Mathematical Logic HW7

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1. Let L, S, and P be the following predicates.

   $L(x) = x$ is a lecturer at JCT.
   $S(x) = x$ is a student at JCT.
   $P(x, y) = x$ teaches y.

   (b) $\exists a(L(a) \wedge \forall b(S(a) \rightarrow P(a, b)))$

   (d) $\exists a(S(a) \wedge \forall b(L(b) \rightarrow P(b, a)))$

   (f) $\exists a(L(a) \wedge \forall b(S(b) \rightarrow \neg P(a, b)))$

2. (b) The universe is all the students and courses of Tal College.

   $S(x) = x$ is a student.
   $F(x) = x$ is female.
   $C(x) = x$ is a course.
   $P(x, y) = x$ is in y

   $\forall a(C(a) \rightarrow \forall b((S(b) \wedge P(b, a)) \rightarrow F(b)))$

   (d) The universe everything.

   $P(x) = x$ is a person.
   $K(x, y) = x$ knows y.

   $\exists a(P(a) \wedge \forall b(P(b) \rightarrow K(a, b))) \wedge \exists a(P(a) \wedge \neg \forall b(P(b) \rightarrow K(b, a)))$

3. (a) The following proposition

   $$\forall x(M(x) \rightarrow \exists y \exists z(H(y) \wedge H(z) \wedge F(x, y) \wedge F(x, z)))$$

   is equivalent to saying "Every old person has two young relatives."

   (b) The proposition "One is young if and only if one is not old." can be formalized as follows.

   $$\forall a(H(a) \leftrightarrow \neg M(a))$$

   (d) The proposition "There is a person who is not young and not old, only if every person has relatives." can be formalized as follows.

   $$\exists a(\neg H(a) \wedge \neg M(a)) \rightarrow \forall a \exists b(F(a, b))$$

4. (b) $\exists a(S(a) \wedge \neg \forall b((T(b) \wedge T(a, b)) \rightarrow R(a, b)))$

   (d) $\forall a(S(a) \rightarrow (D(a) \rightarrow \forall b((T(b) \wedge T(a, b)) \rightarrow R(a, b))))$