1 Applications - gradient and sketching

1.1 Maxima and minima

1. For each of the following functions find the x and y co-ordinates of the stationary points. Then classify each of the points as either a local maximum, a local minimum, or a horizontal point of inflection.

   (i) $y = -3x^2 + 12x + 2$ (ii) $y = x^3 - 6x^2 + 12x + 9$ (iii) $y = 3x^3 - 9x^2 + 1$

2. For each of the following functions find the x and y co-ordinates of the stationary points. Then classify each of the points as either a local maximum, a local minimum, or a horizontal point of inflection.

   (i) $y = 5x^2 - 20x + 9$ (ii) $y = 2x^3 - 9x^2 + 12x + 1$ (iii) $y = x^3 + 3x^2 + 3x + 1$

1.2 Graph sketching

1. For $y = 3x^3 - 9x^2$

   (i) find the x and $y-$intercepts

   (ii) find and classify the stationary points

   (iii) sketch, labelling all intercepts, and stationary points.

2. For $y = (x - 1)^3$

   (i) find the x and $y-$intercepts

   (ii) find and classify the stationary points

   (iii) sketch, labelling all intercepts, and stationary points.

1.3 Second derivative

1. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for each of the following:

   (i) $y = 2x^3 - 4x^2 - 5x + 9$ (ii) $y = (x - 2)e^{2x}$

   (iii) $y = x \ln x$ (iv) $y = 8\sqrt{x} - \cos(3x)$.

2. For each of the following functions find the x and y co-ordinates of the stationary points. Then classify each of the points as either a local maximum, or a local minimum using the Second Derivative Test.

   (i) $y = 3x^3 - 9x^2 + 1$ (ii) $y = x^4 - 8x^2 + 10$.

3. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for each of the following:

(i) $y = x^4 - 4x^3 - 7x + 7$

(ii) $y = (4x - 3)e^x$

(iii) $y = x \sin x$

(iv) $y = 16x^{3/4} - 4 \ln x$.

4. For each of the following functions find the x and y co-ordinates of the stationary points. Then classify each of the points as either a local maximum, or a local minimum using the Second Derivative Test.

(i) $y = 2x^3 + 9x^2 - 24x$

(ii) $y = x^3 + 3x^2 - 4$.