# 1 Index laws

## 1.1 Index laws

1. (i) $\frac{x^4 x^3}{x^2 x^8} = \frac{x^{4+3}}{x^{2+8}} = \frac{x^7}{x^{10}} = \frac{1}{x^{10-7}} = \frac{1}{x^3}$.

   (ii) $\frac{5x^3 y^{-1}}{x^{-2} y^4} = \frac{5x^{3-(-2)}}{y^{4-(-1)}} = \frac{5x^{3+2}}{y^{4+1}} = \frac{5x^5}{y^5}$.

   (iii) $\left(\frac{3x^{-2}}{y}\right)(2x^3 y^{-4}) = \frac{3 \times 2x^{-2+3}}{y^{1-(-4)}} = \frac{6x}{y^{1+4}} = \frac{6x}{y^5}$.

   (iv) $\left(\frac{x^2}{xy^{-2}}\right)^3 = \frac{(x^2)^3}{(xy^{-2})^3} = \frac{x^{2 \times 3}}{x^3 y^{-2 \times 3}} = \frac{x^6}{x^3 y^{-6}} = x^{6-3} y^6 = x^3 y^6$.

   (v) Here we a sum involved, as the greater power for $x$ is $-4$, we multiply by $\frac{x^4}{x^4}$
   $$\frac{x^{-2} + 2x^{-4}}{x - 3x^{-3}} = \frac{(x^{-2} + 2x^{-4})x^4}{(x - 3x^{-3})x^4} = \frac{x^{-2} x^4 + 2x^{-4} x^4}{xx^4 - 3x^{-3} x^4} = \frac{x^2 + 2}{x^5 - 3x}.$$

   (vi) Here we have a sum, then we multiply by $\frac{x}{x}$
   $$\frac{7x^2}{x^3 + 4x^{-1}} = \frac{(7x^2)x}{(x^3 + 4x^{-1})x} = \frac{7x^3}{x^4 + 4}.$$

2. (i) $\frac{x^6 x^3}{x^2 x^9} = \frac{x^{6+3}}{x^{2+9}} = \frac{1}{x^{11-9}} = \frac{1}{x^2}$.

   (ii) $\frac{6x^{-4} y}{x^{-2} y^{-3}} = \frac{6y^{1-(-3)}}{x^{-2-(-4)}} = \frac{6y^4}{x^2}$.

   (iii) $\left(\frac{x^{-2}}{4y^2}\right)(2x^5 y^{-2}) = \frac{2x^{-2+5}}{4y^{2-(-2)}} = \frac{x^3}{2y^4}$.

   (iv) $\left(\frac{x^2}{xy^{-2}}\right)^3 = \frac{(x^2)^3}{(xy^{-2})^3} = \frac{x^6}{x^3 y^{-6}} = x^3 y^6$.

   (v) $\frac{x^{-2} + 2x^{-4}}{x - 3x^{-3}} = \frac{x^{-2} + 2x^{-4}}{x - 3x^{-3}} \times \frac{x^4}{x^4} = \frac{x^2 + 2}{x^5 - 3x} = \frac{x^2 + 2}{x(x^4 - 3)}$.

   (vi) $\frac{7x^2}{x^3 + 4x^{-1}} = \frac{7x^2}{x^3 + 4x^{-1}} \times \frac{x}{x} = \frac{7x^3}{x^4 + 4}$

## 1.2 Fractional indices

1. (i) $\frac{x^{4/3} x}{x^{-2/3}} = \frac{x^{4/3+1}}{x^{-2/3}} = x^{\frac{4}{3}+1-(-\frac{2}{3})} = x^{\frac{4}{3}+1+\frac{2}{3}} = x^3$.

   (ii) $\frac{(x^{2/3})^{3/4}}{x^{-5/2}} = \frac{x^{\frac{2}{3} \times \frac{3}{4}}}{x^{-5/2}} = x^{\frac{1}{2}+\frac{5}{2}} = x^3$.

2. (i) $(5 + 2\sqrt{6})(5 - 2\sqrt{6}) = 5^2 - 10\sqrt{6} + 10\sqrt{6} - (2\sqrt{6})^2 = 25 - 24 = 1$.

   (ii) $\sqrt{x}\left(3\sqrt{x} - \sqrt{4x}\right) = 3(\sqrt{x})^2 - \sqrt{x} \times 2\sqrt{x} = 3x - 2x = x$.

3. (i) $\frac{x^{7/4} x^{-1}}{x^{-1/4}} = \frac{x^{\frac{7}{4}-1}}{x^{-\frac{1}{4}}} = x^{\frac{7}{4}-1+\frac{1}{4}} = x^1 = x$.

   (ii) $\frac{(x^{5/3})^{3/4}}{x^{-3/4}} = \frac{x^{\frac{5}{3} \times \frac{3}{4}}}{x^{-\frac{3}{4}}} = x^{\frac{5}{3} \times \frac{3}{4} + \frac{3}{4}} = x^{\frac{5}{4}+\frac{3}{4}} = x^2$.

4. (i) $(8 + 5\sqrt{11})(8 - 5\sqrt{11}) = 8^2 - (5\sqrt{11})^2 = 64 - 25 \times 11 = -211.$

(ii) $3\sqrt{x}\left(6\sqrt{x} - \sqrt{9x}\right) = 18(\sqrt{x})^2 - 3\sqrt{x} \times 3\sqrt{x} = 18x - 9x = 9x.$