Statistics

Homework 3 – Estimation

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1. (a) Given a sample $x_1, \ldots, x_n$, we are to prove that $\forall k \in \{1, \ldots, n\}$,

$$\hat{\mu}_k = \frac{\sum_{i=1}^{k} x_i}{k}$$

is an unbiased estimator of the average μ. To demonstrate this, we must show that $\mathrm{E}[\hat{\mu}_k] = \mu$.

$$\mathrm{E}[\hat{\mu}_k] = \frac{\sum_{i=1}^{k} \mathrm{E}[x_i]}{k} = \frac{\sum_{i=1}^{k} \mu}{k} = \frac{k\mu}{k} = \mu$$

(b) The larger the value of k, the better the estimator $\hat{\mu}_k$ is. This is because when an estimator's variance from its parameter is small, it is a better estimator. The variance of $\hat{\mu}_k$ is

$$\mathrm{Var}(\hat{\mu}_k) = \mathrm{Var}\left(\frac{\sum_{i=1}^{k} x_i}{k}\right) = \frac{1}{k^2}\,\mathrm{Var}\left(\sum_{i=1}^{k} x_i\right)$$

It is clear from this that the larger k is, the smaller $\frac{1}{k^2}$ is, so the smaller $\mathrm{Var}(\hat{\mu}_k)$ is.