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Fat Tails and Nonlinearity

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Diversity Breakdowns and Invisible Vulnerability

For he who is acquainted with the paths of nature, will more readily observe her deviations; and, *vice versa*, he who has learned her deviations will be able more accurately to describe her paths.

Francis Bacon
Novum Organum [1]

The Memo Went Out

If you are involved in financial markets, you have gotten the memo about fat tails by now.

But awareness of extreme events is not enough. Thoughtful investors must understand two interrelated aspects of the market. The first is the statistical properties of price movements, including important deviations from the bell-shaped distribution. Academics, risk managers, and quantitative investors have explored this aspect extensively. Researchers recognized decades ago that the distribution of price changes includes fat tails. [2]

The second aspect, and one often overlooked or misunderstood, is the mechanism that leads to the statistical imprint. Much of the work on the market's statistical properties is divorced from the propagating mechanism, while traditional theories of market efficiency assume the mechanisms. [3] Crucially, understanding the mechanism provides insight into how and why markets fail.

Our focus here is on nonlinearity. Many complex systems, including markets, have critical points where small incremental condition changes lead to large-scale effects. Researchers in both the physical and social sciences have known about these critical points for a long time; so much so that terms like phase transition and tipping point have slipped into our day-to-day language. Still, critical points throw a monkey wrench into our mostly linear cause-and-effect thinking.

Critical points help explain our perpetual surprise at fat-tail events: We don't see them coming because the state change is much greater than the perturbation suggests. Water does not undergo a dramatic change as it drops from 35 to 33 degrees Fahrenheit, but two degrees of additional cooling changes its state from liquid to solid. Likewise, large changes can occur in markets without visible manifestation in asset price change, while small additional changes can flip the price switch.

Critical points are also important for proper counterfactual thinking. [4] For every critical point we do see, how many were lurking but never triggered? Like water temperature dropping to 33 degrees and again rising, there are likely many near-misses in the markets that elude our detection.

We survey three ideas: black swans and why patterns set us up for surprise; the conditions for crowds to be wise and the role of nonlinearity; and, finally, three examples of nonlinearity, including a physical system, an agent-based model, and a recent market dislocation.

Don't Feed the Turkey

Nassim Taleb uses the black swan metaphor to help popularize the fat-tail idea. He defines a black swan as an outlier event that has an extreme impact and that humans seek to explain after the fact. Recent market turmoil fits the definition well.

The black swan reference reflects Karl Popper's criticism of induction. Popper's point is that to understand a phenomenon, we're better off focusing on falsification than on verification. Seeing lots of white swans doesn't prove the theory that all swans are white, but seeing one black swan does disprove it.

Taleb relates the story of a turkey that is fed 1,000 days in a row.[5] The feedings reinforce the turkey's sense of security and well-being, until one day before Thanksgiving an unexpected and uninvited bad event occurs. All of the turkey's experience and feedback is positive until fortune takes a turn for the worse. Recent comments by a senior executive at one of the world's largest banks evoke the turkey story: "Our losses [from instruments based on U.S. subprime mortgages] greatly exceeded the profits we made in this field over several years."[6]

Here's the point: rising asset prices provide investors confirming evidence that their strategy is good and everything is fine. This induction problem lulls investors into a sense of confidence, and sets the stage for the shock when events turn down. That nonlinearity causes sudden change only adds to the confusion.

Rivets, Redundancy, and Diversity Breakdowns

As we and others have argued, investors should recognize the market as a complex adaptive system. Complex means there are lots of investors within the system. Adaptive means investors change their decision rules in response to market conditions. And system means the whole is greater than the sum of the parts—prices emerge from the interaction of the investors within it. Complex adaptive systems are everywhere in natural and social systems.

The "wisdom of crowds" is a colloquial way of describing the market as a complex system. The work on wisdom of crowds shows that when certain conditions are met—diversity, aggregation, and incentives—markets tend to be efficient. Conversely, when one or more of those conditions are violated, markets can and do become inefficient (i.e., price is no longer an unbiased reflection of value).

For a host of social psychological and sociological reasons, diversity is the most likely condition to be violated. Here, too, market observers have recognized the importance of diversity.[7] But what's essential to recognize is the relationship between diversity breakdowns and asset price performance is nonlinear. Diversity can be on the decline as the asset price rises, which makes the market fragile. Like a rubber band that's stretched, the tension builds and the inevitable snapback is often painful.

Considering the market as a complex adaptive system allows us to understand the mechanism by which prices are set. While many questions remain unanswered, we can look to the three conditions for guidance for how well the market is functioning. Further, we can see why most large market moves are internal to the system (endogenous) versus external factors imposed on the market (exogenous). Our estimate is 80 percent of the market's largest moves in the past 60 years are attributable to endogenous activity.

One virtue of viewing the market as a complex adaptive system is that it allows us to study similar systems in order to draw parallels. One example is the rivet-popper hypothesis, proposed by Paul Ehrlich. The idea is you can remove some rivets from an airplane's wing without consequence because the engineers designed the wing with redundancy in mind. But at a certain point the plane becomes unsafe.

Ehrlich used this metaphor to consider the removal of species from an ecosystem. While another species may step in to serve the functional role of the extinct species, beyond a point there are not enough species to go around, and the system collapses. As with markets, the relationship between the species population (biodiversity) and the ecosystem performance is nonlinear. [8]

We'll now turn to three examples of nonlinearity in complex systems. We start with a physical system and move toward today's environment.

London Bridge Is Swaying 'Round

On June 10, 2000, the Millennium Bridge opened to the public with great fanfare. London's first bridge across the Thames in over a century, it had a sleek design—the architect wanted it to look like a "blade of light." [9] However, when thousands of people stepped on the bridge that day, it started to sway from side to side so much that people had to stop or hold on to the rails. Fearing for the public's safety, officials closed the bridge two days later and, following a retrofitting, it reopened in February 2002.

What led to this high-profile failure? People exert a small amount of lateral excitation when they walk. Normally, these excitations cancel out when a group crosses a bridge. However, the Millennium Bridge initially had insufficient lateral dampeners, which allowed a little swaying when a sufficient number of people were on the bridge. That swaying forced people to change their gait by widening their steps, leading to greater lateral excitation and more swaying. The wobbling and crowd synchrony emerged simultaneously. [10]

The crucial insight is the existence of a critical point. Simulations show that roughly 165 people can walk on the bridge with little impact on the wobble amplitude (see Exhibit 1). But adding just a few more pedestrians causes the amplitude to change dramatically, especially as the feedback between gait adjustment and wobble amplitude kicks in (see the dashed line in Exhibit 1). For the first 165 bridge crossers, there's little wobble and no sense of any potential hazard even though the bridge is on the cusp of a state change.

Exhibit 1: Simulated Outbreak of Wobbling on the Millennium Bridge
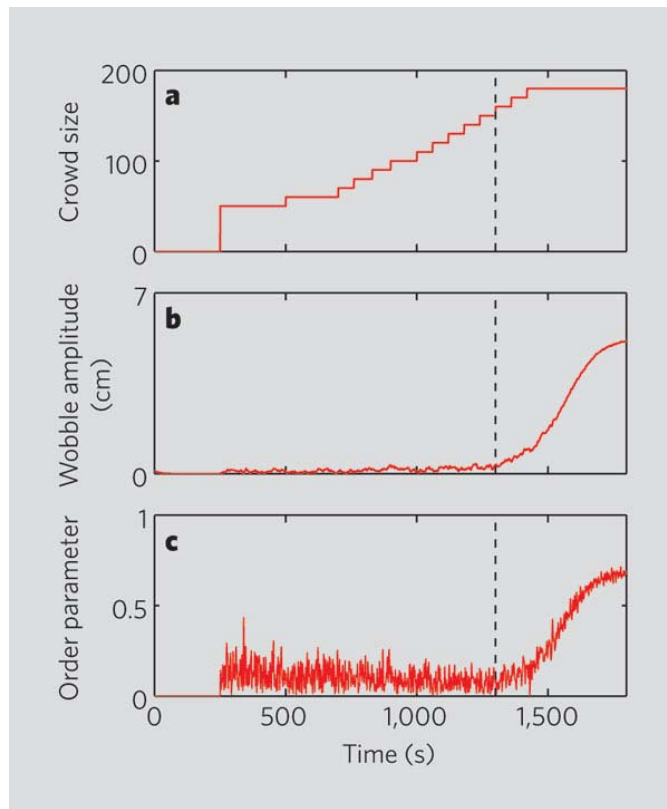
You can imagine testing the bridge with 50, 100, or even 150 people. The harmful wobble lies in the wait, outside of your awareness. The large-scale outcome is due to the internal workings of the system—people walking—not from some external shock. But it is a real risk.

Secret Agent Man Reveals All

Our next example, based on the work of economist Blake LeBaron, is much closer to the real world of markets. [11] LeBaron is a leader in creating agent-based models to analyze economic problems, including asset pricing. These models treat the market as a complex adaptive system with diverse agents, a well-functioning aggregation function, and proper incentives. What makes this work so exciting is that by studying various facets of the system, we can gain insight into how the pricing mechanism works.

LeBaron's model has 1,000 agents, each with well-defined portfolio objectives. He provides the agents with a menu of strategies, which evolve over time. He then lets agents trade, resulting in an asset price. While quite simple, LeBaron's model replicates many empirical features of real markets, including fat-tailed return distributions, volatility patterns, and persistence.

One of the model's most striking findings is the relationship between diversity and asset price. The bottom panel of Exhibit 2 shows a measure of diversity—the degree of rule dispersion. Lower dispersion means less diversity. The top panel shows the asset price.

Exhibit 2: Crashes and Diversity
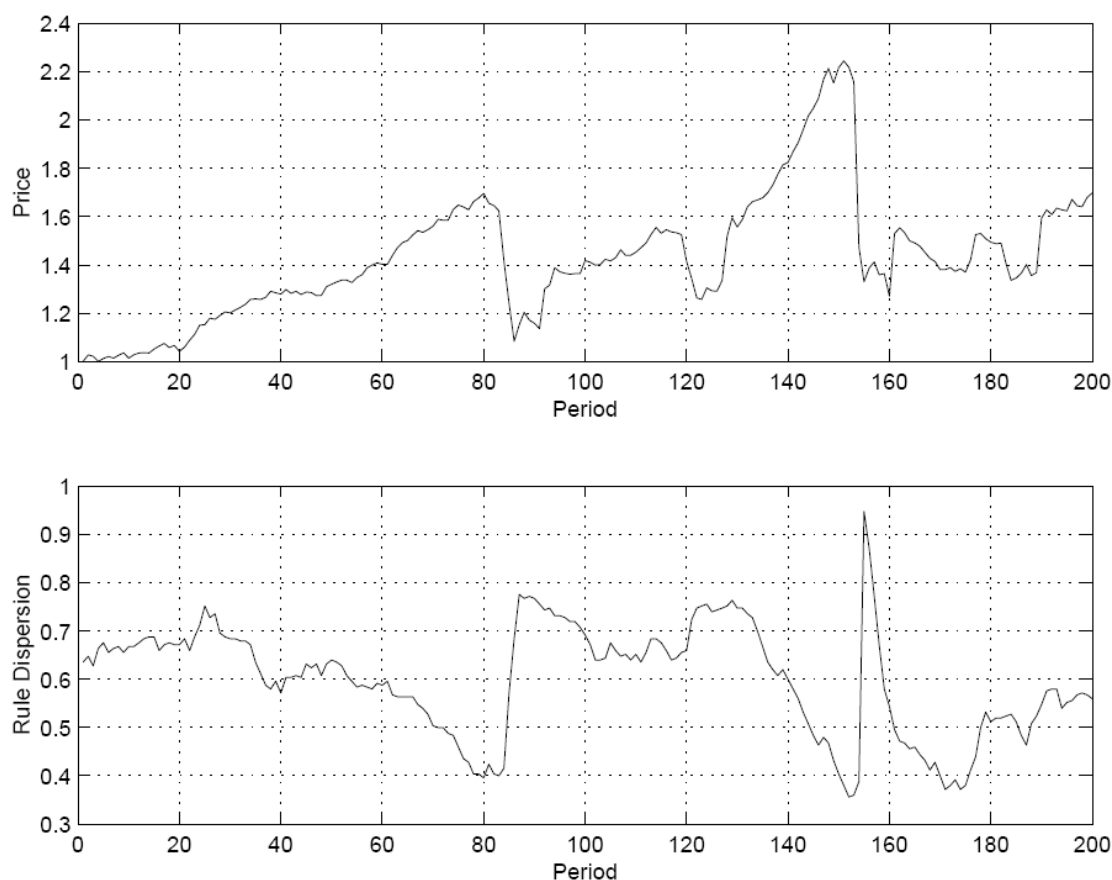
Source: Blake LeBaron, "Financial Market Efficiency in a Coevolutionary Environment," *Proceedings of the Workshop on Simulation of Social Agents: Architectures and Institutions, Argonne National Laboratory and University of Chicago*, October 2000, Argonne 2001, 33-51. Used with permission.

Study the first 80 periods for a moment. Diversity declines steadily, even as the asset price rises. As the asset reaches a short-term price peak, diversity is at its lowest. This is an example of invisible vulnerability, similar to what we saw with the bridge.

LeBaron discusses how crashes happen: [12]

> During the run-up to a crash, population diversity falls. Agents begin to use very similar trading strategies as their common good performance begins to self-reinforce. This makes the population very brittle, in that a small reduction in the demand for shares could have a strong destabilizing impact on the market. The economic mechanism here is clear. Traders have a hard time finding anyone to sell to in a falling market since everyone else is following very similar strategies. In the Walrasian setup used here, this forces the price to drop by a large magnitude to clear the market. The population homogeneity translates into a reduction in market liquidity.

LeBaron's model captures many important themes of real-world markets. Add to this model the use of leverage to enhance the performance of diminishing-return strategies, and the stage is set for large-scale change. This model reveals a concrete link between diversity reduction and asset price changes. [13]

The Pain of 2007

We'll end our tour with a contemporary example of a market-neutral hedge fund. We selected a market-neutral fund in large part because such funds seek to do well in any market environment (i.e., not to be correlated with the market). Exhibit 3 shows the performance of one of these funds for most of 2007. Visually compare Exhibit 3 with the first 100 periods in the top panel of Exhibit 2. Certainly the pattern is similar.

Exhibit 3: Performance of a Market-Neutral Fund, Summer 2007
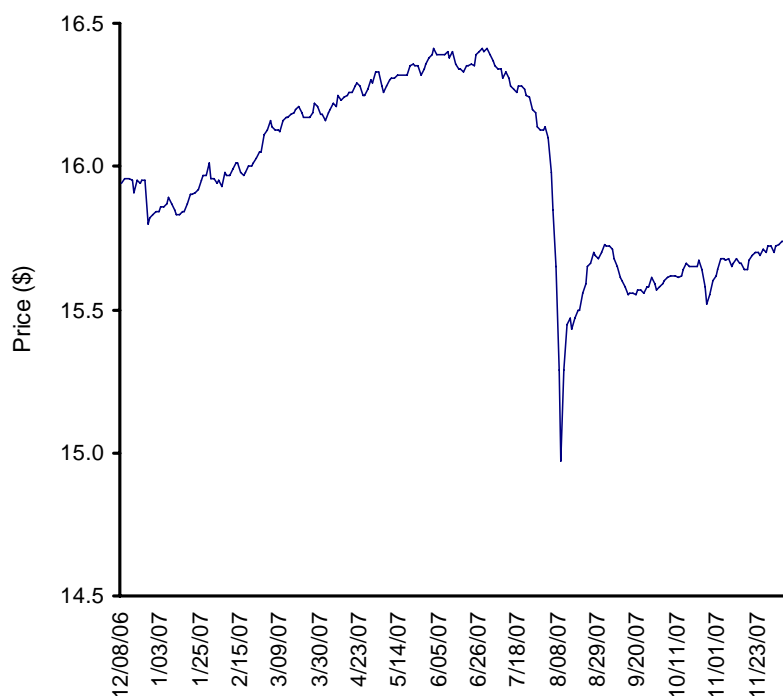
Source: LMCM analysis.

This process has developed numerous times in 2007, leading to some high-profile fund failures. In reality, nothing is really unique about 2007, as we have seen similar events unfold in the past.

Here are the three central points:

- *Avoid the problem of induction.* Humans like to generalize about a system based on a limited sample. The problem of induction reminds us that falsification serves us better than verification. When an asset price is going up, the natural assumption is it will continue to do so. Indeed, prosperity can breed an unhealthy sense of security.

- *Be mindful of diversity breakdowns.* There is no simple way to measure the degree of diversity in the real world. But if a large group of investors seem to all be doing well pursuing a similar strategy, chances are there is a great deal of invisible vulnerability. We saw this in the first quarter of 2000 when many mutual funds were strongly overweight technology shares. More recently, it has been some quantitative trading strategies. But diversity breakdowns do not reveal themselves a little at a time, it is closer to binary.

- *Watch for the nonlinear reaction.* Critical points exist in markets as they do in most complex systems. When diversity levels are low, small perturbations lead to outcomes completely out of proportion with their size. Critical points are endogenous to financial markets, and they can be enhanced through leverage and government policy.

Endnotes

[1] Francis Bacon, *Novum Organum or True Suggestions for the Interpretation of Nature,* Joseph Devey, M.A. ed. (New York: P.F. Collier & Son, 1902), 180.

[2] Benoit Mandelbrot, "The Variation of Certain Speculative Prices," *Journal of Business*, Vol. 36, 4, October, 1963, 394-419.

[3] Didier Sornette, *Why Stock Markets Crash: Critical Events in Complex Financial Systems* (Princeton, NJ: Princeton University Press, 2003), 137.

[4] For a brief discussion on counterfactual thinking, see Michael J. Mauboussin, "Was Harry Potter Inevitable? Cumulative Advantage, Counterfactuals, and the Halo Effect," *Mauboussin on Strategy*, September 7, 2007.

[5] Nassim Nicholas Taleb, *The Black Swan: The Impact of the Highly Improbable* (New York: Random House, 2007), 40-41.

[6] Gonzalo Vina and Sebastian Boyd, "Citi's Losses 'Greatly Exceeded' Profits for Subprime," *Bloomberg.com*, December 4, 2007.

[7] See Roger Lowenstein, *When Genius Failed: The Rise and Fall of Long-Term Capital Management* (New York: Random House, 2000). Also, Richard Bookstaber, *A Demon of Our Own Design: Markets, Hedge Funds, and the Perils of Financial Innovation* (New York: John Wiley & Sons, 2007).

[8] Paul Ehrlich and Brian Walker, "Rivets and Redundancy," *BioScience*, Vol. 48, 5, May 1998, 387.

[9] Deyan Sudjic, ed., *Blade of Light: The Story of London's Millennium Bridge* (London: Penguin Books, Ltd, 2001).

[10] Steven H. Strogatz, Daniel M. Abrams, Allan McRobbie, Bruno Eckhardt, and Edward Ott, "Crowd Synchrony on the Millennium Bridge," *Nature*, Vol. 483, 3, November 2005, 43-44. See http://www.tam.cornell.edu/tam/cms/manage/upload/millennium.pdf.

[11] Blake LeBaron, "Financial Market Efficiency in a Coevolutionary Environment," *Proceedings of the Workshop on Simulation of Social Agents: Architectures and Institutions, Argonne National Laboratory and University of Chicago*, October 2000, Argonne 2001, 33-51.

[12] Ibid., 50.

[13] Amir E. Khandani and Andrew W. Lo, "What Happened to the Quants in August 2007?" *Working Paper*, November 4, 2007. See http://web.mit.edu/alo/www/.