither AB or B?

$p(AB) + p(B) = 35\%$

$20\% + 15\%$

$100 - 35\% = 65\%$

$p = 0.65$

next page

3 victims

b) All 3 have type A?

$0.35 \times 0.35 \times 0.35 = 0.35^3$

$p(A)$

$= 0.0443$

$p = 0.04$

ii) None are type O?

$p(O^c) \rightarrow p(O)^3 = 0.70^3 = 0.34$

$0.35 + 0.15 + 0.20 = 0.70$

$p = 0.34$

iii) At least one is type B?

$p(B) \times p(B^c)^2 = 0.11 = p$

0.15

$0.3 + 0.35 + 0.20 = 0.85$

iv) Only the first victim is type A?

$p(A) \cdot p(A^c)^2 = 0.15 = p$

$0.35 \quad 0.3 + 0.15 + 0.20$

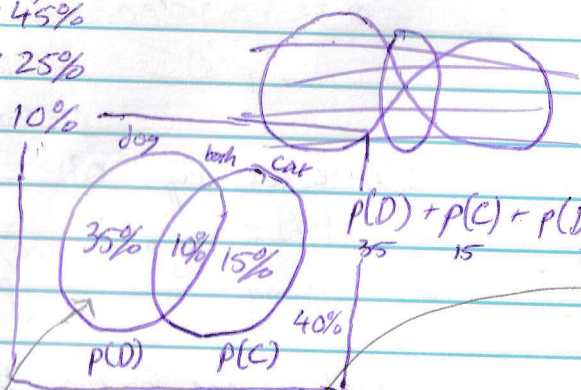
$\hookrightarrow = 0.65$

Q2. Assume that 45% of households have at least one dog, 25% one+ cat, and 10% one+ of both.

Dog: 45%

Cat: 25%

Both: 10%



$P(D) + P(C) + P(D \cap C) = 60\%$

35 15 10

none = 40%

a) Dog but no cat?

$p = 0.35$

b) No dogs/cat

$p = 0.40$

c) Cat or a dog?

$35 + 10 + 15 = 60\%$

$p = 0.60$

d) With a dog, prob. of a cat?

$p(D) \times p(C) \Rightarrow p = 0.05$

$0.35 \quad 0.15$

e) Is owning cats and dogs mutually exclusive?

No. An owner can have both pets at once, meaning it is not mutually exclusive.

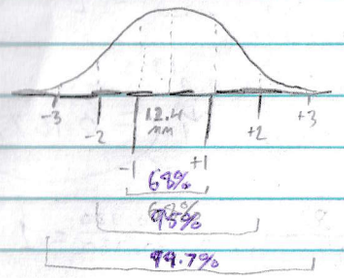
f) Independent events?

Statistically, yes. The ownership of a cat does not influence the ownership of a dog, or vice versa. In reality, it does.

Q3. Thumbnail length of middle-aged men

Mean = 12.4mm σ = 2.7mm

a) Model drawing



b) % of men with a thumbnail < 9.7mm

$z = \frac{x-\mu}{\sigma}$

$\frac{9.7-12.4}{2.7} = -0.97407 = -0.97$

Normal table → 0.1736

= 17.36%

c) % of men w/ thumbnail between 7mm and 15.1mm

$\frac{7-12.4}{2.7} = -2$ $\frac{15.1-12.4}{2.7} = 1$

0.0228 0.8413

1 − 0.0228 = 0.9772

1 − 0.8413 = 0.1587

0.9772 − 0.1587 = 0.8185

→ 81.85%

Q4. $N(\underset{\mu}{12.4}, \underset{\sigma}{2.7})$

a) At least 17mm?

$Z = \frac{17-12.4}{2.7} = 1.704$

4.46% 1 − 0.9554 = 0.0446

b) Between 8mm − 13mm

$\frac{8-12.4}{2.7} = -1.63$ $\frac{13-12.4}{2.7} = 0.22$

1 − 0.0516 = 0.9484

0.5871 1 − 0.5871 = 0.4129

0.9484 − 0.4129 = 0.5355

= 53.55%

Q5. Thumbnail length above 16%, so 84%

$Z = \frac{x-\mu}{\sigma} + \mu \times \sigma \rightarrow Z\sigma + \mu = X$

100 − 16 = 84%

Z-score % lookup on 0.84

→ 0.8389 (Z = 0.99)

$X = 0.99 \times 2.7 + 12.4$

2.673

= 15.073mm

$C = \frac{n!}{x!(n-x)!}$

$p(x=x) = {}^{n}C_{x} \, p^{x} q^{n-x}$

Q6. 80% of learners pass on first try

a) 8 drivers selected

a i) All 8 drivers? x = 8

$0.80^8 = 0.1677$

p = 0.1677 = 0.17

$C = \frac{8!}{8!(8-8)!} = \frac{40320}{40320} = 1$

$1 \times 0.8^8 \times 0.2^{(8-8)} = 0.1677$

ii) 0 or 1 drivers? x ≤ 1

$0.80^2 = 0.64$

$p = 8.448 \times 10^{-5}$ 8.45×10^{-5}

0 driver $C = \frac{8!}{0!(8-0)!} = \frac{40,320}{40,320} = 1$

1 driver $C = \frac{8!}{1!(8-1)!} = \frac{40,320}{5040} = 8$

0 driver $1 \times 0.8^0 \times 0.2^{(8-0)} = 2.56 \times 10^{-6}$

1 driver $8 \times 0.8^1 \times 0.2^{(8-1)} = 8.192 \times 10^{-5}$

0 driver + 1 driver

2.56×10^{-6} 8.192×10^{-5}

$= 8.448 \times 10^{-5}$

iii) At least two? x ≥ 2

$1 - 8.448 \times 10^{-5}$

= 1.00

p = 0.999916

= 0.99

b) 120 drivers

i) Average pass?

$\mu = np$ $\sigma = \sqrt{npq}$

120×0.8 $\sqrt{120 \times 0.8 \times 0.2}$

$\mu = 96$ $\sigma = 4.382$

= 4.38

ii) $z = \frac{101-96}{4.382} = 1.14 \rightarrow 0.8729$

1 − 0.8729 = 0.1271

approx probability that ≥ 100 drivers will pass

Starkingdom result

p = 0.8 n = 120 x_1 = 100

P(x > 100) = 0.1517

= 0.13 = 0.15

Page 5

~~226/272 = 0.8309~~ 226/262

Q7. Random sample of ~~272~~ 262 people; 2 months

- 226 had weight loss of ≥5% $\frac{226}{262} = 0.8626$
- Evidence? of this exercise resulting in ≥5% weight loss in >81% of individuals?

a) H_A: ~~There is evi~~ The majority (>81%) of individuals experienced 5% or greater weight loss.

H_0: There is no significant evidence that the majority (81%) of individuals experienced ≥5% weight loss

b) $H_A: p > 0.81$ $H_0: p \leq 0.81$

one-proportion $\hat{p} \pm z^* \times \sqrt{\frac{\hat{p}\cdot\hat{q}}{n}}$

c) Hypothesis test

~~Area = $\frac{1+0.81}{2}$ = 0.905~~

Assuming 95% confidence:

~~Area = $\frac{1+0.95}{2}$ = 0.975~~ 97.5% → ~~1.96~~ z-table z^*

~~$\hat{p}$ = 0.81 z^* = 1.96~~

~~$\hat{q}$ = 0.19~~

~~P-value = 0.81 ±~~

$\hat{p} = \frac{226}{262} = 0.8626$

$\hat{q} = 1 - \hat{p} = 0.1374$

$z = 2.1703$

P-value = 0.0150

↳ Calculated from Statskingdom with the "one sample proportion test"

Tails: Right ($H_1: p > p_0$)

Significance level: 0.05

Expected proportion: $p_0 = 0.81$

Proportion: 226

$n = 262$

~~$z = \frac{0.81 - 0.8626}{\sqrt{\frac{0.81 \times 0.19}{262}}}$~~

$z = \frac{0.8626 - 0.81}{\sqrt{\frac{0.81 \times 0.19}{262}}}$

→ $z = 2.1703$

d) p-value (0.0150) < alpha (0.05).

The H_0 is rejected, meaning the proportion of people who lost ≥5% of their weight exceeded 0.81.

~~This~~ →

Page 6

Area = $\frac{1 + \text{confidence interval}}{2}$

Q7 continued

e) 90% confidence interval for true proportion.

$\hat{p} \pm z^* \times \sqrt{\frac{\hat{p}\hat{q}}{n}}$ (SE)

$\hat{p} = 0.8626$

$\hat{q} = 0.1374$

$n = 262$

$z^* = 1.645$

Area = $\frac{1 + 0.90}{2} = 0.950$

↓ Lookup z table → z^* = between 1.65 and 1.64 = 1.645 (rough estimate)

$SE = \sqrt{\frac{0.8626 \times 0.1374}{262}}$

$SE = 0.2127$

~~[0.8276,~~ $\hat{p}$ z^* SE

$0.8626 + (1.645 \times 0.2127)$

~~$= 0.8626 + 0.212639$~~ $= 0.8976$

[0.8276, 0.8976]

$\hat{p}$

$0.8626 - (1.645 \times 0.2127)$

$= 0.8276$

We are 90% confident that the true proportion of people who tried this new exercise routine and presented with ≥5% weight loss is between 82.76% and 89.76%

$\hat{p}_{pooled} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$

Q8. Is there a significant difference between the two bank branches' customer satisfaction?

	Satisfied	Not	
Branch A	558	53	$n_1 = 611$
Branch B	496	77	$n_2 = 573$

a) H_0: There is no significant difference between the branch customer satisfaction $p_1 = p_2$

H_1: There is a significant difference between the branch customer satisfaction. $p_1 \neq p_2$

b) $n_1 = 611$ $p_1 = \frac{558}{611} = 0.91325$ $q_1 = 0.08674$ $\alpha = 0.05$

$n_2 = 573$ $p_2 = \frac{496}{573} = 0.86562$ $q_2 = 0.13438$

Area $= \frac{1 + 0.95}{2} = 0.97$

$z^* = 1.96$

$\hat{p}_{pooled} = \frac{611 \times \frac{558}{611} + 573 \times \frac{496}{573}}{611 + 573} = 0.89020$

$\hat{q}_{pooled} = 1 - \hat{p}_{pooled} = 0.109797$

$z = \frac{(p_1 - p_2) - 0}{SE_{pooled}(\hat{p}_1 - \hat{p}_2)}$ $\frac{(0.91325 - 0.86562) - 0}{0.018181}$

$z = 2.6202$

p-value $= 0.00878852 = 0.0088$

$SE_{pooled} = \sqrt{\frac{\hat{p}_{pooled} \cdot \hat{q}_{pooled}}{n_1} + \frac{\hat{p}_{pooled} \cdot \hat{q}_{pooled}}{n_2}}$ SE $= 0.018181$

$SE_{pooled} = \sqrt{\frac{0.89020 \times 0.10979}{611} + \frac{0.89020 \times 0.10979}{573}}$

Q8 continued

p-value calculated from statskingdom "Two sample proportion test calculator"

Tails: Two ($H_1: p_1 \neq p_2$)

Significance value: 0.05

$x_1 = 558$ $n_1 = 611$ $x_2 = 496$ $n_2 = 573$

The p-value (0.0088) is less than the significance level (0.05) meaning the H_0 is rejected and there is a significant difference, between branch A's and B's customer satisfaction

0.01818

c) SE calculated on the previous page.

$z^* = 1.96$ (95% confidence)

$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$

$\frac{558}{611} - \frac{496}{573} \pm 1.96 \times SE = 0.0833$

$\hat{p}_1$ $\hat{p}_2$

$\hookrightarrow -1.96 \times SE = 0.0120$

Confidence interval $\rightarrow [0.0120, 0.0833]$

We are 95% confident that the proportion of customer satisfaction difference is between 1.20% and 8.33%.

d) The differences shown in the confidence interval shows that there is a difference which is consistent with the low p-value showing that there is a significant difference between customer satisfaction for the two branches.