50.021 Artificial Intelligence
Theory Homework 7

Due: every Monday, 4PM before class starts

1. Consider the graph shown in Figure 1. The goal node is G and the start node is S. The values inside the nodes are heuristics values, and the values on the edges are the edge cost.

   (a) Are these heuristics admissible?

   Solution: Yes

   (b) Which of the heuristics is/are *not* consistent?

   Solution: None, all heuristics are consistent.

   For each of the following graph search strategies, write down the state of the frontiers when traversing from S to find G:,

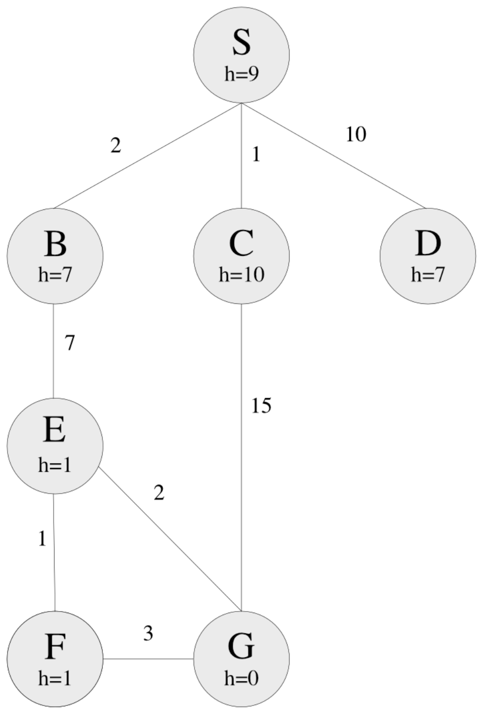

Figure 1: Graph for question 1

   (a) Uniform cost graph search

$$
\begin{aligned}
&[SC(1), SB(2), SD(10)]\\
&[SBE(9), SD(10), SCG(16)]\\
&[SBEF(10), SD(10), SBEG(11), SCG(16)]\\
&[SBEG(11), SBEFG(13), SCG(16)]
\end{aligned}
$$

(b) Greedy graph search

$$[SB(7), SD(7), SC(10)]$$
$$[SBE(1), SD(7), SC(10)]$$
$$[SBEG(0), SBEF(1), SD(7), SC(10)]$$

(c) A* graph search

$$[SB(9), SC(11), SD(17)]$$
$$[SBE(10), SC(11), SD(17)]$$
$$[SBEF(11), SBEG(11), SC(11), SD(17)]$$
$$[SBEG(11), SC(11), SBEFG(13), SD(17)]$$

(d) Best first graph search

$$[SB(7), SD(7), SC(10)]$$
$$[SBE(1), SD(7), SC(10)]$$
$$[SBEG(0), SBEF(1), SD(7), SC(10)]$$

2. Consider the maze shown in Figure 2. As shown on the diagram on the left, two players (Moon and Sun), want to exit a maze via exits E_1 or E_2. At each time step, each player can either **stay** in place or **move** to an *adjacent* free square (north, south east, west, but not diagonally). Assume at each time step they can simultaneously make a move, but a player cannot move into a square that the other player is moving into. Either player can use either exit, but they cannot both use the same exit. When a player moves, it leaves stones to block the path that remained for the next *2 time steps*, as shown in the right figure. During this time, *no player, including itself* can move to that particular square.

   The figure on the left shows one instance where the two players are, and in the figure on the right shows the state of the game *two timesteps* later, where, Moon has moved two steps downwards, and Sun has moved two steps rightwards.

   Our goal is to make both Moon and Sun to get to the exists in as few time steps as possible. The size of the board is as shown: 7×7. Answer the following questions,

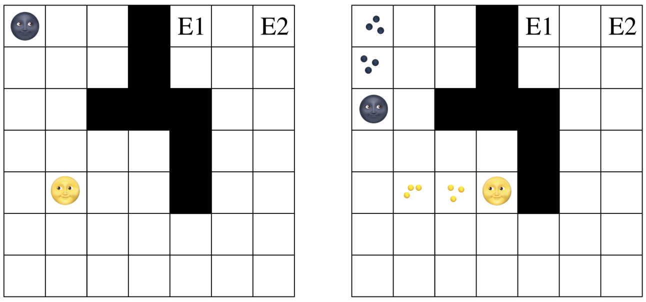

Figure 2: Maze for Question 2

   (a) Write an upper bound on the number of states (i.e. how many different state space must we encode?) Given that,

      i. We can ignore that stones cannot be on walls (bcs then we would have to do a position dependent count),
      ii. We can ignore state reduction due to the fact that a player cannot be on a stone of another player
      iii. We can ignore state reduction due to the fact that stones of a player cannot be on a stone of another player
      iv. We can ignore state reduction in the case when players would be adjacent - that they cannot move into each other

      Give not more than 5 sentences to explain your answer.
      Solution:
      $(7 \times 7)^2 \cdot 5^2$. 49 for each player, and 5 for each of the stone trails.

   (b) At the start of the game, assume that there's no stones around the players. What is the maximum branching factor of each state at the start of the game? What is the maximum branching factor of each state afterwards?
      Solution:
      5×5, and then reduced to 4×4 since one of the options is certainly blocked by the stone trails.

   (c) Let $d(x, y)$ denotes the Manhattan distance between x and y. Consider the following three heuristics defined in terms of the player's location and the exit locations:

      i. $h_1 = \max_{s \in \text{Sun, Moon}} \min\{d(s, E_1), d(s, E_2)\}$.
      This return the maximum over the two players, the distance from the player to the

closest exit.

ii. $h_2 = \max \ \{d(\text{Moon}, E_1), d(\text{Sun}, E_2)\}$.
This assign Moon to exit 1, and Sun to exit 2, and return the maximum distance from either players to its assigned exit.

iii. $h_3 = \min_{(e,e') \in \{(E_1,E_2),(E_2,E_1)\}} \max \ \{d(\text{Moon}, e), d(\text{Sun}, e'\}$.
This returns the max distance from a player to its assigned exit, under the assignment of players to distinct exits which minimizes this quantity.

Which of the above heuristics is/are **not** admissible? Give a counter example with a max of 3 sentences of explanation on each of your choice(s) of answer.

Solution: h_2 is not admissible. Let's say Moon is close to exit 2 but far from exit 1, and Sun is close to exit 1 but far from exit 2. Hence the heuristic will over-estimate since Moon and Sun can choose either exits.

(d) Which of the following statement(s) is/are true? You need not explain your answer.

i. h_1 dominates h_2

ii. h_1 dominates h_3

iii. h_2 dominates h_3 **Solution: T**

iv. h_2 dominates h_1 **Solution: T**

v. h_3 dominates h_1 **Solution: T**

vi. h_3 dominates h_2

vii. $h_1 = h_2$

viii. $h_1 = h_3$

ix. $h_2 = h_3$

The rest is false.