CS 270 - Lab 17

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1 Introduction

You may work in teams of **one** or **two** students. Submit one copy for the entire group.

Write your answers on this lab sheet. Only what is written on this lab sheet will be graded.

This lab is due at the end of the class period. You may not continue to work on it once class has ended.

This lab contains 4 questions.

Grading

- 25 points - Putting everyones names on this page
- 20 points - Earned for each correct question (Answer is fully correct)
- 5 points - Earned for **partial credit** any question

No additional point amounts can be earned. You cannot earn 7 points on a question for example.

The maximum score for a lab is 100. If you get everything correct, that adds up to 105 points but will be reduced to 100.

A question will be marked correct as long as it covers all requirements of the question. It does not need to be perfect, but must be fully correct. A single typo or very minor issue where the intention is clear and all requirements are met would still earn full points.

We want you to complete questions fully, not try to earn partial credit on multiple questions. You may ask your Professor/Course assistant questions during lab.

Labs must be done in the presence of an instructor and/or course assistant or credit will not be given.

Partners should alternate each class day which person is physically typing and submitting the lab.

Do not split up the problems or you risk not finishing on time due to the cumulative nature of the questions.

Enter the name of the student in the group

Member 1 (submitter): Elan Rubin

Member 2: Rufayda Sanjida

Question 1 :

There has been a murder. In this lab, you will use a SAT solver to determine the identity of the murderer or murderers.

Here is what you know:

Dr. Jimmy Corpse has been found dead in his mansion. Based on information from the security system, it is clear that only three people were present in the mansion at the time of death. Each person has given a statement.

- **Maid**: "The Butler is innocent but the Gardener is guilty."
- **Butler**: "Exactly one of us is a murderer."
- **Gardener**: "The Maid and I are both innocent."

A suspect will lie if and only if they were involved in the murder. Innocent people will always tell the truth; murderers always lie. We need to translate each statement into Boolean Logic. We start with the Maid.

Maid: "The Butler is innocent but the Gardener is guilty."

If the Maid is the murder, then they will also lie in their statement. If the maid is innocent they will give a truthful statement.

We will use M, G, and B for the three suspects. If a suspect is innocent, the variable is true. This statement is written as $M \iff (B \wedge \neg G)$. This is not in CNF format for the SAT solver.

Ask `https://www.wolframalpha.com` to convert this expression to CNF for you. In Wolfram, you would enter (M) XNOR (B **AND NOT** G). Note that wolfram uses XNOR for the Bi-Conditional. Enter the expression and hit enter. Scroll down the result page and it will tell you the CNF expression. The Maid's statement in CNF is given below.

$$(\mathrm{M} \vee \neg B \vee G) \wedge (\neg M \vee B) \wedge (\neg M \vee \neg G)$$

Hint: You may want to get these answers checked by a TA. You will use them in the next questions.

(a) Write the Gardener's statement in as a boolean expression.

G ⇔ M∧G

(b) Using wolfram alpha convert the Gardener's statement to CNF. Write it below.

¬G∨M

(c) Write the Butler's statement in as a boolean expression.

B ⇔ ((B∧¬M∧G) ∨ (B∧M∧¬G) ∨ (¬B∧M∧G))

(d) Using wolfram alpha convert the Butler's statement to CNF. Write it below.

(¬B ∨ G ∨ M) ∧ (¬G ∨ ¬M)

Question 2 :

Using your CNF expression from the previous question. Write all 3 statements as one DiMACS file.

Use $M = 1$, $B = 2$, and $G = 3$.

Use https://www.msoos.org/2013/09/minisat-in-your-browser/

(a) Write your DiMACs code below.

```
c gardner: ¬GvM
c butler: (¬B ∨ G ∨ M) ∧ (¬G ∨ ¬M)
c maid: (M ∨ ¬B ∨ G) ∧ (¬M ∨ B) ∧ (¬M ∨ ¬G)
c m 1, b 2, g 3
c -------------------------

p cnf 3 6
1 -3 0
-2 3 1 0
-3 -1 0
1 -2 3 0
-1 2 0
-1 -3 0
```

(b) What answer does the SAT solver give you (something line v -1 ...)

s SATISFIABLE
c conflicts: 0
v -1 -2 -3 0

(c) Express this DIMACS answer back in terms of the context of the problem by saying who the SAT Solver declares as the murderer(s). Then verify that each person's statements are consistent with the Truthness/Falseness of their innocence/guilt. (Recall that murderers always lie and innocent people always tell the truth).

in this case, the SAT solver found that all 3 members of the group were part of the murder

maid: butler is innocent, gardner is guilty - not true
butler: exactly one is a murderer - not true
gardener: maid and gardner are innocent - not true

everyone being a murderer makes sense, because they all lied

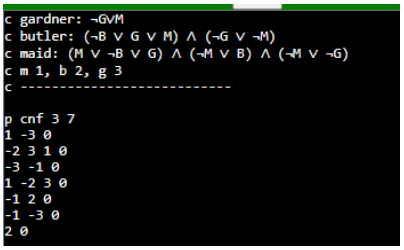
Question 3 :

In the previous question, the SAT solver told you that everyone was the murder.

New evidence comes to light. The detective finds evidence that the Butler was on a video chat during the time of the murder and could not have done it. Add one new statement that the Butler is innocent by adding one more line to the DiMACs.

Use `https://www.msoos.org/2013/09/minisat-in-your-browser/`

(a) Write your updated DiMACs code below.



```
c gardner: ¬GvM
c butler: (¬B v G v M) ∧ (¬G v ¬M)
c maid: (M v ¬B v G) ∧ (¬M v B) ∧ (¬M v ¬G)
c m 1, b 2, g 3
c --------------------------

p cnf 3 7
1 -3 0
-2 3 1 0
-3 -1 0
1 -2 3 0
-1 2 0
-1 -3 0
2 0
```

(b) What answer does the SAT solver give you (something line v -1 ...)

s SATISFIABLE
c conflicts: 0
v 1 2 -3 0

(c) Interpret the answer in terms of the murder mystery. First, explain what the SAT Solver's output means in terms of the murder mystery. Then explain how this relates to the Truthness/Falseness of their innocence/guilt.

this answer says that the maid is innocent, the butler is innocent, and the gardener is the murderer

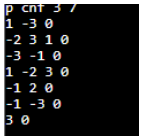
maid: butler is innocent, gardner is guilty - true
butler: exactly one is a murderer - true
gardener: maid and gardner are innocent - not true

only the gardener being the murderer makes sense, because he was the only one who told a lie

Question 4 :

Instead of the Butler having an alibi, do you think it's possible instead for the Gardener to have an alibi?

(a) Adjust the DIMACs code from Q2 but this time try to make it as if it were the Gardener that has the alibi instead of the Gardener



(b) What answer does the SAT solver give you?

s UNSATISFIABLE
c conflicts: 0

(c) Interpret the answer in terms of the murder mystery. First, explain what the SAT Solver's output means in terms of the murder mystery. Then explain how this relates to the Truthness/Falseness of their innocence/guilt.

the gardener definitely does not have an alibi, as the problem becomes unsatisfiable if he is innocent