Differential Equations

Homework 1 – Ordinary Differential Equations

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March 9, 2020

We are to solve the following linear ordinary differential equation and initial condition.

$$y'(x) + 4y(x) = 2e^{3x} + 3\cos(2x) + 2x - 1 \tag{1}$$
$$y(0) = 2$$

We will first state the homogeneous equation.

$$y_h'(x) + 4y_h(x) = 0$$

Rearranging will give us the following.

$$y_h'(x) = -4y_h(x)$$
$$y_h(x) = -\frac{1}{4}y_h'(x)$$

Now we can find a solution to our homogeneous equation.

$$y_h(x) = Ae^{-4x}$$

Once we have that, we can begin working on our particular solution. First we write it in the form

$$y_p(x) = Be^{3x} + C\sin(2x) + D\cos(2x) + Ex + F \tag{2}$$

Now we will differentiate $y_p(x)$.

$$y_p'(x) = 3Be^{3x} + 2C\cos(2x) - 2D\sin(2x) + E \tag{3}$$

Now, after substituting equations 2 and 3 into equation 1, we can rearrange and equate the coefficients.

$$\begin{aligned}3Be^{3x} + 2C\cos(2x) - 2D\sin(2x) + E + 4(Be^{3x} + C\sin(2x) + D\cos(2x) + Ex + F)\\ = 2e^{3x} + 3\cos(2x) + 2x - 1\end{aligned}$$

$$7Be^{3x} + (2C + 4D)\cos(2x) + (4C - 2D)\sin(2x) + 4Ex + E + 4F = 2e^{3x} + 3\cos(2x) + 2x - 1$$

When comparing the coefficients of the each term, we obtain the following system of equations.

$$\begin{aligned}
7B = 2 \quad &\Rightarrow \quad B = \frac{2}{7}\\
2C + 4D = 3 \quad &\Rightarrow \quad C = \frac{3}{2} - 2D\\
4C - 2D = 0 \quad &\Rightarrow \quad D = \frac{3}{5}, C = \frac{3}{10}\\
4E = 2 \quad &\Rightarrow \quad E = \frac{1}{2}\\
E + 4F = -1 \quad &\Rightarrow \quad F = -\frac{3}{8}
\end{aligned}$$

Thus we have a solution for $y_p(x)$.

$$y_p(x) = \frac{2}{7}e^{3x} + \frac{3}{10}\sin(2x) + \frac{3}{5}\cos(2x) + \frac{1}{2}x - \frac{3}{8}$$

Finally we have a family of solutions for $y(x)$ by adding the homogeneous and the particular solutions.

$$y(x) = Ae^{-4x} + \frac{2}{7}e^{3x} + \frac{3}{10}\sin(2x) + \frac{3}{5}\cos(2x) + \frac{1}{2}x - \frac{3}{8}$$

All that remains is to take the initial condition into account to obtain one single solution from our family of solutions.

$$y(0) = 2 = A + \frac{2}{7} + \frac{3}{5} - \frac{3}{8}$$
$$\Rightarrow A = \frac{417}{280} \approx 1.49$$

Therefore the final solution for $y(x)$ is

$$y(x) = \frac{417}{280}e^{-4x} + \frac{2}{7}e^{3x} + \frac{3}{10}\sin(2x) + \frac{3}{5}\cos(2x) + \frac{1}{2}x - \frac{3}{8}$$