

# Differential Equations

## Homework 1 – Ordinary Differential Equations

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We are to solve the following linear ordinary differential equation and initial condition.

$$\begin{aligned}y'(x) + 4y(x) &= 2e^{3x} + 3\cos(2x) + 2x - 1 \\ y(0) &= 2\end{aligned}\tag{1}$$

We will first state the homogeneous equation.

$$y'_h(x) + 4y_h(x) = 0$$

Rearranging will give us the following.

$$\begin{aligned}y'_h(x) &= -4y_h(x) \\ y_h(x) &= -\frac{1}{4}y'_h(x)\end{aligned}$$

Now we can find a solution to our homogeneous equation.

$$y_h(x) = Ae^{-4x}$$

Once we have that, we can begin working on our particular solution. First we write it in the form

$$y_p(x) = Be^{3x} + C\sin(2x) + D\cos(2x) + Ex + F\tag{2}$$

Now we will differentiate  $y_p(x)$ .

$$y'_p(x) = 3Be^{3x} + 2C\cos(2x) - 2D\sin(2x) + E\tag{3}$$

Now, after substituting equations 2 and 3 into equation 1, we can rearrange and equate the coefficients.

$$\begin{aligned}3Be^{3x} + 2C\cos(2x) - 2D\sin(2x) + E + 4(Be^{3x} + C\sin(2x) + D\cos(2x) + Ex + F) \\ = 2e^{3x} + 3\cos(2x) + 2x - 1\end{aligned}$$

$$7Be^{3x} + (2C + 4D)\cos(2x) + (4C - 2D)\sin(2x) + 4Ex + E + 4F = 2e^{3x} + 3\cos(2x) + 2x - 1$$

When comparing the coefficients of the each term, we obtain the following system of equations.

$$\begin{aligned}7B &= 2 &\Rightarrow & B = \frac{2}{7} \\ 2C + 4D &= 3 &\Rightarrow & C = \frac{3}{2} - 2D \\ 4C - 2D &= 0 &\Rightarrow & D = \frac{3}{5}, C = \frac{3}{10} \\ 4E &= 2 &\Rightarrow & E = \frac{1}{2} \\ E + 4F &= -1 &\Rightarrow & F = -\frac{3}{8}\end{aligned}$$

Thus we have a solution for  $y_p(x)$ .

$$y_p(x) = \frac{2}{7}e^{3x} + \frac{3}{10}\sin(2x) + \frac{3}{5}\cos(2x) + \frac{1}{2}x - \frac{3}{8}$$

Finally we have a family of solutions for  $y(x)$  by adding the homogeneous and the particular solutions.

$$y(x) = Ae^{-4x} + \frac{2}{7}e^{3x} + \frac{3}{10}\sin(2x) + \frac{3}{5}\cos(2x) + \frac{1}{2}x - \frac{3}{8}$$

All that remains is to take the initial condition into account to obtain one single solution from our family of solutions.

$$\begin{aligned} y(0) = 2 &= A + \frac{2}{7} + \frac{3}{5} - \frac{3}{8} \\ \Rightarrow A &= \frac{417}{280} \approx 1.49 \end{aligned}$$

Therefore the final solution for  $y(x)$  is

$$y(x) = \frac{417}{280}e^{-4x} + \frac{2}{7}e^{3x} + \frac{3}{10}\sin(2x) + \frac{3}{5}\cos(2x) + \frac{1}{2}x - \frac{3}{8}$$