Data Structures 1 - HW2 - Abraham Murciano

1) $5000 = \Theta(1)$

$\lg^3 n$ $\quad \exists\, c, N \text{ s.t. } \forall n > N,\ c\lg^3 n > 5000$

$c = 5000 \quad N = 2. \quad 5000\lg^3 n > 5000$

when $n > 2$.

$3^{\log n}$ $\quad \exists\, c_1, c_2, N\ \forall n > N,\ c_1 3^{\log n} > c_2 \lg^3 n$

$c_1 = 1 \quad c_2 = 0.03 \quad N = 115 \quad 3^{\log n} > 0.03\lg^3 n$

when $n > 115$

$\frac{n}{\log_3 n}$

$n^{1.2}$

$n\lg^2 n$

1.2^n

$(x+1)!$

n^n

2) 1] $f(n) = (n+1)! \qquad g(n) = n! + 1$

$f(n) = (n+1) \times n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$

$g(n) = \qquad n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1 + 1$

$\forall n > 2,\ (n+1)! > n! + 1$

$(n+1)\, n! > n! + 1$

$\Rightarrow f(n) = \Omega(g(n))$

2)

2] $f(n) = n^{\log n}$ $\quad$ $g(n) = n^{2\log n}$

$g(n) = (n^{\log n})^2 = f(n)^2$

$\forall f(n) > 1,$ $\quad$ $g(n) > f(n)$

$\Rightarrow n^{\log n} > 1$

$n \neq 1$

$\Rightarrow \forall n > 1 \quad n^{\log n} < n^{2\log n}$

$\Rightarrow f(n) = O(g(n))$

3] $f(n) = n^3 - 4n^2$ $\quad$ $g(n) = n^{2\sqrt{n}}$

$f(n) = \Theta(n^3)$

$\forall n > N \quad n^3 < n^{2\sqrt{n}}$

$3 < 2\sqrt{n}$

$n > \frac{9}{4}$

$\Rightarrow \forall n > \frac{9}{4} \quad n^3 < \cancel{2}\, n^{2\sqrt{n}}$

$\Rightarrow f(n) = O(g(n))$

3)

	1 (ascending)	2 (descending
`i = 1`	1	1
`while (i < n)`	n	$\frac{1}{6}n^3 + \frac{5}{6}n$
`    if (A[i] > A[i+1])`	$n-1$	$\frac{1}{6}n^3 + \frac{5}{6}n - 1$
`        A[i] ← A[i] + A[i+1]`	0	$\frac{1}{2}n^2 - \frac{1}{2}n$
`        A[i+1] ← A[i] - A[i+1]`	0	$\frac{1}{2}n^2 - \frac{1}{2}n$
`        A[i] ← A[i] - A[i+1]`	0	$\frac{1}{2}n^2 - \frac{1}{2}n$
`        i = 1`	0	$\frac{1}{2}n^2 - \frac{1}{2}n$
`    else`		
`        i++`	$\overset{n}{n-1}$	$\frac{1}{6}n^3 - \frac{1}{2}n^2 + \frac{4}{3}n - 1$

1) ascending order: $1 + n + (n-1) + (n-1) = 3n - 1 = \Theta(n)$

2) descending order: $1 + (\frac{1}{6}n^3 + \frac{5}{6}n)2 - 1 + 4(\frac{1}{2}n^2 - \frac{1}{2}n) + \frac{1}{6}n^3 - \frac{1}{2}n^2 + \frac{4}{3}n - 1$

$= \frac{1}{3}n^3 + \frac{5}{3}n + 2n^2 - 2n + \frac{1}{6}n^3 - \frac{1}{2}n^2 + \frac{4}{3}n - 1$

$= \frac{1}{2}n^3 + \frac{3}{2}n^2 + n - 1 = \Theta(n^3)$

4) c) $f(n) = O(f(n)^2)$ only for when $f(n)$ is a function for which there exists N, such that for all $n > N$, $f(n) \geq 1$

so statement is false.