##Q1

The double exponential (Laplace) distribution is given by formula:

$$DE(\mu,\alpha) = \frac{\alpha}{2} exp(-\alpha|x-\mu)$$

where, x is a real number,μ is a location parameter, and α is a real positive number.

The cumulative density function (CDF) of a continuous random variable X is defined as:

$$F(x) = \int_{-\infty}^{x} f\,(x)\,dx,\ -\infty < x < \infty$$

The cumulative distribution for the double exponential distribution: For x>μ:

$$F(x) = \int_{-\infty}^{x} \alpha\,/2\ e^{-\alpha(x-\mu)}\,dx,\ x > \mu$$

$$F(x) = 1 - \int_{x}^{\infty} \alpha\,/2\ e^{-\alpha(x-\mu)}\,dx$$

Integrating with respect to x,

$$F(x) = 1 - 1/2\ e^{-\alpha(x-\mu)}$$

For [x≤mu]

$$F(x) = \int_{-\infty}^{x} \alpha\,/2\ e^{\alpha(x-\mu)}\,dx,\ x \le \mu$$

After integrating,

$$F(x) = 1/2\ e^{\alpha(x-\mu)}$$

The inverse CDF: Below illustrates how to solve the equation F(x)=U for x

For U>1/2,

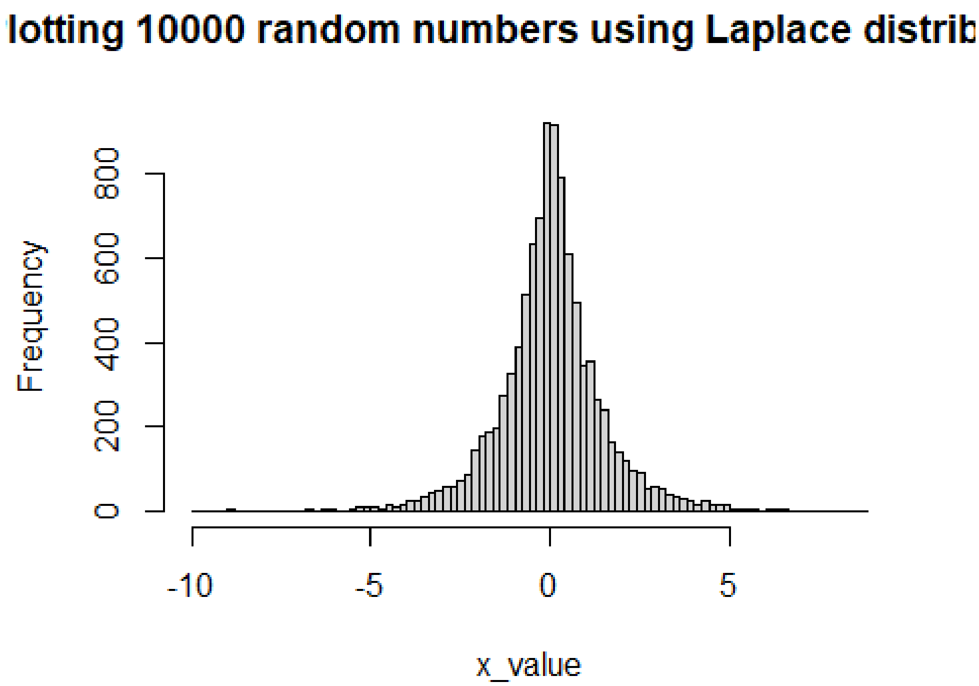
$$U = 1 - 1/2\ e^{\alpha(x-\mu)}$$

Solving for x,

$$x = \mu - ln(2-2U)/\alpha$$

$$U = 1/2\ e^{\alpha(x-\mu)}$$

$$x = \mu + ln(2U)/\alpha$$



The histogram supports the results because it closely matches the Laplace distribution plot. This means that the generated random numbers are accurate.

##Q2

$$c * g_y(x) \geq f_x(x)$$

for all x

Where,

$$g_y(x)$$

is proposal density,

$$c * g_y(x)$$

is the majoring function,

$$f_x(x)$$

is the target density.

$f_y(x) \sim \mathcal{DE}(0,1)$.

$$g_y(x) = \frac{1}{2}e^{-|x|}$$

$f_x(x) \sim \mathcal{N}(0,1)$ is

$$f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Evaluating the c,

$$c \geq \frac{f_x(x)}{g_y(x)}$$

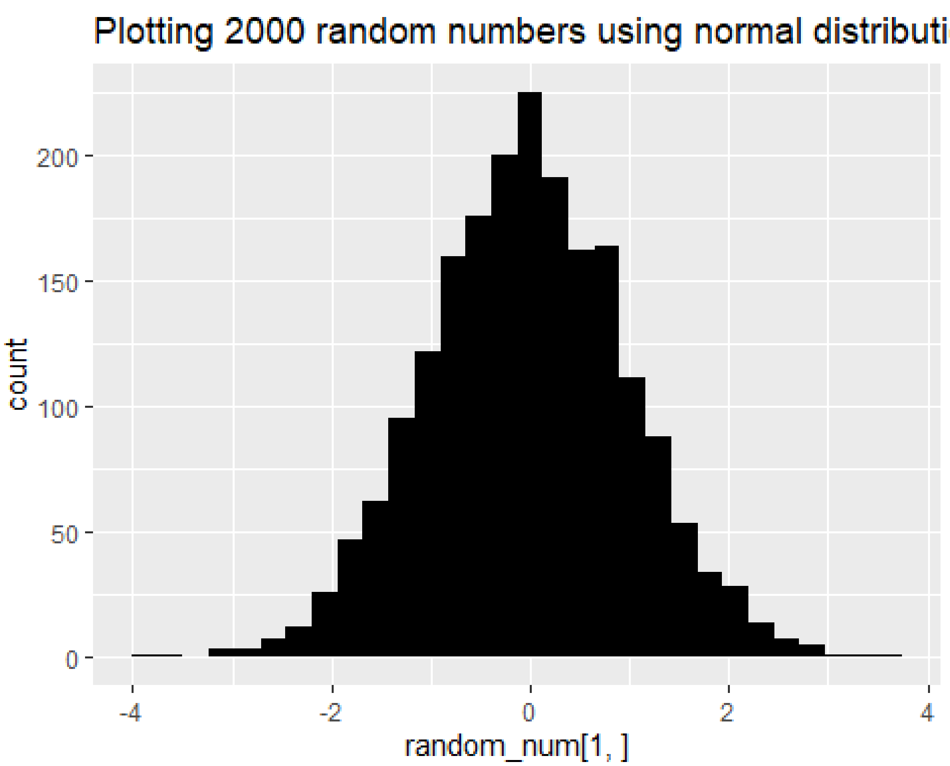
$$c \geq \sqrt{\frac{2}{\pi}} e^{-\frac{x^2}{2}} + |x|$$

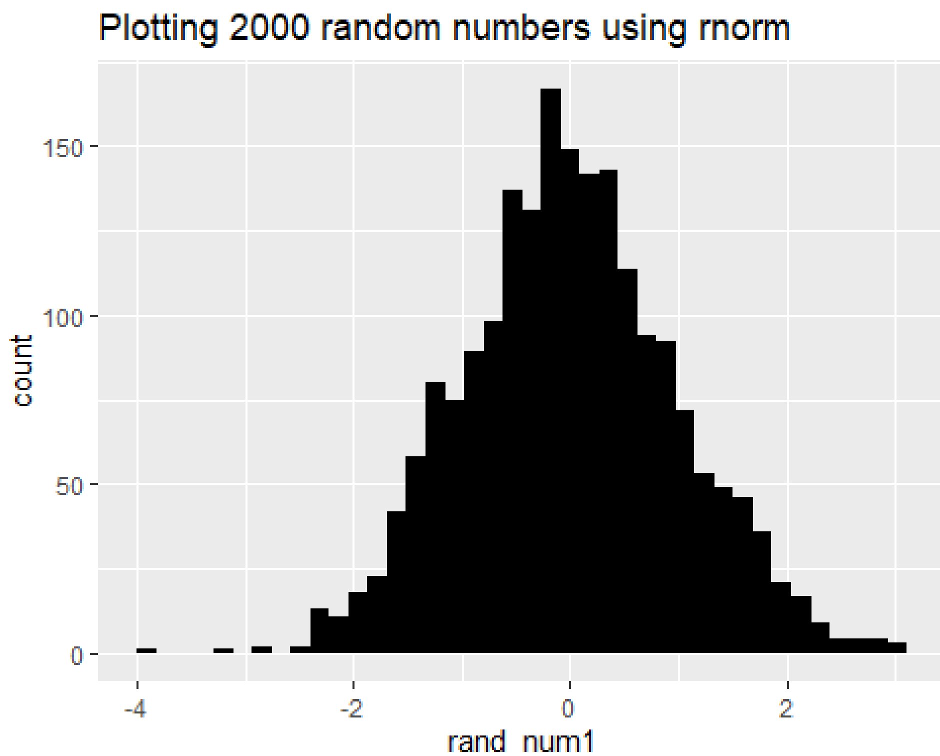
we obtain maximum at x=1 and by setting it to zero. C will be:

$$\sqrt{\frac{2}{\pi}} e^{\frac{1}{2}} = \sqrt{\frac{2e}{\pi}}$$

```
## Warning in geom_histogram(aes(random_num[1, ]), fill = "black", bis = 40):
## Ignoring unknown parameters: `bis`

## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```






```
## [1] "Average rejection rate R= 0.243284146802876"

## [1] "Expected rejection rate ER= 0.23982654946686"
```

The expected rejection rate and average rejection rate are very close, showing only slight differences. When we look at the histogram plots of two methods—one using the acceptance/rejection technique and the other using the rnorm() function—they look quite similar and balanced.