Q1

The Babylonian's seemed to have solved quadratic equations roughly 3600 years ago, or about 1600BC (McMillan, 1984).

Q2

The wages, days worked, and days to build canals for irrigation and transportation were all reasons for the development of the quadratic equations. The construction of canals required knowing the areas of rectangles, a key problem that a quadratic formula could solve (Roberston and O'Connor, 2000).

Q3

$x^2 + px = q$, or $x^2 + px - q = 0$

a) $x^2 + 2x = 10; q > 5$

b) $x = \sqrt{(\frac{p}{2})^2 + q} - \frac{p}{2}$

c) $x = \frac{-p \pm \sqrt{p^2 - 4aq}}{2a}$

d) $\sqrt{\frac{2}{3}^2 + 10} - \frac{2}{2} = \sqrt{\frac{94}{9}} - 1 = 2.2317$

e) $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-10)}}{2(1)}$
-> $\frac{-2 \pm \sqrt{4+40}}{2} = \frac{-2 \pm \sqrt{44}}{2}$
$x^+ = \frac{-2 + \sqrt{44}}{2} = 2.316$
$x^- = \frac{-2 - \sqrt{44}}{2} = -4.316$

f) The Babylonian solution gave a strong approximation of the quadratic formula solution, being off by approximately 0.085. The Babylonian solution did not include the a value that the quadratic does, but that did not have an effect in this specific equation anyway.
Interestingly enough, we subtract $\frac{b}{2}$ from the $\sqrt{}$ and they do too. A big difference is the lack of the $\pm$ symbol, meaning they would never arrive at two solutions given the one equation like we do. We also divide the majority of the equation by $2a$ while they do not, instead placing variables inside fractions. In addition, the divisions are calculated before the square root.

g) They may not have known any other way to solve the quadratic, or if they did, their method is relatively precise for the work that they would need in ancient times and therefore not need another method of calculation. Perhaps they could not find another way in amongst all the equation clutter, or maybe were not given enough resources to pursue the mathematics further. The true reason is likely lost to history.

Q4

Solve $2x^2 + 3x - 10$ using the Babylonian method.

The paper was inaccessible to me through many attempted avenues. I used a video by YouTuber Polar Pi (2020) as a guide instead.

Number line -> Stages v	Start	Midpoint	End
1	x	$x+x+\frac{3}{2}$ $=(2x+\frac{3}{2})\div 2$ $=x+\frac{3}{4}$	$x+\frac{3}{2}$
2	$=u-\frac{3}{4}$		$=u+\frac{3}{4}$
3		$(u-\frac{3}{4})(u+\frac{3}{4})$ $u=\pm\frac{\sqrt{89}}{4}$	
4	$x=3.108$		$x=1.608$

1 - working out
$2x^2+3x-10=0$
-> $2x^2+3x=10$, divide by 2
-> $x^2+\frac{3}{2}x=5$
-> $x\left(x+\frac{3}{2}\right)=5$

2 - working out
Taking x as u and u as the midpoint, the start is $u-\frac{3}{4}$ and the end $u+\frac{3}{4}$

3 - working out
Using the start, end, and original equation:
$(u+\frac{3}{4})(u-\frac{3}{4})$
-> $u^2-\frac{3}{4}^2$
-> $u^2-\frac{9}{16}=5$
-> $u^2=\frac{80}{16}+\frac{9}{16}$
-> $u^2=\frac{89}{16}$
-> $u=\pm\sqrt{\frac{89}{16}}=\pm\frac{\sqrt{89}}{4}$

4 - working out
From the start/end:
$x-\frac{3}{4}=\frac{\sqrt{89}}{4}$
-> $x=\frac{\sqrt{89}+3}{4}$
-> $x=3.108$
and
$x+\frac{3}{4}=\frac{\sqrt{89}}{4}$
-> $x=\frac{\sqrt{89}-3}{4}$
-> $x=1.608$

Comparison against the quadratic formula

The solutions using the quadratic formula are:
$\frac{\sqrt{89}}{4}-\frac{3}{4}=1.608$

and
$-\frac{\sqrt{89}}{4} - \frac{3}{4} = -3.108$
The Babylonian method provided identical unsigned values to the quadratic formula.

Method using swapping out variables (unused answer)

$2x^2 + 3x - 10$ into $u^2 + bu = ac$
u appears to be equivalent to x
-> $x^2 + 3x = 2 \times 10$
-> $x^2 + 3x = 20$

$x = \sqrt{(\frac{p}{2})^2 + q} - \frac{p}{2}$

$x = \sqrt{(\frac{3}{2})^2 + 20} - \frac{3}{2}$
-> $\sqrt{\frac{9}{4} + \frac{80}{4}} - \frac{3}{2}$
-> $x = \frac{\sqrt{89}}{2} - \frac{3}{2} = 3.2169$

This one is off by a significant amount this time, off by 0.108.
It would be identical to the quadratic solution if the $\frac{89}{4}$ wasn't square-rooted.

References

McMillan R D; (1984); *Babylonian Quadratics*; The Mathematics Teacher; 77(1):63-65; https://doi.org/10.5951/MT.77.1.0063

Polar Pi; (21 Jan 2019); 'The Babylonian Method for solving Quadratics (Works on ALL Quadratics)'; [video]; Polar Pi; YouTube, accessed 22 March 2024

Robertson E F and O'Connor J J; (2000); An overview of Babylonian mathematics; Accessed 16 March 2024