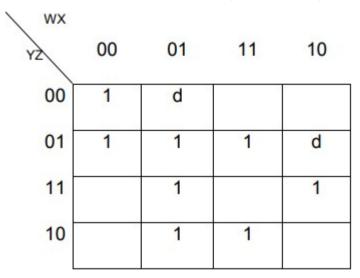# Digital Systems HW 5

Karnaugh Maps

1. Write the minimal expression corresponding to the following map. Indifferent variables are represented by the letter 'd' (don't care).

YZ \ WX	00	01	11	10
00	1	d		
01	1	1	1	d
11		1		1
10		1	1	

$w'y'+w'x+y'z+xyz'+wx'z$

2. Given the following function:
$F(a,b,c,d) = \Sigma(1,2,3,11,12,13,14,15)+ d(5,7,9)$

   a) Make a Karnaugh Map.

	a'b'	a'b	ab	ab'
c'd'	0	0	1	0
c'd	1	0	1	0
cd	1	0	1	1
cd'	1	0	1	0

   b) $ab + acd + a'b'd + a'b'c$

3. Reduce the following expressions to SOP and POS forms using Karnaugh maps:

   a) $x'+xyz'+(x+x'y'z)(x(x'+y'+z)')$

	x'y'	x'y	xy	xy'
z'	1	1	1	0
z	1	1	0	0

   SOP: $x'+yz'$ POS: $(x'+z')(x'+y)$

   b) $(x+xy)(x+x'z)(y'+yz')+(x(y'+z'))'$

	x'y'	x'y	xy	xy'
z'	1	1	1	1
z	1	1	0	1

   SOP: $x'+y'+z'$ POS: $x'+y'+z'$

4. Reduce the following function to SOP and POS forms using Karnaugh maps:
   $F(a,b,c,d,e) = \pi(4,6,7,9,11,12,13,14,15,20,22,25,27,28,30) \cdot \pi_\Phi(1,5,29,31)$

a'	b'c'	b'c	bc	bc'	a	b'c'	b'c	bc	bc'
d'e'	1	0	0	1	d'e'	1	0	0	1
d'e	Φ	Φ	0	0	d'e	1	1	Φ	0
de	1	0	0	0	de	1	1	Φ	0
de'	1	0	0	1	de'	1	0	0	1

SOP: b'c' + e'c' + ab'e

a	b+c	b+c'	b'+c'	b'+c	a'	b+c	b+c'	b'+c'	b'+c
d+e	1	0	0	1	d+e	1	0	0	1
d+e'	Φ	Φ	0	0	d+e'	1	1	Φ	0
d'+e'	1	0	0	0	d'+e'	1	1	Φ	0
d'+e	1	0	0	1	d'+e	1	0	0	1

POS: (a+c')(c'+e)(b'+e')

Quine-Mcclusky Tables

1. Reduce the following function to SOP and POS forms using the Quine-Mcclusky method:

$F_{abcde} = \Pi(6, 7, 14, 15, 17, 19, 21, 23, 25, 29) \cdot \Pi_\phi(1, 5, 9, 13, 18, 22, 30)$
$= \Sigma(0,2,3,4,8,10,11,12,16,20,24,26,27,28,31) + \Sigma_\phi(1,5,9,13,18,22,30)$

0	00000	0,1	0000-	0,1,2,3	000--	0,1,2,3,8,9,10,11	0-0--
1	00001	0,2	000-0	0,1,4,5	00-0-	0,1,4,5,8,9,12,13	0--0-
2	00010	0,4	00-00	0,1,8,9	0-00-	0,2,8,10,16,18,24,26	--0-0
4	00100	0,8	0-000	0,2,8,10	0-0-0	0,4,8,12,16,20,24,28	---00
8	01000	0,16	-0000	0,2,16,18	-00-0	16,18,20,22,24,26,28,30	1---0
16	10000	1,3	000-1	0,4,8,12	0--00		
3	00011	1,5	00-01	0,4,16,20	-0-00		
5	00101	1,9	0-001	0,8,16,24	--000		
9	01001	2,3	0001-	1,3,9,11	0-0-1		
10	01010	2,10	0-010	1,5,9,13	0--01		
12	01100	2,18	-0010	2,3,10,11	0-01-		
18	10010	4,5	0010-	2,10,18,26	--010		
20	10100	4,12	0-100	4,5,12,13	0-10-		
24	11000	4,20	-0100	4,12,20,28	--100		
11	01011	8,9	0100-	8,9,10,11	010--		
13	01101	8,10	010-0	8,9,12,13	01-0-		
22	10110	8,12	01-00	8,10,24,26	-10-0		
26	11010	8,24	-1000	8,12,24,28	-1-00		
28	11100	16,18	100-0	16,18,20,22	10--0		
27	11011	16,20	10-00	16,18,24,26	1-0-0		
30	11110	16,24	1-000	16,20,24,28	1--00		
31	11111	3,11	0-011	10,11,26,27	-101-		
		5,13	0-101	18,22,26,30	1--10		
		9,11	010-1	20,22,28,30	1-1-0		
		9,13	01-01	24,26,28,30	11--0		
		10,11	0101-	26,27,30,31	11-1-		
		10,26	-1010				
		12,13	0110-				
		12,28	-1100				
		18,22	10-10				
		18,26	1-010				
		20,22	101-0				
		20,28	1-100				
		24,26	110-0				

		24,28	11-00					
		11,27	-1011					
		22,30	1-110					
		26,27	1101-					
		26,30	11-10					
		28,30	111-0					
		27,31	11-11					
		30,31	1111-					

	0	2	3	4	8	10	11	12	16	20	24	26	27	28	31
10,11,26,27						x	x					x	x		
26,27,30,31												x	x		x
0,1,2,3,8,9,10,11	x	x	x		x	x	x								
0,1,4,5,8,9,12,13	x			x	x			x							
0,2,8,10,16,18,24,26	x	x			x	x			x		x	x			
0,4,8,12,16,20,24,28	x			x	x			x	x	x	x			x	
16,18,20,22,24,26,28,30									x	x	x	x		x	

SOP: F = abd + a'c' + d'e'

		group	d	Group	d	Group	d
1	00001	1,5	4	1,5,9,13	4,8	1,5,9,13,17,21,25,29	4,8,16
5	00101	1,9	8	1,5,17,21	4,16		
6	01010	1,17	16	1,9,17,25	8,16		
9	01001	5,7	2	5,7,13,15	2,8		
17	10001	5,13	8	5,7,21,23	2,16		
18	10010	5,21	16	5,13,21,29	8,16		
7	00111	6,7	1	6,7,14,15	1,8		
13	01101	6,14	8	6,7,22,23	1,16		
14	01110	6,22	16	6,14,22,30	8,16		
19	10011	9,13	4	9,13,25,29	4,16		
21	10101	9,25	16	17,19,21,23	2,4		
22	10110	17,19	2	17,21,25,29	4,8		
25	11001	17,21	4	18,19,22,23	1,4		
15	01111	17,25	8				
23	10111	18,19	1				
29	11101	18,22	4				
30	11110	7,15	8				
		7,23	16				

		13,15	2					
		13,29	16					
		14,15	1					
		14,30	16					
		19,23	4					
		21,23	2					
		21,29	8					
		22,23	1					
		22,30	8					
		25,29	4					

		6	7	14	15	17	19	21	23	25	29
	5,7,13,15		x		x						
	5,7,21,23		x					x	x		
0-01-	6,7,14,15	x	x	x	x						
	6,7,22,23	x	x								
	6,14,22,30	x		x							
10--0	17,19,21,23					x	x	x	x		
	18,19,22,23						x		x		
---01	1,5,9,13,17,21,25,29					x		x		x	x

POS: (a+c+d')(a'+b+e)(d+e')

2. Given the function:
   f(W,X,Y,Z) = Σ(0,1,3,5,8,10,11,12) + Σ$_\phi$(9,14)
   a) What are the prime implicants of the function?

		Group	d	Group	d
0	0000	0,1	1	0,1,8,9	1,8
1	0001	0,8	8	1,3,9,11	2,8
8	1000	1,3	2	8,9,10,11	1,2
3	0011	1,5	4	8,10,12,14	2,4
5	0101	1,9	8		
9	1001	8,9	1		
10	1010	8,10	2		
12	1100	8,12	4		
11	1011	3,11	8		
14	1110	9,11	2		
		10,11	1		
		10,14	4		
		12,14	2		

PIs are (0,1,8,9), (1,3,9,11), (8,9,10,11), (8,10,12,14) and (1,5)

b) What are the Essential Prime Implicants (EPI) of the function?

EPIs are (0,1,8,9), (1,3,9,11), (8,10,12,14) and (1,5)

c) What is the minimal expression in SOP form?

x'y'+x'z+wz'+w'y'z