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Size Matters

The Kelly Criterion and the Importance of Money Management

To suppose that safety-first consists in having a small gamble in a large number of different [companies] where I have no information to reach a good judgment, as compared with a substantial stake in a company where one's information is adequate, strikes me as a travesty of investment policy.

John Maynard Keynes
Letter to F.C. Scott, February 6, 1942 [1]

Pressing the Edge

As an investor, maximizing wealth over time requires you to do two things: find situations where you have an analytical edge; and allocate the appropriate amount of capital when you do have an edge. While Wall Street dedicates a substantial percentage of time and effort trying to gain an edge, very few portfolio managers understand how to size their positions to maximize long-term wealth.

A simple example illustrates the point. Assume you can participate in a coin toss game where heads pays $2 and tails costs $1. You start with a $100 bankroll and can play for 40 rounds. What betting strategy will allow you to achieve the greatest probability of the most money at the end of the fortieth round? [2]

We'll get to the answer in a moment, but let's consider the obvious extremes: if you bet too little, you won't take advantage of a clearly positive expected-value opportunity. On the other hand, if you bet everything, you risk losing all of your money. Money management is all about determining the right amount of capital to allocate to an investment opportunity, given the edge and the frequency of such opportunities.

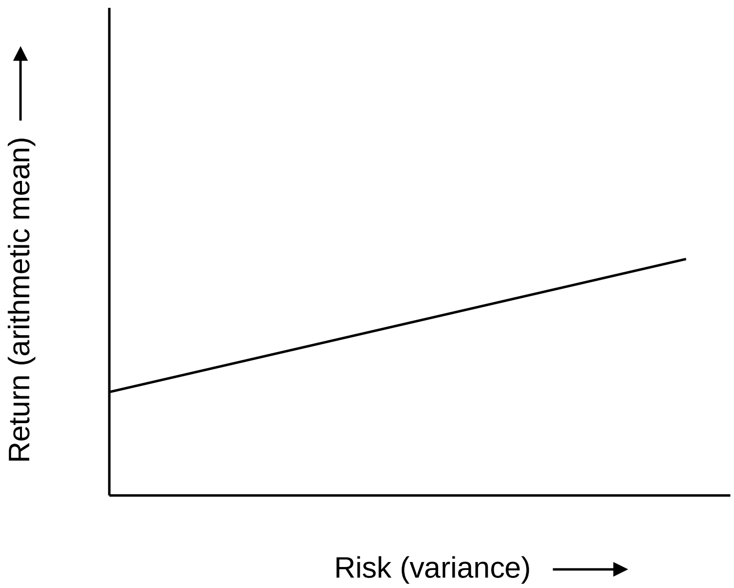
Position size is extremely important in determining equity portfolio returns. Two portfolio managers with the same list and number of stocks can generate meaningfully different results based on how they allocate the capital among the stocks. Great investors don't stop with finding attractive investment opportunities; they know how to take maximum advantage of the opportunities. As Charlie Munger says, good investing combines patience and aggressive opportunism.

Morningstar data reveal that most investors don't operate this way. U.S. domestic diversified funds have 77 positions (median) and the top 10 holdings represent just over one-quarter of the portfolio (median). Further, 35 percent of mutual funds have 100 or more positions and a 94 percent median correlation with the S&P 500 Index. Whether attributable to incentives or suboptimal strategy—and we suspect both are at play—most active managers do little to distinguish themselves.

The Mean/Variance Way

So how best to allocate capital, either across asset classes or within an asset class? The classic answer comes from the concept of mean/variance efficiency, first formalized by Harry Markowitz in 1952. [3] The premise is that risk and reward are related linearly (see Exhibit 1). The mean is the average arithmetic return from an asset or portfolio. Variance measures how spread distribution points are from the average.

Exhibit 1: Mean/Variance Model of Risk and Reward



Source: LMCM.

A risk averse investor seeks the highest return for a given level of risk. For all portfolios with a given level of risk, the investor will select the one with the highest return. And for an assumed level of return, the investor prefers the one with the least risk. No optimal portfolio exists since different individuals have different risk preferences, but portfolios away from the efficient frontier—the best reward for a given level of risk—are suboptimal. Mean/variance is powerful because if you specify the function that accurately expresses your utility, you can find a portfolio that's right for you.

But what if you ask the asset allocation question a different way: How do you maximize the likelihood that you'll have the most money at the end of a particular period? As it turns out, mean/variance doesn't answer that question.

Shannon, Chance, and The Kelly Criterion

Bell Labs scientist Claude Shannon is well known for developing information theory—essentially, the necessary properties and systems for transmitting intelligence. Before Shannon, most engineers tried to understand the information problem by focusing on a message's meaning. Shannon's insight was that information is related to chance. As author William Poundstone notes, "Information exists only when the sender is saying something that the recipient doesn't already know and can't predict. Because true information is unpredictable, it is essentially a series of random events like spins of a roulette wheel or rolls of a dice." [4]

As an example, Poundstone points to a television commercial depicting a wife asking her husband to bring home "shampoo." The husband, misunderstanding her, shows up with "Shamu," the killer whale. Neither the wife's request nor the husband's misunderstanding is surprising. The commercial captures our attention because the husband acts on a highly improbable and unpredictable request without further information.

For Shannon, the incompressible part of a message relates to its unpredictability. The less probable a message, the more bandwidth it requires. A request to bring home Shamu undoubtedly demands more bandwidth than a routine demand for shampoo.

Shannon's theory also considers equivocation—the chance the message is wrong—and shows you must subtract equivocation from the channel capacity to determine the information rate. More reliable information leads to a higher information rate for a given channel capacity. Most of the information channels we use today, including phones, television, the Internet, operate using Shannon's ideas.

What does any of this have to do with optimal bet size? Shannon's colleague at Bell Labs, John Kelly, recognized another application for information theory's ideas: gambling.[5] Information in a betting setting is something the market does not already know. Consistent with the idea of equivocation, true information is also probabilistic.

Kelly imagined a system where you have an edge; a set of expectations that differs from those of the market. He then developed a formula, based on Shannon's work, showing the exact amount of your bankroll you should bet in order to maximize your capital over the long term. Consistent with the theory, the maximum rate of return comes when you know something the market doesn't.

We can express the Kelly formula a number of ways. We'll follow Poundstone's exposition:[6]

$$\frac{Edge}{Odds} = f$$

Here, *edge* is the expected value of the financial proposition, *odds* reflect the market's expectation for how much you win if you win, and *f* represents the percentage of your bankroll you should bet. Note that in an efficient market, there is no edge because the odds accurately represent the probabilities of success. Hence, bets based on the market's information have zero expected value (this before the costs associated with betting) and an *f* of zero.

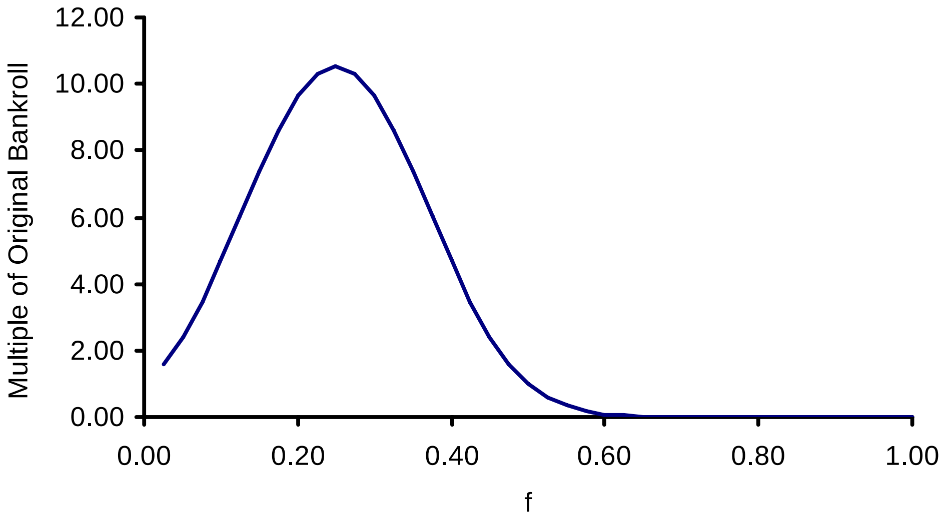
Let's go back and answer our opening coin-toss question using the Kelly formula. The payoff scheme, a $2 win for a heads and a $1 loss for a tails, suggests 2-to-1 odds. Since we're dealing with a fair coin, we know the tosses will be 1-to-1.[7] So we recognize something the market doesn't: heads will show up more often than the payoff scheme suggests.

Solving the formula, edge is $0.50 (expected value, or 50 percent x $2 + 50 percent x -$1) and odds are $2 (the amount you win if you win). The optimal amount to bet is 25 percent of your bankroll in each round. Said differently, betting 25 percent will lead to a greater accumulation of wealth, on average, than any other betting strategy.

$$\frac{\$0.50}{\$2.00} = f = 25\%$$

Exhibit 2 shows wealth outcomes based on a range of *f* values for 40 rounds. Betting too little leaves a substantial amount of money on the table, while betting too much leads to near-certain ruin. The latter point bears emphasis: if there is a probability of loss, even with a positive expected value economic proposition, betting too much *reduces* your expected wealth. Such overbetting may have been the source of demise for a number of high-profile hedge funds.[8]

Exhibit 2: The Kelly Formula Solves for the Optimal Betting Strategy



Source: Vince, 16 and LMCM.

Though basic, this illustration draws out two crucial points for investors of all stripes:

- An intelligent investor needs an edge (a view different than that of the market); and
- An investor needs to properly allocate capital to maximize value when an investment idea does appear.

The Kelly formula contributes to a larger concept known as the Kelly Criterion, or Kelly system. [9] Based on information theory, the Kelly Criterion says an investor should choose the investment(s) with the highest geometric mean return. This strategy is distinct from those based on mean/variance efficiency. Importantly, however, you can calculate geometric mean using the same arithmetic mean and variance from mean/variance models. [10]

Mathematician and investor Ed Thorp is probably the Kelly Criterion's most visible advocate and successful practitioner. In the early 1960s, Thorp developed a system of card counting to improve a player's odds in the card game blackjack and complemented it with the Kelly system to optimize wealth building. [11] Thorp went on to co-found Princeton-Newport Partners, delivering 20 percent annual compounded returns, with a 6 percent standard deviation, over a 20-year span via various investment strategies.

In his book, *The Mathematics of Gambling*, Thorp explains the Kelly system's attractive features:[12]

1. *The chance of ruin is "small."* Because the Kelly system is based on proportional bets, losing all of your capital is theoretically impossible (assuming money is infinitely divisible). Even so, a small chance of a significant drawdown remains.

2. *The Kelly system is highly likely to grow a bankroll faster than other systems*. Provided comparably attractive opportunities continue to appear, there is a high probability the system will generate a bankroll that exceeds other systems by a determinable multiple.

3. *You tend to reach a specified level of winnings in the least average time*. If you have a financial end goal in mind and continuous opportunities, the Kelly system will likely allow you to achieve the objective in a shorter time than other systems.

In short, the Kelly system has proven to be both theoretically sound and useful for practitioners. Still, the most enthusiastic supporters for the approach (information theorists, mathematicians, gamblers, and traders) do not include mainstream economists. We now turn to some of the more practical constraints with the Kelly system, and we contrast the Kelly system with mean/variance efficiency.

Practical Considerations with the Kelly Criterion and Mean/Variance

Under ideal conditions the Kelly Criterion is clearly a powerful concept. Using the Kelly formula's optimal betting strategy in our coin-toss example is unquestionably valuable. The real world, however, presents a great deal more complexity than a coin toss or blackjack table. In the stock market an investor faces many more outcomes than a gambler in a casino. That said, the Kelly Criterion works well when you parlay your bets, face repeated opportunities, and know what the underlying distribution looks like.

We now take a look at these conditions, using the opportunity to compare the Kelly Criterion to mean/variance efficiency.

Parlaying bets. You can approach financial opportunities with one of two betting strategies: bet the same amount each time or reinvest your winnings. As it turns out, what you look for will be very different based on which strategy you select.

Kelly recognized this, writing: "suppose the gambler's wife allowed him to bet one dollar each week but not to reinvest his winnings. He should then maximize his expectation (expected value of capital) on each bet." [13] In other words, if you employ the first strategy, you should focus on average payout calculated with the *arithmetic* mean. In this case, the mean/variance approach is the way to go.

In contrast, the Kelly Criterion assumes you parlay your bets, and says you should choose the opportunities with the highest *geometric* means.

As an illustration of the difference between arithmetic and geometric returns, consider the following stock price changes (this may be reminiscent of the late 1990s and early 2000s):

T_0	T_1	T_2
\$100	\$200	\$20

What is the arithmetic average return from T_0 to T_2? The answer is simply the sum of the changes (100 percent + -90 percent = 10 percent) divided by the number of periods (2). The arithmetic average is 5 percent (10 percent/2).

In contrast, the geometric average is the product of the changes (2.0 x .1) to the Nth root (2) minus 1.

$$= \sqrt{2.0 \times 0.1} - 1 = -55.3\%$$

In this case, the arithmetic average shows 5 percent while the geometric average is negative 55 percent. Notably, the geometric mean is always less than or equal to the arithmetic mean. The greater the variance, the larger the difference between the arithmetic and geometric mean. Additionally, if a series contains a single payoff of zero, the geometric mean is always zero. Play a game with a zero payoff long enough and you are assured ruin.

Exhibit 3 reproduces three series of payoffs with varying arithmetic returns, variances, and geometric returns that Poundstone uses in *Fortune's Formula*:

Exhibit 3: Payoff Series Including Mean and Variance

	A		B		C	
	Probability	Payoff	Probability	Payoff	Probability	Payoff
	50%	$ 1.00	50%	$ 2.00	50%	$ 3.00
	50%	$ 2.00	17%	$ -	50%	$ 0.50
			17%	$ 1.00		
			17%	$ 3.00		
Arithmetic mean		$ 1.50		$ 1.67		$ 1.75
Variance		$ 0.30		$ 1.07		$ 1.88
Geometric mean		$ 1.41		$ -		$ 1.22

Source: Poundstone, 198.

If you bet the same amount every time, like Kelly's once-a-week gambler, you should focus on the arithmetic means. Mean/variance doesn't determine the best series because individuals may have different preferences. Both the risk and returns rise for these series as you move from left to right. Determine your risk preference and you can settle on the best strategy for you. Clearly, though, the highest expected payoff is with series C.

In contrast, the parlay bettor using the Kelly Criterion will always choose series A. According to Poundstone's calculation, starting with $1 and reinvesting profits each week for a year leads to an expected fortune of over $67 million. The same strategy with series C amasses an expected value of just under $38,000.

Series B has a favorable arithmetic mean, but the geometric mean is zero. This happens because one of the payoffs is zero, which means you will lose all of your money with this strategy *given enough trials*.

Leaving aside the technical details of the Kelly Criterion, the central message for investors is that standard mean/variance analysis *does not* deal with the compounding of investments. If you seek to compound your wealth, then maximizing geometric returns should be front and center in your thinking.

Repeated trials. Both the Kelly Criterion and mean/variance approaches assume lots of trials, or financial propositions. The probabilistic nature of most market-based financial propositions means you need a substantial number of observations to reasonably assure you capture the system's signal, versus short-term noise.

Know the distribution. Long-term stock market investing differs from casino games, or even trading, because outcomes vary much more than a simple model suggests. Any practical money management system faces the challenge of correcting for more complicated real-world distributions. [14]

Substantial empirical evidence shows that stock price changes do not fall along a normal distribution. [15] Actual distributions contain many more small change observations and many more large moves than the simple distribution predicts. These tails play a meaningful role in shaping total returns for assets, and can be a cause of substantial financial pain for investors who do not anticipate them.

As a result, mean and variance insufficiently express the distribution and mean/variance can at best crudely approximate market results. Notwithstanding this, practitioners assess risk and reward using a majority of analytical tools based on faulty mean/variance metrics.

So the mean/variance approach has two major strikes against it. First, it doesn't work for parlayed bets (even though most investors do reinvest). Second, it doesn't consider the verity of non-normal distributions. Yet most mainstream economists *still* argue that maximizing geometric returns is the wrong way to allocate capital. Why?

Neoclassical Economic Objections to the Kelly Criterion

One of the most vocal critics of geometric mean maximization happens to be one of the most well-known and well-regarded economists in the world: MIT's Paul Samuelson. Poundstone notes that Samuelson likes to describe the Kelly Criterion as a fallacy. In a 1971 paper on the topic, Samuelson provides a theorem and what he calls its false corollary: [16]

> *Theorem*: If one acts to maximize the geometric mean at every step, if the period is "sufficiently long," "almost certainly" higher terminal wealth and terminal utility will result than from any other decision rule.
>
> From this indisputable fact, it is tempting to believe in the truth of the following false corollary:
>
> *False Corollary*: If maximizing the geometric mean almost certainly leads to a better outcome, then the expected utility of its outcomes exceeds that of any other rule, provided T is sufficiently large.

How do economists reconcile the apparently conflicting ideas that maximizing geometric mean will almost certainly result in higher wealth (theorem) with the notion that this approach is possibly inferior to other strategies (corollary)?

Perhaps the clearest explanation of the mainstream economics case comes from Mark Rubinstein.[17] First, he notes the geometric mean maximization strategy does not assure that you will end up with more wealth than other strategies. Since the approach is based on probability, there remains a very small chance an investor will do poorly. This low-probability, high-impact scenario may violate an individual's utility function.

Second, success of geometric mean maximization depends on investors staying in the market for the long run. If an investor needs access to the funds in the near-term, the benefits of compounding do not apply.

Third, the system assumes the investment payoffs remain steady and the investment opportunities set is large enough to accommodate a rising asset base. Shifting investment payoffs undermine the system.

Finally, Rubinstein invokes the macro-consistency test: to judge a strategy's superiority, ask what would happen if *everyone* tried to follow it. His point is all investors cannot apply the geometric mean strategy successfully.

So who's right, the Kelly camp or the Samuelson camp?

One way to understand the difference of opinion is to distinguish between normative and positive arguments. Normative arguments stem from a view of how the world should be, while positive arguments reflect how things are and will likely be in the foreseeable future. Economists dismiss the strategy of maximizing geometric means based on a normative argument. Investors should have specific utility functions and act consistently with those functions. Since the small chance of a large loss will violate an individual's utility function, geometric mean maximization is not right for everyone (Rubinstein's first point).

A positive argument is based on how people actually behave. Very few people take the time to quantify their utility functions, and those functions shift over time and with varying circumstances.

Most investing decisions are made by professional investment managers who must serve a diverse group of fund holders. Thorp notes that when he explains the Kelly Criterion to investors they say, "Yeah, sounds good to me, I want that." [18]

Economic historian Philip Mirowski gives a more scathing denouncement of the economic field. He suggests economists have little interest in what people really do—that's more the realm of psychology, and they don't add much when suggesting how people should act. He writes: [19]

> [N]eoclassicals have wavered between claiming that they were describing actual behavior and claiming that they were prescribing what rational behavior should be. Their contempt for psychology has always given lie to the first claim, so of necessity, they have eventually retreated to the second. *This second position is untenable, however, because it conflicts with the ideology of the scientist as a detached and value-neutral observer as it commits the transgression of defining rationality in a post-hoc manner in order to conform to the mathematical model of utility.* (Emphasis added.)

There are two other problems with utility theory and investing. The first comes from the father of mean/variance analysis, Harry Markowitz. In his famous *Portfolio Selection*, Markowitz advocates the geometric mean maximization approach. In spite of arguments by Jan Mossin (one of the founders of the capital asset pricing model) and Samuelson in the 1960s, Markowitz reconfirmed his endorsement of the geometric mean maximization strategy in the preface to his second edition published in 1970. Markowitz suggests utility-maximizing man "acts absurdly" over the long term: [20]

> I concluded . . . that the investor who is currently reinvesting everything for "the long run" should maximize the expected value of the logarithm of wealth. Mossin and Samuelson have each shown that this is not true for a wide range of functions relating to utility of wealth at the end of the last period, T. The fascinating Mossin-Samuelson result, combined with the straightforward arguments supporting the earlier conclusions, seemed paradoxical at first. I have since returned to the view . . . that for large T, the Mossin-Samuelson man acts absurdly, like a player who would pay an unlimited amount for the St. Petersburg game . . . the terminal utility function must be bounded to avoid this absurdity; and the [maximization of mean geometric return] argument applies when utility of wealth is bounded.

The second problem comes from Kahneman and Tversky's prospect theory. Utility theory considers gains and losses in the context of the investor's total wealth (broad frame). In contrast, prospect theory considers gains and losses versus isolated components of wealth, like changes in a specific stock or portfolio price (narrow frame). Experimental studies show that investors use price, or changes in price, as a reference point when evaluating financial transactions. Investors pay attention to the narrow frame. Utility theory does not explain how people behave. [21]

Even if you agree the utility argument is not persuasive enough to suggest abandoning the Kelly strategy, Rubinstein makes some points worth considering carefully. For a geometric mean maximization system to work, an investor has to participate in the markets over the long term. In addition, the portfolio manager must be able to systematically identify investment edges—points of view different than that of the market and with higher expected returns.

Finally, since by definition not all market participants can have an edge, not all investors can use a Kelly system. In fact, most financial economists believe markets to be efficient. For them, a discussion of optimal betting strategy is moot because no one can systematically gain edges.

Based on our observations of behavior, portfolio structure, and incentives, we conclude that very few investors are organized to take advantage of the principle of mean geometric return maximization. Here's why.

Why Many Money Managers Focus on Arithmetic Returns

As we noted, geometric mean maximization requires an investor to be in the market over the long haul. If capital is free to come and go, however, as is the case with an open-end mutual fund, the portfolio manager may not have the luxury of thinking long-term. Even if geometric mean maximization is the best way to go, market realities may compel a short-term focus.

The reasoning is straightforward: an open-end portfolio with poor short-run performance faces the very real prospect of losing assets. In turn, portfolio managers have a strong incentive to focus on the investment ideas they perceive will do well in the short term, even at the expense of ideas offering higher rates of return over the long term. Geometric mean maximization simply does not make sense for a portfolio manager in this short-term mindset.

If an open-end fund structure encourages this short-term perspective, why aren't more funds closed-end? (The assets in open-end mutual funds are 25 times larger than those in closed-end funds.) The obvious first answer is that investors don't want to lock up their money; they prefer the flexibility to reallocate capital in the case a portfolio manager performs poorly.

But Jeremy Stein argues the dominance of open-end funds reflects both the preference of investors and the desires of the mutual fund companies. [22] In a closed-end fund, changes in asset level are solely a function of results. In contrast, in an open-end fund the potential for inflows balances the risk of outflows. Most fund managers recognize that the upside of positive flows, especially if results are good, more than offsets the risk of outflows.

Many fund companies understand the best way to favorably tilt the inflow/outflow equation is to operate within the near-term consensus; the focus shifts to delivering acceptable results over sequential short-term periods, even at the cost of higher, albeit lumpier, long-term returns. The high portfolio turnover rate (averaging around 100 percent in the last couple of years) we see supports this view. In case after case, short-term incentives discourage portfolio managers from adopting the Kelly system.

Loss Aversion and the Kelly Criterion

Poundstone highlights another important feature of the Kelly system: the returns are more volatile than other systems. While the Kelly system offers the highest probability of the most wealth after a long time, the path to the terminal wealth resembles a roller coaster. The higher the percentage of your bankroll you bet (*f* from the Kelly formula) the larger your drawdowns.

Another important lesson from prospect theory—and a departure from standard utility theory—is individuals are loss averse. [23] Specifically, people regret losses roughly two to two and a half times more than similar-sized gains. Naturally, the longer the holding period in the stock market the higher the probability of a positive return because stocks, in aggregate, have a positive expected value. Loss aversion can lead investors to suboptimal decisions, including the well-documented disposition effect.

Investors checking their portfolios frequently, especially volatile portfolios, are likely to suffer from myopic loss aversion. [24] The key point is that a Kelly system, which requires a long-term perspective to be effective, is inherently very difficult for investors to deal with psychologically.

It is possible to reduce the strategy's volatility by taking partial Kelly positions. Naturally, these positions also reduce expected return.

What Does All This Mean for Equity Portfolio Management?

So what lessons should equity investors draw from this discussion? A few points emerge:

- *Edge is key*. Recall the foundation of Kelly's model rests on having a view that is different, and more correct, than that of the market. Having an edge requires understanding the market's perspective. As Poundstone writes, "The stock ticker is like a tote board. It gives the public odds. A trader who wants to beat the market must have an edge, a more accurate view of what bets on stocks are really worth." [25]

  One way for equity investors to think about edge is finding situations where the stock's rate of return is likely to be higher than the market anticipates. A stock's excess rate of return is a function of its percentage discount to fair value—the margin of safety—and how long it takes the market to close the price-to-value gap.

- *Greater opportunity suggests a larger bet*. Finding an edge only gets you part of the way to maximizing long-term wealth. Appropriately sizing the position is the other part. A distinct minority of investors are skilled at position sizing, while most investors—again, generally reflecting agency costs—are satisfied to perform in line with their investment benchmark. One good, albeit convenient, example of the first group is Warren Buffett. In the mid-1960s, Buffett allocated close to one-quarter of his assets into one stock, American Express, when he was convinced the security offered superior return prospects. Note, too, that Berkshire Hathaway is essentially a closed fund. [26]

- *Mean/variance is not the best way to think about maximizing long-term wealth if you are reinvesting your investment proceeds*. If you face a one-time financial decision, you want to maximize your arithmetic mean. But with repeated favorable opportunities—either through time or diversification—chances are you will do better in the long term by maximizing geometric mean. Mean/variance may be deeply embedded in the investment industry's lexicon, but it doesn't do as good a job at building wealth as a Kelly-type system.

- *Applying the Kelly Criterion is hard psychologically*. Assuming you do have an investment edge and a long-term horizon, applying the Kelly system is still hard because of loss aversion. Most investors face institutional and psychological constraints in applying a Kelly-type system.

Endnotes

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[3] Harry M. Markowitz, "Portfolio Selection," *Journal of Finance*, vol. 12., 1, 1952, 77-91.
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[5] J.L. Kelly, Jr., "A New Interpretation of Information Rate," *Bell System Technical Journal*, 1956, 917-926.
[6] Poundstone, 72.
[7] For a good discussion of how to understand odds, see David Sklansky, *Getting the Best of It, 2nd ed.* (Henderson, NV: Two Plus Two Publishing, 1997).
[8] See Nicholas Chan, Mila Getmansky, Shane M. Hass, and Andrew W. Lo, "Systemic Risk and Hedge Funds," *NBER Working Paper*, August 1, 2005. Low volatility periods, as we have just experienced, tempt investors to use more leverage to generate higher returns. According to the Kelly system, this overbetting leads not to higher return but ruin. We also know that volatility is clustered. So overbetting coming into a period of higher volatility may cause financial distress for specific investment firms. Also see Markowitz (1959) for a demonstration of the deleterious impact of overbetting.
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[26] Roger Lowenstein, *Buffett: The Making of an American Capitalist* (New York: Random House, 1995), 82.

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