Thus we get a contradiction. Hence the argument

EXAMPLE 2.64 Prove that the following premises are inconsistent:

(a) $P \rightarrow (Q \rightarrow R), S \rightarrow (Q \wedge \neg R), P \wedge S$

(b) $P \rightarrow Q, Q \vee R \rightarrow S, S \rightarrow \neg P, P \wedge \neg R.$

Solution:

(a)

1.	$P \rightarrow (Q \rightarrow R)$	Premise (i)
2.	$\neg P \vee (Q \rightarrow R)$	Line 1 and I_{12}(iii)
3.	$P \wedge S$	Premise (iii)

	Line 3 and simplification
4. P	Lines 2, 4 and DS
5. $Q \to R$	Line 3 and simplification
6. S	Premise (ii)
7. $S \to (Q \wedge \neg R)$	Lines 6, 7 and MP
8. $Q \wedge \neg R$	Line 8 and simplification
9. Q	Lines 5, 9 and MP
10. R	Line 8 and simplification
11. $\neg R$	Lines 10, 11
12. $R \wedge \neg R$	Line 12
13. **F**	

Hence the given premises are inconsistent.

(a)

1. $P \to Q$	Premise (i)
2. $P \wedge \neg R$	Premise (iv)
3. P	Line 2 and simplification
4. Q	Lines 1, 3 and MP
5. $\neg R$	Line 2 and simplification
6. $Q \vee R \to S$	Premise (ii)
7. $S \to \neg P$	Premise (iii)
8. $Q \vee R \to \neg P$	Lines 6, 7 and HS
9. $\neg (Q \vee R)$	Lines 3, 8 and MT
10. $\neg Q \wedge \neg R$	Line 9 and De Morgan's law
11. $\neg Q$	Line 10 and simplification
12. $Q \wedge \neg Q$	Lines 4, 11 and conjunction
13. **F**	Line 12

Hence the given premises are inconsistent.

EXAMPLE 2.65 Test the validity of the following argument:

If the advertisement is successful, then the sales of the product will go up. Either the advertisement is successful or the production of the product will be stopped. The sales of the product will not go up. Therefore the production of the product will be stopped.

Solution: Let

A be "The advertisement is successful"

P be "The sales of the product will go up"

S be " The production of the product will be stopped".

Then the premises are

(i) $A \to P$ (ii) $A \vee S$ (iii) $\neg P$

The conclusion is S.

1. $A \to P$	Premise (i)
2. $\neg P \to \neg A$	Line 1 and I_{14}
3. $\neg P$	Premise (iii)
4. $\neg A$	Lines 2, 3 and MP
5. $A \vee S$	Premise (ii)
6. S	Line 4, 5 and DS

Hence the argument is valid.

EXAMPLE 2.70 Show that from

(i) $\exists x\,(F(x) \wedge G(x)) \rightarrow \forall y\,(M(y) \rightarrow N(y))$

(ii) $\exists y\,[M(y) \wedge \neg N(y)]$

the conclusion $\forall x\,(F(x) \rightarrow \neg G(x))$ follows.

Solution: The argument is as follows:

1.	$\exists y\,(M(y) \wedge \neg N(y))$	Premise (ii)
2.	$M(c) \wedge \neg N(c)$	Line 1 and EI
3.	$\neg(\neg M(c) \vee N(c))$	Line 2 and De Morgan's law
4.	$\neg(M(c) \rightarrow N(c))$	Line 3 and I_{12} (vi)
5.	$\exists y\,(\neg(M(y) \rightarrow N(y))$	Line 4 and EG
6.	$\neg(\forall y\,(M(y) \rightarrow N(y))$	Line 5 and I_{18}
7.	$\exists x\,(F(x) \wedge G(x)) \rightarrow \forall y\,(M(y) \rightarrow N(y))$	Premise (i)
8.	$\neg(\exists x\,(F(x) \wedge G(x))$	Lines 6, 7 and MT
9.	$\forall x\,(\neg(F(x) \wedge G(x))$	Line 8 and I_{17}
10.	$\neg(F(c) \wedge G(c))$	Line 9 and UI
11.	$\neg(F(c)) \vee \neg(G(c))$	Line 10 and De Morgan's law
12.	$F(c) \rightarrow \neg G(c)$	Line 11 and I_{12} (vi)
13.	$\forall x\,(F(x) \rightarrow \neg G(x))$	Line 12 and UG

Hence the conclusion is valid.

EXAMPLE 2.71 Prove that $\forall x\,(S(x) \rightarrow \neg P(x))$ follows from $\forall x\,(P(x) \rightarrow Q(x))$ and $\forall x\,(S(x) \rightarrow \neg Q(x))$.

Solution: We assume $S(x)$ as an additional premise.

1.	$\forall x\,(P(x) \rightarrow Q(x))$	Premise (i)
2.	$\forall x\,(S(x) \rightarrow \neg Q(x))$	Premise (ii)
3.	$S(c) \rightarrow \neg Q(c)$	Line 2 and UI
4.	$S(c)$	Additional premise
5.	$\neg Q(c)$	Lines 3, 4 and MP
6.	$P(c) \rightarrow Q(c)$	Line 1 and UI
7.	$\neg P(c)$	Line 6 and MT
8.	$S(c) \rightarrow \neg P(c)$	Lines 4, 7 and CP (RI_{10})
9.	$\forall x\,(S(x) \rightarrow \neg P(x))$	Line 8 and US

EXAMPLE 2.78 Test the validity of the following argument:

$$\begin{array}{l} \forall x\, \forall y\, A(x, y) \\ \exists x\, \forall y\, [A(x, y) \rightarrow B(y, x)] \\ \hline \therefore\ \exists z\, \forall y\, B(y, z) \end{array}$$

Solution:

1. $\exists x\, \forall y\, [A(x, y) \rightarrow B(y, x)]$	Premise (ii)
2. $\forall y\, [A(x, y) \rightarrow B(y, x)]$	Line 1 and UI
3. $\forall x\, \forall y\, A(x, y)$	Premise (i)
4. $\forall y\, A(x, y)$	Line 3 and UI
5. $A(x, y)$	Line 4 and UI
6. $A(x, y) \rightarrow B(y, x)$	Line 2 and UI
7. $B(y, x)$	Lines 5, 6 and MP
8. $\forall y\, B(y, x)$	Line 7 and UG
9. $\exists z\, \forall y\, B(y, z)$	Line 8, EG

Hence the argument is valid.