50.021 Artificial Intelligence

Constraint Satisfaction Problem

Suppose you are in charge in scheduling for ISTD classes that happen on Mondays, Tuesdays, and Thursdays. There are 5 classes that happen on these days and 3 professors who will be teaching these classes. Each professor can only teach one class at a time.

The classes are:

1. **C1**: Introduction to Algorithm, 8:00AM - 9:00AM
2. **C2**: Introduction to Artificial Intelligence, 8:30AM - 9:30AM
3. **C3**: Networks and Security, 9:00AM - 10:00AM
4. **C4**: Graphics, 9:00AM - 10:00AM
5. **C5**: Machine Learning, 9:30AM - 1030AM

The professors are:

- A: Professor Moriarty, who can teach classes C3 and C4
- B: Professor X, who can teach classes C2, C3, C4, and C5
- C: Professor Dumbledore, who can teach classes C1, C2, C3, C4, and C5

(1) Assume that there is one variable per-class, each with a domain (professors). Write down these variables, domains, and constraints.
Solution: $(C1 : C), (C2 : B, C), (C3 : A, B, C), (C4 : A, B, C), (C5 : B, C)$
Based on the timings, the constraints are: $C1 \neq C2, C2 \neq C3, C3 \neq C4, C4 \neq C5, C2 \neq C4, C3 \neq C5$.

(2) Draw the constraint graph associated with the CSP in part (1).
Solution: The CSP graph is as shown in Figure 1.

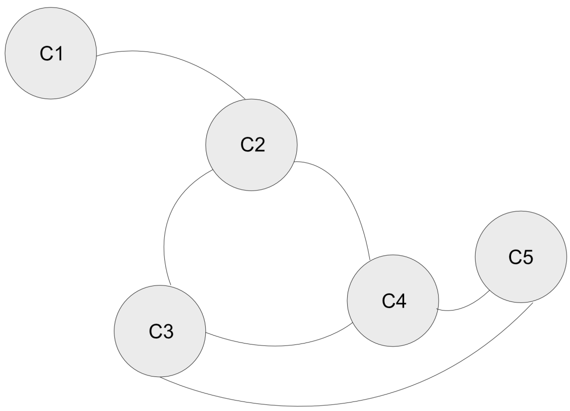

Figure 1: Graph for question 2

(3) Perform vanilla backtracking search (no pruning) on your graph in part (2) to get a viable solution. Show your steps.

Assume that we assign variables and domains lexicographically.

$$C1 : C$$
$$C1 : C, C2 : B$$
$$C1 : C, C2 : B, C3 : A$$
$$C1 : C, C2 : B, C3 : A, C4 : A, \text{ violation, backtrack}$$
$$C1 : C, C2 : B, C3 : A, C4 : B, \text{ violation, backtrack}$$
$$C1 : C, C2 : B, C3 : A, C4 : C$$
$$C1 : C, C2 : B, C3 : A, C4 : C, C5 : B$$

(4) Show the domains of the variables after running arc-consistency on this initial graph (after having already enforced any unary constraints).
Solution: $(C1 : C), (C2 : B), (C3 : A, C), (C4 : A, C), (C5 : B, C)$

(5) Perform backtrack search with arc consistency from the initialization after arc-consistency in (4). Show your steps.
Assume that we assign variables and domains lexicographically.

$$C1 : C, \text{ domain unchanged}$$
$$C1 : C, C2 : B, \text{ domain changed: } (C5 : C)$$
$$C1 : C, C2 : B, C3 : A, \text{ domain changed: } (C4 : C)$$
$$C1 : C, C2 : B, C3 : A, C4 : C$$
$$C1 : C, C2 : B, C3 : A, C4 : C.C5 : B$$

(6) The constraint graph in part (2) has an *almost tree* structure. Choose a variable such that when intializing it with a value, the remaining graph would have no cycles. There might be more than one choice.
Then initialize your chosen variable with a value, run tree CSP and see if it finds a solution, if no solution is found, try out all possible values for this variable until you find a solution. Show your steps.
Solution: One can choose to determine a solution for either C4 or C3 to get a tree structured constraint graph, and then run tree CSP. As an answer, we choose to pick C3: A first. The graph becomes $C1 \rightarrow C2 \rightarrow C4 \rightarrow C5$. The domain changed is $(C4 : B, C)$.

The tree CSP checks from C5, then C4, then C2, then C1. There is no values removed from the domain while doing `RemoveInconsistent`. Then assign C to C1. This automatically sets C2 as B. Next, C4 is C, and finally, C5 is B.