Differential Equations

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1 Section 1

A. Given a differential equation and a function $y(x)$, we are to check that y is a solution to the differential equation.

1. For the following differential equation and function $y(x)$,

$$\frac{dy}{dx} = 3y, \qquad y = 4e^{3x}$$

Since we have y, it can be differentiated to find $\frac{dy}{dx}$, then verify that it in fact is equal to $3y$.

$$\frac{dy}{dx} = 12e^{3x} = 3 \cdot 4e^{3x} = 3y$$

3. For the following differential equation and function $y(x)$,

$$\frac{d^2y}{dx^2} + 16y = 0, \qquad y = \sin(4x)$$

This time we must differentiate y twice to find $\frac{d^2y}{dx^2}$.

$$\frac{dy}{dx} = 4\cos(4x) \Rightarrow \frac{d^2y}{dx^2} = -16\sin(4x)$$

Now we can substitute this back into the original equation, and it quite obviously satisfies it.

$$-16\sin(4x) + 16\sin(4x) = 0$$

5. For the following differential equation and function $y(x)$,

$$\frac{dy}{dx} + 2xy = 1, \qquad y = e^{-x^2} \int_0^x e^{t^2}\,dt + ce^{-x^2}$$

we must find $\frac{dy}{dx}$.

$$\begin{aligned}\frac{dy}{dx} &= e^{-x^2}e^{x^2} - 2xe^{-x^2}\int_0^x e^{t^2}\,dt - 2cxe^{-x^2} \\ &= e^0 - 2x\left(e^{-x^2}\int_0^x e^{t^2}\,dt + ce^{-x^2}\right) \\ &= 1 - 2xy\end{aligned}$$

Now we can substitute this into the original differential equation, and we see that is in fact satisfies it.

$$\frac{dy}{dx} + 2xy = 1 - 2xy + 2xy = 1$$

B. **2.** We are given the following differential equation.

$$\frac{dy}{dx} = 4xe^{2x}$$

To find a solution, both sides can be integrated.

$$y = 4\int xe^{2x}dx$$

Now integration by parts can be applied, with $f(x) = x$ and $g'(x) = e^{2x}$. This implies that $f'(x) = 1$ and $g(x) = \frac{1}{2}e^{2x}$. Therefore, by the rule of integration by parts,

$$\int xe^{2x}dx = \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx$$

The next step is to calculate the integral of e^{2x}. To do so, we will substitute $u = 2x$.

$$\int e^{2x}dx = \frac{1}{2}\int e^{u}du = \frac{1}{2}\cdot\frac{e^u}{\ln(e)} = \frac{1}{2}e^{2x} + c$$

Therefore we have

$$\begin{aligned} y &= 4\left(\frac{1}{2}xe^{2x} - \frac{1}{2}\left(\frac{1}{2}e^{2x}\right)\right) + c = 2xe^{2x} - e^{2x} + c \\ &= e^{2x}(2x-1) + c \end{aligned}$$

The graphs for y when $c = 0$, 1, and -6 can be seen in

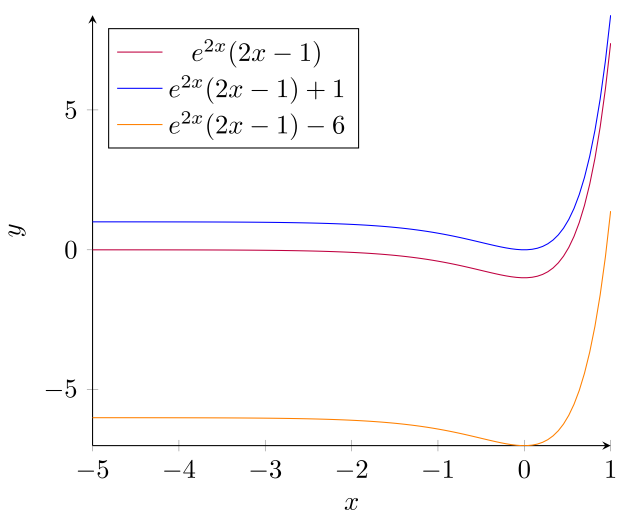

Figure 1: $y = e^{2x}(2x-1) + c$ when $c = 0, 1,$ or -6

4.