Bayesian Learning Computer Lab 1
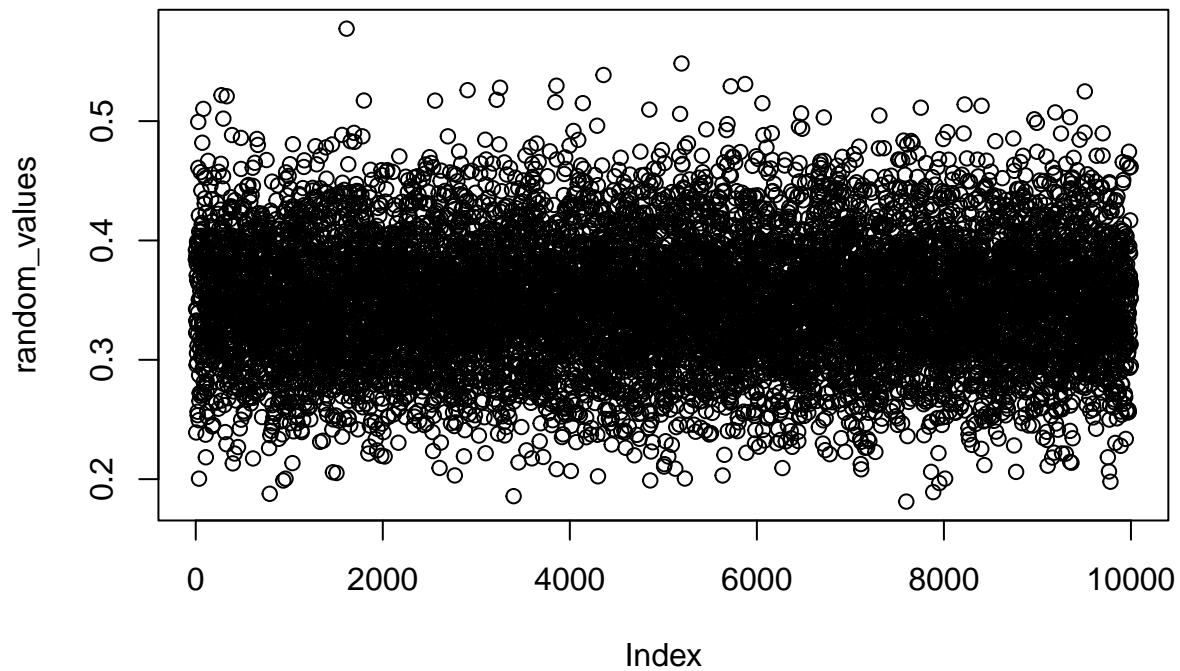
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Assignment 1. Daniel Bernoulli

a)

```r
set.seed(12345)
alpha_new = 8+22
beta_new = 8+70-22
random_values <- rbeta(n = 10000, shape1 = alpha_new, shape2 = beta_new)
plot(random_values)
```

```r
mean_list <- numeric()
std_list <- numeric()
for (i in 1:length(random_values)){
  mean_list[i] <- mean(random_values[1:i])
  std_list[i] <- sd(random_values[1:i])
}
plot(mean_list, ylab="Mean", xlab="Number of samples")
abline(h = alpha_new/(alpha_new+beta_new), col='red')
```
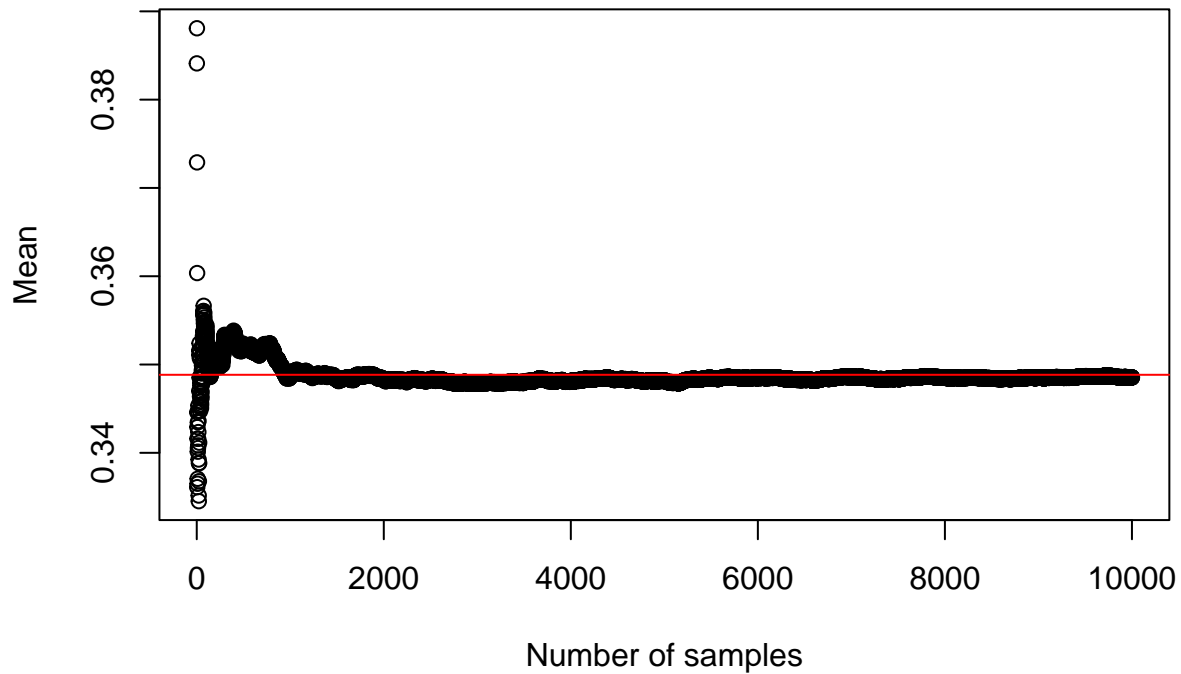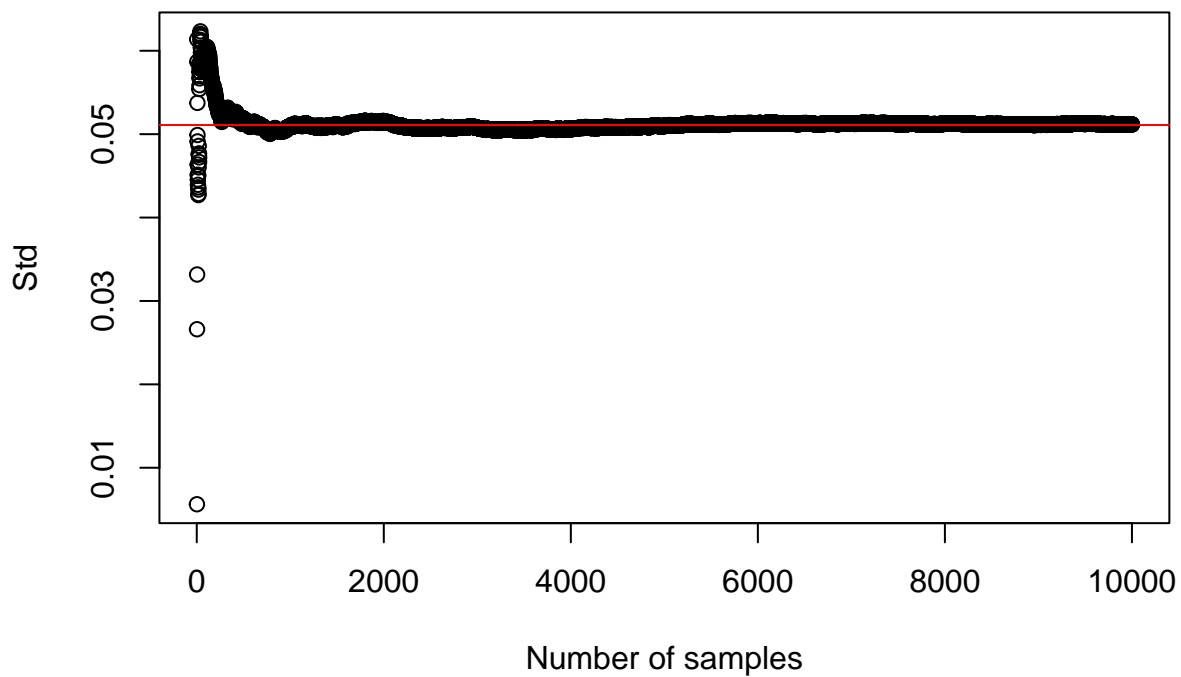


```r
plot(std_list, ylab="Std", xlab="Number of samples")
abline(h = (alpha_new*beta_new/((alpha_new+beta_new)^2*(alpha_new+beta_new+1)))^0.5, col='red')
```

The red lines in the last two plots represent the true values of mean and sd.

b)

```r
set.seed(12345)
exact_value <- 1 - pbeta(q = 0.3, shape1 = alpha_new, shape2 = beta_new)
compute_value <- sum(random_values>0.3)/10000
cat("The compute value and the exact value are",compute_value, 'and', exact_value,', respectively.')
```

```
## The compute value and the exact value are 0.8294 and 0.8285936 , respectively.
```

c)

```r
phi <- random_values/(1-random_values)
hist(phi,breaks = 100, freq = FALSE)
```
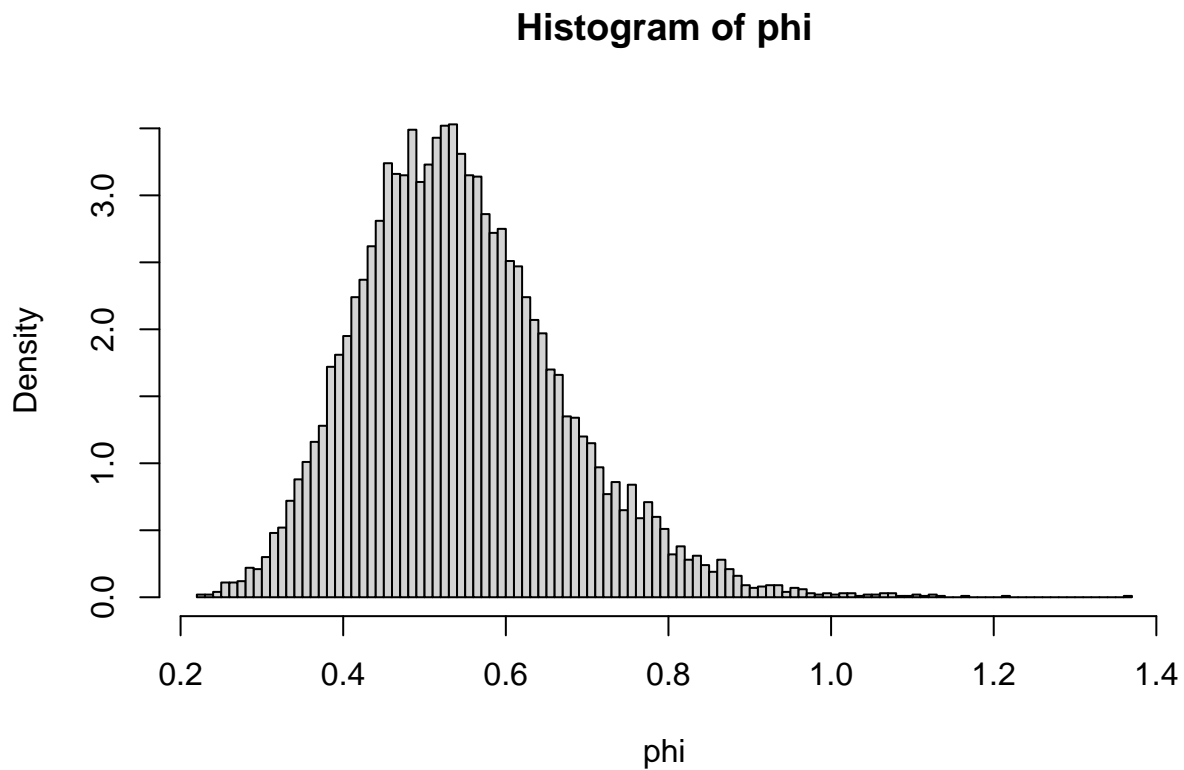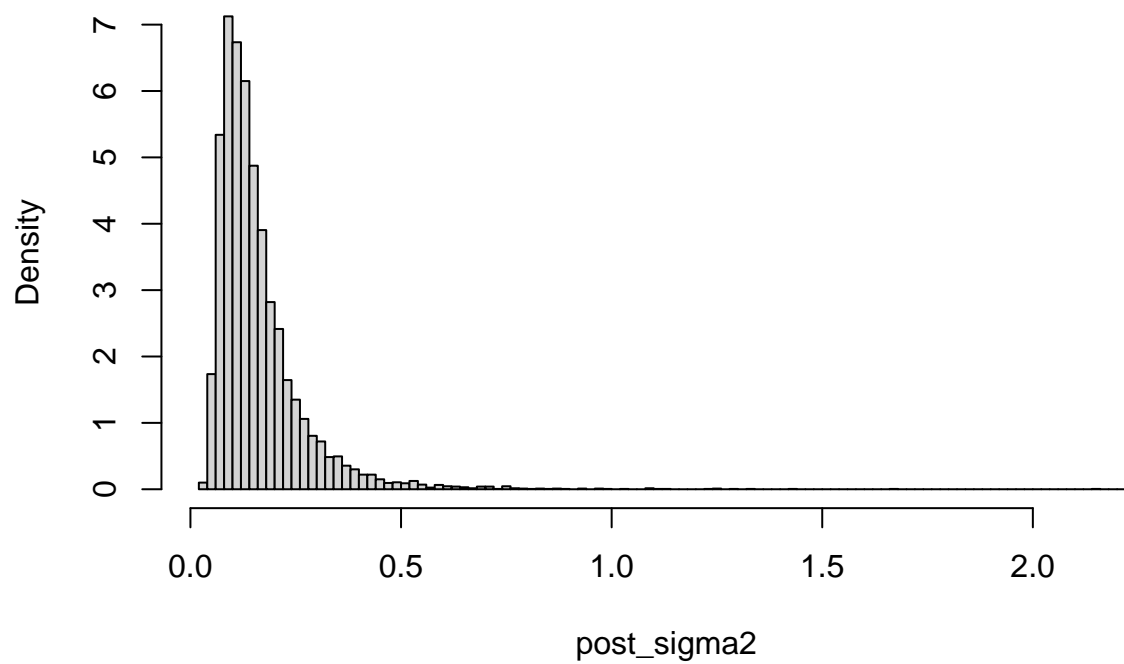
Histogram of phi

Assignment 2. Log-normal distribution and the Gini coefficient.

a)

```r
samples <- c(33,24,48,32,55,74,23,17)
post_sigma2 <- 1/rchisq(10000, length(samples), sum((log(samples)-3.6)^2)/length(samples))
hist(post_sigma2, breaks = 100, freq = FALSE)
```
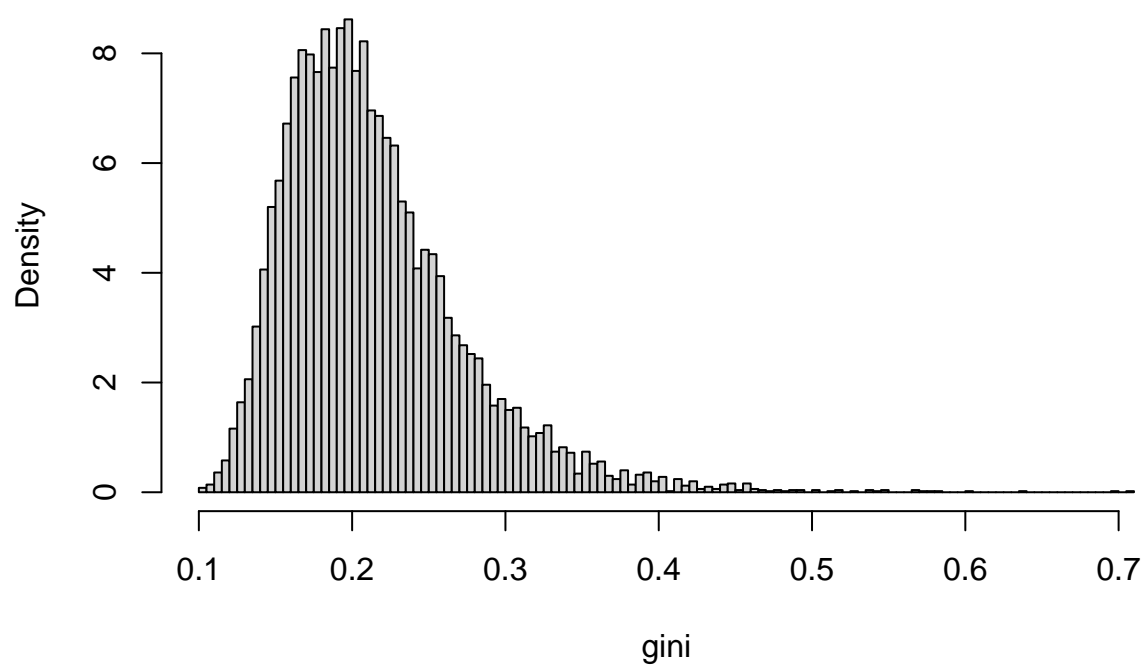
Histogram of post_sigma2

b)

```r
gini <- 2 * pnorm(sqrt(post_sigma2/2)) - 1
hist(gini, breaks = 100, freq = FALSE)
```

Histogram of gini

c)

```r
quantile(gini, c(0.025,1-0.025))
```

```
##      2.5%     97.5%
## 0.1321501 0.3578281
```

d)

```r
library(coda)
```

```
## Warning:   'coda' R 4.3.3
```

```r
HPDinterval(as.mcmc(gini),prob=0.95)
```

```
##          lower     upper
## var1 0.1227501 0.3311466
## attr(,"Probability")
## [1] 0.95
```

```
df <- data.frame(x=density(gini)$x,y=density(gini)$y)
df_sorted <- df[order(df$y, decreasing = TRUE), ]
df_sorted$cum_pro <- cumsum(df_sorted$y)/sum(df$y)
```

Assignment 3. Bayesianinference.

a)

Prior:

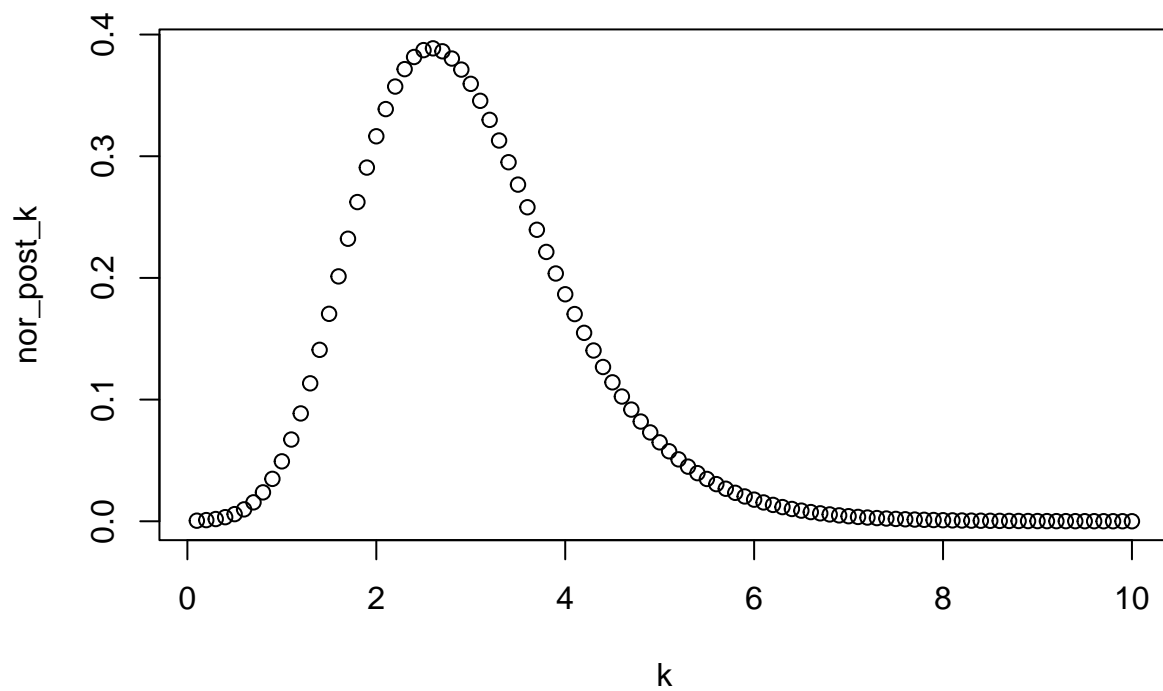$$\kappa \sim \frac{1}{2}e^{-\frac{1}{2}\kappa}$$

Likelihood:

$$y|\mu,\kappa \sim \frac{exp[\kappa\sum_{i=1}^{n}\cos(y_i-\mu)]}{[2\pi I_0(\kappa)]^n}$$

Posterior:

$$\kappa|\mu,y \sim \frac{exp[\kappa(\sum_{i=1}^{n}\cos(y_i-2.4)-0.5)]}{[I_0(\kappa)]^n}$$

```
samples <- c(-2.79,2.33,1.83,-2.44,2.23,2.33,2.07,2.02,2.14,2.54)
k <- seq(0.1,10,0.1)
post <- function(k){
  exp(k*(sum(cos(samples-2.4))-0.5))/(besselI(k,0))^length(samples)
}
post_k <- sapply(k, post)
nor_post_k <- post_k/(sum(post_k)*0.1)
plot(k, nor_post_k)
```

b)

```
optim_post_k <- function(k){
  -exp(k*(sum(cos(samples-2.4))-0.5))/(besselI(k,0))^length(samples)
}
optim_k <- optim(2.5, optim_post_k, method = "BFGS")
optim_k$par
```

```
## [1] 2.586444
```