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Beampattern matching in colocated MIMO radar using transmit covariance matrix design

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ABSTRACT

The common problem in the multiple-input multiple-output (MIMO) radar is that the beampattern matching realized by the optimization-based algorithms would lead to high computational complexity. Using the presented transmit covariance matrix design (TCMD) algorithm, the efficiency of MIMO radar beampattern matching can be improved drastically. In this paper, the designed covariance matrix (CM) is constructed as an iterative form, and the appropriate power factor is designed in closed-form to realize accurate spatial power control and guarantee the positive semidefiniteness of the design CM. Moreover, the beampattern matching is realized by successively adjusting the spatial power at the directions where the specifications are not satisfied. Numerical results of accurate spatial power control and beampattern matching are carried out to show the effectiveness and efficiency of the presented TCMD algorithm.

1. Introduction

It is well known that the colocated multiple-input multiple-output (MIMO) radar emits different waveforms through each transmit antenna, leading to improved degrees of freedoms (DoFs) and better detection performance compared with the traditional phased-array radar [1–3]. The main issue in MIMO radar is transmit beampattern design, which can be achieved by exploiting the transmit waveform design [4]. In the existing literatures, the transmit waveform design can be generally classified into two categories: direct waveform design [4–8], design covariance matrix (CM) [9–13] and then generate the waveforms based on the designed CM. Unlike the direct waveform design techniques, the waveform design via the CM can reduce the computational complexity [14]. Therefore, this paper attempts to obtain an approximation of the desired beampatterns via CM design.

The authors in [9] give the earliest in-depth survey of transmit beamforming for MIMO radar, and a constrained least-squares problem is formulated to approach the ideal beampattern. Moreover, the semidefinite programming algorithm is introduced in [10] to improve the efficiency of CM design. In addition, the authors in [11] introduce two algorithms to realize MIMO radar beampattern matching. Specifically, the omnidirectional beampattern design (OMBD) algorithm is proposed to match the desired beampattern; and the beampattern design with sidelobe control (SCBD) algorithm including the 1-, 2- and infinity-norm minimization criteria is proposed to synthesize beampattern with sidelobe control. The major drawback to the above described methods is the use of optimization techniques [15], and the computational complexity is very high for large antenna arrays. Furthermore, the closed-form algorithms are developed in [16] to reduce the computational complexity of CM design. However, these methods serve as an approximation to the desired beampattern. In [17], the authors propose a discrete-Fourier-transform (DFT)-based method to realize desired beampattern matching with low complexity. However, it follows from [6,14] that the performance of the DFT-based method is slightly poorer in the scenario of the small number of antennas. To account for knowledge inaccuracies on the array manifold, the robust transmit beampattern design for MIMO radar is discussed in [18]. The peak sidelobe level is considered as figure of merit, and polynomial time procedures are devised to synthesize the desired optimal MIMO waveform covariance matrices. Moreover, the robust design of MIMO space-time transmit code and space-time receive filter is studied in [19], and an iterative method is proposed to improve the worst-case signal-to-noise ratio.

This paper focuses on the MIMO radar beampattern design with known array manifold, and a transmit covariance matrix design (TCMD) algorithm is proposed to realize efficient MIMO radar beampattern matching. In summary, the contributions of this paper are listed as follows:

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1. A new form of the designed CM is constructed, and the spatial power at a specific direction can be controlled accurately in a simple manner.
2. A closed-form expression of the power factor is derived to guarantee that the designed CM is positive semidefinite and has equal diagonal elements.
3. A new scheme is proposed to realize MIMO radar beampattern matching efficiently.

More specifically, the designed CM at the current step $\mathbf{R}_k$ is expressed as a linear combination of the CM at the previous step $\mathbf{R}_{k-1}$ and an appended CM. The appended CM is constructed as a dyadic product $\mathbf{a}_k\mathbf{a}_k^{\mathrm{H}}$ multiplied by a power factor β_k, where $\mathbf{a}_k$ is the steering vector associated with the spatial power control angle θ_k. The matrix $\mathbf{R}_k$ is updated via a simple procedure to guarantee the positive semidefiniteness of the design CM. The performance of beampattern matching is close to that realized by optimization methods. Simulation results validate the efficiency advantage of the TCMD algorithm.

The rest of this paper is organized as follows. The preliminaries of MIMO radar are introduced in Section 2. Section 3 introduces the TCMD algorithm and presents a beampattern matching scheme. The simulation results are provided in Section 4, and the conclusion is given in Section 5.

Notations: The bold upper-case and lower-case letters represent matrices and vectors, respectively. In particular, $(\cdot)^{\top}$, $(\cdot)^{\mathrm{H}}$, and $(\cdot)^*$ are the transpose, complex conjugate transpose, and conjugate, respectively. $|\cdot|$ is the absolute value, and $\|\cdot\|$ denotes the Euclidean norm of a vector. $\delta(\cdot)$ denotes the Kronecker delta function, and $j=\sqrt{-1}$. $\mathrm{tr}(\cdot)$ stands for the trace of a square matrix, and $\mathbb{S}_+^N$ is the set of order $N\times N$ positive semidefinite matrices.

2. Preliminaries

Consider a colocated MIMO radar system with N transmit antennas, and all elements are omnidirectional. $\mathbf{x}(l)=[x_1(l),x_2(l),\cdots,x_N(l)]^{\top}$ is the baseband signal transmitted at the lth time sample, where $x_n(l)$ denotes the distinct waveform transmitted by the nth antenna. The transmit signal in each antenna consists of L samples, and $L\geq N$.

Under the assumption that the transmitted probing signals are narrow-band and that the propagation is nondispersive, the baseband signal arriving at direction θ is $\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{x}(l)$ [10,18], where $\mathbf{a}(\theta)=\left[e^{j\zeta_1(\theta)},e^{j\zeta_2(\theta)},\cdots,e^{j\zeta_N(\theta)}\right]^{\top}$ is the steering vector associated with θ, and $\zeta_n(\theta)$ is the phase delay of the nth antenna. Moreover, the transmit beampattern in a specific direction θ is written as

$$P(\theta)=\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{R}\mathbf{a}(\theta), \tag{1}$$

where the waveform covariance matrix (CM) of the radar signal is expressed as

$$\mathbf{R}=\frac{1}{L}\sum_{l=1}^{L}\mathbf{x}(l)\mathbf{x}^{\mathrm{H}}(l). \tag{2}$$

It is widely known that the MIMO radar achieves higher DoFs than the traditional phased-array radar, and the existing literature [1,3] indicates that the transmit CM can be designed to improve the system performance of MIMO radar.

The MIMO radar beampattern matching problem is to design the CM that the corresponding beampattern can realize the following goals:

1. Transmit beampattern in the mainlobe region is imposed to have the minimum variation with the ideal one.
2. Spatial beampattern in the sidelobe region prefers to below a given bound.

It follows from [4] that, once the CM is synthesized, the transmit waveform can be obtained. Therefore, this paper concentrates on designing the waveform CM, and an algorithm is presented to achieve efficient beampattern matching.

3. TCMD algorithm

This section introduces the CM design algorithm to achieve accurate spatial power control. Moreover, a beampattern matching scheme is applied to realize efficient MIMO radar transmit beampattern synthesis.

3.1. Designed CM formulation

It follows from (2) that the transmit CM is the sum of several dyadic product $\mathbf{x}(l)\mathbf{x}^{\mathrm{H}}(l)$. Motivated by the form of $\mathbf{R}$, the designed CM is constructed as an iterative form:

$$\mathbf{R}_k=\mathbf{R}_{k-1}+\beta_k\mathbf{a}_k\mathbf{a}_k^{\mathrm{H}}, \tag{3}$$

where the designed CM at the kth step is a linear combination of the previous CM $\mathbf{R}_{k-1}$ and $\mathbf{a}_k\mathbf{a}_k^{\mathrm{H}}$ multiplied by the designed power factor β_k, and $\mathbf{a}_k$ is the steering vector associated with the spatial power control angle θ_k. The initial CM is set as $\mathbf{R}_0=\beta_0\left(\mathbf{a}_0\mathbf{a}_0^{\mathrm{H}}+\mathbf{I}_N\right)$, where $\mathbf{I}_N$ is an identity matrix of order N. $\mathbf{a}_0$ is the steering vector associated with direction θ_0, and θ_0 is the main-beam center of the ideal beampattern P_d. Moreover, we let $\beta_0=1/\left(\|\mathbf{a}_0\|^4+\|\mathbf{a}_0\|^2\right)$ to normalize the spatial power at θ_0.

The normalized spatial power of the ideal MIMO radar beampattern is defined as:

$$L_d(\theta,\theta_0)\triangleq\frac{P_d(\theta)}{P_d(\theta_0)} \tag{4}$$

to quantify the beampattern performance at direction θ. Moreover, we let

$$\mathbf{a}_k^{\mathrm{H}}\mathbf{R}_k\mathbf{a}_k=L_d(\theta_k,\theta_0) \tag{5}$$

to achieve accurate spatial power control at direction θ_k, and the designed power factor is obtained as:

$$\begin{aligned}&\mathbf{a}_k^{\mathrm{H}}\mathbf{R}_{k-1}\mathbf{a}_k+\beta_k\|\mathbf{a}_k\|^4=L_d(\theta_k,\theta_0)\\&\Rightarrow\beta_k=\frac{L_d(\theta_k,\theta_0)-\mathbf{a}_k^{\mathrm{H}}\mathbf{R}_{k-1}\mathbf{a}_k}{\|\mathbf{a}_k\|^4}.\end{aligned} \tag{6}$$

Note that the obtained β_k in (6) might lead to a result that the corresponding transmit CM is not a member of the positive semidefinite matrices. It follows from the *Proposition 5* in [20] that, if $\mathbf{R}_{k-1}\in\mathbb{S}_+^N$, then $\mathbf{R}_k\in\mathbb{S}_+^N$ if and only if

$$\beta_k\geq-\frac{1}{\mathbf{a}_k^{\mathrm{H}}\mathbf{R}_{k-1}^{-1}\mathbf{a}_k}. \tag{7}$$

The theoretical derivation in [21] also shows that $\beta_k=-1/\mathbf{a}_k^{\mathrm{H}}\mathbf{R}_{k-1}^{-1}\mathbf{a}_k$ is the best power factor guaranteeing the positive semidefiniteness of $\mathbf{R}_k$. Therefore, by considering the accurate spatial power control and the physically justified requirement of $\mathbf{R}_k$, the designed power factor is

$$\beta_k'=\begin{cases}\beta_k, \beta_k\geq-\frac{1}{\mathbf{a}_k^{\mathrm{H}}\mathbf{R}_{k-1}^{-1}\mathbf{a}_k}\\-\frac{1}{\mathbf{a}_k^{\mathrm{H}}\mathbf{R}_{k-1}^{-1}\mathbf{a}_k}, \text{otherwise.}\end{cases} \tag{8}$$

Moreover, the finally obtained CM is normalized to satisfy the power budget

$$\hat{\mathbf{R}}=\sigma\mathbf{R}_k, \tag{9}$$

where the normalized factor is

$$\sigma=\frac{P_0}{N\mathbf{R}_k(1,1)}. \tag{10}$$

Note that the designed CM has the following characteristics:

1. The power transmitted by the different antenna is the same if the steering vectors are all constant modulus, simplifying the real system design.
2. Designed CM is a member of the Hermitian positive semidefinite matrices.

3.2. Beampattern matching scheme

The beampattern matching is realized by successively adjusting the spatial power at the directions where the specifications are not satisfied, and the key step of beampattern matching is selecting the spatial power control angle.

Considering the different constraints of beampattern matching problem in the mainlobe and sidelobe regions, the synthesized beampattern at the previous step is considered to determine the mainlobe spatial power control angle. Specifically, the maximum beampattern deviation in the mainlobe region is defined as:

$$D_k(\theta) \triangleq \left||L_d(\theta)| - \mathbf{a}^{\mathrm{H}}(\theta)\mathbf{R}_{k-1}\mathbf{a}(\theta)\right|, \forall\theta \in \Theta_{\text{Main}}, \tag{11}$$

and the spatial power control angle in the mainlobe region is

$$\theta_{\text{Main},k} = \arg\max\{D_k(\theta)\} \tag{12}$$

to minimize the maximum beampattern deviation, where Θ_{Main} denotes the mainlobe region. Moreover, the sidelobe spatial power control angle at the kth step is

$$\theta_{\text{Side},k} = \arg\max\left\{\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{R}_{k-1}\mathbf{a}(\theta)\right\}, \theta \in \Theta_{\text{Peak}} \tag{13}$$

to control the spatial power in the set of peak sidelobe locations Θ_{Peak}.

Considering the mainlobe and sidelobe constraints of transmit beampattern, the matching problem in the mainlobe region is firstly addressed to reduce the variation between the designed beampattern and the ideal one, guaranteeing the radiation performance of the MIMO radar system. And then the beampattern in the sidelobe region is synthesized to realize the low sidelobe level. The proposed MIMO radar beampattern matching scheme is summarized in Algorithm 1. Specifically, the maximum iteration steps for mainlobe and sidelobe beampattern matching are K_{Main} and K_{Side}, respectively. The maximum mainlobe deviation of the normalized beampattern is ε, and the main peak to the sidelobe level is η. The above steps are repeated until the desired beampattern is synthesized or the maximum iteration steps are satisfied. After designing the CM, the waveforms can be easily synthesized via [4].

Algorithm 1 Proposed Beampattern Matching Scheme.

Initialization: $k = 1$, θ_0, $\mathbf{R}_0$, P_d, K_{Main}, K_{Side}
Mainlobe Beampattern Matching:
1: **while** $|D_k(\theta)| > \varepsilon, \forall\theta \in \Theta_{\text{Main}} || k \le K_{\text{Main}}$ **do**
2: Select the spatial power control angle θ_k from (12).
3: Set the normalized spatial power at θ_k via (5).
4: Obtain the designed power factor β_k' via (8).
5: Design CM via (3).
6: Update the iteration index, let $k = k + 1$.
7: **end while**
Sidelobe Beampattern Matching:
8: **while** $\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{R}_{k-1}\mathbf{a}(\theta) > \eta, \forall\theta \in \Theta_{\text{Side}} || k \le K_{\text{Main}} + K_{\text{Side}}$ **do**
9: Select the response control angle θ_k from (13).
10: Set the normalized spatial power at θ_k via (5).
11: Obtain the designed power factor β_k' via (8).
12: Design CM via (3).
13: Update the iteration index, let $k = k + 1$.
14: **end while**
15: Obtain the final CM via (9).
Output: The designed CM $\mathbf{R}$

The major computational cost of the TCMD algorithm lies in the power factor design in (8). Therefore, the complexity at each iteration is $O(N^3)$ floating-point operations, and the total complexity is $O(KN^3)$ floating-point operations, where K is the number of iterations. Moreover, in the simulation results, we observed that the TCMD algorithm could address the MIMO radar beampattern matching problem within ten steps. However, it is hard to theoretically prove that the TCMD algorithm converges to the global optimum.

4. Simulation results

Some numerical examples are provided in this section to evaluate the performance of the TCMD algorithm compared to the omnidirectional beampattern design (OMBD) and beampattern design with sidelobe control (SCBD) algorithms in [11], and the alternating direction method of multipliers (ADMM) algorithm [6]. Consider the uniformly half-wavelength spaced linear array with $N = 10$ elements, $L = 32$ and the total transmit power $P_0 = 1$. The mainlobe maximum pattern deviation is $\varepsilon = 0.1$, and the main peak to the sidelobe level is $\eta = 0.1$ (-20 dB). The sampling interval in the angular sector $[-90^\circ, 90^\circ]$ is 0.1°. Moreover, all the examples are performed on a PC equipped with a CPU Ryzen 7 5800X at 4.6 GHz and 32 GB RAM.

4.1. Accurate spatial power control

The TCMD algorithm is performed to realize beampattern matching, the ideal beampattern in the mainlobe region $\Theta_{\text{Main}} = [-30^\circ, 30^\circ]$ is 0 dB, and the sidelobe level in $\Theta_{\text{Side}} = \{[-90^\circ, -30^\circ) \cup (30^\circ, 90^\circ]\}$ is less than -20 dB. The desired pattern is obtained after implementing eight steps, and the intermediate results are shown in Fig. 1. Specifically, the spatial power control angle and the power factor at the first step are -23.6° and 0.0091, respectively. The synthesized result in Fig. 1(a) illustrates that the TCMD algorithm can realize accurate spatial power control. The mainlobe beampattern matching is realized at the fourth step, the previous and synthesized results are shown in Fig. 1(b). Fig. 1(c) and (d) give the simulation results at the fifth step and the eighth step, showing that the TCMD algorithm can realize accurate sidelobe control.

4.2. Symmetric beampattern matching

This subsection compares the beampattern designed by the TCMD algorithm with that of the OMBD and ADMM algorithms, and the ideal pattern is given in the last subsection.

Figure 2 (a) reveals the simulation results of normalized beampattern (obtained in dB). The results illustrate that the tested algorithms can obtain the beampattern with good mainlobe ripple. In addition, the numerical results of the OMBD, ADMM, and TCMD algorithms are shown in Fig. 2(b), and the maximum mainlobe ripple (MMR) is defined as

$$\text{MMR} \triangleq \max\{P(\theta_i)\} - \min\left\{P(\theta_j)\right\}, \theta_i, \theta_j \in \Theta_{\text{Main}} \tag{14}$$

to measure the beampatterns synthesized by the tested algorithms. Specifically, the MMRs of the beampatterns associated with OMBD, ADMM, and TCMD algorithms are 0.22, 0.22, and 0.19, respectively, indicating that the beampattern obtained by the TCMD algorithm has less mainlobe ripple. Moreover, the sidelobe level obtained by the ADMM algorithm is higher than that of the OMBD and TCMD algorithms. The numerical results show that the beampattern matching performance of the TCMD algorithm is close to that realized by optimization methods.

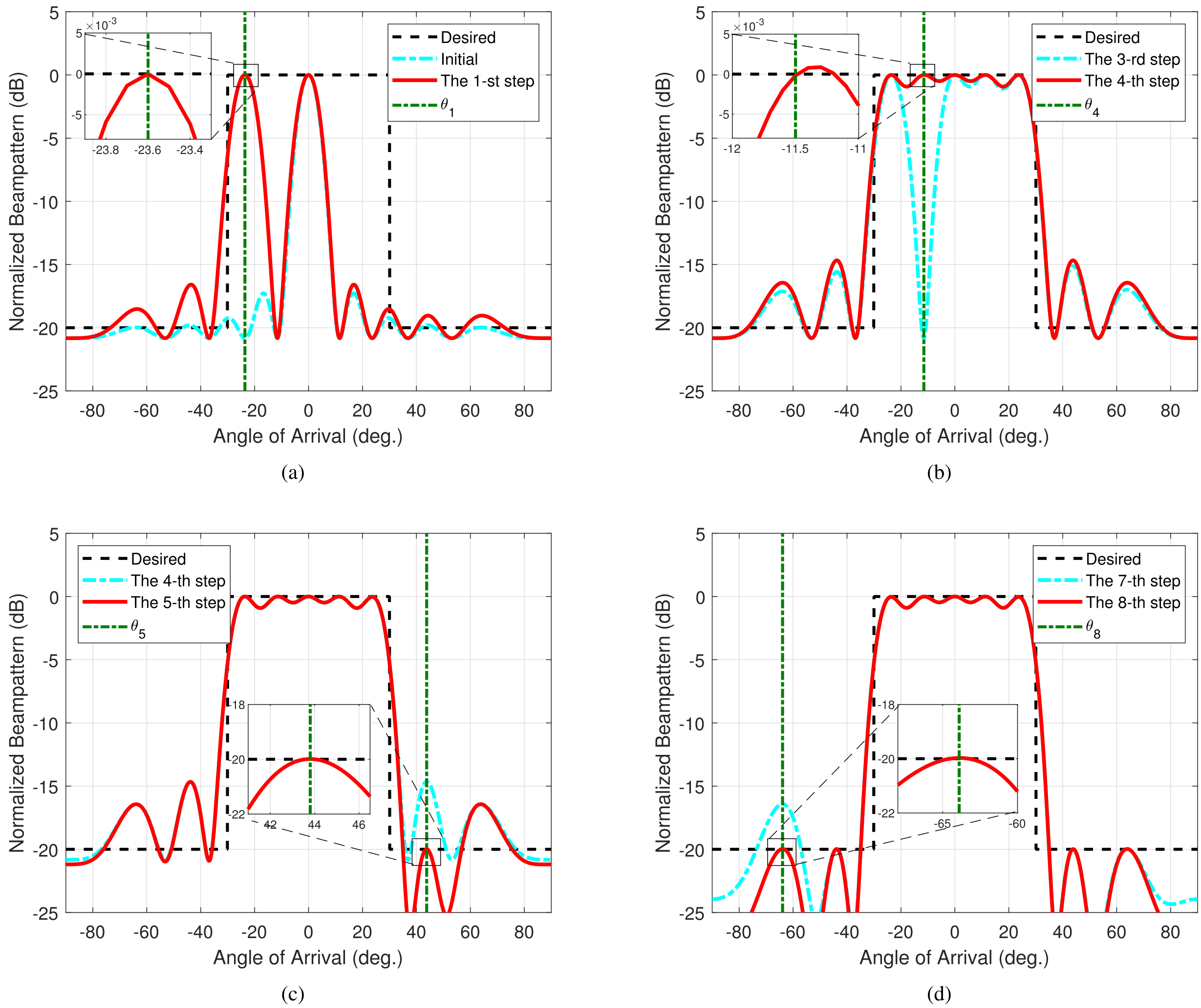

Fig. 1. Illustration of the TCMD algorithm on spatial power control, the desired beampattern is symmetric of width 60-degrees. (a) Synthesized pattern at the first step; (b) Synthesized pattern at the fourth step; (c) Synthesized pattern at the fifth step; (d) Synthesized pattern at the eighth step.

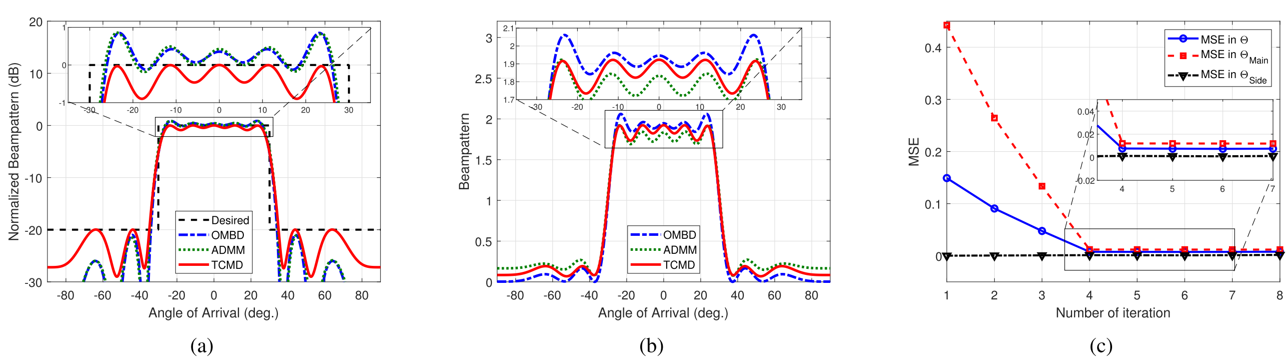

Fig. 2. The comparison of the synthesized single mainlobe beampatterns, the desired beampattern is symmetric of width 60-degrees. (a) Normalized beampatterns of OMBD, ADMM, and TCMD algorithms; (b) Synthesized beampatterns of OMBD, ADMM, and TCMD algorithms; (c) MSE performance of the proposed TCMD algorithm.

Define the mean-squared error (MSE) as

$$\mathrm{MSE} = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{P(\theta_t)}{P(\theta_0)} - L_d(\theta_t, \theta_0)\right|^2, \tag{15}$$

where the number of sampling points $T = 1801$. The curves of the MSEs in Θ, Θ_{Main}, and Θ_{Side} versus the iteration number are shown in Fig. 2(c) to quantify the performance of the beampattern designed by the TCMD algorithm. It indicates that the TCMD algorithm converges to the desired solution with the increase of the iteration number.

4.3. Asymmetric beampattern matching

Next, we consider synthesizing the flat-top main-beam pattern with a low notch level. Let $\Theta_{\mathrm{Main}} = [-15^\circ, 15^\circ]$ with the desired level 0 dB, the notch area is $\Theta_{\mathrm{Notch}} = [35^\circ, 55^\circ]$ with the upper bound -25 dB, and the sidelobe level is lower than -20 dB in the

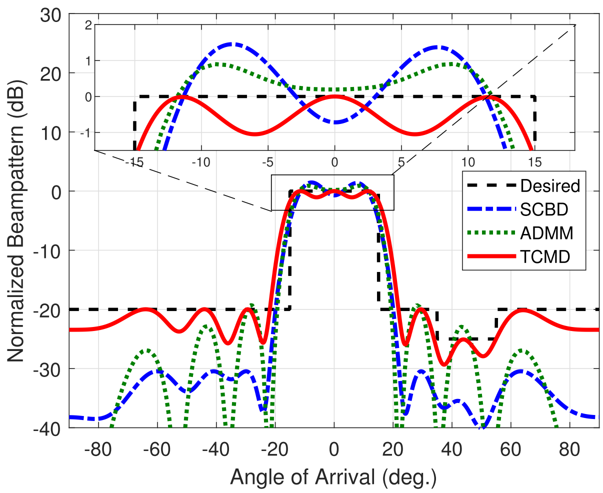

Fig. 3. Comparison between our TCMD algorithm and the methods proposed in [11] and [6], the desired beampattern is asymmetric.

Table 1
Complexity, MSE, and Computation time (seconds) Comparison.

	Complexity of each iteration	MSE		Computation time	
		Fig. 2	Fig. 3	Fig. 2	Fig. 3
Ref [11]	$O\left(Q^{3/2}N^2\right)$ [a]	0.0062	0.0115	11.35 s	39.62 s
ADMM [6]	$O\left(N^3L^3\right)$	**0.0061**	**0.0089**	24.86 s	30.24 s
TCMD	$\boldsymbol{O\left(N^3\right)}$	0.0078	0.0126	**0.10 s**	**0.14 s**

[a] The number of constraints is Q, and the number of sensors is N. Typically, $N < Q$. The computational complexity of the convex programming algorithm is $O\left(Q^{3/2}N^2\right)$ [22].

rest of the area. The SCBD algorithm in [11] and the ADMM algorithm in [6] are compared, and the normalized beampatterns (obtained in dB) are displayed in Fig. 3. The TCMD algorithm realizes less mainlobe ripple after nine steps, and all the constraints are respected. The SCBD algorithm obtains lower sidelobe values than that of the ADMM and TCMD algorithms. The ADMM algorithm obtains undesired notch levels.

4.4. Measurement results comparison

Moreover, Table 1 summarizes the complexity, MSE, and computation time of the tested algorithms. It is shown that the MSE performance of the TCMD algorithm is close to that of the convex optimization-based OMBD and SCBD algorithms in [11], and the TCMD algorithm has lower complexity and computation time. The advantage of the ADMM algorithm is that it can realize the direct waveform design. However, it needs more runtime than the TCMD algorithm. More specifically, the maximum runtime of the TCMD algorithm in the simulation examples is less than 0.14 s, meaning that the TCMD algorithm is efficient for MIMO radar beampattern matching and sidelobe control.

5. Conclusions

This paper presents a novel closed-form algorithm to synthesize MIMO radar transmit covariance matrix (CM). The designed CM is constructed as an iterative form, and the presented transmit covariance matrix design (TCMD) algorithm can realize accurate spatial power control. The positive semidefiniteness of the designed CM is guaranteed, and the average power constraint on CM is fulfilled. Numerical results demonstrate that the TCMD algorithm can synthesize desired beampatterns with a small number of iterations, and the computational complexity of the TCMD algorithm is much lower than that of the optimization-based methods.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.sigpro.2021.108440.

CRediT authorship contribution statement

Xiaoyu Ai: Conceptualization, Methodology, Software, Validation, Writing – original draft. **Lu Gan:** Supervision, Writing – review & editing.

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