CS270 Homework 2

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Question 1 : 5 points

Write the following English Sentence as formal logic.

”I’ll get into outerspace only if I’m abducted by aliens.”

Let S mean ”I’ll get into outerspace” and Let A mean ”I’m abducted by aliens”.

$S \Rightarrow A$

Question 2 : 5 points

Write the following English Sentence as formal logic.

”If there are no eggs left, then I will need to stop at the store.”

Let E mean ”there are eggs left” and Let S mean ”I need to stop at the store”.

$\neg E \Rightarrow S$

Question 3 : 5 points

Write the following English Sentence as formal logic.

”There are eggs left or I need to stop at the store.”

Let E mean ”there are eggs left” and Let S mean ”I need to stop at the store”.

$E \vee S$

Question 4 : 5 points

Prove by Truth Table that your answer to Question 2 and your answer to Question 3 are logically

equivalent. (Q2 $\Leftarrow\Rightarrow$ Q3)

E	S	Q2 Output	Q3 Output
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

Question 5 : 10 points

(A ∧ B) ∴ ¬(¬A ∨ ¬B)

Proof:

Construct a proof for the argument: $A \wedge B \therefore \neg(\neg A \vee \neg B)$

1	$A \wedge B$	
2	A	∧E 1
3	B	∧E 1
4	$\neg A \vee \neg B$	
5	$\neg A$	
6	$\bot$	¬E 2, 5
7	$\neg B$	
8	$\bot$	¬E 3, 7
9	$\bot$	∨E 4, 5–6, 7–8
10	$\neg(\neg A \vee \neg B)$	¬I 4–9

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

CHECK PROOF START OVER

Question 6 : 10 points

A ∨ (B ∧ C) ∴ (A ∨ B) ∧ (A ∨ C)

Proof:

Construct a proof for the argument: $A \vee (B \wedge C) \therefore (A \vee B) \wedge (A \vee C)$

1	$A \vee (B \wedge C)$	
2	$\quad A$	
3	$\quad A \vee B$	∨I 2
4	$\quad A \vee C$	∨I 2
5	$\quad (A \vee B) \wedge (A \vee C)$	∧I 3, 4
6	$\quad B \wedge C$	
7	$\quad B$	∧E 6
8	$\quad C$	∧E 6
9	$\quad A \vee B$	∨I 7
10	$\quad A \vee C$	∨I 8
11	$\quad (A \vee B) \wedge (A \vee C)$	∧I 9, 10
12	$(A \vee B) \wedge (A \vee C)$	∨E 1, 2–5, 6–11

NEW LINE NEW SUBPROOF

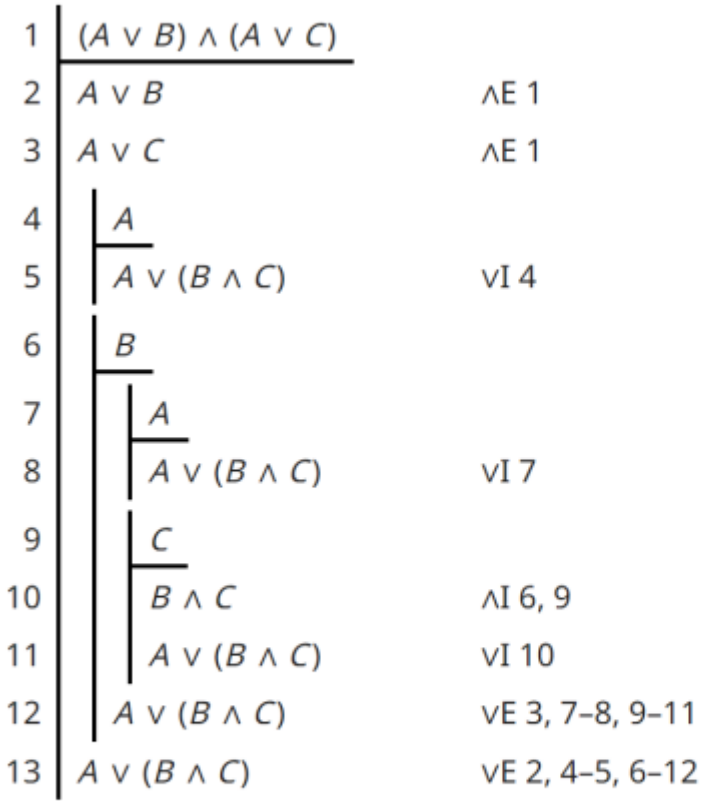
☺ Congratulations! This proof is correct.

CHECK PROOF START OVER

Question 7 : 10 points

(A ∨ B) ∧ (A ∨ C) ∴ A ∨ (B ∧ C)

Construct a proof for the argument: $(A \vee B) \wedge (A \vee C) \therefore A \vee (B \wedge C)$



NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

Question 8 : 15 points

¬(A ∧ B) ∴ ¬A ∨ ¬B

Construct a proof for the argument: $\neg(A \wedge B) \therefore \neg A \vee \neg B$

1	$\neg(A \wedge B)$	
2	$\neg(\neg A \vee \neg B)$	
3	$\neg A$	
4	$\neg A \vee \neg B$	∨I 3
5	$\perp$	¬E 2, 4
6	A	IP 3–5
7	$\neg B$	
8	$\neg A \vee \neg B$	∨I 7
9	$\perp$	¬E 2, 8
10	B	IP 7–9
11	$A \wedge B$	∧I 6, 10
12	$\perp$	¬E 1, 11
13	$\neg A \vee \neg B$	IP 2–12

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

Question 9 : 15 points

(¬A ∨ ¬B) ∴ ¬(A ∧ B)

Construct a proof for the argument: $\neg A \vee \neg B \therefore \neg(A \wedge B)$

1	$\neg A \vee \neg B$	
2	$\neg A$	
3	$A \wedge B$	
4	A	$\wedge$E 3
5	$\perp$	$\neg$E 2, 4
6	$\neg(A \wedge B)$	$\neg$I 3–5
7	$\neg B$	
8	$A \wedge B$	
9	B	$\wedge$E 8
10	$\perp$	$\neg$E 7, 9
11	$\neg(A \wedge B)$	$\neg$I 8–10
12	$\neg(A \wedge B)$	$\vee$E 1, 2–6, 7–11

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

Question 10 : 20 points

$\therefore (\neg A \Longrightarrow B) \Longleftrightarrow (A \vee B)$

Construct a proof for the argument: $\therefore (\neg A \rightarrow B) \leftrightarrow (A \vee B)$

1	$A \vee B$	
2	$\neg A$	
3	A	
4	$\bot$	¬E 2, 3
5	B	X 4
6	B	
7	B	∨E 1, 3–5, 6–6
8	$\neg A \rightarrow B$	→I 2–7
9	$\neg A \rightarrow B$	
10	$\neg(A \vee B)$	
11	$\neg A$	
12	B	→E 9, 11
13	$A \vee B$	∨I 12
14	$\bot$	¬E 10, 13
15	A	IP 11–14
16	$A \vee B$	∨I 15
17	$\bot$	¬E 10, 16
18	$A \vee B$	IP 10–17
19	$(\neg A \rightarrow B) \leftrightarrow (A \vee B)$	↔I 1–8, 9–18

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.