Complexity Reduction in Beamforming of Uniform Array Antennas for MIMO Radars

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***Abstract*— Covariance matrix design and beamforming in multiple-input multiple-output (MIMO) radar systems have always been a time-consuming task with a substantial number of unknown variables in the optimization problem to be solved. Based on the radar and target conditions, beamforming can be a dynamic process and in real-time scenarios, it is critical to have a fast beamforming. In this paper, we propose a beampattern matching design technique that is much faster compared to the well-known traditional semidefinite quadratic programming (SQP) counterpart. We show how to calculate the covariance matrix of the probing transmitted signal to obtain the MIMO radar desired beampattern, using a facilitator library. While the proposed technique inherently satisfies the required practical constraints in covariance matrix design, it significantly reduces the number of unknown variables used in the minimum square error (MSE) optimization problem, and therefore reduces the computational complexity considerably. Simulation results show the superiority of the proposed technique in terms of complexity and speed, compared with existing methods. This superiority is enhanced by increasing the number of antennas.**

***Index Terms*— Covariance matrix design, real-time beamforming, multiple-input multiple-output (MIMO) radar, semidefinite quadratic programming (SQP), minimum square error (MSE).**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) radar systems have been attracting considerable attention recently due to their performance advantages such as higher spatial resolution, improved target identifiability, waveform diversity and flexibility to design a variety of transmit beampatterns [1], [2], [3]. They have the potential to significantly improve radar remote sensing performance in a number of important applications including airborne surface surveillance, over-the-horizon radars [4], [5], [6] and tracking multiple targets [7]. The concept of MIMO systems is not new. MIMO techniques have experienced great success in other radio frequency (RF) systems, mostly in wireless communications. The important enabler features for both radar and communication systems to benefit from MIMO techniques are generally the same, however, the performance metrics and implementation approaches are quite different. In communications systems, MIMO antennas enable improved channel capacity in complex propagation and scattering environments dominated by multipath propagation. Similar to MIMO communications that can develop wireless network and improve the capability of communication, MIMO radar systems can achieve high performance in signal processing [8], [9]. MIMO radars have numerous advantages over uniform phased array (UPA) radar counterpart; namely decreasing sidelobe levels (SLLs) in designing beampatterns, reducing the signal-to-interference-plus-noise ratio (SINR) and controlling cross-correlations. Radars with UPA antennas transmit fully correlated waveforms with possibly different phases and amplitudes, therefore they are considered to be single-input single output (SISO) systems. In MIMO radars, the waveforms that are transmitted by different antennas can be orthogonal or have a percentage of orthogonality and this degree of freedom provides MIMO radars some features that not present in SISO systems such as a wide field of view (FOV) and the ability to create virtual arrays [10], [11].

MIMO radars can be classified into two types: widely distributed and collocated MIMO radars. In widely distributed antennas, array elements are physically separated with a large space, enough for each antenna to see different targets' radar cross-sections (RCSs). These radars can enhance spatial diversity as well as detection. In collocated MIMO radars the transmitting antennas are located at small distances from each other, and their transmitted waveforms can be completely independent (or orthogonal), partially correlated or completely correlated. There are other categories of MMO radar systems such as monostatic, bistatic and multistatic. Monostatic MIMO radars use unique antennas as transmitters and receivers. In bistatic radars, the transmitters are in one place and the receivers are in another place, and multistatic radars are those that have several transmitters in one place and several receivers in another place. In this paper, the focus is on collocated and monostatic MIMO radars.

In MIMO radars, waveform design plays a significant role in achieving desired advantages in numerous applications such as beamforming with integrated sidelobe levels (ISLs) constraints [12], antenna selection [13], spectrally compatible applications [14], [15] and etc. The collocated MIMO radar

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waveform design can be divided into two categories; one on the receiver side for achieving maximum output SINR [11], [16], [17] which can enhance the target detection performance and suppress signal-dependent interferences. The other is to design MIMO radar waveforms with desirable transmit beampattern to control the radiation power distribution on the transmitter side. The second category itself can be classified into two subclasses of direct and indirect waveform design. In direct waveform design, the aim is to design transmitted signals from antenna to obtain desired characteristics. Therefore, the main aim in optimization problem is to design the probing waveform directly, however, indirect waveform design refers to design of the covariance matrix in the first step, then deriving the waveforms from the achieved covariance matrix in the second step. Therefore, the waveform covariance matrix design problem is considered as the main aim by proposing the transmit beampattern design metrics to match a desired transmit beampattern or to solve the minimum square error (MSE) problem and minimize a penalty term via optimization methods. The penalty term is the cross-correlation of composing different direction signals for defined angles in the region of interest. In direct waveform design problems, constant modulus (CM) and positive semidefinite (PSD) constraints are the practical constraints that mostly considered in waveform design optimization problem. CM is a practical constraint for increasing the efficiency of power amplifiers (PAs) used behind the antennas in MIMO radars [18]. It applies equivalent waveform transmission from antenna elements in each time (or sample). In indirect waveform design problems, PSD constraint is essential for the realization of waveforms and the constraint of equality of the diagonal elements of the covariance matrix which is necessary for avoiding the destructive effects on the PAs operation in the saturation region. For instance in the direct waveform design problem, [19] considered waveform design in space domain by selecting appropriate phases of the waveforms from all MIMO radar antenna elements to obtain the desired beampattern. In their proposed algorithm, the waveform transmitted from an antenna element must be coded in time domain to be orthogonal or nearly orthogonal. In [20], the binary transmit waveforms are designed via minimizing the beampattern integrated sidelobe to mainlobe ratio and likelihood ascent search (LAS) method is used as a solution for optimization problem. Additionally, [21] applies two algorithms based on the alternating direction method of multipliers (ADMM) method to obtain the desired beampattern in directly probing waveform design. The ADMM algorithm is a distributed optimization approach with the numerical robustness of the augmented Lagrangian method. Based on the ADMM framework, non-convex polynomial functions and non-convex multi-constraint problems could be solved [22].

Indirect waveform design is considered in the literature such as [23] where the covariance matrix design is stated as a semidefinite quadratic programming (SQP) problem for obtaining the elements of a square-root matrix of the covariance matrix based on parametrizing coordinate of a hypersphere with considering the practical constraints. In [24] Toeplitz matrices are proposed for MSE problem with low computational complexity while fulfilling the necessary constraint. Also in [25] the iterative methods are utilised for designing the covariance matrix and used the barrier method as a simple technique for solving convex optimization and synthesise BPSK waveforms which satisfy CM constraint. Reference [26] used to convert a constraint problem to an unconstraint one and extract the covariance matrix. In [27] and [28] a closed-form method is presented to design the covariance matrix for a uniform linear array that uses the discrete Fourier transform (DFT) by reducing the complexity of the iterative methods. Also in [29] and [30] several covariance matrices are proposed that satisfy PSD and the equality of the diagonal elements of the covariance matrix constraints and then BPSK waveforms which realise these covariance matrices are generated. To this end, synthesising transmit waveform under practical constraint (CM or peak to average power ratio (PAPR)) after obtaining covariance matrix is reviewed in [25], [30], and [31]. Reference [32] applies cyclic algorithm (CA) to generate CM waveform with specific covariance matrix. In [33] an effective algorithm is proposed based on multi-variable optimization problem of designing the CM transmitter waveform for a collocated MIMO radar to meet a desired output beampattern with both indirect and direct waveform design methods. Waveform covariance matrix design problem in different scenarios is solved in [34] by well-known SQP method, where different covariance matrices are generated in different applications. Two important form of objects that are solved by CVX [35] and considered in [34] are beampattern matching design problem and minimum sidelobe level beampattern design problem. However, in [23], [34], [35], and [36] the focus is on minimising the MSE problem between the achieved beampattern and the desired beampattern, and little attention is paid to SLL suppression, mainlobe ripple constraints and wide beampattern design.

In this paper, we focus on the indirect waveform design in collocated and monostatic MIMO radar. We propose a new simple and fast approach for solving MSE problem to design the covariance matrix under practical constraints. The main contribution of this paper is to provide a fast and real-time covariance matrix design method to achieve the desired beampattern with appropriate specification in the mainlobe and SLLs, using our proposed facilitator libraries. We introduce the facilitator libraries that include a set of the covariance matrices which are designed based on simple UPA structure as UPA library and minimum SLL beampattern design problem solution [34] as SLL library. In the proposed technique, the desired covariance matrix in MSE problem can be written as a linear combination of stored covariance matrices in the library with corresponding coefficients. Then we propose an algorithm to find the coefficients as unknown variables by solving an optimization problem to achieve the best covariance matrix in beampattern matching design problem. In this approach, we will show that we can achieve a significant reduction in the number of unknown variables, a considerable decrease in the computational complexity and a great reduction in the time consumption compared with well-known counterpart SQP and when there is a large number of antennas, this difference becomes even more significant. The novel aspects of the proposed technique can be highlight as follows:

- Beampattern design in radar systems should be fast and therefore, providing a method for fast and real-time waveform design is one of the real-time system requirements. The proposed technique reduces the number of variables in the optimization problem and hence reduces the complexity and increases the speed of algorithm compared with other methods in the literature.
- PSD constraint and the uniform elemental transmit power constraint for proposed covariance matrix are already satisfied for all matrices stored in the proposed facilitator libraries, and these conditions also exist for the linear combinations of the called covariance matrices of the libraries. Therefore, by establishing these two practical conditions we can omit them in the optimization problem and this, significantly facilitates the solution of the corresponding optimization problem.

The rest of the paper is arranged as follows. The problem formulation is presented in Section II. The proposed technique in optimal design is explained in Sections III. Simulation results are illustrated in Section IV and finally conclusions are drawn in Section V.

*N*otation: Bold upper case letters $\mathbf{X}$ and lower case letters x, respectively denote matrices and vectors. Conjugate transposition, conjugate and transposition of a matrix denoted by $(.)^H$, $(.)^*$ and $(.)^T$, respectively, and statistical expectation is denoted by E{.}. The *(m,n)*th element of a matrix is denoted by $\mathbf{X}_{mn}$. $\mathbf{X} \geq 0$ shows the matrix $\mathbf{X}$ is PSD.

II. Problem Formulation

Consider a collocated and monostatic MIMO radar system with M number of transmit antennas that are placed in distance d (it could be assumed half wavelength). Let be the discrete time radar waveform radiated of mth antenna for $m = 1, \ldots, M$. Also, $n = 1, \ldots, N$ denotes the number of samples of each radar waveform transmitted by each antenna. It is assumed that the transmitted signals are narrow-band and the propagation is non-dispersive. The steering vector of the array is defined as follows

$$a(\theta) = [1\ e^{j2\pi \frac{d}{\lambda}\sin(\theta)}\ \ldots\ e^{j2\pi \frac{(M-1)d}{\lambda}\sin(\theta)}]^T, \tag{1}$$

where λ is the carrier signal wavelength and the baseband transmitted signal vector at each time for the nth sample can be written as

$$x(n) = [x_1(n)\ x_2(n)\ \ldots\ x_M(n)\,]^T. \tag{2}$$

Under the assumption that the transmit antenna of the MIMO radar systems is calibrated, that is, $a(\theta)$ is a known function of θ, the received signal by a target located at angle θ can be given by

$$r(n;\theta) = a^T(\theta)x(n). \tag{3}$$

Therefore, the transmit beampattern of the transmitted signal at the location θ is given by

$$P(\theta) = E\{a^T(\theta)x(n)x^H(n)a^*(\theta)\} = a^H(\theta)\mathbf{R}a(\theta), \tag{4}$$

where $\mathbf{R} = \mathrm{E}\{x(n)x^{\mathrm{H}}(n)\}$ is the covariance matrix of the transmitted waveforms. In the indirect method, where the beampattern design problem is to synthesize covariance matrix, the equality of the diagonal elements of covariance matrix and PSD constraint as practical constraints can be expressed respectively as

$$\begin{aligned} R_{mm} &= \frac{c}{M}, \qquad m = 1, \ldots, M, \\ \mathbf{R} &\geq 0, \end{aligned} \tag{5}$$

where R_{mm} denotes the *(m,m)*th element of $\mathbf{R}$ and c is the total transmit power of the array. These practical constraints are essential for efficiency of PAs in MIMO radars. Next the problem of finding $\mathbf{R}$ in indirect method for achieving desired beampattern can be expressed as two states: beampattern matching design and minimum sidelobe level beampattern design [34] which are described as follows.

A. Beampattern Matching Design

In the beampattern matching design problem, the goal is maximising the total spatial power at a number of given target locations, or match achieved beampattern with a desired one as $P_d(\theta)$ which is known by MSE problem. We assume that a grid of target locations is over $\{\theta_k\}$ where $k = 1, \ldots, K$, with the total number of given target locations K. The aim is to choose the best covariance matrix $\mathbf{R}$ such that the achieved beampattern by (4) matches to the desired beampattern $P_d(\theta)$ over the range of interests. Therefore, the main problem is considered as

$$\begin{aligned} \min_{\mathbf{R}} \quad & \frac{1}{K}\sum_{k=1}^{K} \omega_k [P_d(\theta_k) - a^H(\theta_k)\mathbf{R}a(\theta_k)]^2 \\ s.t \quad & R_{mm} = \frac{c}{M}, \qquad m = 1, \ldots, M \\ & \mathbf{R} \geq 0 \end{aligned} \tag{6}$$

where $\omega_k \geq 0$ is the weight for the kth space grid point. If we assume that the covariance matrix for M antennas is symmetric, there would be $\frac{M^2-M}{2}$ complex unknown variables to solve (6) by CVX toolbox [35].

B. Minimum Sidelobe Level Beampattern Design

In some applications in MIMO radar systems, the beampattern design problem is considered as minimising the SLL in a certain angle range, when pointing the MIMO radars toward single angular (only like θ_0). Minimum SLL beampattern design problem, with PSD constraint and the uniform elemental transmit power constraint for covariance matrix, can be expressed as

$$\begin{aligned} \min_{t,\mathbf{R}} \ & -t \\ s.t \ & \\ & a^H(\theta_0)\mathbf{R}a(\theta_0) - a^H(\mu_l)\mathbf{R}a(\mu_l) \geq t,\ \forall \mu_l \in \Omega \\ & a^H(\theta_1)\mathbf{R}a(\theta_1) = 0.5a^H(\theta_0)\mathbf{R}a(\theta_0) \\ & a^H(\theta_2)\mathbf{R}a(\theta_2) = 0.5a^H(\theta_0)\mathbf{R}a(\theta_0) \\ & \mathbf{R} \geq 0 \\ & R_{mm} = \frac{c}{M},\ m = 1, \ldots, M \end{aligned} \tag{7}$$

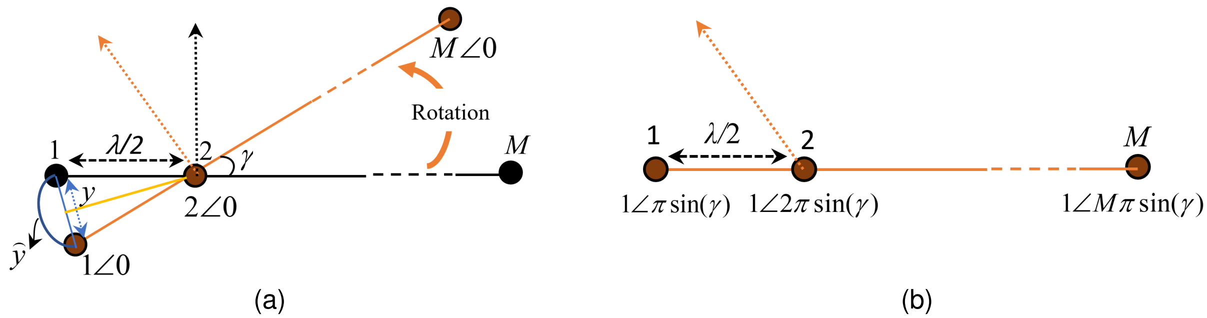

Fig. 1. (a) An example of UPA antennas and beampattern configuration and mechanical rotated systems, (b) electrical rotated UPA as a substitution.

where $\theta_2 - \theta_1 (\theta_1 < \theta_0 < \theta_2)$ determines the 3 dB main bandwidth and is a discrete angle range that covers the side-lobe region of interest. While this problem is a semi-definite program (SDP) and can, therefore, be efficiently solved numerically, it does not seem that have a closed-form solution. Therefore, the goal here is to choose an $\mathbf{R}$ that makes minimum sidelobe beampattern in (4). This problem can be solved by CVX toolbox [35].

III. Proposed Method in Optimal Design

In this section, we describe our method for covariance matrix design using the proposed library. In a given beam-pattern matching design problem with a desired beampattern $P_d(\theta)$, our focus would be to solve the MSE problem to obtain the covariance matrix in a simpler and faster method and get $P_d(\theta)$ using two facilitator libraries. Our proposed libraries include a set of certain covariance matrices. These stored covariance matrices satisfied practical constraints and can be created based on specific parameters like focusing on mainlobe or reduction of sidelobe. In the following the libraries instruction is explained conceptually and then the way of using them to solve MSE problem will be presented in two scenarios using first proposed library as a UPA Library (say UPA-LIB) and the second proposed library as a SLL library (say SLL-LIB), respectively. Details of the proposed method and its significant impact on reducing computational complexity have also been examined.

A. First Scenario: Beampattern Matching Design With UPA-LIB

Starting from the UPA with M number of antennas that are spaced half wavelength. Beampattern in different directions could be simply achieved with mechanical rotation of elements. Let consider Fig. 1a as an example of mechanical rotation of UPA elements with rotation angle γ and the arc length that we consider equivalent with $\widehat{y}$. According to the geometry of Fig. 1a, $\widehat{y}$ is estimated by $\frac{\lambda}{2}\sin(\gamma)$. If we consider electrical rotation or phase changes in each UPA element as instead of mechanical rotation, therefore the phase difference for the mth element is equivalent $\Delta\phi = m\frac{2\pi}{\lambda}\widehat{y}$, where $m = 1, \ldots, M$. The electrical rotation of UPA elements is depicted in Fig. 1b. For small value of γ, phase difference equivalents with $\Delta\phi = m\pi\sin(\gamma)$. Let us consider the amplitude and phase of transmitted signal from each element in mechanical rotation as $1\angle 0$ while in electrical rotation it changes to $1\angle m\pi\sin(\gamma)$ for the mth antenna element. It means that according to Fig. 1b for $1, \ldots, M$ number of antennas, the excitations are $1\angle\pi\sin(\gamma), \ldots., 1\angle M\pi\sin(\gamma)$. Next, we present a covariance matrix as

$$\mathbf{R} = \begin{bmatrix} e^{j\pi\sin(\gamma)} \\ \vdots \\ e^{j\pi M\sin(\gamma)} \end{bmatrix} \left[e^{-j\pi\sin(\gamma)} \ \ldots \ e^{-j\pi M\sin(\gamma)} \right],$$
$$R_{l,k} = e^{j\pi(l-k)\sin(\gamma)}, \qquad k, l = 1, \ldots, M \tag{8}$$

where $R_{l,k}$ represents each element in covariance matrix $\mathbf{R}$. The value of the γ can be substituted for angles that include in the range of the radar's FOV. Let's consider $\{\tilde{\theta}_i\}_{i=1}^{I}$ to represent the angles of radar's FOV where I indicates the number of angles in FOV. Based on simulation experiences, it is recommended to select a minimum distance of one degree between the angles in FOV angles. By substituting $\{\tilde{\theta}_i\}_{i=1}^{I}$ for γ in (8), the corresponding covariance matrix can be calculated for each angle in FOV range. Therefor, in M number of antennas, the covariance matrix calculated using in (8) for each angle in range $\{\tilde{\theta}_i\}_{i=1}^{I}$ is referred to as $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$. Next, we store the calculated covariance matrices, denoted as $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$ and consider them as members of a library. The instruction of our first proposed library, known as UPA-LIB, is based on this procedure. The procedure of finding $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$ using (8), for any number of antennas and for any angle range in radar FOV as $\{\tilde{\theta}_i\}_{i=1}^{I}$ is very fast. It also should be noted that the covariance matrices $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$ in (8) satisfy practical constraints, such as the equality of its diagonal elements and PSD. The procedure for constructing UPA-LIB is summarized in Algorithm 1.

Algorithm 1 Proposed Algorithm to Create UPA-LIB

1. **Input**: M and $\{\tilde{\theta}_i\}_{i=1}^{I}$
2. **Output**: $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$
3. **for**: $i = 1$ to $i = I$ do
4. Import $\tilde{\theta}_i$ (from angel range $\{\tilde{\theta}_i\}_{i=1}^{I}$).
5. Substitute $\tilde{\theta}_i$ for γ in (8).
6. Calculate $\mathbf{R}$ in (8) and call it $\tilde{\mathbf{R}}_i$.
7. Save $\tilde{\mathbf{R}}_i$.
8. **end**
9. Export $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$.

Next, the collection of saved covariance matrices $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$, which are considered as members of UPA-LIB, is utilized to solve the MSE problem to achieve the desired beampattern. Therefore, in the proposed technique "beampattern matching design with UPA-LIB" we call stored covariance matrices from UPA-LIB as $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$ and consider unknown coefficients as $\{\beta_i\}_{i=1}^{I}$ for these covariance matrices. These unknown coefficients are considered as variables in the proposed optimization problem. Next, we propose new covariance matrix as the weighted linear summation of called covariance matrices in the form of $\sum_{i=1}^{I}\beta_i\tilde{\mathbf{R}}_i$. The goal is to find variables $\{\beta_i\}_{i=1}^{I}$ to find new covariance matrix and reach desired beampattern in presented optimization problem using UPA-LIB as

$$\begin{aligned} \min_{\beta_i} \quad & \frac{1}{K}\sum_{k=1}^{K}\omega_k[P_d(\theta_k) - a^H(\theta_k)(\sum_{i=1}^{I}\beta_i\tilde{\mathbf{R}}_i)a(\theta_k)]^2 \\ s.t \quad & \beta_i \geq 0, \qquad i = 1,\ldots,I \end{aligned} \tag{9}$$

where $\omega_k \geq 0$ shows the weight for the kth space grid in $k = 1,\ldots,K$, $\tilde{\mathbf{R}}_i$ is the ith called covariance matrix from UPA-LIB and β_i is the ith corresponding variable. It is noteworthy that (9) is an optimization problem in quadratic form, which has a closed-form solution. Solving this optimization problem does not necessarily guarantee the positive definite property of the covariance matrix, hence a constraint of positive coefficients has been added to the problem, which naturally requires numerical methods for its solution and a closed-form is not available for its solution. The equality diagonal elements and PSD constraints for covariance matrix, in comparison with (6), is omitted here because these two practical constraints are already satisfied in UPA-LIB and positive coefficients $\{\beta_i\}_{i=1}^{I}$. It means that if $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$ meet these two constraints, then the linear summation of multiplication coefficients and called covariance matrices that are considered as $\sum_{i=1}^{I}\beta_i\tilde{\mathbf{R}}_i$ would be satisfied these two constraints. The variables are optimized using the CVX toolbox [35] and this leads to the desired beampattern. The procedure of the proposed algorithm of finding the covariance matrix in beampattern matching problem with UPA-LIB is given in Algorithm 2.

Algorithm 2 Proposed Algorithm to Find Covariance Matrix in Beampattern Matching Problem Using UPA-LIB

1. **Input**: M, $P_d(\theta)$ and $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$ (from UPA-LIB)
2. **Output**: $\mathbf{R} = \sum_{i=1}^{I}\beta_i\tilde{\mathbf{R}}_i$ and $P(\theta)$
3. Consider variables $\{\beta_i\}_{i=1}^{I}$.
4. **for**: $i = 1$ to $i = I$ do
5. Import $\tilde{\mathbf{R}}_i$ from UPA-LIB.
6. Calculate $\sum_{i=1}^{I}\beta_i\tilde{\mathbf{R}}_i$.
7. **end**
8. Calculate $\{\beta_i\}_{i=1}^{I}$ via (9).
9. Calculate $\mathbf{R} = \sum_{i=1}^{I}\beta_i\tilde{\mathbf{R}}_i$ and $P(\theta)$ via (4).

We will show in simulation results in section IV that the number of unknown variables $\{\beta_i\}_{i=1}^{I}$ in our technique is considerably lower than other counterpart techniques and it makes our optimization problem to be simple and fast. In order to prove our claim, we will compare our proposed method in Algorithm 2, with the well-known beampattern matching design problem (SQP method) in [34] where, for M number of antennas, $\frac{M^2-M}{2}$ complex variables must be determined in MSE problem while in our proposed technique, variables $\{\beta_i\}_{i=1}^{I}$ are real and their number is considerably lower than mentioned counterpart. The reducing of the number of variables and consequently the reduction of computational complexity, are more evident when the number of antennas increases. We will compare the performance of the proposed algorithm by runnng over 100 trials in simulations in section IV. The time consumed in our first proposed technique is composed of two terms. The first term t_0 is related to the UPA-LIB creation time that can be done simply and very fast. The second term t_i is adjusted for applying Algorithm 2 in each turn. Therefore, the average computational time is $\sum_{i=1}^{I}\frac{t_0+t_i}{100} \simeq t_i$.We will show that our proposed technique computational time is significantly lower than its counterpart. In sections III-C and IV, more details have been addressed.

B. Second Scenario: Beampattern Matching Design With SLL-LIB

In this section, we introduce another advanced library called SLL-LIB. The SLL-LIB consists of a set of covariance matrices, which are obtained by solving the minimum SLL problem as defined in (7). Each member of SLL-LIB represents a covariance matrix that satisfies the minimum SLL criteria. Unlike UPA-LIB, where the covariance matrices are obtained quickly through a closed-form relation (8), the covariance matrices in SLL-LIB are obtained by solving the more time-consuming optimization problem (7). Therefore, to save time, covariance matrices are generated only once across a wide range of angles, while ensuring a minimum distance of one degree between each angle and utilize them offline for different real-time beampattern matching design problem. It means that for every number of antennas, an offline SLL-LIB could be created and then can be used in different simulations for solving any real-time beampattern matching design problem in order to achieve the desired characteristic of lower SLLs compared with UPA-LIB. Let us consider the angles as $\{\hat{\theta}_j\}_{j=1}^{J}$ where J is the number of angles. We state θ_0 in (7) as centre phase in angle range of $\{\hat{\theta}_j\}_{j=1}^{J}$ for M antennas and consider $\theta_2 - \theta_1 = \sin^{-1}(\frac{1}{M})$ and $\Omega = [-90^\circ \sim (\theta_0 - \sin^{-1}(\frac{3}{M}))] \cup [(\theta_0 + \sin^{-1}(\frac{3}{M})) \sim 90^\circ]$. Therefore, we calculate all the corresponding covariance matrices by (7) and store them as $\{\hat{\mathbf{R}}_j\}_{j=1}^{J}$. The collections of these saved covariance matrices create SLL-LIB that is applied to solve the MSE problem. The procedure for constructing SLL-LIB is summarized in Algorithm 3.

It is important to emphasize that the covariance matrices $\{\hat{\mathbf{R}}_j\}_{j=1}^{J}$ adhere to practical constraints, including the equality of its diagonal elements and PSD. Once the SLL-LIB has been established and stored, our next objective is to utilize it to solve the proposed technique "beampattern matching design problem with SLL-LIB". The procedure for solving MSE problem is similar to the beampattern matching design with UPA-LIB. We call stored covariance matrices from SLL-LIB which are considered as $\{\hat{\mathbf{R}}_j\}_{j=1}^{J}$ and introduce unknown coefficients as $\{\eta_j\}_{j=1}^{J}$ corresponding to these

Algorithm 3 Proposed Algorithm to Create SLL-LIB

Input: M and $\{\hat{\theta}_j\}_{j=1}^{J}$
Output: $\{\hat{\mathbf{R}}_j\}_{j=1}^{J}$
for: $j = 1$ to $j = J$ do
  Import $\hat{\theta}_j$ (from angel range $\{\hat{\theta}_j\}_{j=1}^{J}$).
  Substitute $\hat{\theta}_j$ for θ_0 in (7).
  Calculate $\theta_2 - \theta_1 = \sin^{-1}(\frac{1}{M})$ in (7).
  Calculate Ω in (7),

$$\Omega = [-90^\circ \sim (\theta_0 - \sin^{-1}(\frac{3}{M}))] \cup [(\theta_0 + \sin^{-1}(\frac{3}{M})) \sim 90^\circ].$$

  Calculate $\mathbf{R}$ in (7) and call it $\hat{\mathbf{R}}_j$.
  Save $\hat{\mathbf{R}}_j$.
end
Export $\{\hat{\mathbf{R}}_j\}_{j=1}^{J}$.

covariance matrices and state the proposed covariance matrix as a linear combination of them in the form of $\sum_{j=1}^{J} \eta_j \hat{\mathbf{R}}_j$. These coefficients are considered as unknown variables in the proposed problem. Similar to UPA-LIB, the practical constraints for covariance matrix in (5) are omitted. It means that in both cases, beampattern matching design with UPA-LIB and SLL-LIB, the covariance matrices have definite positive elements and the main diagonal elements are equal. Therefore, their linear combinations with positive real coefficients ensure the final covariance matrix is PSD and its diagonal elements are equal. The formation of new beampattern matching design problem can be stated as (10) and solved by CVX.

$$\begin{aligned} \min_{\eta_j} \quad & \frac{1}{K}\sum_{k=1}^{K} \omega_k [P_d(\theta_k) - a^H(\theta_k)(\sum_{j=1}^{J} \eta_j \hat{\mathbf{R}}_j) a(\theta_k)]^2 \\ s.t \quad & \eta_j \geq 0, \qquad j = 1, \ldots, J \end{aligned} \tag{10}$$

where $\hat{\mathbf{R}}_j$ is the jth called covariance matrix from SLL-LIB and η_j is the jth coefficient. $\sum_{j=1}^{J} \eta_j \hat{\mathbf{R}}_j$ defined as linear combination of weighted called covariance matrix as our desired final covariance matrix. The only constraint in (10) is that η_j must be PSD. The pseudo code for the second proposed algorithm to find covariance matrix in beampattern matching problem by SLL-LIB is given in Algorithm 4.

Algorithm 4 Proposed Algorithm to Find Covariance Matrix in Beampattern Matching Problem Using SLL-LIB

Input: M, $P_d(\theta)$ and $\{\hat{\mathbf{R}}_j\}_{j=1}^{J}$ (from SLL-LIB)
Output: $\mathbf{R} = \sum_{j=1}^{J} \eta_j \hat{\mathbf{R}}_j$ and $P(\theta)$.
Consider variables $\{\eta_j\}_{j=1}^{J}$.
for: $j = 1$ to $j = J$ do
  Import $\hat{\mathbf{R}}_j$ from SLL-LIB.
  Calculate $\sum_{j=1}^{J} \eta \hat{\mathbf{R}}_j$.
end
Calculate $\{\eta\}_{j=1}^{J}$ via (10).
Calculate $\mathbf{R} = \sum_{j=1}^{J} \eta_j \hat{\mathbf{R}}_j$ and $P(\theta)$ via (4).

C. Computational Complexity Analysis

In order to calculate the complexity of a problem, it is executed multiple times (100 times here) for different values of input. The change in the average execution time is considered as a measure of complexity based on the number of inputs. Fig. 2 shows the complexity for the proposed method compared to the SQP method in two scenarios UPA-LIB and SLL-LIB. It should be noted that the complexity obtained through SQP method matches the complexity mentioned in the reference [37] and is approximately $\mathcal{O}(M^{3.5})$. Additionally, the complexity of the proposed method has been reported in the worst case $\mathcal{O}(M^{1.18})$ and $\mathcal{O}(M^{1.25})$ for beampattern matching design with UPA-LIB and SLL-LIB, respectively. A significant reduction in complexity is observed in the proposed method compared to the SQP method.

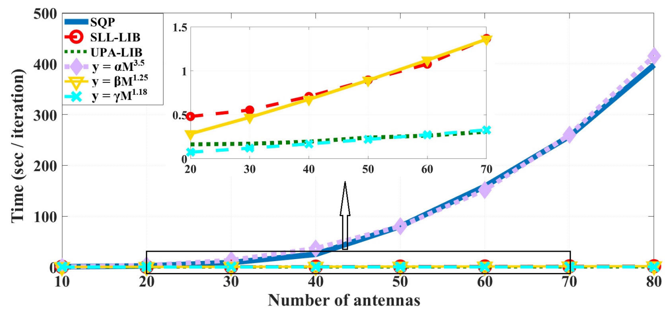

Fig. 2. Complexity result between SQP and our proposed methods.

IV. Simulation Results

In this section, several numerical simulations are conducted to assess the performance of the proposed algorithm in real-time beampattern matching design problem with the proposed libraries. In all simulations, we assume an array with half-wavelength element spacing and the range of angle to be $[-90^\circ \sim 90^\circ]$ with the resolution of 1° which gives $K = 181$ grid points and assume $c = 1$. For all simulations, the desired beampattern is $P_d(\theta)$. we evaluate the performance of the proposed algorithm by conducting over 100 trials in simulation and provide the timetables for each simulation. Besides, our personal computer that these simulations are conducted on has the configurations of 64-bit Intel i7-8550U CPU and 16GB RAM.

A. Test on Beampattern Matching Design With UPA-LIB

In this subsection, experiments are conducted to testify the performance of the proposed method using UPA-LIB. For comparison purposes, we consider the well-known counterpart, i.e., the SQP method based on beampattern matching design method in [34] to design the covariance matrix. In the first simulation, we consider a desired beampattern matching problem as follows

$$P_d^1(\theta) = \begin{cases} 1 & \theta \in \{[-20, -10] \cup [10, 20]\} \\ 0 & otherwise, \end{cases} \tag{11}$$

which can be considered as two mainlobes with centres at $\theta = -15^\circ$ and $\theta = 15^\circ$ and each beamwidth is 10°. Also, we considered $M = 16$ and $M = 30$. Fig. 3 shows the beampattern results of the first simulation for the proposed method with UPA-LIB as a comparison with the stated counterpart.

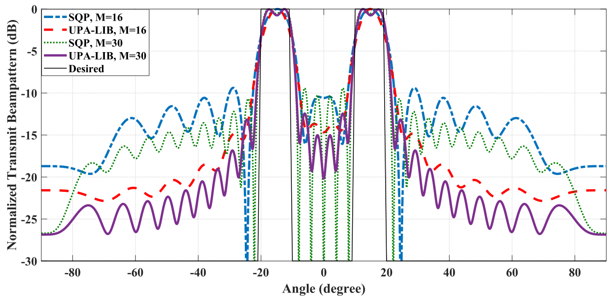

Fig. 3. Beampattern design results in Simulation 1.

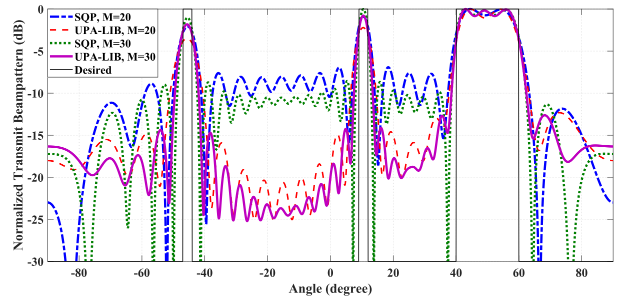

Fig. 4. Beampattern design results in Simulation 2.

TABLE I

COMPARISON OF RUNTIMES IN SIMULATION. 1 AND THE NUMBER OF UNKNOWN VARIABLES

	Runtimes (seconds)		Number of unknown variables	
Method	UPA-LIB	SQP	UPA-LIB (real)	SQP (complex)
$M = 16$	2.3	5.16	41	120
$M = 30$	2.4	56.7	41	435

TABLE II

COMPARISON OF RUNTIMES IN SIMULATION. 2 AND THE NUMBER OF UNKNOWN VARIABLES

	Runtimes (seconds)		Number of unknown variables	
Method	UPA-LIB	SQP	UPA-LIB (real)	SQP (complex)
$M = 20$	2.78	12.5	108	190
$M = 30$	3.26	49.17	108	435

As can be seen, our proposed method has lower SLLs and the same mainlobes ripples.

Furthermore, comparison of the runtimes and number of unknown variables of two algorithms in two different number of antennas are provided in Table I. Comparatively, these parameters in the UPA-LIB design algorithm change are considerably lower than SQP. All those imply that the proposed algorithm is more applicable especially to large number of antennas. It is important to mention that the unknown variables in SQP method are complex (in asymmetric beampattern design), while in the presented method, the coefficients are real, and this is another advantage of our proposed method.

For the second simulation, we consider synthesizing an asymmetric desired beampattern is defined as follows

$$P_d^2(\theta) = \begin{cases} 1 & \theta \in \{[-47, -44] \cup [9, 12] \cup [40, 60]\} \\ 0 & \quad otherwise, \end{cases} \tag{12}$$

which can be considered as three mainlobes with centres at $\theta = -45.5°$, $\theta = 10.5°$ and $\theta = 50°$ and each beamwidth is $3°$, $3°$ and $20°$. The number of antennas considered $M = 20$ and $M = 30$. Fig. 4 shows the beampattern results of the second simulation for the proposed method in two different number of antennas for SQP and desired beampattern. It is shown that our method synthesizes a beampattern which matches to the desired beampattern better than SQP with providing lower SLLs (around 10dB) in region between two narrow band mainlobes.

Comparison of the runtimes and number of unknown variables of the proposed method and SQP are shown in Table II. Significantly decrease of these parameters in our method rather than SQP is evident. As can be seen, in high number of antennas, the number of unknown variables in our proposed method is approximately a quarter of SQP and the runtime is decreased around 93%.

In the simulations 3 and 4, we consider a desired beampattern matching problem for ideal orthogonal MIMO and simple narrow band phase array at $\theta = 0°$ as (13) and (14) respectively as follows

$$P_d^3(\theta) = \begin{cases} 1 & \theta \in \{[-90, 90]\} \\ 0 & otherwise, \end{cases} \tag{13}$$

$$P_d^4(\theta) = \begin{cases} 1 & \theta \in \{[-1, 1]\} \\ 0 & otherwise. \end{cases} \tag{14}$$

The beampattern simulation results for the proposed method with UPA-LIB and its counterpart for ideal orthogonal MIMO and narrow linear phase array as a singlelobe beampattern are shown in Fig. 5a and Fig. 5b, respectively. Fig. 5a demonstrates that our method achieves comparable mainlobe ripple in ideal orthogonal MIMO beamforming when compared to the SQP method. Fig. 5b shows that our proposed method exhibits lower SLLs in the singlelobe beampattern. This indicates that our method possesses the capability to effectively suppress interference because the direct relation between lower SLL and improved interference suppression is evident. Due to its remarkable capability to significantly reduce SLL in the singlelobe compared to SQP, our method effectively suppresses interference in the multilobe beampatterns.

According to Table III, by significantly reducing the calculation time and greatly reducing the number of unknown variables, especially in Simulation 4, we have been able to reach the usual covariance matrices for MIMO orthogonal and singlelobe beampattern in the literature [10].

As it can be seen in orthogonal MIMO with maximum beamwidth, the number of variables is less than half of SQP. The number of variables in the proposed method will decrease by beamwidth reduction.

In order to gain a better insight of the efficiency of our proposed method in a more complex beampattern scenario with a large number of antennas, we consider a beampattern matching problem in the simulation 5 as follows

$$P_d^5(\theta) = \begin{cases} 1 & \theta \in \{[-40, -35] \cup [-25, -15] \cup [-5, 5] \\ & \qquad \cup [15, 20] \cup [40, 50]\} \\ 0 & \quad otherwise, \end{cases} \tag{15}$$

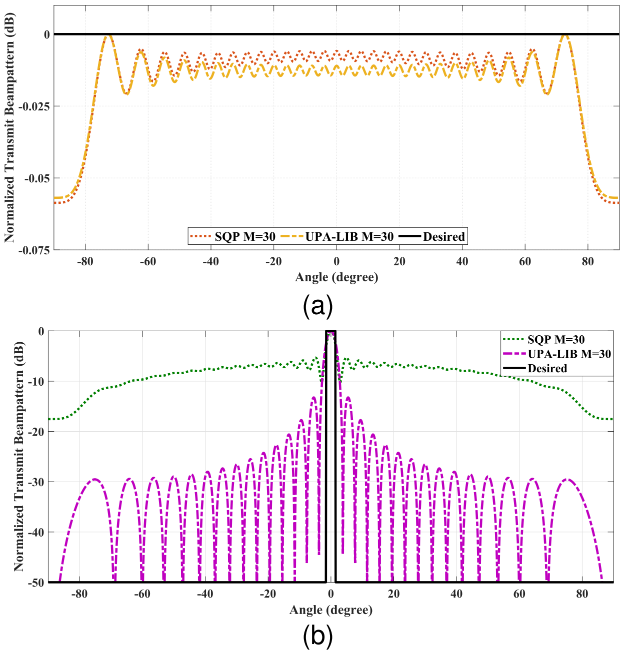

Fig. 5. (a) Beampattern design results in orthogonal MIMO in Simulation 3 and (b) narrow band phase array in Simulation 4.

TABLE III
COMPARISON OF RUNTIMES IN SIMULATION. 3 AND 4 THE NUMBER OF UNKNOWN VARIABLES

	Runtimes (seconds)		Number of unknown variables	
Method	UPA-LIB	SQP	UPA-LIB (real)	SQP (complex)
UPA (in $\theta = 0°$)	1.3	35.4	3	435
Orthogonal MIMO	3.7	30.5	181	435

which can be considered as five asymmetric mainlobes with centres at $\theta = -37.5°$, $\theta = -20°$, $\theta = 0°$, $\theta = 17.5°$ and $\theta = 45°$. The number of antennas considered $M = 30$ and $M = 50$. Fig. 6 shows the results of the fifth simulation for the proposed method with UPA-LIB as well as the stated counterpart. As reflected in Fig. 6, the proposed method beampattern has comparable ripple in the mainlobes region and lower SLLs relative to SQP. Also, the runtimes and number of unknown variables in the optimization problem are provided in Table IV. As seen earlier, the runtime and number of unknown variables of UPA-LIB design algorithm change significantly and is considerably lower than SQP especially to large-number of antennas. It is obvious from Table IV that the number of unknown variables is reduced 92% in $M = 50$ case which considerably increases the speed of solving the corresponding optimization problem.

B. Test on Beampattern Matching Design With SLL-LIB

In this subsection, we simulate the performance of the second proposed algorithm using SLL-LIB. For comparison purposes, we implement the SQP method and our first proposed method with UPA-LIB. In the following simulations we created libraries for $M = 6, 10$ and 25 number of antennas (based on the second scenario in section III) and used of them offline. In the simulation 6, the desired beampattern is

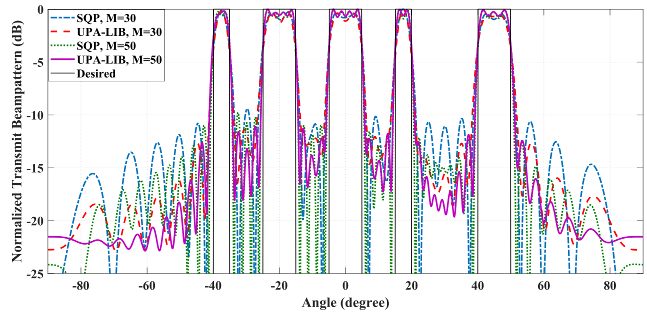

Fig. 6. Beampattern design results in Simulation. 5.

TABLE IV
COMPARISON OF RUNTIMES IN SIMULATION. 5 AND THE NUMBER OF UNKNOWN VARIABLES

	Runtimes (seconds)		Number of unknown variables	
Method	UPA-LIB	SQP	UPA-LIB (real)	SQP (complex)
$M = 30$	3.9	57.7	91	435
$M = 50$	5.39	544	91	1225

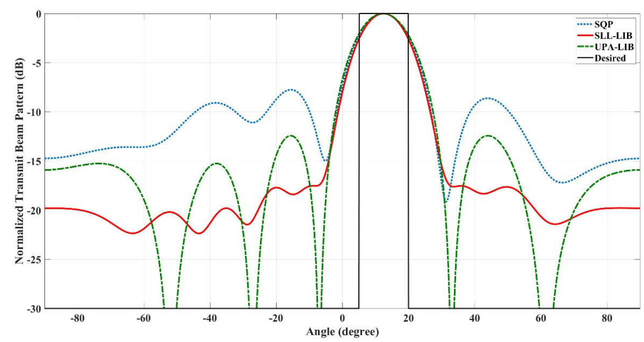

Fig. 7. Beampattern design results in Simulation 6, $M = 6$.

considered as follows

$$P_d^6(\theta) = \begin{cases} 1 & \theta \in \{[5, 20]\} \\ 0 & otherwise, \end{cases} \tag{16}$$

which can be considered as one mainlobe with centres at $\theta = 12.5°$ and beamwidth is $\theta = 15°$. The number of antennae considered $M = 6$. Fig. 7 shows the beampattern results of this simulation for the proposed method with SLL-LIB, UPA-LIB as well as stated counterpart. As can be seen, this method has lower SLLs and the same mainlobe ripple compared to the SQP and UPA-LIB.

In the simulation 7, the wide band and symmetric desired beampattern is considered as follows

$$P_d^7(\theta) = \begin{cases} 1 & \theta \in \{[-30, 30]\} \\ 0 & otherwise, \end{cases} \tag{17}$$

which can be considered as wide symmetric beampattern with centres at $\theta = 0°$ and beamwidth is $\theta = 60°$. The number of antennae is considered $M = 10$. Fig. 8 shows the results of the beammpatern for the proposed method with UPA-LIB and SLL-LIB as well as stated counterpart.

As can be observed, the performance improvement of the provided enhanced library is considerable. As the last scenario in the simulation 8, the desired beampattern is considered as

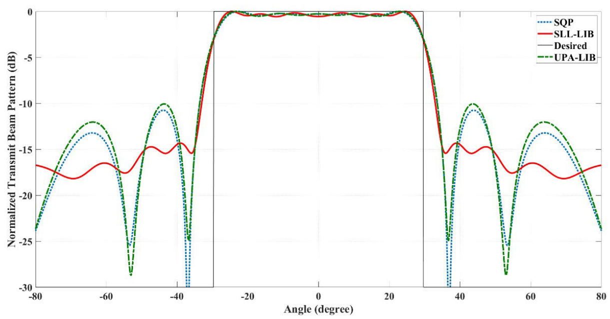

Fig. 8. Beampattern design results in Simulation 7, $M = 10$.

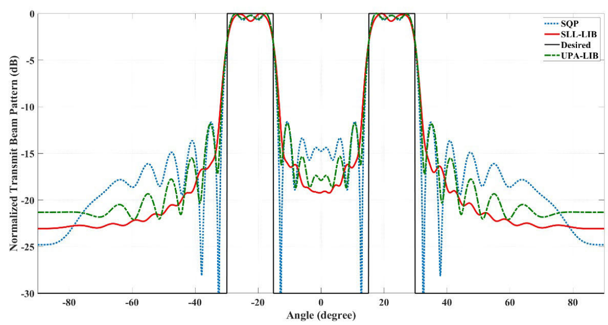

Fig. 9. Beampattern design results in Simulation 8, $M = 25$.

TABLE V
COMPARISON OF RUNTIMES IN SIMULATION. 6, 7 AND 8 AND THE NUMBER OF UNKNOWN VARIABLES

Method	Runtimes (seconds)			Number of unknown variables		
	SLL-LIB	UPA-LIB	SQP	SLL-LIB (real)	UPA-LIB (real)	SQP (complex)
$M = 6$	1.9	1.75	1.66	21	21	15
$M = 10$	1.98	2.15	5.65	61	61	120
$M = 25$	2.1	2.2	23.7	61	61	300

follows

$$P_d^8(\theta) = \begin{cases} 1 & \theta \in \{[-30, -15] \cup [15, 30]\} \\ 0 & otherwise, \end{cases} \tag{18}$$

which can be considered as two mainlobes with centres at $\theta = -22.5°$ and $\theta = 22.5°$ and each beamwidth is $\theta = 15°$. The number of antennas is considered $M = 25$. Fig. 9 shows the comparison results. In the proposed method with SLL-LIB, we can reduce the SLLs while achieving the appropriate level of the ripples in mainlobes. Furthermore, the runtimes and the number of unknown variables for the simulation 6, 7 and 8 are provided in Table V. Comparatively, the runtime of the optimization algorithm for SLL-LIB-design is lower than SQP. All those imply that the proposed algorithm is more applicable especially for large number of antennas. Furthermore, the SLL-LIB design algorithm has an equivalent number of unknown variables as the UPA-LIB beampattern matching design, resulting in a considerable 80% reduction compared to the SQP method.

V. CONCLUSION

A novel approach for covariance matrix design in collocated MIMO radar has been presented. The innovation of this approach is to solve the beampattern matching problem using facilitator libraries as UPA-LIB and SLL-LIB. We proposed a new covariance matrix design technique using the facilitator library members in a reduced number of unknown variables in MSE problem and therefore in a significantly lower time. We showed in different scenarios and simulation cases that this technique outperforms its well-known counterpart (SQP) in terms of computational complexity and consumed time, while keeping an acceptable beampattern matching level, thus making the system more affordable for real-time scenarios. The proposed technique is a straightforward algorithm with high speed which makes it an appropriate and practical method for real-time applications where fast beamforming for a large number of real-time beampatterns in different angles and large number of antennas, is essential. This can have various applications in future researchs such as automated driving systems, health monitoring structure, displacement measurements and mimo radar image reconstruction, where MIMO radar based high-speed beamforming is required.

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