Automata & Formal Languages

Homework 1 – Deterministic Finite Automata

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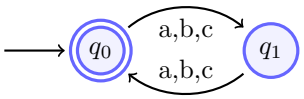
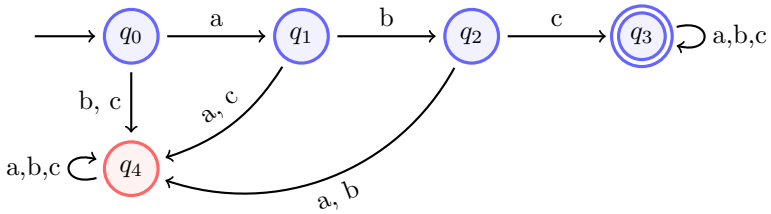
1. $\mathcal{L} = \{w \in \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}^* : (\exists n \in \mathbb{N} : |W| = 2n)\}$



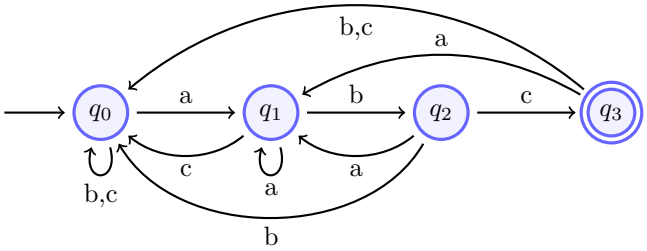
2. $\mathcal{L} = \{w \in \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}^* : \mathrm{abc} \sqsubseteq w\}$



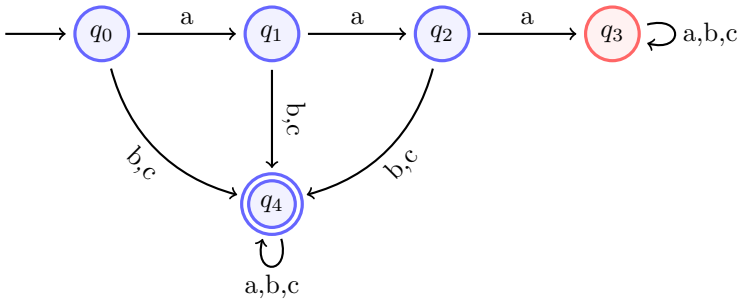
3. $\mathcal{L} = \{w \in \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}^* : \mathrm{abc} \sqsupseteq w\}$

For this language, the word "aaaabc" belongs in the language, but "aaa" does not. For the string "aaaabc" the DFA accepts it as follows.

$q_0\mathrm{a} \to q_1\mathrm{a} \to q_1\mathrm{a} \to q_1\mathrm{a} \to q_1\mathrm{b} \to q_2\mathrm{c} \to q_3$



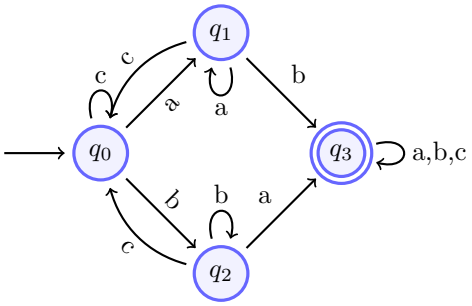
4. $\mathcal{L} = \{w \in \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}^* : \mathrm{aaa} \not\sqsubseteq w\}$



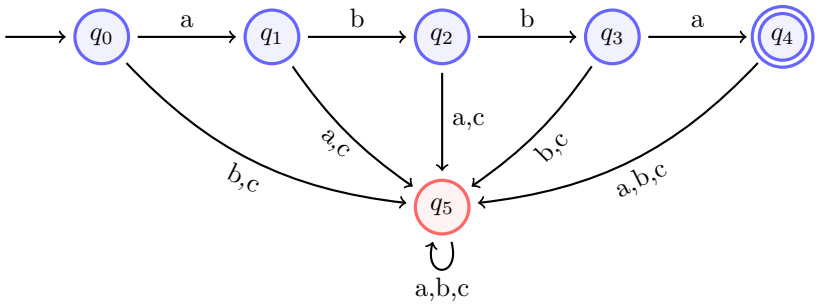
The transition table for this automata is as follows.

	a	b	c
$\rightarrow q_0$	q_1	q_4	q_4
q_1	q_2	q_4	q_4
q_2	q_3	q_4	q_4
q_3	q_3	q_3	q_3
q_4	q_4	q_4	q_4

5. $\mathcal{L} = \{w \in \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}^* : (\exists u, v \in \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}^* : w = u \circ \mathrm{ab} \circ v \vee w = u \circ \mathrm{ba} \circ v)\}$



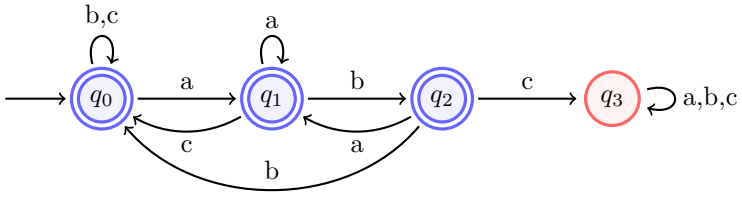
6. $\mathcal{L} = \{\text{abba}\}$ where $\Sigma = \{\text{a}, \text{b}, \text{c}\}$



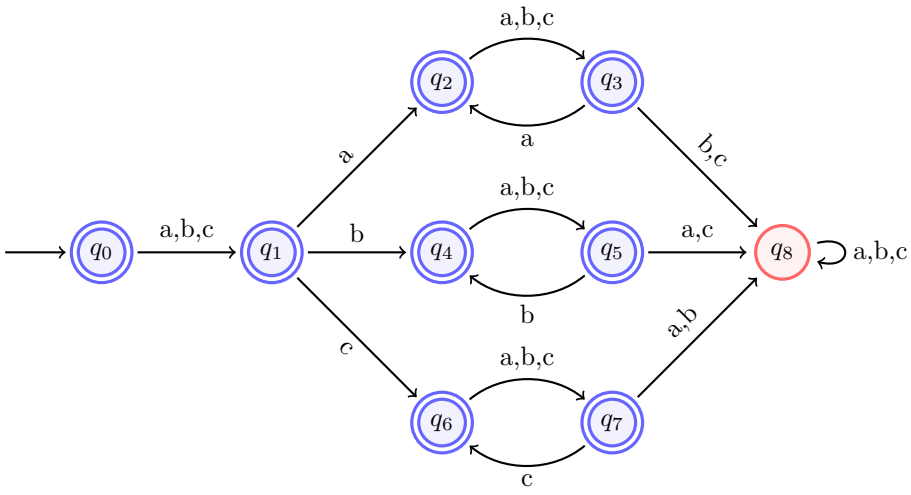
7. $\mathcal{L} = \{w \in \{\text{a}, \text{b}, \text{c}\}^* : \forall u, v \in \{\text{a}, \text{b}, \text{c}\}^*, w \neq u \circ \text{abc} \circ v\}$

For this language, the string "cbabcba" does not belong in the language, but "cbacba" does. The DFA processes the accepted word as follows.

$q_0\text{c} \to q_0\text{b} \to q_0\text{a} \to q_1\text{c} \to q_0\text{b} \to q_0\text{a} \to q_1$



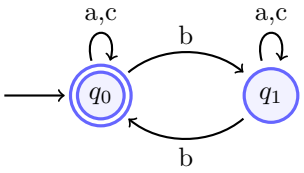
8. $\mathcal{L} = \{w \in \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}^* : w \text{ has the same symbol in all even positions}\}$



The transition table for this automata is as follows.

	a	b	c
$\rightarrow q_0$	q_1	q_1	q_1
q_1	q_2	q_4	q_6
q_2	q_3	q_3	q_3
q_3	q_2	q_8	q_8
q_4	q_5	q_5	q_5
q_5	q_8	q_4	q_8
q_6	q_7	q_7	q_7
q_7	q_8	q_8	q_6
q_8	q_8	q_8	q_8

9. $\mathcal{L} = \{w \in \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}^* : (\exists n \in \mathbb{N} : \#_{\mathrm{b}}(w) = 2n)\}$



10. $\mathcal{L} = \{\text{abaa, abbaa}\}$ where $\Sigma = \{\text{a, b}\}$