Mathematical Logic HW4

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1. (b) $A \to B$ from $\{E, A \to D, D \to (E \to B)\}$.

1.	E	premise
2.	$A \to D$	premise
3.	$D \to (E \to B)$	premise
4.	$A \to (E \to B)$	2, 3, hypothetical syllogism
5.	A	assumption
6.	$E \to B$	4, 5, modus ponens
7.	B	1, 6, modus ponens
8.	$A \to B$	3-5, conditional proof

(d) $A \to G$ from $\{\neg(B \to G) \to \neg A, A \to B\}$

1.	$\neg(B \to G) \to \neg A$	premise
2.	$A \to B$	premise
3.	$A \to (B \to G)$	1, transposition
4.	A	assumption
5.	B	2, 4, modus ponens
6.	$B \to G$	3, 4, modus ponens
7.	G	5, 6, modus ponens
8.	$A \to G$	4-7, conditional proof

(f) $A \vee B$ from $\{A \wedge B\}$

1.	$A \wedge B$	premise
2.	A	1, conjunction elimination
3.	$A \vee B$	2, disjunction introduction

(h) $\neg A$ from $\{A \to (B \vee C), B \to D, \neg C \vee E, (D \vee E) \to \neg A\}$

1.	$A \to (B \vee C)$	premise
2.	$B \to D$	premise
3.	$\neg C \vee E$	premise
4.	$(D \vee E) \to \neg A$	premise
5.	A	assumption
6.	$B \vee C$	1, 5, modus ponens
7.	B	assumption
8.	D	2, 7, modus ponens
9.	$D \vee E$	8, disjunction introduction
10.	$\neg A$	4, 9, modus ponens
11.	C	assumption
12.	E	3, 12, disjunctive syllogism
13.	$D \vee E$	13, disjunction introduction
14.	$\neg A$	4, 14, modus ponens
15.	$\neg A$	6, 7-10, 11-14, disjunction elimination
16.	$\bot$	5, 15, contradiction introduction
17.	$\neg A$	5-16, negation introduction

(j) $(y \to z) \to \neg y$ from $\{r \leftrightarrow s, \neg(r \wedge s), z \to (y \to (r \vee s))\}$

1.	$r \leftrightarrow s$	premise
2.	$\neg(r \wedge s)$	premise
3.	$z \to (y \to (r \vee s))$	premise
4.	$r \to s$	1, biconditional elimination
5.	$s \to r$	1, biconditional elimination
6.	z	assumption
7.	$y \to (r \vee s)$	3, 6, modus ponens
8.	y	assumption
9.	$r \vee s$	7, 8, modus ponens
10.	s	assumption
11.	s	10, reiteration
12.	s	4, 9, 10-11, disjunction elimination
13.	r	assumption
14.	r	13, reiteration
15.	r	5, 9, 13-14, disjunction elimination
16.	$r \wedge s$	12, 15, conjunction introduction
17.	y	assumption
18.	$\neg(r \wedge s)$	2, reiteration
19.	$\neg y$	8-16, 17-18 negation introduction
20.	y	assumption
21.	z	6, reiteration
22.	$y \to z$	20-21, conditional proof
23.	$y \to z$	22, reiteration
24.	$\neg y$	19, reiteration
25.	$(y \to z) \to \neg y$	23-24 conditional proof
26.	$\neg z$	assumption
27.	$y \to z$	assumption
28.	$\neg y$	26, 27, modus tollens
29.	$(y \to z) \to \neg y$	27-28, conditional proof
30.	$z \vee \neg z$	axiom
31.	$(y \to z) \to \neg y$	6-25, 26-29, 30, disjunction elimination

2. (b) Proof for $\neg\neg x \to x$. By deduction, if $\{\neg\neg x\} \vdash x$, then it must be that $\neg\neg x \to x$.

1.	$\neg\neg x$	assumption
2.	$\neg\neg x \to (\neg x \to \neg\neg x)$	axiom 1
3.	$\neg x \to \neg\neg\ x$	1, 2, modus ponens
4.	$\neg x \to \neg\neg x \to ((\neg x \to \neg x) \to x)$	axiom 3
5.	$(\neg x \to \neg x) \to x$	3, 4, modus ponens
6.	$\neg x \to ((\neg x \to \neg x) \to \neg x)$	axiom 1
7.	$\neg x \to ((\neg x \to \neg x) \to \neg x) \to ((\neg x \to (\neg x \to \neg x)) \to (\neg x \to \neg x))$	axiom 2
8.	$(\neg x \to (\neg x \to \neg x)) \to (\neg x \to \neg x)$	6, 7, modus ponens
9.	$\neg x \to (\neg x \to \neg x)$	axiom 1
10.	$\neg x \to \neg x$	8, 9, modus ponens
11.	x	5, 10, modus ponens
12.	$\neg\neg x \to x$	1-11, deduction

3. (b) We must prove $\vdash \neg\alpha \to (\alpha \to \beta)$.

1.	$\neg\alpha$	assumption
2.	$\neg\alpha \to (\neg\beta \to \neg\alpha)$	axiom 1
3.	$\neg\beta \to \neg\alpha$	1, 2, modus ponens
4.	$(\neg\beta \to \neg\alpha) \to ((\neg\beta \to \alpha) \to \beta)$	axiom 3
5.	$(\neg\beta \to \alpha) \to \beta$	3, 4, modus ponens
6.	α	assumption
7.	$\alpha \to (\neg\beta \to \alpha)$	axiom 1
8.	$\neg\beta \to \alpha$	6, 7, modus ponens
9.	β	5, 8, modus ponens
10.	$\alpha \to \beta$	6-9, deduction
11.	$\neg\alpha \to (\alpha \to \beta)$	1-10, deduction

(d) We must prove $\vdash (\neg\beta \to \neg\alpha) \to (\alpha \to \beta)$.

1.	$\neg\beta \to \neg\alpha$	assumption
2.	$(\neg\beta \to \neg\alpha) \to ((\neg\beta \to \alpha) \to \beta)$	axiom 3
3.	$(\neg\beta \to \alpha) \to \beta$	1, 2, modus ponens
4.	$\alpha \to (\neg\beta \to \alpha)$	axiom 1
5.	α	assumption
6.	$\neg\beta \to \alpha$	4, 5, modus ponens
7.	β	3, 6, modus ponens
8.	$\alpha \to \beta$	5-7, deduction
9.	$(\neg\beta \to \neg\alpha) \to (\alpha \to \beta)$	1-8, deduction

4. (a) We must prove $\vdash a \to (b \to a)$.

1.	a	assumption
2.	b	assumption
3.	a	1, reiteration
4.	$b \to a$	2-3, conditional proof
5.	$a \to (b \to a)$	1-4, conditional proof

(b) We must prove $(a \to (b \to c)) \to ((a \to b) \to (a \to c))$.

1.	$a \to (b \to c)$	assumption
2.	$a \to b$	assumption
3.	a	assumption
4.	b	2, 3, modus ponens
5.	$b \to c$	1, 3, modus ponens
6.	c	4, 5, modus ponens
7.	$a \to c$	3-6, conditional proof
8.	$(a \to b) \to (a \to c)$	2-7, conditional proof
9.	$(a \to (b \to c)) \to ((a \to b) \to (a \to c)$	1-8, conditional proof

(c) We must prove $(\neg b \to \neg a) \to ((\neg b \to a) \to b)$.

1.	$\neg b \to \neg a$	assumption
2.	$\neg b \to a$	assumption
3.	b	1, 2, negation introduction
4.	$(\neg b \to a) \to b$	2-3, conditional proof
5.	$(\neg b \to \neg a) \to ((\neg b \to a) \to b)$	1-4, conditional proof