机器学习实验二

姓名：任薏霖

学号：2011897

专业：物联网工程

实验要求

题目：回归模型 基本要求：

1. 构造线性回归模型，并采用批量梯度下降和随机梯度下降进行优化；输出训练集和测试集的均方误差（MSE），画出MSE收敛曲线。
2. 对于批量梯度下降和随机梯度下降，采用不同的学习率并进行MSE曲线展示，分析选择最佳的学习率。

中级要求： 探究回归模型在机器学习和统计学上的差异。

高级要求： 编程实现岭回归算法，求解训练样本的岭回归模型，平均训练误差和平均测试误差（解析法、批量梯度下降法和随机梯度下降法均可）。

导入需要的包

In [1]:

```
#导入
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import copy
```

数据集预处理

In [2]:

```
data = pd.read_csv("winequality-white.csv")
data
```

Out[2]:

	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	free sulfur dioxide	total sulfur dioxide	density	pH	sulphates	al
0	7.0	0.27	0.36	20.7	0.045	45.0	170.0	1.00100	3.00	0.45	
1	6.3	0.30	0.34	1.6	0.049	14.0	132.0	0.99400	3.30	0.49	
2	8.1	0.28	0.40	6.9	0.050	30.0	97.0	0.99510	3.26	0.44	
3	7.2	0.23	0.32	8.5	0.058	47.0	186.0	0.99560	3.19	0.40	
4	7.2	0.23	0.32	8.5	0.058	47.0	186.0	0.99560	3.19	0.40	
...	...	...	...	...	...	...	...	...	...	...	
4893	6.2	0.21	0.29	1.6	0.039	24.0	92.0	0.99114	3.27	0.50	
4894	6.6	0.32	0.36	8.0	0.047	57.0	168.0	0.99490	3.15	0.46	
4895	6.5	0.24	0.19	1.2	0.041	30.0	111.0	0.99254	2.99	0.46	
4896	5.5	0.29	0.30	1.1	0.022	20.0	110.0	0.98869	3.34	0.38	
4897	6.0	0.21	0.38	0.8	0.020	22.0	98.0	0.98941	3.26	0.32	

4898 rows × 12 columns

In [3]:

```
#检查数据集中是否含有空值
print(data.isnull().sum())
```

```
fixed acidity           0
volatile acidity        0
citric acid             0
residual sugar          0
chlorides               0
free sulfur dioxide     0
total sulfur dioxide    0
density                 0
pH                      0
sulphates               0
alcohol                 0
quality                 0
dtype: int64
```

In [4]:

```
quality = data[['quality']]
#提取data数据的最后一列 quality
t = data.iloc[:, 0:11]
#对数据进行标准化
d = (t - t.min()) / (t.max() - t.min())
print(d)
```

```
      fixed acidity  volatile acidity  citric acid  residual sugar  chlorides  \
0          0.307692          0.186275     0.216867        0.308282   0.106825
1          0.240385          0.215686     0.204819        0.015337   0.118694
2          0.413462          0.196078     0.240964        0.096626   0.121662
3          0.326923          0.147059     0.192771        0.121166   0.145401
4          0.326923          0.147059     0.192771        0.121166   0.145401
...             ...               ...          ...             ...        ...
4893       0.230769          0.127451     0.174699        0.015337   0.089021
4894       0.269231          0.235294     0.216867        0.113497   0.112760
4895       0.259615          0.156863     0.114458        0.009202   0.094955
4896       0.163462          0.205882     0.180723        0.007669   0.038576
4897       0.211538          0.127451     0.228916        0.003067   0.032641

      free sulfur dioxide  total sulfur dioxide   density        pH  \
0                0.149826              0.373550  0.267785  0.254545
1                0.041812              0.285383  0.132832  0.527273
2                0.097561              0.204176  0.154039  0.490909
3                0.156794              0.410673  0.163678  0.427273
4                0.156794              0.410673  0.163678  0.427273
...                   ...                   ...       ...       ...
4893             0.076655              0.192575  0.077694  0.500000
4894             0.191638              0.368910  0.150183  0.390909
4895             0.097561              0.236659  0.104685  0.245455
4896             0.062718              0.234339  0.030461  0.563636
4897             0.069686              0.206497  0.044342  0.490909

      sulphates   alcohol
0      0.267442  0.129032
1      0.313953  0.241935
2      0.255814  0.338710
3      0.209302  0.306452
4      0.209302  0.306452
...         ...       ...
4893   0.325581  0.516129
4894   0.279070  0.258065
4895   0.279070  0.225806
4896   0.186047  0.774194
4897   0.116279  0.612903

[4898 rows x 11 columns]
```

In [5]:

```
#将标准化的特征值表与quality进行合并
data = pd.concat([d, quality], axis = 1)
pd.set_option('display.max_columns', 50)
pd.set_option('display.width', 1000)

data.to_csv('wine_data.csv') #将处理好的数据另存为新的csv文件

print(data)
```

```
      fixed acidity  volatile acidity  citric acid  residual sugar  chlorides  free 
sulfur dioxide  total sulfur dioxide  density        pH  sulphates  alcohol  quali
ty
0          0.307692          0.186275     0.216867        0.308282   0.106825
0.149826             0.373550   0.267785  0.254545  0.267442  0.129032        6
1          0.240385          0.215686     0.204819        0.015337   0.118694
0.041812             0.285383   0.132832  0.527273  0.313953  0.241935        6
2          0.413462          0.196078     0.240964        0.096626   0.121662
0.097561             0.204176   0.154039  0.490909  0.255814  0.338710        6
3          0.326923          0.147059     0.192771        0.121166   0.145401
0.156794             0.410673   0.163678  0.427273  0.209302  0.306452        6
4          0.326923          0.147059     0.192771        0.121166   0.145401
0.156794             0.410673   0.163678  0.427273  0.209302  0.306452        6
...             ...               ...          ...             ...        ...
...                   ...       ...       ...       ...       ...      ...
4893       0.230769          0.127451     0.174699        0.015337   0.089021
0.076655             0.192575   0.077694  0.500000  0.325581  0.516129        6
4894       0.269231          0.235294     0.216867        0.113497   0.112760
0.191638             0.368910   0.150183  0.390909  0.279070  0.258065        5
4895       0.259615          0.156863     0.114458        0.009202   0.094955
0.097561             0.236659   0.104685  0.245455  0.279070  0.225806        6
4896       0.163462          0.205882     0.180723        0.007669   0.038576
0.062718             0.234339   0.030461  0.563636  0.186047  0.774194        7
4897       0.211538          0.127451     0.228916        0.003067   0.032641
0.069686             0.206497   0.044342  0.490909  0.116279  0.612903        6

[4898 rows x 12 columns]
```

In [6]:

```
df = pd.read_csv("wine_data.csv")
df = df.iloc[:, 1:13]
df
```

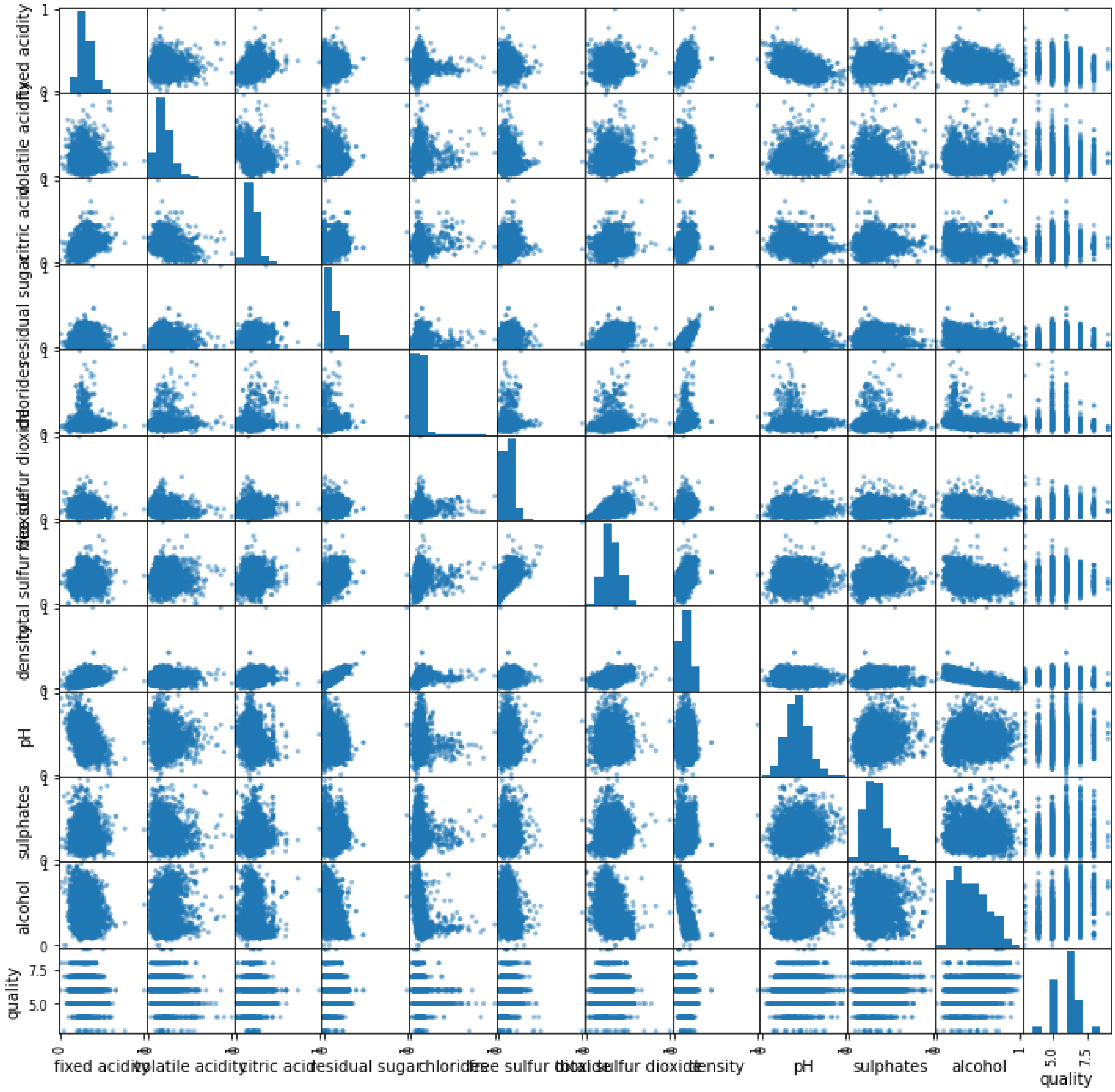
Out[6]:

	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	free sulfur dioxide	total sulfur dioxide	density	pl
0	0.307692	0.186275	0.216867	0.308282	0.106825	0.149826	0.373550	0.267785	0.25454
1	0.240385	0.215686	0.204819	0.015337	0.118694	0.041812	0.285383	0.132832	0.52727
2	0.413462	0.196078	0.240964	0.096626	0.121662	0.097561	0.204176	0.154039	0.49090
3	0.326923	0.147059	0.192771	0.121166	0.145401	0.156794	0.410673	0.163678	0.42727
4	0.326923	0.147059	0.192771	0.121166	0.145401	0.156794	0.410673	0.163678	0.42727
...	...	...	...	...	...	...	...	...	.
4893	0.230769	0.127451	0.174699	0.015337	0.089021	0.076655	0.192575	0.077694	0.50000
4894	0.269231	0.235294	0.216867	0.113497	0.112760	0.191638	0.368910	0.150183	0.39090
4895	0.259615	0.156863	0.114458	0.009202	0.094955	0.097561	0.236659	0.104685	0.24545
4896	0.163462	0.205882	0.180723	0.007669	0.038576	0.062718	0.234339	0.030461	0.56363
4897	0.211538	0.127451	0.228916	0.003067	0.032641	0.069686	0.206497	0.044342	0.49090

4898 rows × 12 columns

In [7]:

```
from pandas.plotting import scatter_matrix
scatter_matrix(data, figsize = (12, 12));
```

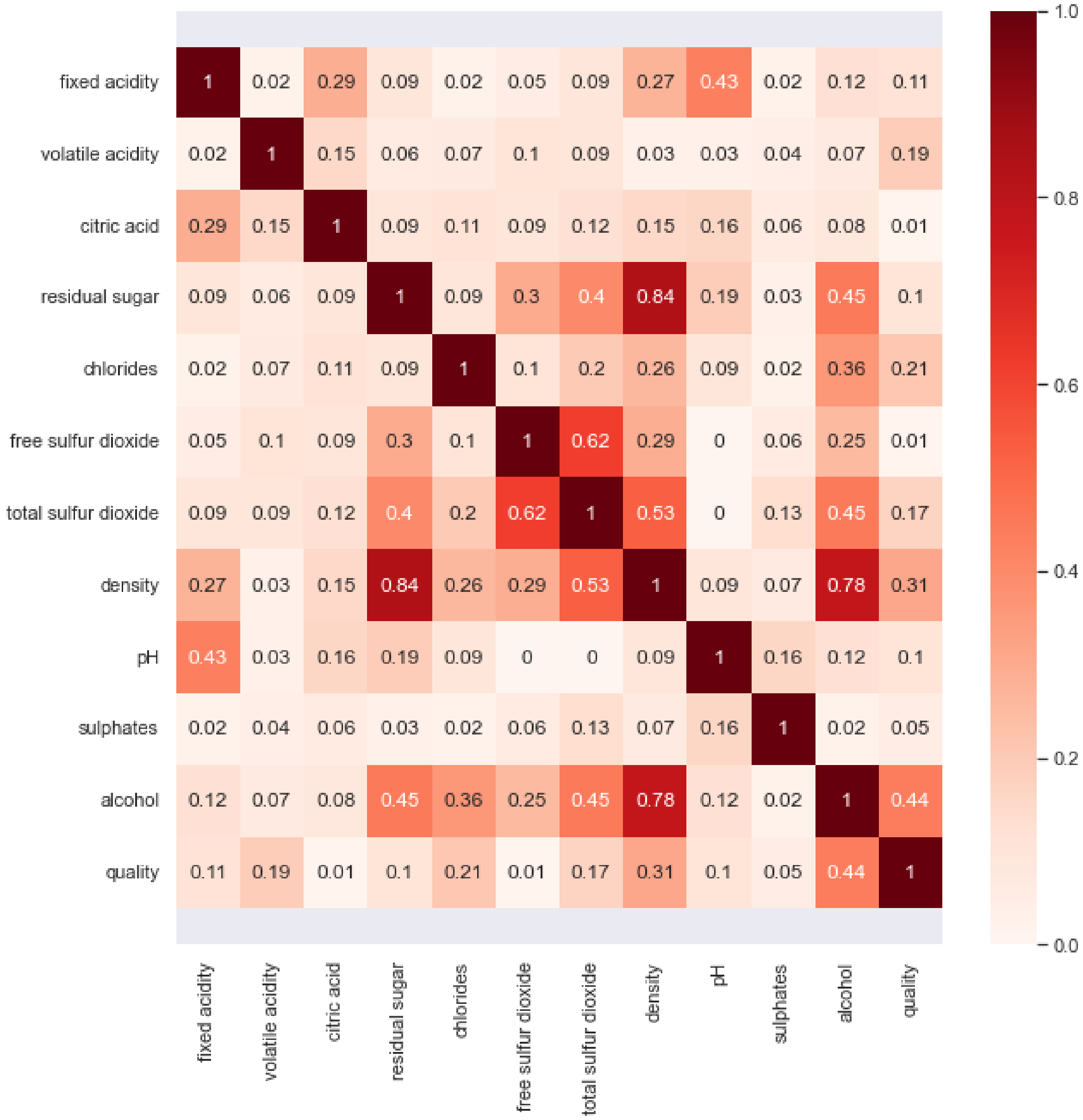


In [8]:

```
correlation_matrix = np.absolute(data.corr().round(2))
sns.set(rc = {'figure.figsize':(10, 10)})
ax = sns.heatmap(correlation_matrix, annot = True, cmap = 'Reds')
bottom, top = ax.get_ylim()
ax.set_ylim(bottom + 0.5, top - 0.5)
```

Out[8]:

(12.5, -0.5)



基本要求

分割数据集为训练集和测试集

In [9]:

```
# 这里注意一个小trick：回归系数会比特征x多一维，为了向量相乘方便，可以在训练集X左侧添加全为1的一列
data = pd.concat([pd.DataFrame(np.ones(data.shape[0]), columns=['x0']), data], axis=1)
#分割训练集
train_data = data.sample(frac = 0.8, random_state = 0, axis = 0)
#分割测试集
text_data = data[~data.index.isin(train_data.index)]
```

In [10]:

```
train_x = train_data.iloc[:, :12]
train_y = train_data.iloc[:, 12:]

text_x = text_data.iloc[:, :12]
text_y = text_data.iloc[:, 12:]
```

In [11]:

```
train_x
```

Out[11]:

	x0	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	free sulfur dioxide	total sulfur dioxide	density	
2762	1.0	0.336538	0.235294	0.210843	0.012270	0.121662	0.020906	0.357309	0.102757	0.4
42	1.0	0.307692	0.225490	0.156627	0.104294	0.178042	0.090592	0.350348	0.159823	0.3
1419	1.0	0.365385	0.058824	0.445783	0.015337	0.091988	0.087108	0.218097	0.086563	0.3
3664	1.0	0.115385	0.205882	0.325301	0.078221	0.077151	0.181185	0.338747	0.051089	0.5
2125	1.0	0.211538	0.196078	0.132530	0.177147	0.115727	0.139373	0.357309	0.165606	0.4
...	...	...	...	...	...	...	...	...	...	
2845	1.0	0.259615	0.137255	0.174699	0.104294	0.056380	0.048780	0.180974	0.115674	0.3
3384	1.0	0.288462	0.147059	0.180723	0.097393	0.103858	0.139373	0.394432	0.144399	0.4
2056	1.0	0.288462	0.127451	0.162651	0.269172	0.097923	0.135889	0.317865	0.250434	0.5
4016	1.0	0.326923	0.313725	0.144578	0.121166	0.136499	0.149826	0.329466	0.176403	0.4
1622	1.0	0.259615	0.352941	0.295181	0.108896	0.106825	0.048780	0.371230	0.165606	0.3

3918 rows × 12 columns

In [12]:

```
text_x
```

Out[12]:

	x0	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	free sulfur dioxide	total sulfur dioxide	density	
0	1.0	0.307692	0.186275	0.216867	0.308282	0.106825	0.149826	0.373550	0.267785	0.2
7	1.0	0.307692	0.186275	0.216867	0.308282	0.106825	0.149826	0.373550	0.267785	0.2
21	1.0	0.250000	0.225490	0.228916	0.035276	0.086053	0.059233	0.215777	0.078851	0.4
24	1.0	0.269231	0.186275	0.246988	0.010736	0.127596	0.048780	0.308585	0.154039	0.6
25	1.0	0.307692	0.166667	0.192771	0.128834	0.109792	0.188153	0.547564	0.161751	0.4
...	...	...	...	...	...	...	...	...	...	
4877	1.0	0.201923	0.450980	0.000000	0.003067	0.068249	0.034843	0.169374	0.110854	0.4
4884	1.0	0.259615	0.245098	0.228916	0.118098	0.115727	0.229965	0.382831	0.150569	0.3
4885	1.0	0.269231	0.254902	0.240964	0.115031	0.109792	0.229965	0.373550	0.150954	0.3
4890	1.0	0.221154	0.254902	0.174699	0.024540	0.080119	0.080139	0.211137	0.043763	0.3
4895	1.0	0.259615	0.156863	0.114458	0.009202	0.094955	0.097561	0.236659	0.104685	0.2

980 rows × 12 columns

In [13]:

```
train_x = train_x.to_numpy()
train_y = train_y.to_numpy()
text_x = text_x.to_numpy()
text_y = text_y.to_numpy()
```

In [14]:

```
#初始化回归系数
t_theta = np.random.randn(train_x.shape[1], 1)
t_theta_T = t_theta.T
#t_theta
t_coef = t_theta
t_coef
```

Out[14]:

```
array([[ 0.42023214],
       [ 0.70132557],
       [-0.36896723],
       [-0.93551505],
       [ 2.68712197],
       [-1.15655271],
       [-1.19983376],
       [ 1.56516553],
       [-0.39509006],
       [-0.0829456 ],
       [ 1.8432112 ],
       [-0.67693911]])
```

随机梯度下降

In [15]:

```
#n_epoch 训练次数
#l_rate  训练步长

def Random_grad(x, y, theta, alpha, iters):
    """
    :param alpha: 梯度下降学习率/步长
    :param iters: 训练次数
    :param theta: 一般初始设置为[[0, 0, 0]], 初值对梯度下降收敛速度影响大
    :param x: 二维matrix, 特征向量集
    :param y: 二维matrix, 结果集
    :return:
    """
    ept=0.001 #精度
    loss=1 #定义一个损失 方便进入循环体 后来表示两次迭代损失函数的差异
    i = 0
    numsSample = x.shape[0]
    while i < iters and loss > ept:
        t = np.random.randint(0, numsSample) #随机抽取一个样本

        # for t in range(numsSample): 这样其实也是遍历了所有数据
        #     partial=X[i:i+1,:].T.dot((X[i:i+1,:].dot(theta)-y[i,:]).reshape(1,1)) #损失函数关于the
        #     theta=theta-alpha*partial
        partial = x[t:t + 1, :].T.dot((x[t:t + 1, :].dot(theta) - y[t, :]).reshape(1, 1)) #损失函数
        theta = theta - alpha * partial
        i += 1
        loss = (1 / (2 * x.shape[0])) * np.sum((x.dot(theta) - y)**2) #计算两次迭代之间的差异(损失函
    return theta
```

In [16]:

```
#定义梯度下降学习率数组
alpha = [0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1]
for i in range(0, 6):
    theta = Random_grad(train_x, train_y, t_coef, alpha[i], 50)
    print("theta: ", theta.T)
```

```
theta:  [[ 0.64117277  0.76951532 -0.32887876 -0.88875952  2.70617244 -1.13160216
  -1.17387405  1.6295605  -0.36691614  0.01118072  1.90804008 -0.58092432]]
theta:  [[ 1.0361759   0.88387187 -0.24543832 -0.80540513  2.74324197 -1.08405337
  -1.12296383  1.7474914  -0.30951146  0.17678115  2.04198457 -0.43784596]]
theta:  [[ 2.04201411  1.1667543  -0.04091602 -0.60973334  2.80806197 -0.97396596
  -1.01325458  2.05378787 -0.19268649  0.66117554  2.32558568  0.00917404]]
theta:  [[ 2.77111459  1.39309478  0.0999471  -0.43131788  2.87706751 -0.90778103
  -0.93106209  2.23657804 -0.10795136  0.88019252  2.61458475  0.38476068]]
theta:  [[ 3.35272691  1.46947834  0.31477483 -0.43135669  2.96961774 -0.72574218
  -0.91038727  2.34579119 -0.07465442  1.08970173  2.70844453  0.96509797]]
theta:  [[ 4.19725614  1.69425041 -0.12135489 -0.46544628  2.92223825 -0.94425512
  -0.76536505  2.42852127 -0.1074964   1.44815032  2.06980816  1.40078086]]
```

批量梯度下降

In [17]:

```
#n_epoch 训练次数
#l_rate  训练步长n

def Batch_grad(x, y, theta, alpha, iters):
    """
    :param alpha: 梯度下降学习率/步长
    :param iters: 训练次数
    :param theta: 一般初始设置为[[0, 0, 0]]，初值对梯度下降收敛速度影响大
    :param x: 二维matrix，特征向量集
    :param y: 二维matrix，结果集
    :return:
    """
    ept=0.001 #精度
    loss=1 #定义一个损失 方便进入循环体 后来表示两次迭代损失函数的差异
    i = 0
    while i < iters and loss > ept:
        partial = (1 / x.shape[0]) * x.T.dot(x.dot(theta) - y) #损失函数关于theta的偏导数
        theta = theta - alpha * partial
        i += 1
        loss = (1 / (2 * x.shape[0])) * np.sum((x.dot(theta) - y) ** 2)
    return theta
```

In [18]:

```
alpha = [0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1]
for i in range(0, 6):
    theta = Batch_grad(train_x, train_y, t_coef, alpha[i], 50)
    print("theta: ", theta.T)
```

```
theta:  [[ 0.65293635  0.76901197 -0.32449091 -0.88863515  2.70638815 -1.13175642
  -1.17330259  1.63266506 -0.36576376  0.01667554  1.91463586 -0.57634773]]
theta:  [[ 1.06236573  0.88793027 -0.24648902 -0.80619183  2.74000341 -1.08822666
  -1.12672592  1.75090583 -0.31451944  0.19186753  2.03974655 -0.3982898 ]]
theta:  [[ 2.04701494  1.1723608  -0.06118889 -0.60827361  2.81829268 -0.98442138
  -1.0156435   2.03055358 -0.19447808  0.61240971  2.33557698  0.0396178 ]]
theta:  [[ 3.00908956  1.44100266  0.10613463 -0.41693947  2.8797821  -0.88810068
  -0.91252324  2.27597435 -0.09600919  1.01854192  2.59458483  0.52422233]]
theta:  [[ 3.21833119  1.45508138  0.07458012 -0.38428264  2.82437719 -0.88923779
  -0.91385811  2.19973977 -0.16135535  1.08193584  2.50694989  0.88570875]]
theta:  [[ 3.30437703  1.3603129  -0.10948512 -0.38442525  2.67124519 -0.92031955
  -0.95014964  1.90506386 -0.35685674  1.03539812  2.14718749  1.48555242]]
```

预测函数

In [19]:

```
#row  输入数值
#coef 回归模型参数
def predict(x, theta):
    y = 0
    for i in range(len(x) - 1):
        y += float(theta[i]) * float(x[i])
    return y
```

均方误差

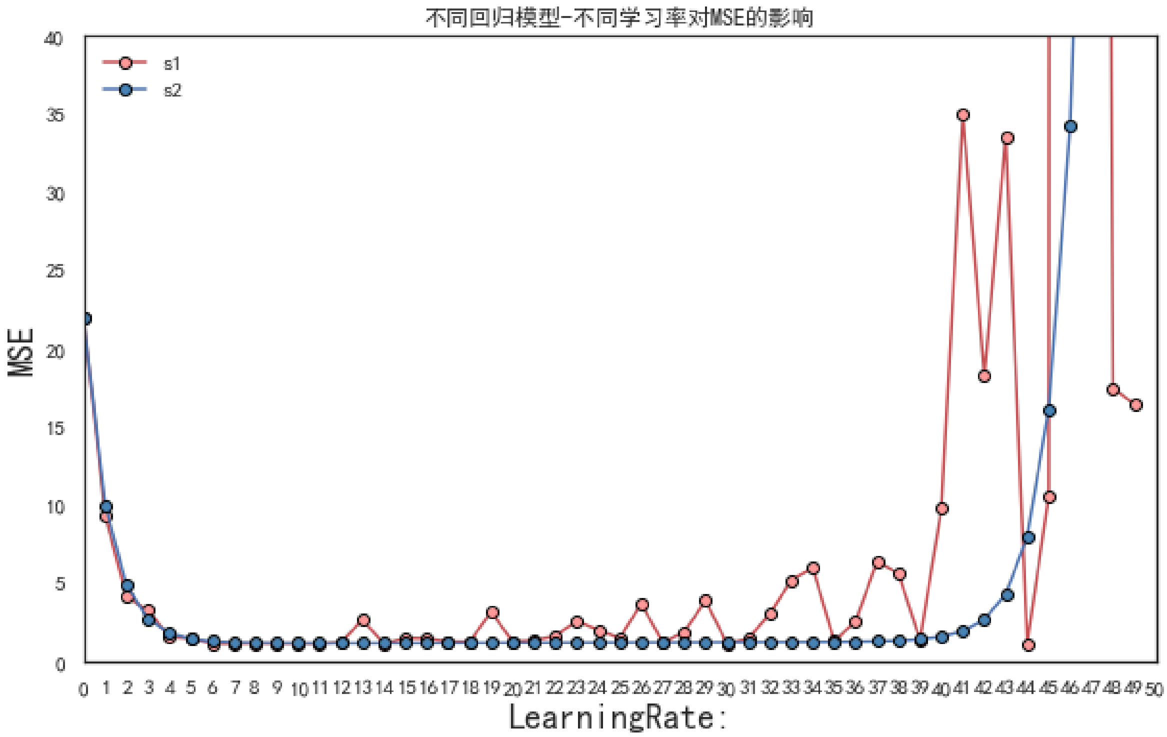
In [20]:

```
#计算均方误差
def MSE(theta, x, y):
    loss = 0
    for i in range(len(x)):
        temp = predict(x[i], theta) - y[i]
        loss += temp ** 2
    return loss / len(y)
```

可视化

In [21]:

```
#结果可视化--学习率
MSE_Random = []
MSE_Descent= []

plt.style.use('seaborn-ticks')
fig = plt.figure(figsize=(10,6))
#alpha = [0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1]
for i in range(0,50):
    MSE_Random.append(MSE(Random_grad(train_x, train_y, t_coef, i / 40, 10), train_x, train_y))
    MSE_Descent.append(MSE(Batch_grad(train_x, train_y, t_coef, i / 40, 10), train_x, train_y))

plt.xlabel('LearningRate:', fontsize=18)
plt.ylabel('MSE', fontsize=18)

x_major_locator = plt.MultipleLocator(1)
ax = plt.gca()
ax.xaxis.set_major_locator(x_major_locator)

plt.xlim(0, 50)
plt.ylim(0, 40)

plt.plot(range(0, 50), MSE_Random, 'r', label='s1', marker = "o", markeredgecolor = 'black', markerf
plt.plot(range(0, 50), MSE_Descent, 'b', label='s2', marker = "o", markeredgecolor = 'black', marker

plt.legend()
plt.rcParams['font.sans-serif'] = ['SimHei'] # 设置字体，不然中文无法显示
#plt.rcParams['image.cmap'] = 'gray' # 设置 颜色 style

#plt.grid()

plt.title("不同回归模型-不同学习率对MSE的影响")

plt.show()
```

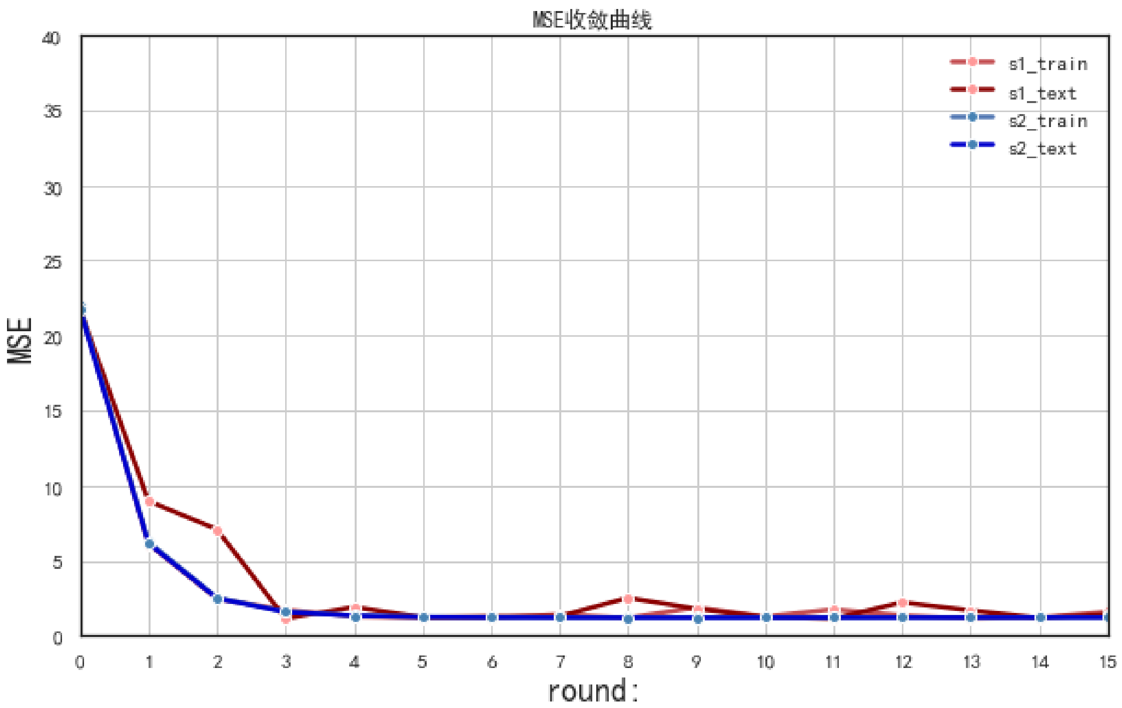


由上图可知，随着学习率的增大，批量梯度下降和随机梯度下降的回归模型的MSE值均有大幅度降低，并在一定程度内保持平衡，但是随机梯度下降法易出现抖动和噪音；当学习率>1时，每次迭代中目标函数可能不会减少，所以可能不会收敛。

因此，通过上图曲线，可以判断当学习率为0.3时，效果较好。

In [22]:

```
#结果可视化--不同回归算法--学习率根据上问选择0.3
MSE_Random_train = []
MSE_Random_text = []
MSE_Batch_train= []
MSE_Batch_text= []

fig = plt.figure(figsize=(10,6))
plt.style.use('seaborn-ticks')

for i in range(0,50):
    MSE_Random_train.append(MSE(Random_grad(train_x, train_y, t_coef, 0.3, i), train_x, train_y))
    MSE_Random_text.append(MSE(Random_grad(text_x, text_y, t_coef, 0.3, i), text_x, text_y))
    MSE_Batch_train.append(MSE(Batch_grad(train_x, train_y, t_coef, 0.3, i), train_x, train_y))
    MSE_Batch_text.append(MSE(Batch_grad(text_x, text_y, t_coef, 0.3, i), text_x, text_y))


plt.xlabel('round:',fontsize=18)
plt.ylabel('MSE',fontsize=18)

x_major_locator = plt.MultipleLocator(1)
ax = plt.gca()
ax.xaxis.set_major_locator(x_major_locator)

plt.xlim(0, 15)
plt.ylim(0,40)

plt.plot(range(0,50), MSE_Random_train, 'r', label='s1_train', marker = "o", markeredgecolor = 'whit
plt.plot(range(0,50), MSE_Random_text, 'darkred', label='s1_text', marker = "o", markeredgecolor = '

plt.plot(range(0,50), MSE_Batch_train, 'b', label='s2_train', marker = "o", markeredgecolor = 'white
plt.plot(range(0,50), MSE_Batch_text, 'mediumblue', label='s2_text', marker = "o", markeredgecolor =

plt.legend()

plt.grid()

plt.rcParams['font.sans-serif'] = ['SimHei'] # 设置字体，不然中文无法显示

plt.title("MSE收敛曲线")

plt.show()
```



中级要求

探究回归模型在机器学习和统计学的差异

1. 回归模型应用于机器学习：

1）机器学习的目的是构建一个可重复预测的模型，从而确定预测结果的可行性。

2）机器学习算法的评价准确性可通过测试数据集来验证。

3）机器学习不基于假设，机器学习没有连续性分割边界的限制。同样我们也并不需要假设自变量或因变量的分布。

2. 回归模型应用于统计学：

1）如果试图证明数据变量间的关系具有统计学意义，会使用统计模型，因为统计关系更易关注变量间关系，而非预测。

2）对于统计模型来说，基于置信区间的回归参数分析，重要性测试以及其他测试可以用于评价该模型的有效性。

3）统计模型基于一系列的假设。例如线性回归模型假设：（1）自变量和因变量线性相关；（2）同方差；（3）波动均值为0；（4）观测样本相互独立；（5）波动服从正态分布。Logistics回归同样拥有很多的假设。即使是非线性回归也要遵守一个连续的分割边界的假设。

高级要求

岭回归算法

In [23]:

```
#岭回归算法
def Ridge(x, y, lam):# 可设置岭系数为0.2
    #lam 可初设为0.2
    xMat = np.mat(x)
    yMat = np.mat(y)

    xTx = xMat.T * xMat  # 矩阵乘法  xMat.shape
    rxTx = xTx + np.eye(xMat.shape[1]) * lam  # 岭回归求解的括号的部分
    # 计算矩阵的值,如果值为0，说明该矩阵没有逆矩阵
    if np.linalg.det(rxTx) == 0.0:
        print("This matrix cannot do inverse")
        return
    # xTx.I为xTx的逆矩阵
    theta = rxTx.I * xMat.T * yMat
    return theta
```

In [24]:

```
#计算平均误差
def avg_loss(theta, x, y):
    loss = 0
    for i in range(len(x)):
        temp = y[i] - predict(x[i], theta)
        loss += temp
    return loss / len(y)
```

In [25]:

```
# 生成50个值作为label的候选值，此处是alphas
# linspace默认生成50个值，若想生成100个，可以修改为(0.001,1,100)
alphas_to_test = np.linspace(0.001,1)
alphas_to_test
```

Out[25]:

```
array([0.001     , 0.02138776, 0.04177551, 0.06216327, 0.08255102,
       0.10293878, 0.12332653, 0.14371429, 0.16410204, 0.1844898 ,
       0.20487755, 0.22526531, 0.24565306, 0.26604082, 0.28642857,
       0.30681633, 0.32720408, 0.34759184, 0.36797959, 0.38836735,
       0.4087551 , 0.42914286, 0.44953061, 0.46991837, 0.49030612,
       0.51069388, 0.53108163, 0.55146939, 0.57185714, 0.5922449 ,
       0.61263265, 0.63302041, 0.65340816, 0.67379592, 0.69418367,
       0.71457143, 0.73495918, 0.75534694, 0.77573469, 0.79612245,
       0.8165102 , 0.83689796, 0.85728571, 0.87767347, 0.89806122,
       0.91844898, 0.93883673, 0.95922449, 0.97961224, 1.        ])
```

In [26]:

```
avg_train_loss = []
for i in range(0, 50):
    thetas = Ridge(train_x, train_y, alphas_to_test[i])

    avg_train_loss = avg_loss(Ridge(train_x, train_y, alphas_to_test[i]), train_x, train_y)
    avg_text_loss = avg_loss(Ridge(text_x, text_y, alphas_to_test[i]), text_x, text_y)

    print("theta:", thetas)
    print("avg_train_loss:", avg_train_loss)
    print("avg_text_loss:", avg_text_loss)
```

```
theta: [[ 5.49449189]
 [ 0.62107259]
 [-1.97398603]
 [ 0.01209971]
 [ 5.21492112]
 [ 0.05788709]
 [ 0.80887554]
 [ 0.01057385]
 [-7.43377416]
 [ 0.72509743]
 [ 0.56491596]
 [ 1.28719773]]
avg_train_loss: [0.52247141]
avg_text_loss: [0.30246122]
theta: [[ 5.47480416e+00]
 [ 5.61805448e-01]
 [-1.97802913e+00]
 [ 8.87293554e-03]
 [ 5.02563301e+00]
```

In [27]:

```python
avg_train_loss = []
for i in range(0, 50):
    loss = avg_loss(Ridge(train_x, train_y, alphas_to_test[i]), train_x, train_y)
    avg_train_loss.append(loss)
avg_train_loss
```

Out[27]:

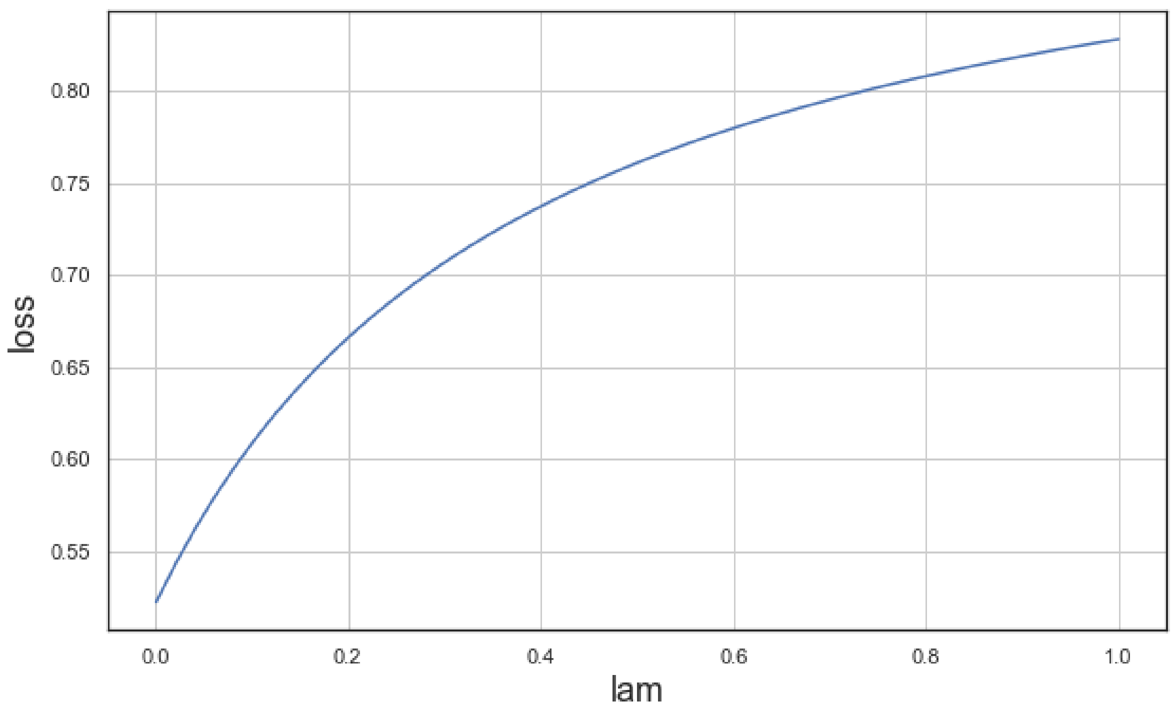
```
[array([0.52247141]),
 array([0.54356387]),
 array([0.56266182]),
 array([0.58003553]),
 array([0.59590854]),
 array([0.61046735]),
 array([0.62386877]),
 array([0.63624563]),
 array([0.64771116]),
 array([0.65836251]),
 array([0.66828348]),
 array([0.67754675]),
 array([0.68621566]),
 array([0.69434568]),
 array([0.70198558]),
 array([0.70917839]),
 array([0.71596228]),
 array([0.72237117]),
 array([0.72843534]),
 array([0.73418188]),
 array([0.73963512]),
 array([0.74481696]),
 array([0.74974719]),
 array([0.7544437]),
 array([0.75892272]),
 array([0.76319902]),
 array([0.76728607]),
 array([0.77119614]),
 array([0.7749405]),
 array([0.77852946]),
 array([0.78197248]),
 array([0.78527829]),
 array([0.78845493]),
 array([0.79150981]),
 array([0.7944498]),
 array([0.79728125]),
 array([0.80001006]),
 array([0.8026417]),
 array([0.80518126]),
 array([0.80763348]),
 array([0.81000279]),
 array([0.8122933]),
 array([0.81450888]),
 array([0.81665314]),
 array([0.81872945]),
 array([0.82074098]),
 array([0.82269071]),
 array([0.82458143]),
 array([0.82641577]),
 array([0.82819621])]
```

In [28]:

```
#确定准确率较高值所对应的岭回归系数
fig = plt.figure(figsize=(10,6))
plt.style.use('seaborn-ticks')

plt.xlabel('lam',fontsize=18)
plt.ylabel('loss',fontsize=18)

# x_major_locator = plt.MultipleLocator(1)
# ax = plt.gca()
# ax.xaxis.set_major_locator(x_major_locator)

# plt.xlim(0, 15)
# plt.ylim(0,40)

# plt.plot(range(0,50), MSE_Random_train, 'r', label='s1_train', marker = "o", markeredgecolor = 'wh
# plt.plot(range(0,50), MSE_Random_text, 'darkred', label='s1_text', marker = "o", markeredgecolor =

# plt.plot(range(0,50), MSE_Batch_train, 'b', label='s2_train', marker = "o", markeredgecolor = 'whi
# plt.plot(range(0,50), MSE_Batch_text, 'mediumblue', label='s2_text', marker = "o", markeredgecolor
#plt.legend()

plt.grid()

#plt.rcParams['font.sans-serif'] = ['SimHei'] # 设置字体，不然中文无法显示

#plt.title("岭回归系数曲线")

# plt.show()

plt.plot(alphas_to_test, avg_train_loss)
```

Out[28]:

```
[<matplotlib.lines.Line2D at 0x1b783a27850>]
```



通过上述图像，确定岭回归系数lam = 0.1

In [29]:

```
theta = Ridge(train_x, train_y, 0.1)
theta
```

Out[29]:

```
matrix([[ 5.41284398e+00],
        [ 3.81128708e-01],
        [-1.98914735e+00],
        [-3.00093950e-04],
        [ 4.44251614e+00],
        [ 8.31431155e-03],
        [ 8.64516340e-01],
        [-3.07396294e-02],
        [-5.83993347e+00],
        [ 6.09215084e-01],
        [ 5.27678204e-01],
        [ 1.49866854e+00]])
```

In [30]:

```
#岭回归模型下的平均训练误差和平均测试误差
avg_train_loss = avg_loss(Ridge(train_x, train_y, 0.1), train_x, train_y)
avg_text_loss = avg_loss(Ridge(text_x, text_y, 0.1), text_x, text_y)
print("avg_train_loss:", avg_train_loss, ", avg_text_loss:", avg_text_loss)
```

```
avg_train_loss: [0.60844357] , avg_text_loss: [0.6494829]
```