

For clustering vertices, the statistical decision theoretical problem statement is as follows:

1. A graph of a fixed size: $G_n = (V, E, Y)$:
 - (a) V is the number of vertices.
 - (b) E is the number of edges.
 - (c) Y is vector labels $\{0, 1\}^n$. These are two distinct groups and we don't know them.
2. $SBM_n^k(\rho, \beta, \dots)$ for $k = 2$:
 - (a) $\rho \in \delta_2$ (2 simplex)
 - (b) $\beta \in (0, 1)^{2 \times 2}$
3. $A = \{y \in \{0, 1\}^n\}$. This is the set of cluster assignments.
4. $l : G_n \times A \rightarrow R_+$; $l = \sum_{i=1}^n \Theta \{\hat{y}_i = y\}$
5. $R = A \times A \times P$ (functional on true data, action class, model distribution)