Discrete Mathematics

Homework 7

Abraham Murciano

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4. (a) i. $4! \times 4! = 576$

   ii. $4! \times 4! \times 2 = 1152$

   iii. $4! \times 4! \times 2 = 1152$

   iv. $8! = 40320$

   (b) i. $5! \times 4! \times 3! \times 3! = 103,680$

   ii. $5! \times 4! \times 5! = 345,600$

   iii. $5! \times 4! \times 4! = 69,120$

   iv. $12! = 479,001,600$

6. (a) $^{12}C_4 = 495$

   (b) $^{3}C_1 \times {}^{9}C_3 = 252$

   (c) $^{12}C_4 - {}^{9}C_4 = 369$

24. Twenty men can be split into four teams of five players each in $\binom{20}{5,5,5,5}$ ways. This is equal to 11,732,745,024.

25. (a) $\binom{4+7-1}{7} = 120$

    (b) $7^4 = 2401$

26. (a) $\binom{7+7-1}{7} = 1716$

    (b) $\binom{7+10-1}{10} = 8008$

27. (b) $\binom{9+15-1}{15} = 490,314$

    (c) $\binom{10+20-1}{20} - \binom{10+15-1}{15} = 8,707,501$

    (e) i. $\binom{9+16-1}{16} = 735,471$

    ii. $\binom{10+21-1}{21} - \binom{10+16-1}{16} = 12,264,175$

28. (a) $(m+1)^n$

    (b) $m!$

    (c) $^{5}C_1 \times {}^{4}C_1 \times 3^3 = 540$

29. (b) There are infinitely many solutions, since there is no lower bound on the variables d and e.

    (d) We must find the number of solutions for te following equation given that all the variables are nonnegative, and a, b, c are odd and d, e are even.

$$a + b + c + d + e = 15$$

Due to the parity restrictions, we have the following equations, where all the variables x' are nonnegative.

$$\begin{aligned}
a &= 2a' + 1 \\
b &= 2b' + 1 \\
c &= 2c' + 1 \\
d &= 2d' \\
e &= 2e'
\end{aligned}$$

We can now substitute each variable from the original equation with an equivalent value from another of the equations, to obtain

$$\begin{aligned}
&2a' + 1 + 2b' + 1 + 2c' + 1 + 2d' + 2e' = 15 \\
&2a' + 2b' + 2c' + 2d' + 2e' = 18 \\
&a' + b' + c' + d' + e' = 9
\end{aligned}$$

Therefore the number of solutions is:

$$\binom{5+9-1}{9} = 715$$

31. (a) i. $$\binom{3+30-1}{30} = 496$$

ii. $$\binom{3+40-1}{40} - \binom{3+29-1}{29} = 396$$

(b) $$\binom{3+300-1}{300} - 3\binom{3+120-1}{120} = 23,308$$

32. (a) $4^3 = 64$ (b) ${}^4C_3 = 4$

33. (a) $$\binom{6+10-1}{10} = 3003$$ (b) $$3003 - \binom{6+10-6-1}{4} = 2877$$

36. Lay the n 1's in a row. There are $n+1$ places where any number of 0's could be inserted; $n-1$ places are between two 1's, and two places are on either end of the row of 1's, as shown in figure 1.

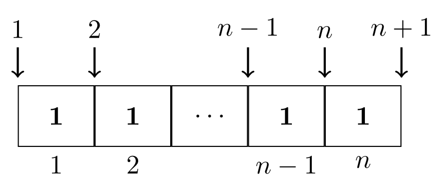

Figure 1: Row of n 1's with $n+1$ spaces to insert 0's

However, since no two 1's may be adjacent, there must be at least one 0 in all the spots 2 through n. Now that we have accounted for the positions of $n-1$ of the 0's, there are $m-(n-1) = m-n+1$ remaining 0's to place, with $n+1$ positions to place them. This gives us the number of ways to arrange n 1's and m 0's with no two 1's being adjacent to be equal to:

$$\binom{(n+1)+(m-n+1)-1}{m-n+1} = \binom{m+1}{m-n+1} = \binom{m+1}{(m+1)-(m-n+1)} = \binom{m+1}{n}$$