

# INF8953DE (Fall 2021) : Reinforcement Learning - Assignment 1

Amine EL AMERI - Matricule: 2164634

```
In [ ]: import numpy as np
import scipy as sp
import matplotlib.pyplot as plt
```

## 1 Bandit Problem

Q1

```
In [ ]: class Bandit():

    def __init__(self):
        self.numberOfArms = 10
        self.mean = 0
        self.variance = 1
        self.stdDeviation = np.sqrt(self.variance)
        self.q_star = [np.random.normal(self.mean, self.stdDeviation) for i in range(10)]
        self.q_star_a_star = max(self.q_star) # best action value
        self.Q = [0]*10 # estimates of action values
        self.nTimes = [0]*10 # number of times each action is selected

        self.H_t = [0]*10 # preferences of the actions
        self.alpha_GB = 0

    def pull(self, action):
        R = np.random.normal(self.q_star[action-1], self.stdDeviation) # reward
        self.nTimes[action-1] += 1
        self.Q[action-1] = self.Q[action-1] + (R-self.Q[action-1])/self.nTimes[action-1]
        return R
```

```
In [ ]: banditQ1 = Bandit()
banditQ1.pull(7)
```

```
Out[ ]: -0.23639739164512186
```

Q2

```
In [ ]: def epsilon_greedy(self, epsilon):
    randNumber = np.random.uniform(0,1)
    if randNumber >= epsilon:
        knownRewards = [i for i in self.Q if i is not None]
        if len(knownRewards) == 0: # this is our first pull
            return self.pull(np.random.randint(1,10))
        else:
            greedyAction = self.Q.index(max(knownRewards)) + 1
            return self.pull(greedyAction)
    else:
        return self.pull(np.random.randint(1,10))
```

```
Bandit.epsilon_greedy = epsilon_greedy
```

In [ ]:

```
def Q2Plots(nRuns, nPulls, epsilons):

    fig, axs = plt.subplots(2)

    for eps in epsilons:
        bandits = [None]*nRuns # list of bandit instances
        x = []
        y_reward = []
        y_regret = []
        for pull in range(nPulls):
            avgReward = 0
            avgRegret = 0
            for run in range(nRuns): # each run (all the pulls associated) is done by 1 bandit
                if bandits[run] == None:
                    bandits[run] = Bandit()
                    reward = bandits[run].epsilon_greedy(eps)
                    avgReward += reward # average reward for a pull across runs
                    avgRegret += bandits[run].q_star_a_star - reward # average regret for a pull
                else:
                    reward = bandits[run].epsilon_greedy(eps)
                    avgReward += reward
                    avgRegret += bandits[run].q_star_a_star - reward

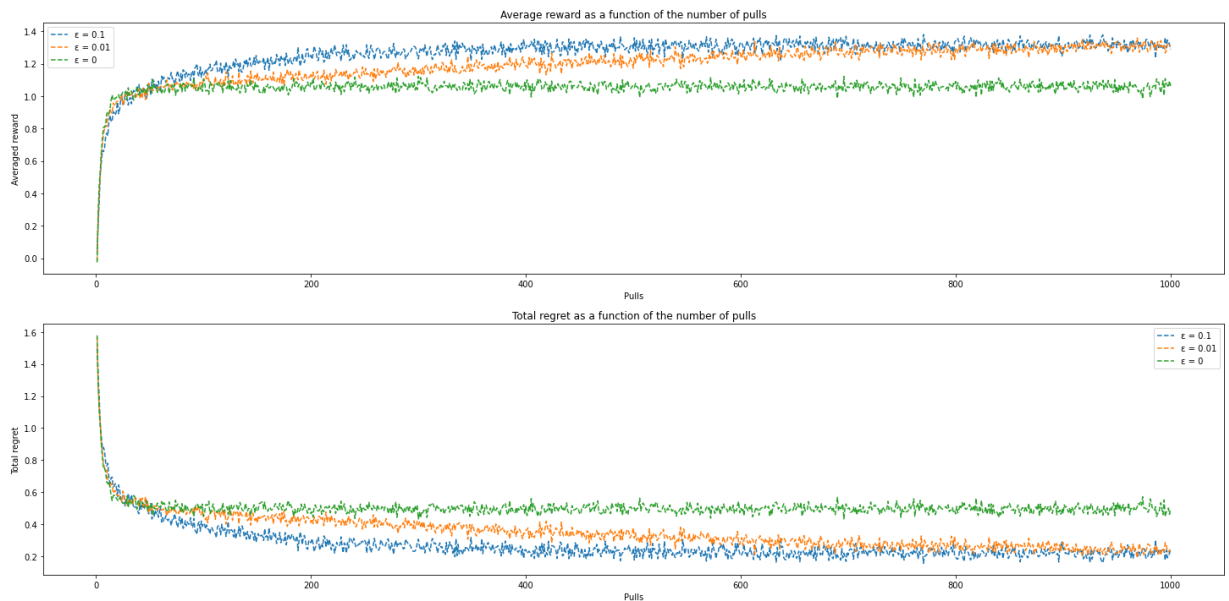
            avgReward /= nRuns
            avgRegret /= nRuns
            x.append(pull+1)
            y_reward.append(avgReward)
            y_regret.append(avgRegret)

        axs[0].plot(x, y_reward, ls='--')
        axs[1].plot(x, y_regret, ls='--')

        axs[0].set_xlabel("Pulls")
        axs[0].set_ylabel("Averaged reward")
        axs[0].set_title("Average reward as a function of the number of pulls")
        axs[0].legend(["\u03B5 = 0.1", "\u03B5 = 0.01", "\u03B5 = 0"])

        axs[1].set_xlabel("Pulls")
        axs[1].set_ylabel("Total regret")
        axs[1].set_title("Total regret as a function of the number of pulls")
        axs[1].legend(["\u03B5 = 0.1", "\u03B5 = 0.01", "\u03B5 = 0"])

plt.rcParams['figure.figsize'] = [25, 12]
Q2Plots(2000, 1000, [0.1, 0.01, 0])
```

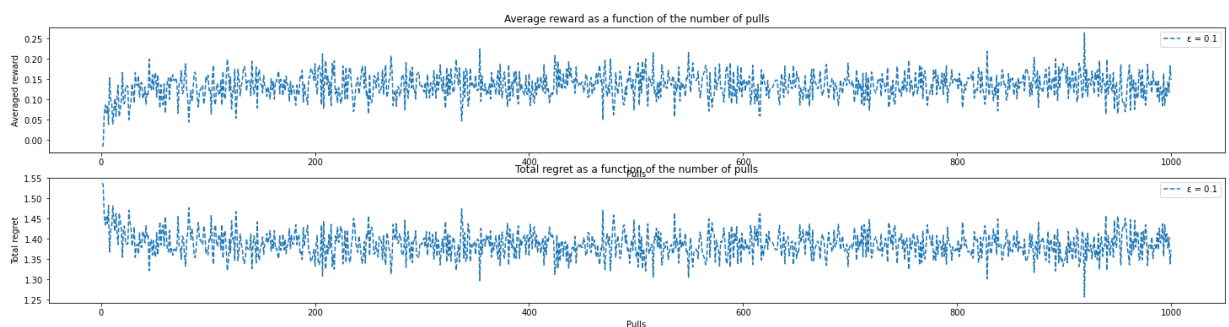


I expected the epsilon greedy method with the biggest  $\epsilon$  in  $\{0, 0.01, 0.1\}$  to be the best, because it does more exploration and thus has more chance to discover the action with the biggest reward quickly and then exploit it. This is indeed the case here when looking at the plots (eps=0.1 better than eps=0.01 better than eps=0).

### Q3

The epsilon greedy method with  $\epsilon = 0.9$  is not a good strategy because even if it will discover the best action quickly (because of doing exploration 90% of the time), exploiting this action only 10% of the time is not enough. This method will always be taking a big number of random (thus not optimal) decisions.

```
In [ ]: plt.rcParams['figure.figsize'] = [25, 6]
Q2Plots(2000, 1000, [0.9])
```



Indeed, even after 900+ pulls, the plot shows that epsilon greedy with 0.9 still takes random and not optimal actions. The average reward is not comparable to what the same algorithm with  $\epsilon = 0.1$  or  $\epsilon = 0.01$  could win.

### Q4

```
In [ ]: def optimisticInitialValue_withPlot(nRuns, nPulls, initValues):

    fig, ax = plt.subplots()

    for initValue in initValues:
        bandits = [None]*nRuns # list of bandit instances
        x = []
        y_reward = []
        for pull in range(nPulls):
```

```

avgReward = 0
for run in range(nRuns): # each run (all the pulls associated) is done by 1 b

    if bandits[run] == None:
        bandits[run] = Bandit()
        bandits[run].Q = [initValue]*10 # initializing the new bandit instance wi

    action = np.argmax(bandits[run].Q) + 1
    reward = bandits[run].pull(action)
    avgReward += reward # average reward for a pull across runs

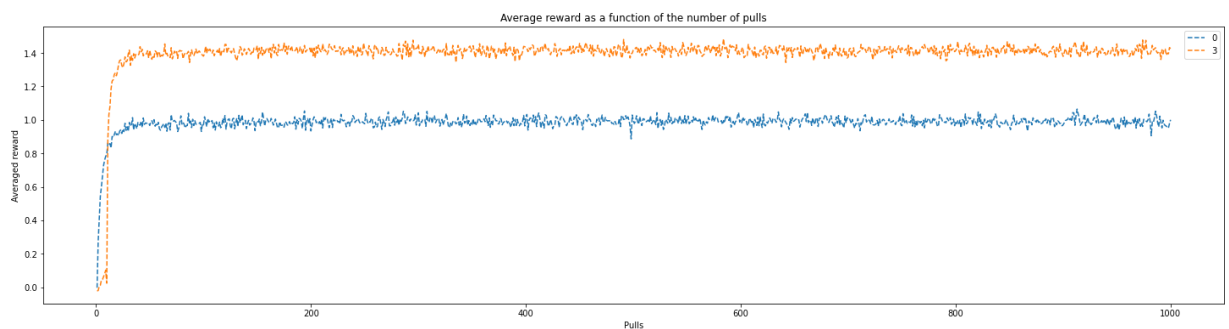
avgReward /= nRuns
x.append(pull+1)
y_reward.append(avgReward)

ax.plot(x, y_reward, ls='--', label = initValue)

ax.set_xlabel("Pulls")
ax.set_ylabel("Averaged reward")
ax.set_title("Average reward as a function of the number of pulls")
ax.legend(loc='best')

plt.rcParams['figure.figsize'] = [25, 6]
optimisticInitialValue_withPlot(2000, 1000, [0, 3])

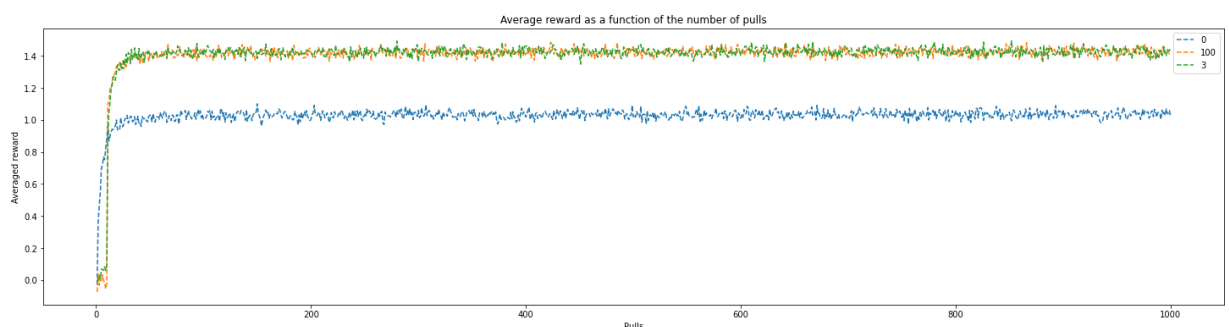
```



I expected the algorithm with optimistic setting to have a better average reward because the optimistic initial values makes the algorithm explore more before starting the exploitation. This is indeed what we see in the plots.

## Q5

In [ ]: `optimisticInitialValue_withPlot(2000, 1000, [0, 100, 3])`



I expected the plots of the two settings  $Q1(a) = 3$  and  $Q1(a) = 100$  to be different at the beginning ( $Q1(a) = 100$  will explore for more time) but to converge after a certain number of

pulls to the same average award, since they will both have a good estimate of the action values, and thus will exploit the same action. And this is what we see in the plots.

## Q6

```
In [ ]: def UCB(self, c):
    if 0 in self.nTimes: # if an action have  $N_t(a) = 0$ , we choose it
        return self.pull(self.nTimes.index(0) + 1)
    else:
        UCBAction = np.argmax( np.array(self.Q) + c*np.sqrt(np.log(sum(self.nTimes))) /
        return self.pull(UCBAction)

Bandit.UCB = UCB
```

```
In [ ]: def UCB_Plots(nRuns, nPulls, c_array):

    fig, ax = plt.subplots()

    for c in c_array:
        bandits = [None]*nRuns # list of bandit instances
        x = []
        y_reward = []
        for pull in range(nPulls):
            avgReward = 0
            for run in range(nRuns): # each run (all the pulls associated) is done by 1 b

                if bandits[run] == None:
                    bandits[run] = Bandit()

                reward = bandits[run].UCB(c)
                avgReward += reward # average reward for a pull across runs

            avgReward /= nRuns
            x.append(pull+1)
            y_reward.append(avgReward)

        ax.plot(x, y_reward, ls='--', label = c)

    ax.set_xlabel("Pulls")
    ax.set_ylabel("Averaged reward")
    ax.set_title("Average reward as a function of the number of pulls")
    ax.legend(loc='best')

plt.rcParams['figure.figsize'] = [25, 6]
UCB_Plots(2000, 1000, [0.2, 1, 5])
```



$c$  controls the level of exploration, I think the setting  $c=1$  will have better average reward than  $c=0.2$  and  $c=5$ , because it's in the middle, and so it will do enough exploration but also enough

exploitation. In the plots we see that  $c=0.2$  and  $c=1$  are very close after 100 to 300 pulls, but that  $c=1$  start to become slightly better after 400 pulls.  $c=5$  is not at the level of the other 2, because it gives too much importance to exploration.

## Q7

I expect the gradient bandit algorithm with baseline to be better than without baseline, and the parameter  $\alpha = 0.1$  to be better than  $\alpha = 0.5$

In [ ]:

```
import math

def pull_Gradient_Bandit(self, isBaseline):

    exp_Ht = np.exp(np.array(self.H_t))
    PI_a = exp_Ht/sum(exp_Ht)

    action = np.random.choice(10, 1, p=PI_a)[0] + 1

    R_avg = sum(self.Q)/10 # average reward
    R = np.random.normal(self.q_star[action-1], self.stdDeviation) # reward
    self.nTimes[action-1] += 1
    self.Q[action-1] = self.Q[action-1] + (R-self.Q[action-1])/self.nTimes[action-1]

    if isBaseline == True:
        baseline = R_avg
    else:
        baseline = 0

    for i in range(10):
        if i == action - 1:
            self.H_t[i] += self.alpha_GB*(R - baseline)*(1 - PI_a[i])
        else:
            self.H_t[i] -= self.alpha_GB*(R - baseline)*PI_a[i]
    return R

Bandit.pull_Gradient_Bandit = pull_Gradient_Bandit
```

In [ ]:

```
def Gradient_Bandit_Plots(nRuns, nPulls, alpha, baseline, meanNormal):

    fig, ax = plt.subplots()

    #for i in range(len(alpha_array)):
    bandits = [None]*nRuns # list of bandit instances
    x = []
    y_reward = []
    for pull in range(nPulls):
        avgReward = 0
        for run in range(nRuns): # each run (all the pulls associated) is done by 1 bandit
            if bandits[run] == None:
                bandits[run] = Bandit()
                bandits[run].q_star = [np.random.normal(meanNormal, 1) for i in range(10)] #
                bandits[run].q_star_a_star = max(bandits[run].q_star) # best action value
                bandits[run].alpha_GB = alpha

            reward = bandits[run].pull_Gradient_Bandit(baseline)
            avgReward += reward # average reward for a pull across runs

        avgReward /= nRuns
        x.append(pull+1)
```

```

y_reward.append(avgReward)

lbl = "alpha=" + str(alpha) + " " + "baseline=" + str(baseline)
ax.plot(x, y_reward, ls='--', label = lbl)

ax.set_xlabel("Pulls")
ax.set_ylabel("Averaged reward")
ax.set_title("Average reward as a function of the number of pulls")
ax.legend(loc='best')

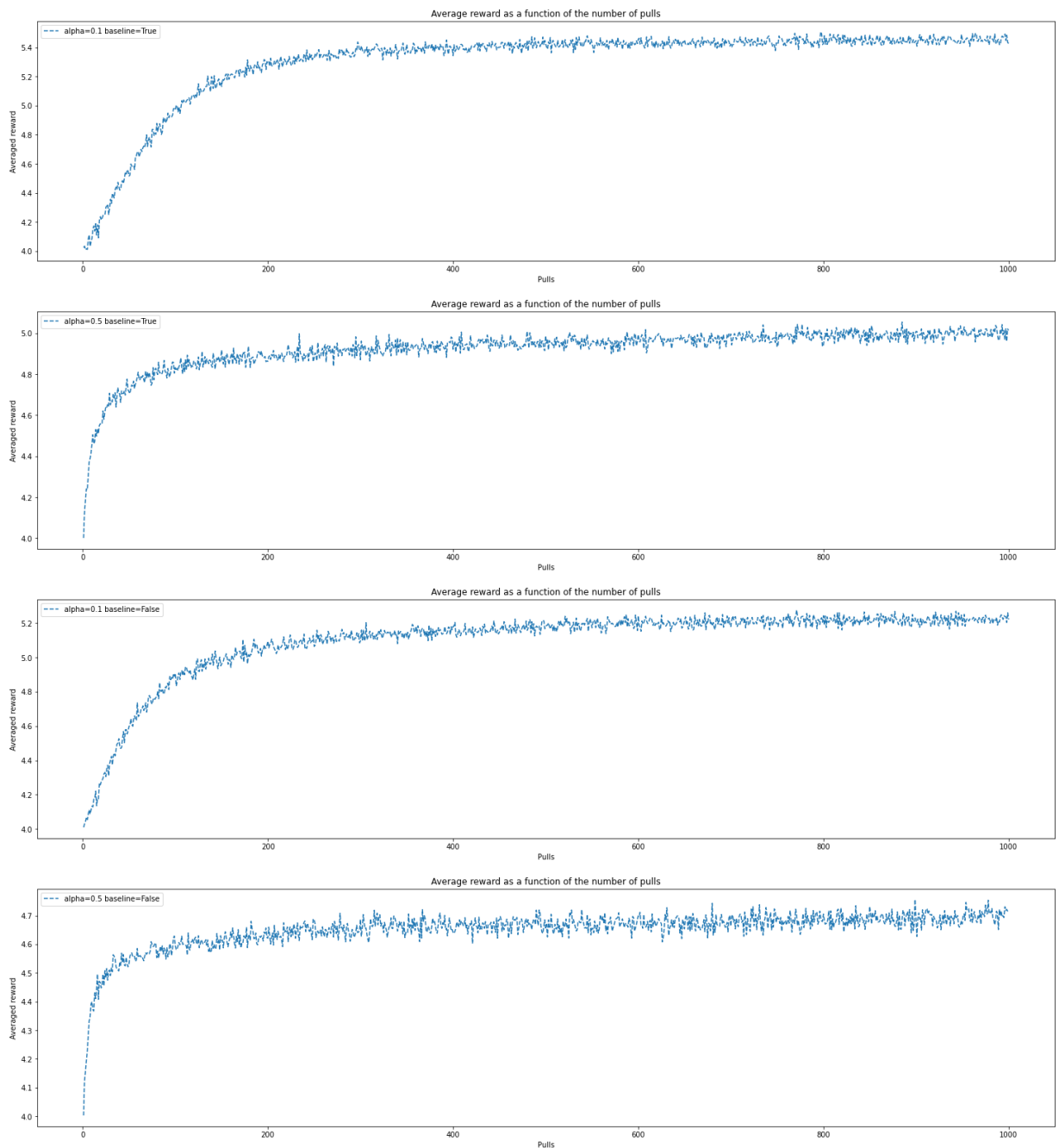
```

In [ ]:

```

plt.rcParams['figure.figsize'] = [25, 6]
Gradient_Bandit_Plots(2000, 1000, 0.1, True, meanNormal = 4)
Gradient_Bandit_Plots(2000, 1000, 0.5, True, meanNormal = 4)
Gradient_Bandit_Plots(2000, 1000, 0.1, False, meanNormal = 4)
Gradient_Bandit_Plots(2000, 1000, 0.5, False, meanNormal = 4)

```



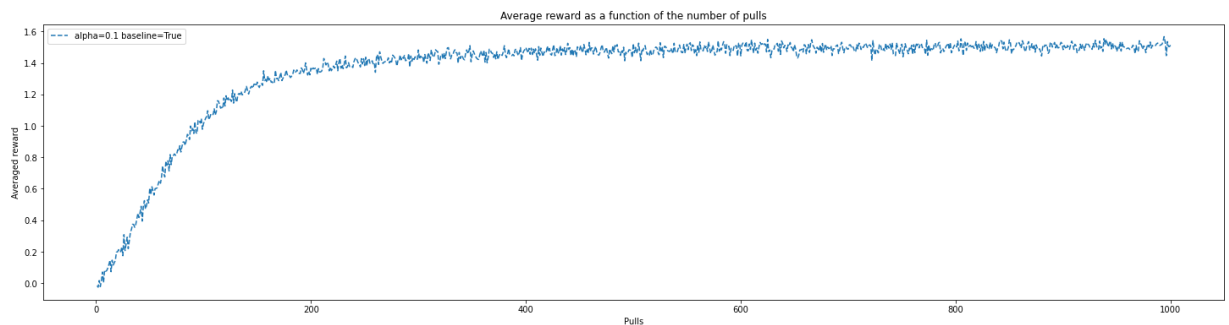
Indeed we see that ( $\alpha = 0.1$  with baseline) have the best average reward, and that ( $\alpha = 0.5$  without baseline) have the worst. However ( $\alpha = 0.1$  without baseline) and ( $\alpha = 0.5$  with baseline) are pretty close.

## Q8

First we should plot the gradient bandit with mean 0 for the distribution of the action values, to have a fair comparison with the other plots

In [ ]:

```
Gradient_Bandit_Plots(2000, 1000, 0.1, True, meanNormal = 0)
```



Now we can summarize: (we take the best parameters for each method)

- **Epsilon greedy (with eps = 0.1):** goes up to avg\_reward = 1 in less than 20 pulls, and converges to **avg\_reward = 1.25**
- **Optimistic initial value (with initValue = 3):** goes up to avg\_reward = 1.3 in less than 20 pulls, and converges to **avg\_reward = 1.4**
- **UCB (with c = 1):** goes up to avg\_reward = 1.1 in less than 20 pulls, and converges to **avg\_reward = 1.5**
- **Gradient Bandit (with baseline and alpha = 0.1):** needs approx 100 pulls to go up to avg\_reward = 1, and converges to **avg\_reward = 1.55**

So in terms of the **average award** at the convergence: **Gradient Bandit > UCB > Optimistic initial value > Epsilon greedy**

But in terms of the **speed of convergence**: **Optimistic initial value > UCB > Epsilon greedy** but are all fast and almost equivalent, unlike **Gradient Bandit** which is slow.

Since **regret** is correlated to the average award, then the ranking in terms of regret is the same as the ranking in terms of average award, and is **Gradient Bandit better than UCB better than Optimistic initial value better than Epsilon greedy**

## 2 Dynamic Programming

## Q1

Since

- Every action deterministically cause the corresponding state transition
- The agent follows the equiprobable random policy
- And we have an undiscounted, episodic task
- The reward is -1 on all transitions until the terminal state

Then  $p(s', r|s, a) = 1$ ,  $\pi(a|s) = \frac{1}{4}$ ,  $\gamma = 1$ , and  $r = -1$

So the Bellman equations for the state value and the action value functions can be simplified to:



$$v_{\pi}(s) = \sum_{a=1}^4 \frac{1}{4} \sum_{s'} (-1 + v_{\pi}(s')) = \sum_{s'} (-1 + v_{\pi}(s'))$$

$$q_{\pi}(s, a) = -1 + v_{\pi}(s')$$

We want to solve the system

$$\begin{cases} v_{\pi}(0) = v_{\pi}(24) = 0 \\ v_{\pi}(s) = \sum_{s'} (-1 + v_{\pi}(s')) \quad \forall s \in 1, \dots, 23 \end{cases} \quad (1)$$

In [ ]:

```
from scipy.optimize import fsolve

def myBellmanSystem(v):
    F = np.zeros(25)

    for s in range(0, 25):
        if (s == 0) or (s == 24):
            F[s] = v[s]
        elif s == 4:
            F[4] = v[4] + 4 - (v[3] + v[9] + 2*v[4])
        elif s == 20:
            F[20] = v[20] + 4 - (v[15] + v[21] + 2*v[20])
        elif s in range(1, 4):
            F[s] = v[s] + 4 - (v[s-1] + v[s+1] + v[s+5] + v[s])
        elif s in range(5, 16, 5):
            F[s] = v[s] + 4 - (v[s-5] + v[s+1] + v[s+5] + v[s])
        elif s in range(9, 20, 5):
            F[s] = v[s] + 4 - (v[s-5] + v[s-1] + v[s+5] + v[s])
        elif s in range(21, 24):
            F[s] = v[s] + 4 - (v[s-1] + v[s+1] + v[s-5] + v[s])
        else:
            F[s] = v[s] + 4 - (v[s-1] + v[s+1] + v[s-5] + v[s+5])

    return F

zeros = np.zeros(25)
state_value_function = fsolve(myBellmanSystem, zeros)

print(f"\n v_pi(16) = {np.around(state_value_function[16], 1)}")
print(f" v_pi(12) = {np.around(state_value_function[12], 1)}")

v_equiprobable_policy = np.around(state_value_function, 1).reshape(5, 5)
print(v_equiprobable_policy) # state value function for the equiprobable policy

v_pi(16) = 0.0
v_pi(12) = 1.3
[[ 0.  -5.4  1.3  8.   2.7]
 [ 9.4  2.7  1.3 -0.  -6.7]
 [ 1.3  1.3  1.3  1.3  1.3]
 [-6.7  0.   1.3  2.7  9.4]
 [ 2.7  8.   1.3 -5.4  0. ]]
```

Now that we know that  $v_{\pi}(16) = 0$  and  $v_{\pi}(12) = 1.3$

We can calculate:

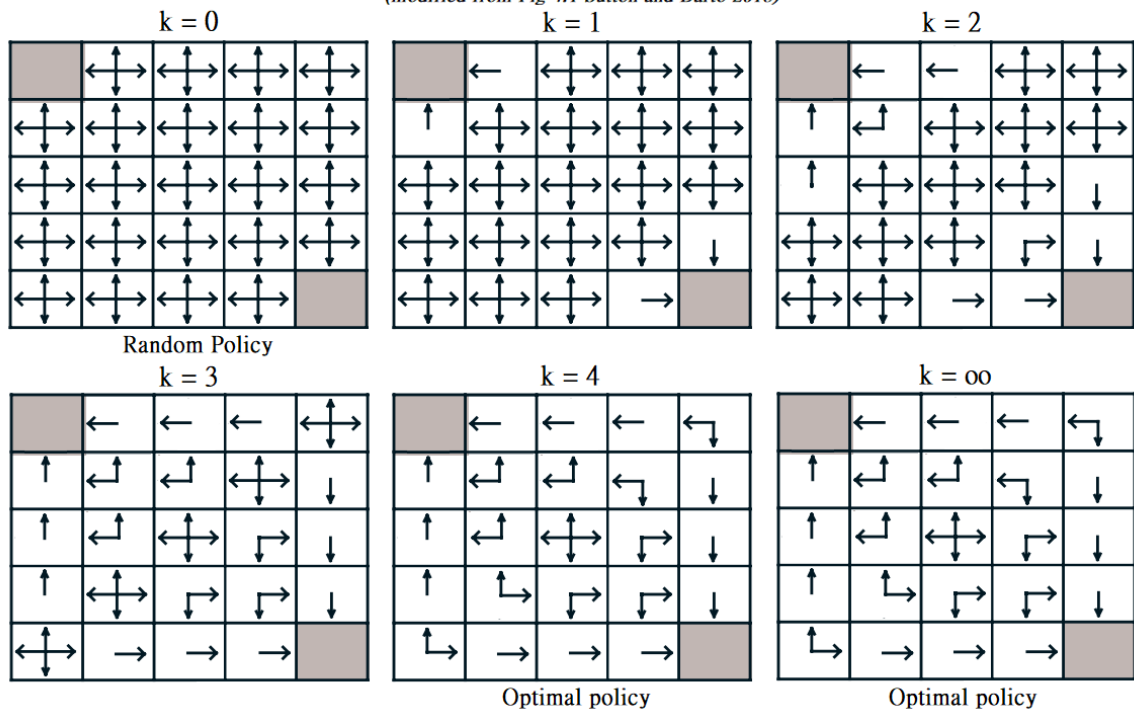
$$q_{\pi}(11, \text{down}) = -1 + v_{\pi}(16) = -1 + 0 = -1$$

$$q_{\pi}(7, \text{down}) = -1 + v_{\pi}(12) = -1 + 1.3 = 0.3$$

Q2

```
In [ ]: from IPython.display import Image
from IPython.core.display import HTML
Image(url= "https://i.imgur.com/E7RGaY4.png", width=800, height=500)
# another link: https://imgur.com/a/7776EeL
```

Out[ ]: Amine EL AMERI - INF8953DE (Fall 2021) - Assignment 1 - Dynamic Programming - Question 2  
(modified from Fig 4.1 Sutton and Barto 2018)



Q3

Q3-a

```
In [ ]: !pip install git+https://github.com/zafarali/emdp.git
```

```
Collecting git+https://github.com/zafarali/emdp.git
  Cloning https://github.com/zafarali/emdp.git to /tmp/pip-req-build-nt4n9dkc
  Running command git clone -q https://github.com/zafarali/emdp.git /tmp/pip-req-build-nt4n9dkc
Requirement already satisfied: numpy>=1.9.1 in /usr/local/lib/python3.7/dist-packages (from emdp==0.0.5) (1.19.5)
```

```
In [384... import emdp.gridworld as gw
from emdp import actions
# Knowing that actions.LEFT = 0, actions.RIGHT = 1, actions.UP = 2, actions.DOWN = 3

def build_gridworld():
    size = 5
    gamma = 0.99
    terminal_states = [(0, 0), (4, 4)]

    P = gw.build_simple_grid(size, terminal_states, p_success=1)

    R = np.zeros((P.shape[0], P.shape[1]))
    R[:, :] = -1

    p0 = np.ones(P.shape[0])/P.shape[0]

    return gw.GridWorldMDP(P, R, gamma, p0, terminal_states, size)
```

```
In [385... def policy_evaluation(policy, state_values_entry):
    mdp = build_gridworld()
```

```
MDPactions = [actions.LEFT, actions.RIGHT, actions.UP, actions.DOWN] # which is eq
gamma = mdp.gamma

state_values = state_values_entry.copy()

while (True):
    state_values_old = state_values.copy()
    for s in range(1, 24):
        mdp.set_current_state_to(mdp.unflatten_state(np.eye(26)[s]))
        action = policy[s]
        state, reward, done, _ = mdp.step(int(action))
        next_s = np.where(state == 1)[0][0]
        state_values[s] = reward + gamma * state_values[next_s]

    D = abs(state_values_old - state_values).max()
    if D < 1e-4:
        break

return state_values
```

In [220...

```
def one_policy_improvement_round(state_values):
    mdp = build_gridworld()
    gamma = mdp.gamma
    MDPactions = [actions.LEFT, actions.RIGHT, actions.UP, actions.DOWN]
    improvedPolicy = []

    for s in range(1, 24):
        all_pi_s = []
        for action in MDPactions:
            mdp.set_current_state_to(mdp.unflatten_state(np.eye(26)[s]))
            state, reward, done, _ = mdp.step(action)
            next_s = np.where(state == 1)[0][0]
            all_pi_s.append(reward + gamma * state_values[next_s])

        greedyAction = MDPactions[np.argmax(np.array(all_pi_s))]
        improvedPolicy.append(greedyAction)

    return np.array([0] + improvedPolicy + [0])
```

In [425...

```
def policy_iteration():
    size = 5

    initial_random_policy = np.random.randint(low = actions.LEFT, high = actions.DOWN,
        initial_state_values = np.zeros(size * size)

    policy = initial_random_policy.copy()
    state_values = initial_state_values.copy()

    iterations = 0
    while True:
        policy_old = policy.copy()
        state_values = policy_evaluation(policy, state_values)
        policy = one_policy_improvement_round(state_values)
        if np.array_equal(policy, policy_old):
            break

        iterations += 1

    return policy, state_values, iterations
```

In [426...

```
optimal_policy_policy_iteration, optimal_state_values, nIterations = policy_iteratio
```

```
print(f"Optimal policy (policy iteration):\n {optimal_policy_policy_iteration.reshape(5,5)}")
print(f"Optimal state values:\n {optimal_state_values}\n")
print(f"Total number of iterations: {nIterations}\n")
```

Optimal policy (policy iteration):

```
[[0 0 0 0 0]
 [2 0 0 0 3]
 [2 0 0 1 3]
 [2 0 1 1 3]
 [1 1 1 1 0]]
```

Optimal state values:

```
[ 0.      -1.      -1.99     -2.9701    -3.940399 -1.      -1.99
 -2.9701    -3.940399 -2.9701    -1.99     -2.9701    -3.940399 -2.9701
 -1.99     -2.9701    -3.940399 -2.9701    -1.99     -1.      -3.940399
 -2.9701    -1.99     -1.         0.         ]
```

Total number of iterations: 3

In [438...

```
def meanReward_numberTimesteps(policy): #Q3-a
    mdp = build_gridworld()
    nEpisodes = 5
    rewards = [0]*nEpisodes
    timesteps = [0]*nEpisodes

    done = False
    for episode in range(nEpisodes):
        mdp.reset()
        state, reward, done, _ = mdp.step(int(np.random.choice([actions.LEFT, actions.RIGHT])))
        while done == False:
            s = np.where(state == 1)[0][0]
            if s == 25:
                break
            print(policy[s])
            state, reward, done, _ = mdp.step(int(policy[s]))
            rewards[episode] += reward
            timesteps[episode] += 1
            if s == 0 or s == 24:
                break
    print({"mean reward": sum(rewards)/len(rewards), "mean timesteps": sum(timesteps)/len(timesteps)})
```

In [432...

```
def policy_iteration_with_meanReward_numberTimesteps():
    size = 5

    initial_random_policy = np.random.randint(low = actions.LEFT, high = actions.DOWN, size=(size, size))
    initial_state_values = np.zeros(size * size)

    policy = initial_random_policy.copy()
    state_values = initial_state_values.copy()

    iterations = 0
    while True:
        policy_old = policy.copy()
        state_values = policy_evaluation(policy, state_values)
        policy = one_policy_improvement_round(state_values)
        if np.array_equal(policy, policy_old):
            break

        iterations += 1
        print(f"Iteration: {iterations}")
        meanReward_numberTimesteps(policy) # function for Q3-a

    #return policy, state_values, iterations
```

In [436...

```
#if this cell takes more than 5s, it should be re-executed more than once
policy_iteration_with_meanReward_numberTimesteps()
```

```
Iteration: 1
{'mean reward': -4.2, 'mean timesteps': 4.2}
Iteration: 2
{'mean reward': -2.6, 'mean timesteps': 2.6}
Iteration: 3
{'mean reward': -3.4, 'mean timesteps': 3.4}
Iteration: 4
{'mean reward': -2.6, 'mean timesteps': 2.6}
```

In [376...

```
def value_iteration():
    mdp = build_gridworld()
    size = mdp.size
    gamma = mdp.gamma
    MDPActions = [actions.LEFT, actions.RIGHT, actions.UP, actions.DOWN]

    initial_random_state_values = np.random.randint(low = -10, high = 10, size = size)
    initial_random_state_values[0] = initial_random_state_values[24] = 0

    state_values = initial_random_state_values.copy()
    while (True):
        state_values_old = state_values.copy()
        for s in range(1, 24):
            all_v_s = []
            for action in MDPActions:
                mdp.set_current_state_to(mdp.unflatten_state(np.eye(26)[s]))
                state, reward, done, _ = mdp.step(action)
                next_s = np.where(state == 1)[0][0]
                all_v_s.append(reward + gamma * state_values[next_s])

            state_values[s] = max(all_v_s)

        D = abs(state_values_old - state_values).max()
        if D < 1e-2:
            break

    policy = one_policy_improvement_round(state_values)

    return policy
```

In [437...

```
optimal_policy_value_iteration = value_iteration()
print(f"Value iteration's optimal policy:\n {optimal_policy_value_iteration.reshape(
```

```
Value iteration's optimal policy:
[[0 0 0 0 0]
 [2 0 0 0 0]
 [0 0 0 0 0]
 [0 0 0 0 3]
 [0 0 0 1 0]]
```

Q3-b

In [361...

```
def visualize_policy(policy):
    viz = []
    for i in policy:
        if i == 0:
            viz.append("LEFT")
        elif i == 1:
            viz.append("RIGHT")
```

```

elif i == 2:
    viz.append("UP")
else:
    viz.append("DOWN")
viz[0] = viz[24] = "T"
return np.array(viz).reshape(5, 5)

```

In [378...

```

print(f"Policy iteration's optimal policy:\n {visualize_policy(optimal_policy_policy)}")
print(f"Value iteration's optimal policy:\n {visualize_policy(optimal_policy_value_i)}")

```

```

Policy iteration's optimal policy:
[['T' 'LEFT' 'LEFT' 'LEFT' 'LEFT']
 ['UP' 'LEFT' 'LEFT' 'LEFT' 'DOWN']
 ['UP' 'LEFT' 'LEFT' 'RIGHT' 'DOWN']
 ['UP' 'LEFT' 'RIGHT' 'RIGHT' 'DOWN']
 ['RIGHT' 'RIGHT' 'RIGHT' 'RIGHT' 'T']]

```

```

Value iteration's optimal policy:
[['T' 'LEFT' 'LEFT' 'LEFT' 'LEFT']
 ['UP' 'LEFT' 'LEFT' 'LEFT' 'LEFT']
 ['LEFT' 'LEFT' 'LEFT' 'LEFT' 'LEFT']
 ['LEFT' 'LEFT' 'LEFT' 'LEFT' 'DOWN']
 ['LEFT' 'LEFT' 'LEFT' 'RIGHT' 'T']]

```

We observe that the optimal policy obtained by policy iteration is the same as the optimal policy I expected in Q2. The optimal policy obtained by value iteration is only close to it next to the terminal states.