Advanced SQL

05 — Window Functions

Summer 2022

Torsten Grust
Universität Tübingen, Germany

With SQL:2003, the ISO SQL Standard introduced **window functions**, a new mode of row-based computation:

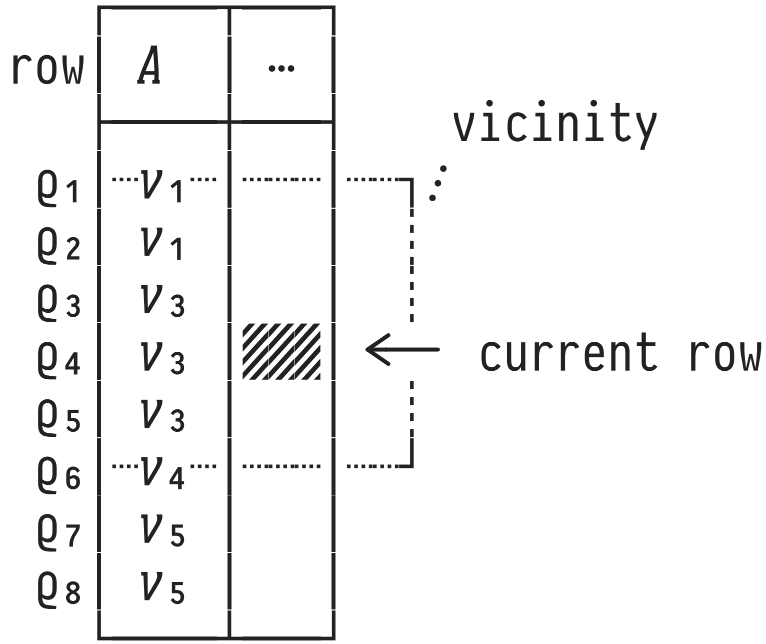
SQL Feature	Mode of Computation
function	row → row
table-generating function	row → table of rows
aggregate	group of rows → row (one per group)
window function	row vicinity → row (one per row)

SQL Modes of Computation

Window functions …

- … are **row-based:** each individual input row r is mapped to one result row,
- … use the **vicinity** around r to compute this result row.

Row Vicinity: Window Frames



- Each row is the **current row** at one point in time.

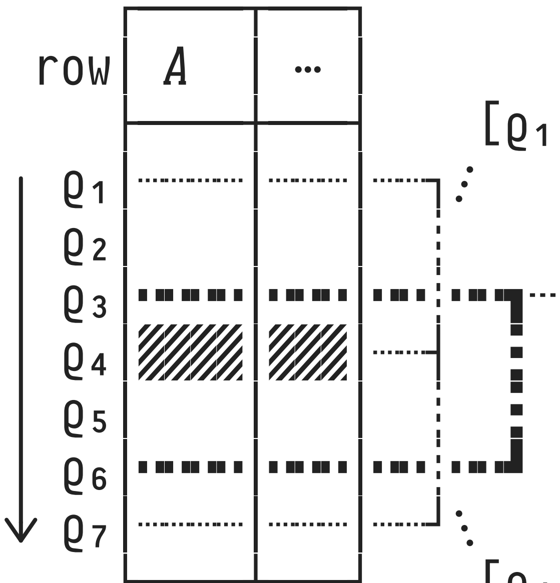
- Row vicinity (**window**, **frame**) is based on either:

  ❶ row **position** (ROWS windows),
  ❷ row **values** v_i (RANGE windows),
  ❸ row **peers** (GROUPS windows).

- As the current row changes, the window *slides* with it.
- ⚠️ Window semantics depend on a defined **row ordering**.

Window Frame Specifications (Variant: ROWS)

$\overbrace{f}^{\text{window function}}$ **OVER (ORDER BY** $\overbrace{e_1,\ldots,e_n}^{\text{ordering criteria}}$ **[ROWS** $\overbrace{\textit{frame}}^{\text{frame specification}}$ **])**



frame:

$[\varrho_1,\ldots,\varrho_4]$: **BETWEEN UNBOUNDED PRECEDING AND CURRENT ROW**

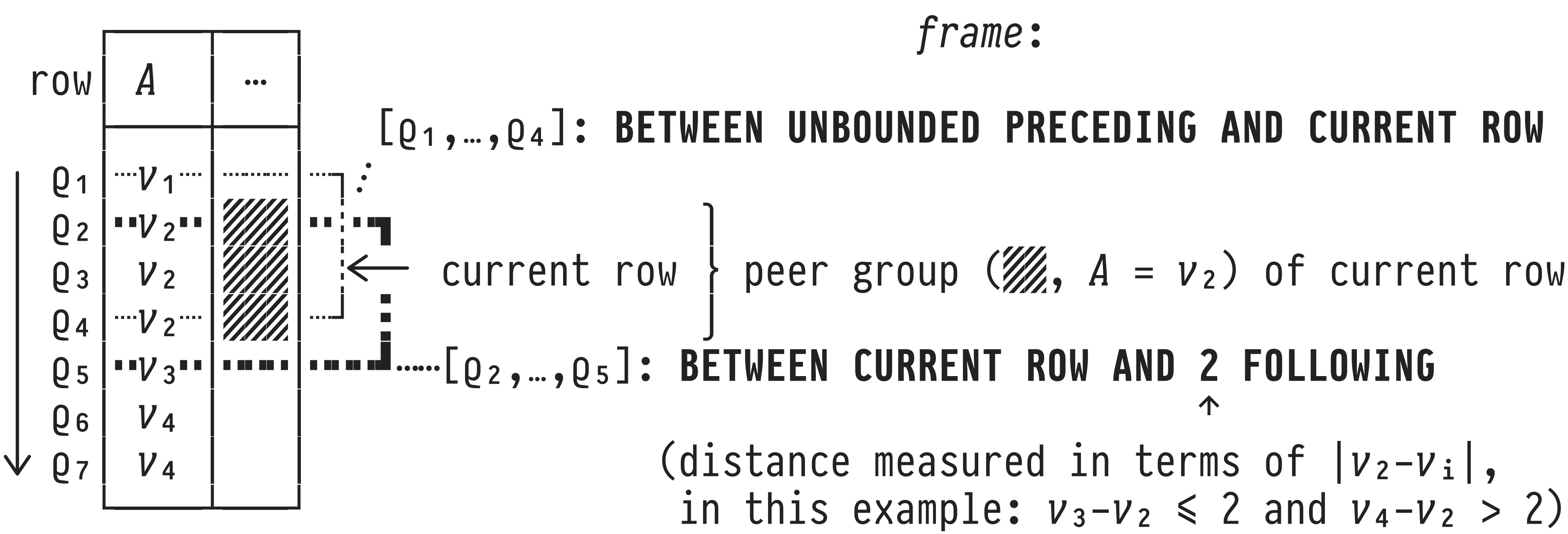
$[\varrho_3,\ldots,\varrho_6]$: **BETWEEN 1 PRECEDING AND 2 FOLLOWING**

(distance from measured in number of rows)

$[\varrho_4,\ldots,\varrho_7]$: **BETWEEN CURRENT ROW AND UNBOUNDED FOLLOWING**

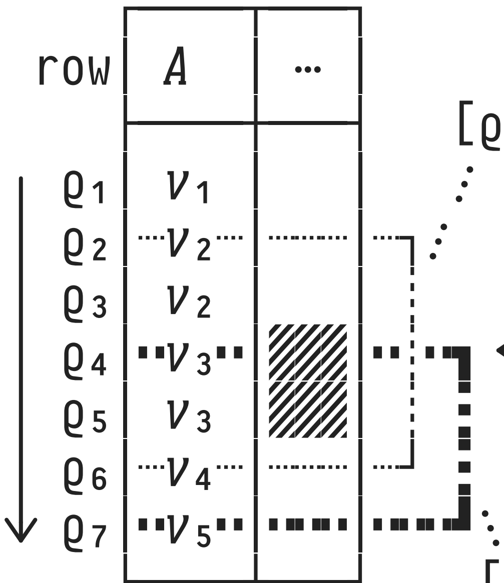
Window Frame Specifications (Variant: RANGE)

$$\overbrace{f}^{\text{window function}}\ \textbf{OVER (ORDER BY}\ \overset{\text{one criterion}}{\downarrow}{A}\ \textbf{[RANGE}\ \overbrace{\textit{frame}}^{\text{frame specification}}\ \textbf{])}$$



Window Frame Specifications (Variant: GROUPS)

$$\overbrace{f}^{\text{window function}}\ \textbf{OVER}\ (\textbf{ORDER BY}\ \overbrace{e_1,\ldots,e_n}^{\text{ordering criteria}}\ [\ \textbf{GROUPS}\ \overbrace{\mathit{frame}}^{\text{frame specification}}\ ])$$



frame:

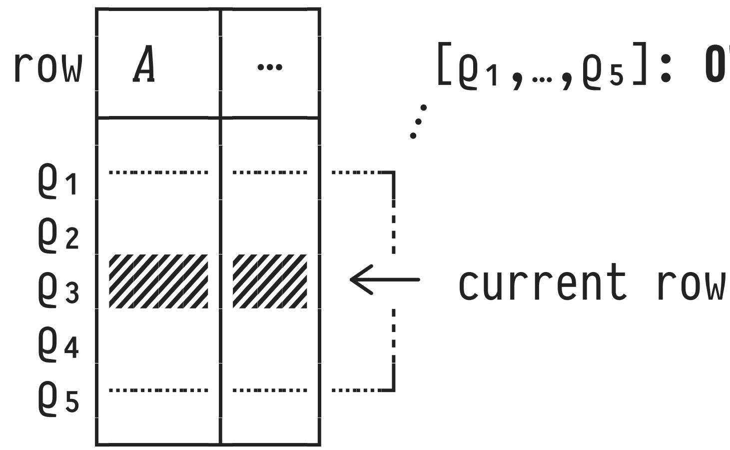
[$\varrho_2,\ldots,\varrho_6$]: **BETWEEN 1 PRECEDING AND 1 FOLLOWING**

(distance measured in number of peer groups)

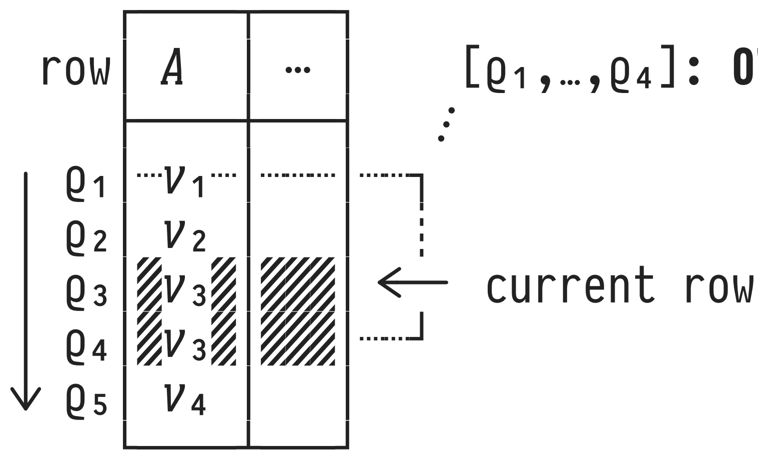
← current row } peer group (▨, $A = v_3$) of current row

[$\varrho_4,\ldots,\varrho_7$]: **BETWEEN CURRENT ROW AND UNBOUNDED FOLLOWING**

Window Frame Specifications: Abbreviations



$[\varrho_1,\ldots,\varrho_5]$: **OVER ()** ≡ all rows form the window frame (for any current row)



$[\varrho_1,\ldots,\varrho_4]$: **OVER (ORDER BY *A*)** ≡

OVER (ORDER BY *A*
RANGE BETWEEN UNBOUNDED PRECEDING
AND CURRENT ROW)

WINDOW Frame Specifications: Exclusion[1]

$\overbrace{f}^{\text{window function}}$ **OVER (ORDER BY** *A frame* $\overbrace{[\ \mathit{exclusion}\ ]}^{\text{exclusion clause}}$**)**

row	*A*	…
ϱ_1	v_1	
ϱ_2	v_2	
ϱ_3	v_3	
ϱ_4	v_3	
ϱ_5	v_3	
ϱ_6	v_4	

← current row

exclusion:

EXCLUDE NO OTHERS	$[\varrho_2, \varrho_3, \varrho_4, \varrho_5]$
EXCLUDE CURRENT ROW	$[\varrho_2, \varrho_3, \quad \varrho_5]$
EXCLUDE GROUP	$[\varrho_2 \qquad]$
EXCLUDE TIES	$[\varrho_2, \quad \varrho_4 \quad]$

[1] **Q:** Which *frame* would lead to a window as shown above?

WINDOW Clause: Name the Frame

Syntactic 🍭: If window frame specifications

1. become unwieldy because of verbose SQL syntax and/or
2. one frame is used multiple times in a query,

add a WINDOW clause to a SFW block to **name the frame**, *e.g.*:

```
SELECT … f OVER w_i … g OVER w_j …
FROM   …
WHERE  …
   ⋮
WINDOW w_1 AS (frame_1), …, w_n AS (frame_n)
ORDER BY …
```

Use SQL Itself to Explain Window Frame Semantics

Regular **aggregates** may act as window functions *f*. All **rows in the frame will be aggregated:**

```
SELECT w.row                               AS "current row",
       COUNT(*)             OVER win AS "frame size",
       array_agg(w.row) OVER win AS "rows in frame"
FROM   W AS w
WINDOW win AS (frame)
```

row	a	b
ϱ_1	1	●
ϱ_2	2	○
ϱ_3	3	○
ϱ_4	3	●
⋮	⋮	⋮

Table W

Q: What is the Chance of Fine Weather on Weekends?

Input: Daily weather readings in sensors:

day	weekday	temp	rain
1	Fri	10	800
2	Sat	12	300
⋮	⋮	⋮	⋮

Table sensors

- The weather is fine on day ***d*** if—**on *d* and the two days prior**—the minimum temperature is above 15°C and the overall rainfall is less than 600ml/m².
- **Expected output:**

weekend?	% fine
f	29
t	43

2 ┊ PARTITION BY: Window Frames Inside Partitions

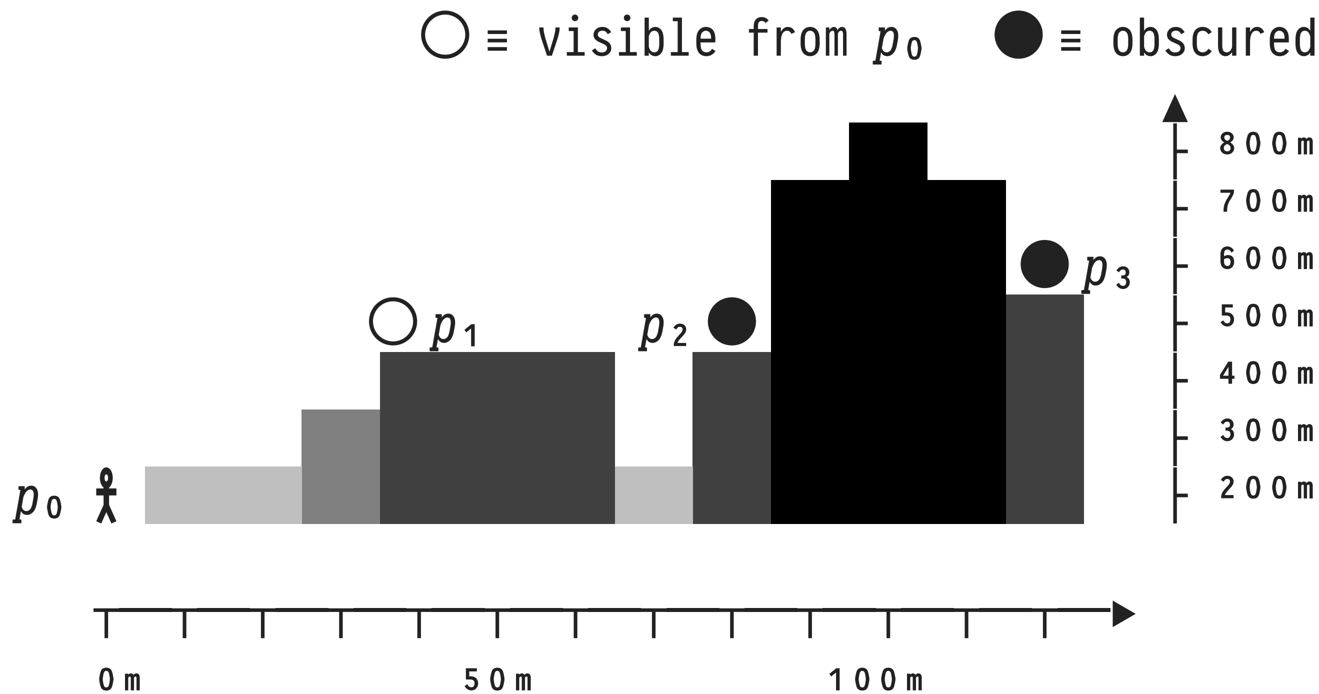
Optionally, we may **partition** the input table *before* rows are sorted and window frames are determined:

$$f\ \mathbf{OVER}\ (\Big[\ \overbrace{\mathbf{PARTITION\ BY}\ p_1,\ldots,p_m}^{\text{all input rows that agree on all } p_i \text{ form one partition}}\ \Big]\ \Big[\ \mathbf{ORDER\ BY}\ e_1,\ldots,e_n\ \Big]\ \Big[\ \mathit{frame}\ \Big])$$

- Note:

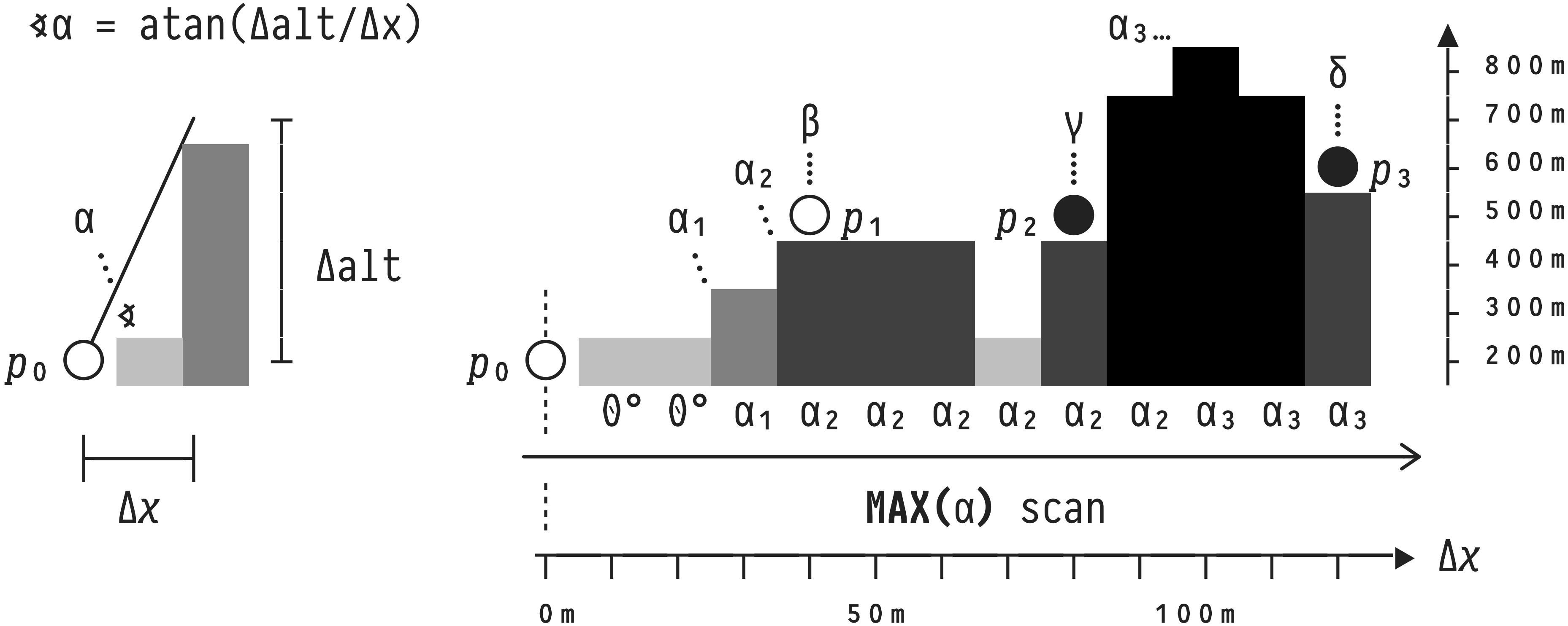
  1. Frames **never cross partitions.**
  2. BETWEEN ⋯ PRECEDING AND ⋯ FOLLOWING **respects partition boundaries.**

Q: Which Spots are Visible in a Hilly Landscape?



- From the viewpoint of p_0 (웃) we can see p_1, but…
  - … p_2 is **obscured** (no straight-line view from p_0),
  - … p_3 is **obscured** (lies behind the 800m peak).

Q: Visible Spots in a Hilly Landscape? — A: MAX Scan!



- We have $0° < \alpha_1 < \alpha_2 < \alpha_3$ and $\beta \geqslant \alpha_2$, $\gamma < \alpha_2$, $\delta < \alpha_3$.

p_1 visible $p_{2,3}$ obscured

Q: Visible Spots in a Hilly Landscape? — A: MAX Scan!

- **Input:** Location of p_0 (here: $x = 0$) and 1D-map of hills:

x	alt
0	200
10	200
⋮	⋮
120	500

Table map

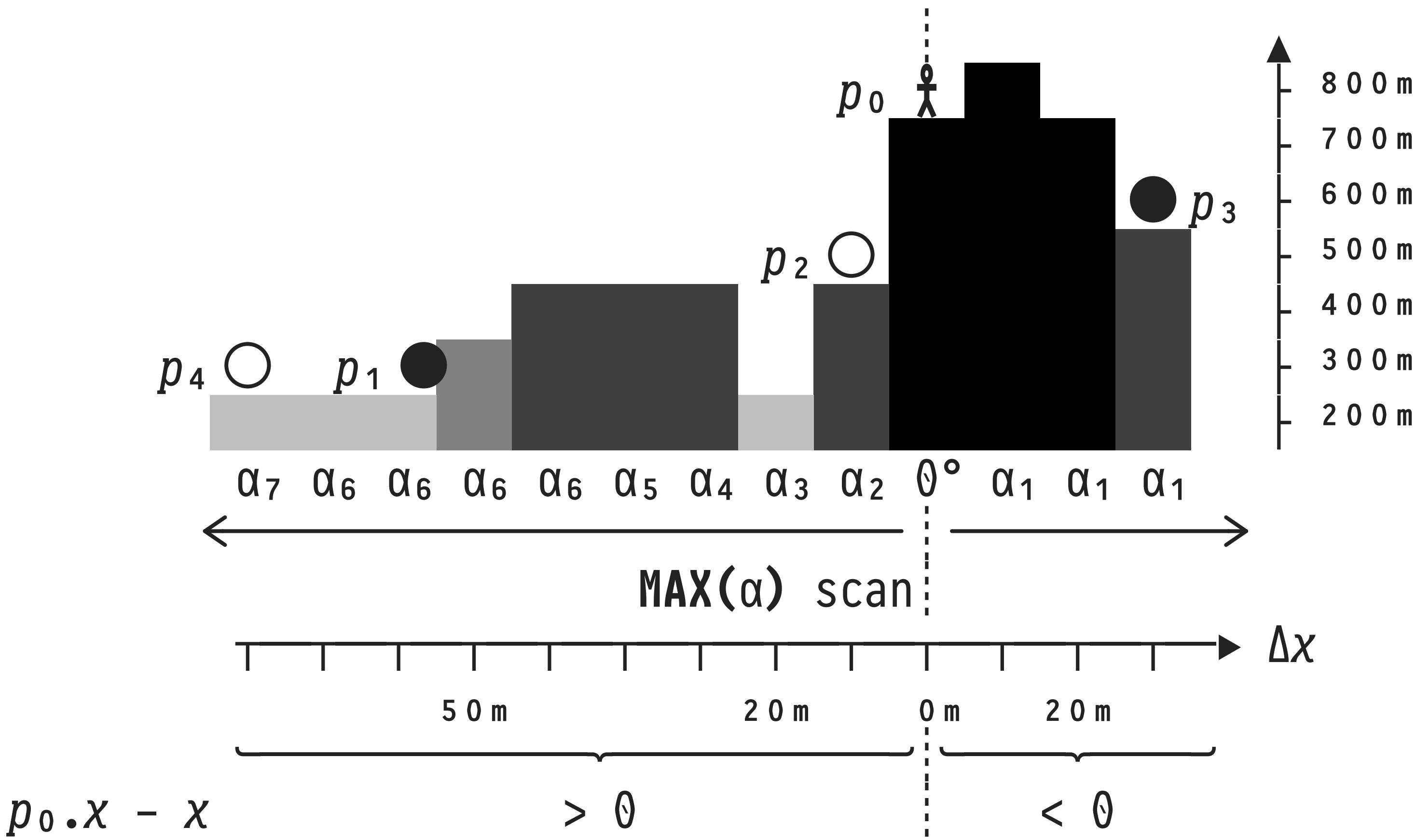
- **Output:** Can p_0 see the point on the hilltop at x?

x	visible?
0	true
10	true
⋮	⋮
120	false

Q: Visible Spots in a Hilly Landscape? — A: MAX Scan!

```
WITH
-- ❶ Angles α (in °) between p₀ and the hilltop at x
angles(x, angle) AS (
  SELECT m.x,
         degrees(atan((m.alt - p0.alt) /
                      abs(p0.x - m.x))) AS angle
  FROM   map AS m
  WHERE  m.x > p0.x
),
-- ❷ MAX(α) scan (to the right of p₀)
max_scan(x, max_angle) AS (
  SELECT a.x,
         MAX(a.angle)
           OVER (ORDER BY abs(p0.x - a.x)) AS max_angle
  FROM   angles AS a
),
⋮
```

Looking Left *and* Right: PARTITION BY



- Need MAX scans left *and* right of p_0 $\Rightarrow$ use PARTITION BY.

Looking Left *and* Right: PARTITION BY

```
WITH
⋮
-- ❷ MAX(α) scan (left/right of p₀)
max_scan(x, max_angle) AS (
  SELECT a.x,                   --   ∈ {-1, 0 ,1}
         MAX(a.angle)           --
           OVER (PARTITION BY sign(p0.x - a.x)
                    ORDER BY abs(p0.x - a.x)) AS max_angle
  FROM   angles AS a  --
                      --       Δx > 0
),
⋮
```

- $\forall$a $\in$ angles: a.x $\neq$ p0.x $\Rightarrow$ We end up with **two** partitions.

Scans are a general and expressive computational pattern:

$$\underbrace{agg(e)}_{(\phi,z,\oplus)}\ \texttt{OVER (ORDER BY}\ e_1,\ldots,e_n\ \texttt{\{ROWS,RANGE,GROUPS\} BETWEEN UNBOUNDED PRECEDING AND CURRENT ROW)}$$

- Available in a variety of forms in programming languages
  - Haskell: scanl $z \oplus xs$, APL: $\oplus\backslash xs$, Python: accumulate.
    scanl $\oplus$ z $[x_1, x_2, \ldots] = [z,\ z \oplus x_1,\ (z \oplus x_1) \oplus x_2,\ \ldots]$

- In parallel programming: *prefix sums* (☞ Guy Blelloch)
  - Sorting, lexical analysis, tree operations, reg.exp. search, drawing operations, image processing, …

Q: Assume *xs* ≡ '((b*2)-4×a×c)*0.5'. What is computed below?

```
SELECT inp.pos, inp.c,
       SUM((array[1,-1,0])[COALESCE(p.oc, 3)])
         OVER (ORDER BY inp.pos) AS d
FROM   string_to_table(:'xs', NULL)
         WITH ORDINALITY AS inp(c,pos),
       LATERAL (VALUES (array_position(array['(',')'],
                                        inp.c))) AS p(oc)
ORDER BY inp.pos;
```

💡 **Hint** (this is the same query expressed in APL):

```
xs ← '((b*2)-4×a×c)*0.5'
+\ (1 ¯1 0)['()'⍳xs]
```

```
    window function
          ↓
          f OVER ([ PARTITION BY p₁,…,pₘ ]
                  [ ORDER BY e₁,…,eₙ ]
                  [ frame ])
```

Three kinds of **window functions** *f*:

1. **Aggregates:** SUM(•), AVG(•), MAX(•), array_agg(•), … process all rows in the frame. ✓

2. **Row Access:** access row by *absolute/relative position* in ordered frame or partition: first/last/n^{th}/n rows away.

3. **Row Ranking:** assign numeric *rank of row* in partition.

6 ¦ LAG/LEAD: Access Rows of the Past and Future

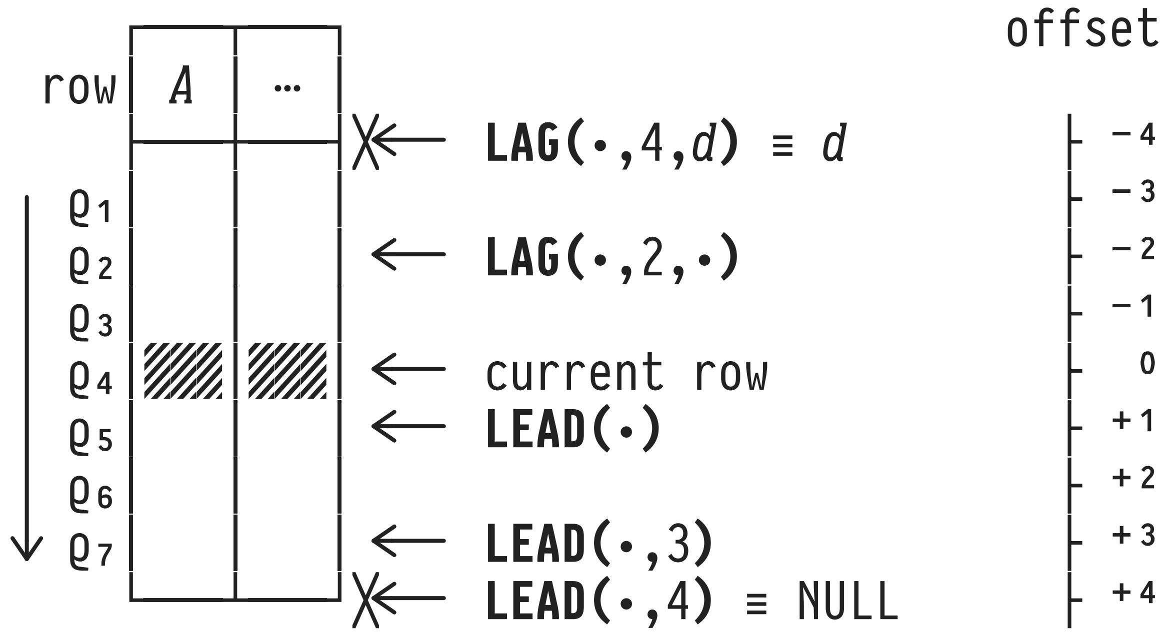
Row access at offset $\pm n$, relative to the current row:

```
-- evaluate e as if we were
-- n rows before the current row
--
     LAG(e,n,d) OVER ([ PARTITION BY p₁,…,pₘ ]
                        ORDER BY e₁,…,eₙ)
```

Note:

- LEAD(e,n,d): ··· n rows **after** the current row ···
- Scope is partition—no row access outside the partition.
- If there is no row at offset $\pm n$ $\Rightarrow$ return value d.

LAG/LEAD: Row Offsets (Assume: No Partitioning, ORDER BY *A*)



- The frame of the current row is irrelevant for LAG/LEAD.
- Default parameters if absent: n ≡ 1, d ≡ NULL.

A March Through the Hills: Ascent or Descent?

```sql
SELECT m.x, m.alt,
       CASE sign(LEAD(m.alt, 1) OVER rightwards - m.alt)
            WHEN -1 THEN '↘' WHEN 1 THEN '↗'
            WHEN  0 THEN '→' ELSE '?'
       END AS climb,
       LEAD(m.alt, 1) OVER rightwards - m.alt AS "by [m]"
FROM   map AS m
WINDOW rightwards AS (ORDER BY m.x)  --  marching right
```

x	alt	climb	by [m]
0	200	→	0
⋮	⋮	⋮	⋮
90	700	↗	100
100	800	↘	-100
110	700	↘	-200
120	500	?	NULL

🔧 Crime Scene: Sessionization

A spy broke into the Police HQ computer system. A log records keyboard activity of user uid at time ts:

uid	ts
👮	05-25-2020 07:25:12
👮	05-25-2020 07:25:18
🕵	05-25-2020 08:01:55
🕵	05-25-2020 08:05:07
🕵	05-25-2020 08:05:30
👮	05-25-2020 08:05:39
🕵	05-25-2020 08:05:46

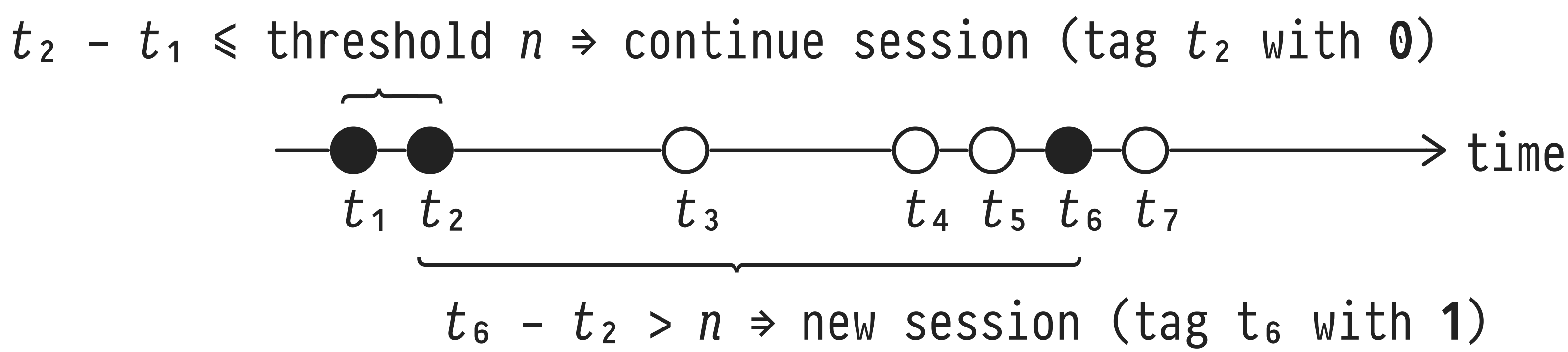
Table log

- **Q:** Can we **sessionize** the log so that investigators can identify *sessions* (≡ streaks of uninterrupted activity)?

Sessionization (Query Plan)

1. Cop and spy sessions happen independently (even if interleaved): partition log into 👮/● and 🕵/○ rows.

2. **Tag** keyboard activities (below: tagging for ●):



3. **Scan** the tagged table and derive session IDs by maintaining a **runnning sum** of *start of session* tags.

Sessionization (Query Plan)

1

uid	ts
●	t_1
●	t_2
○	t_3
○	t_4
○	t_5
●	t_6
○	t_7

2

uid	ts
●	t_1
●	t_2
●	t_6
○	t_3
○	t_4
○	t_5
○	t_7

3

uid	ts	sos
●	t_1	1
●	t_2	0
●	t_6	1
○	t_3	1
○	t_4	1
○	t_5	0
○	t_7	0

← log start

$t_6 - t_2 > n$
⇒ new session

← log start

$t_7 - t_5 \leqslant n$
⇒ continue

4

uid	ts	session
●	t_1	1
●	t_2	1
●	t_6	2
○	t_3	1
○	t_4	2
○	t_5	2
○	t_7	2

- At log start, always begin a new session (sos = 1).
- How to assign *global session IDs* (○'s sessions: 3, 4)?

Image Compression by Run-Length Encoding

Compress image by identifying pixel **runs** of the same color:

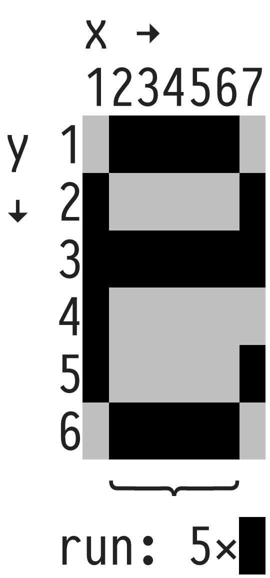


Table **original**

x	y	pixel
1	1	
2	1	
⋮	⋮	⋮
6	6	
7	6	

→

Table **encoding**

run	length	pixel
1	1	
2	5	
⋮	⋮	⋮
12	5	
13	1	

- Here: assumes a row-wise linearization of the pixel map.
- In b/w images we may omit column pixel in table encoding.

Run-Length Encoding (Query Plan)

❶

x	y	pixel	change?
1	1		t 1
2	1		t 1
3	1	≟	f 0
4	1		f 0
5	1		f 0
6	1		f 0
7	1		t 1
⋮	⋮	⋮	⋮ ⋮

❷

x	y	pixel	change?	Σ change?
1	1		1	1
2	1		1	2
3	1		0	2
4	1		0	2
5	1		0	2
6	1		0	2
7	1		1	3
⋮	⋮	⋮	⋮	⋮

run #2 of length 5

❶: `LAG(pixel,1,undefined)`: pixel @ (1,1) always “changes.”

❷: A `SUM()` scan of `change?` yields run identifiers.

7 ¦ FIRST_VALUE, LAST_VALUE, NTH_VALUE: In-Frame Row Access

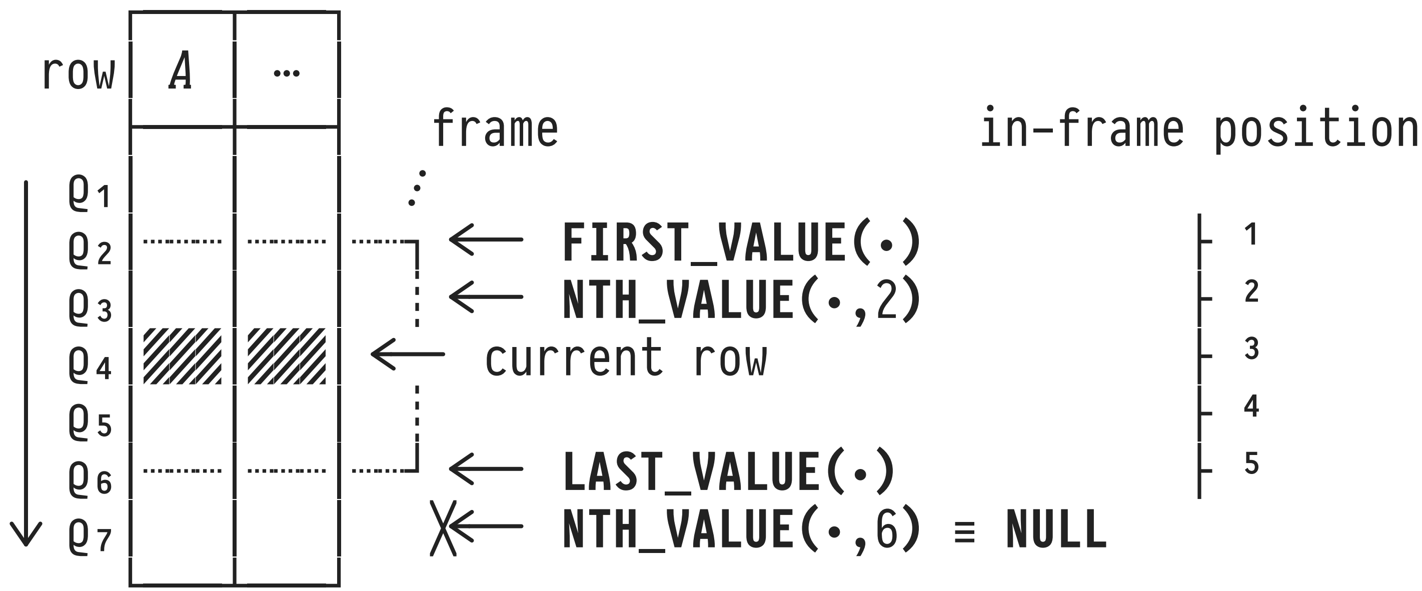
Aggregates reduce *all rows* inside a frame to a single value. Now for something different:

- **Positional access to individual rows** inside a frame is provided by three window functions:

```
-- evaluate expression e as if we were at
-- the first/last/nth row in the frame
--
        FIRST_VALUE(e)  ⎫
        LAST_VALUE(e)   ⎬ OVER (…)
        NTH_VALUE(e,n)  ⎭
```

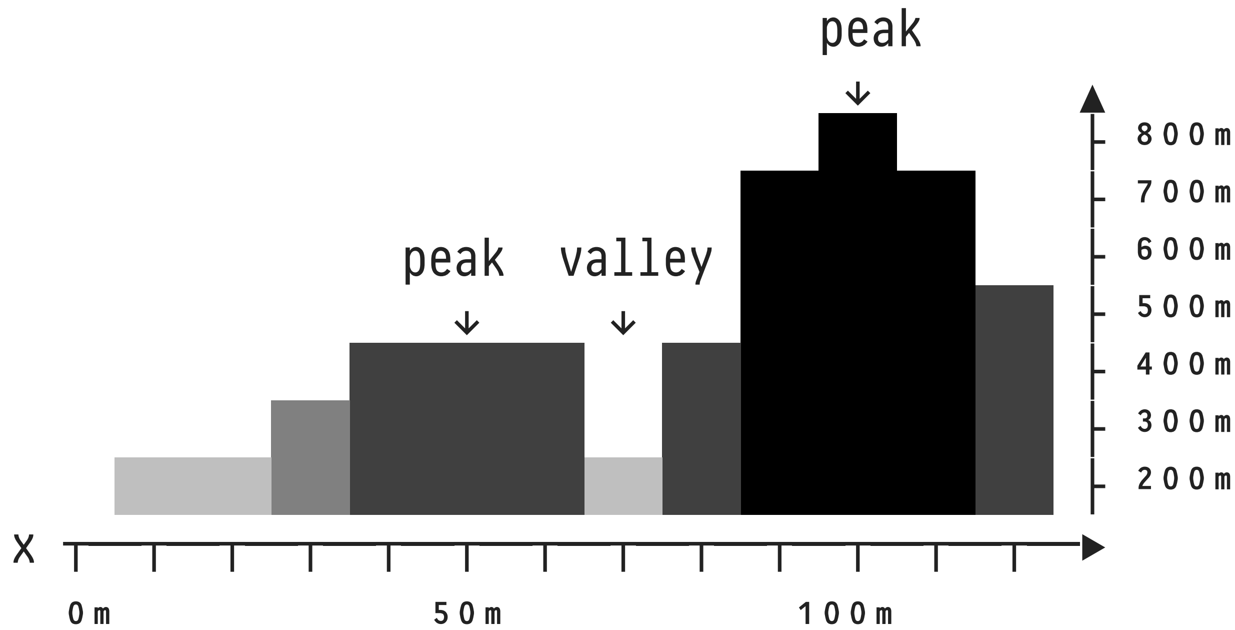
- NTH_VALUE(e,n): No n^{th} row in frame ⇒ return NULL.

In-Frame Row Access



- We have FIRST_VALUE(e) ≡ NTH_VALUE(e,1).

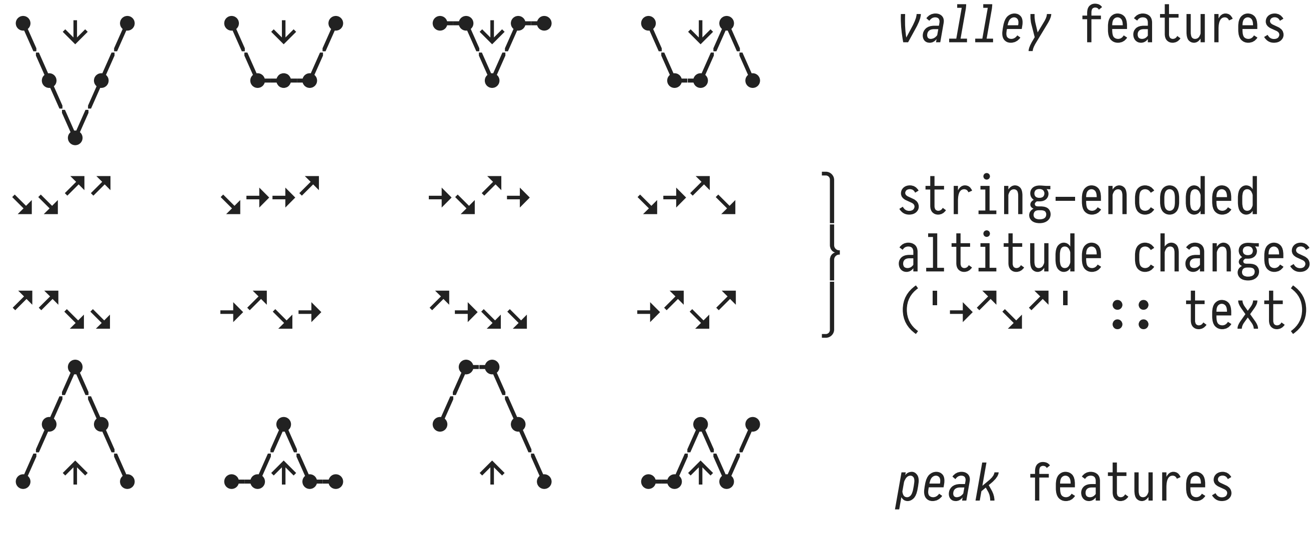
Detecting Landscape Features



- Detect features in a hilly landscape. Attach label $\in$ {peak,valley,-} to every location x.
- Feature defined by relative altitude change **in vicinity**.

Detecting Landscape Features (Query Plan)

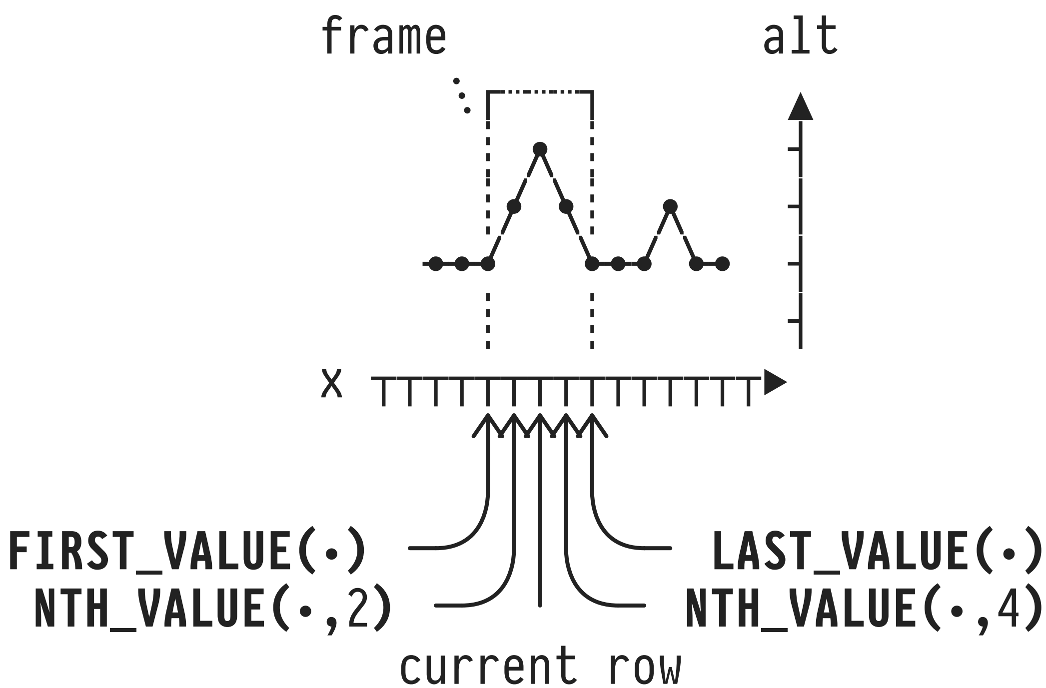
1. Track relative altitude changes in a sliding **x-window of size 5:**



2. **Pattern match** on change strings to detect features.

Altitude Changes in a Sliding Window

- Frame: ROWS BETWEEN 2 PRECEDING AND 2 FOLLOWING (5 rows):



- FIRST_VALUE(alt) < NTH_VALUE(alt,2) ⇒ ascent ('↗').

Altitude Changes in a Sliding Window

```
-- Find slopes in ±2 vicinity around point x
SELECT m.x, slope(
 sign(FIRST_VALUE(m.alt) OVER w-NTH_VALUE(m.alt,2) OVER w),
 sign(NTH_VALUE(m.alt,2) OVER w-m.alt                    ),
 sign(m.alt                    -NTH_VALUE(m.alt,4) OVER w),
 sign(NTH_VALUE(m.alt,4) OVER w-LAST_VALUE(m.alt)  OVER w)
)
FROM   map AS m
WINDOW w AS (ORDER BY m.x ROWS BETWEEN 2 PRECEDING
                                   AND 2 FOLLOWING)
```

- Recall: 1D landscape represented as table `map(x,alt)`.
- UDF encodes altitude changes: `slope(-1,-1,0,1)` ≡ '↗↗→↘'.

Row Pattern Matching (SQL:2016)

SQL:2016 introduced an entirely new SQL construct, **row pattern matching** (MATCH_RECOGNIZE):

1. ORDER BY: Order the rows of a table.
2. DEFINE: Tag rows that satisfy given predicates.
3. PATTERN: Specify a **regular expression over row tags**, find matches in the ordered sequence of rows.
4. MEASURES: For each match, evaluate expressions that measure its features (matched rows, length, …).

⚠️ As of June 2022, not supported by . Currently implemented by Oracle® only.

Row Pattern Matching (SQL:2016)

```
SELECT *
FROM   map
MATCH_RECOGNIZE (
 ORDER BY x
 MEASURES FIRST(x,1)     AS x,
          MATCH_NUMBER() AS feature,
          CLASSIFIER()   AS slope
 ONE ROW PER MATCH
 AFTER MATCH SKIP TO NEXT ROW
 PATTERN (((DOWN DOWN|DOWN EVEN|UP DOWN|EVEN DOWN)…)
 DEFINE UP   AS UP.alt   > PREV(UP.alt),    --
        DOWN AS DOWN.alt < PREV(DOWN.alt),  --  row tags
        EVEN AS EVEN.alt = PREV(EVEN.alt)   --
)
```

Output

x	feature	slope
50	1	DOWN
70	2	UP
100	3	DOWN

8 ┆ Numbering and Ranking Rows

Countless problem scenarios involve the **number** (position) or **rank** of the current row in an *ordered sequence* of rows.

- Family of **window functions to number/rank rows:**

```
ROW_NUMBER()     ⎫
                 ⎪        --  intra-partition ranking ✓
DENSE_RANK()     ⎪        --
RANK()           ⎬  OVER ([ PARTITION BY p₁,…,pₘ ]
                 ⎪         [ ORDER BY e₁,…,eₙ ] )
PERCENT_RANK()   ⎪        --
CUME_DIST()      ⎪        -- ranking w/o ORDER BY ϟ
NTILE(n)         ⎭
```

- Scope is partition (if present)—*frame* is irrelevant.

Numbering and Ranking Rows — *f*() OVER (ORDER BY *A*)

Table W | *f*

row	*A*	ROW_NUMBER	DENSE_RANK	RANK
ϱ1	1	1	1	1
ϱ2	2	2	2	2
ϱ3	**3**	3	3	3
ϱ4	**3**	4	3	3
ϱ5	**3**	5	3	3
ϱ6	4	6	4	6
ϱ7	**6**	7	5	7
ϱ8	**6**	8	5	7
ϱ9	7	9	6	9

Rows that agree on the sort criterion (here: *A*) …

- … number randomly
- … rank equally

Mind the ranking gap (think Olympics)

- In general: DENSE_RANK() ⩽ RANK() ⩽ ROW_NUMBER()

Once More: Find the Top *n* Rows in a Group

species	length	height	legs
⋮	⋮	⋮	∈ {2,4,NULL}

Table dinosaurs

```
  SELECT tallest.legs, tallest.species, tallest.height
  FROM (SELECT d.legs, d.species, d.height,
ROW_NUMBER()····RANK() OVER (PARTITION BY d.legs
                             ORDER BY d.height DESC) AS n
        FROM  dinosaurs AS d
        WHERE d.legs IS NOT NULL) AS tallest
  WHERE tallest.n <= 3
```

- RANK() *vs* ROW_NUMBER(): both OK, but different semantics!
- Need a subquery: window functions *not* allowed in WHERE.

Identify Consecutive Ranges

- What you often encounter in scientific papers 😩:
  "... *as Knuth has shown in [5,2,14,3,1,42,6,10,7,13]* ..."

- What you want to see 😀:
  "... *as Knuth has shown in [1–3,5–7,10,13&14,42]* ..."

Table **citations**

ref
5
2
⋮
13

Output

ref	range
1	r_0
2	r_0
⋮	⋮
42	r_4

references belong to the same range

Identify Consecutive Ranges (Query Plan)

1 ref	2 ref		ROW_NUMBER()			
5	1	-	1	=	0	
2	2	-	2	=	0	range 0 ≡ r_0
14	3	-	3	=	0	
3	5	-	4	=	1	
1	6	-	5	=	1	range 1 ≡ r_1
42	7	-	6	=	1	
6	10	-	7	=	3	range 3 ≡ r_2
10	13	-	8	=	5	range 5 ≡ r_3
7	14	-	9	=	5	
13	42	-	10	=	32	range 32 ≡ r_4

subtract

Numbering and Ranking Rows — *f* OVER (ORDER BY *A*)

row	***A***	**PERCENT_RANK**	**CUME_DIST**	**NTILE(3)**
ϱ1	1	0	$^1/_9$	1
ϱ2	2	$^1/_8$	$^2/_9$	1
ϱ3	**3**	$^2/_8$	$^5/_9$	1
ϱ4	**3**	$^2/_8$	$^5/_9$	2
ϱ5	**3**	$^2/_8$	$^5/_9$	2
ϱ6	4	$^5/_8$	$^6/_9$	2
ϱ7	**6**	$^6/_8$	$^8/_9$	3
ϱ8	**6**	$^6/_8$	$^8/_9$	3
ϱ9	7	$^8/_8$	$^9/_9$	3

Rows that agree on the sort criterion (here: ***A***) rank equally

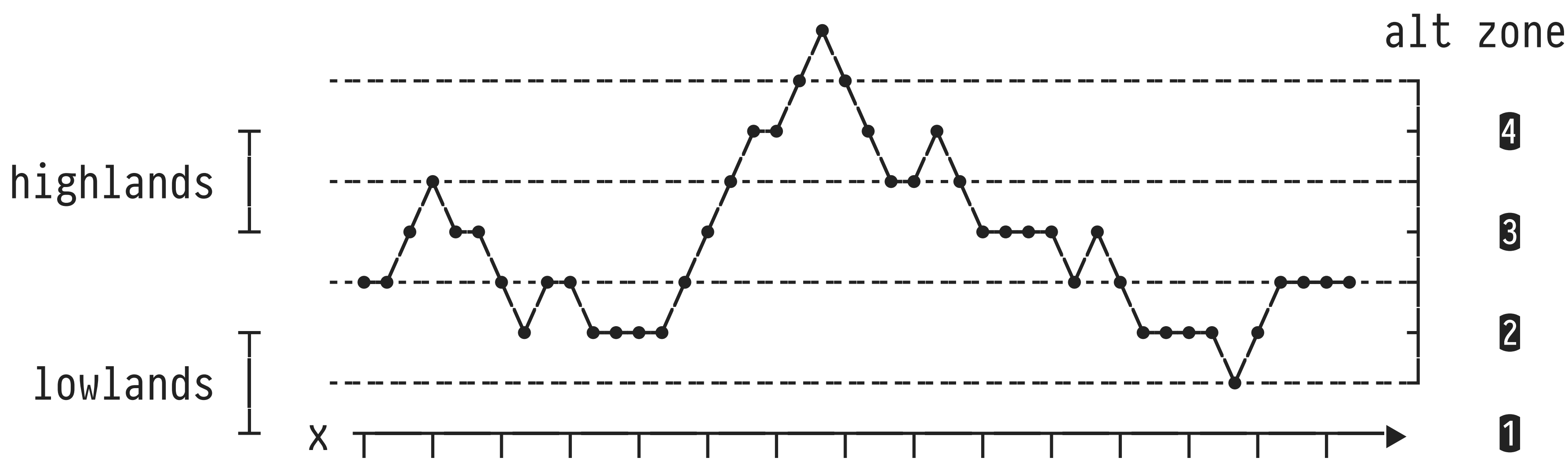
current row is in the n^{th} of 3 chunks of rows

n% of the other rows rank lower than the current row

the current row and lower ranked rows make up *n*% of all rows

Altitudinal Mountain Zones

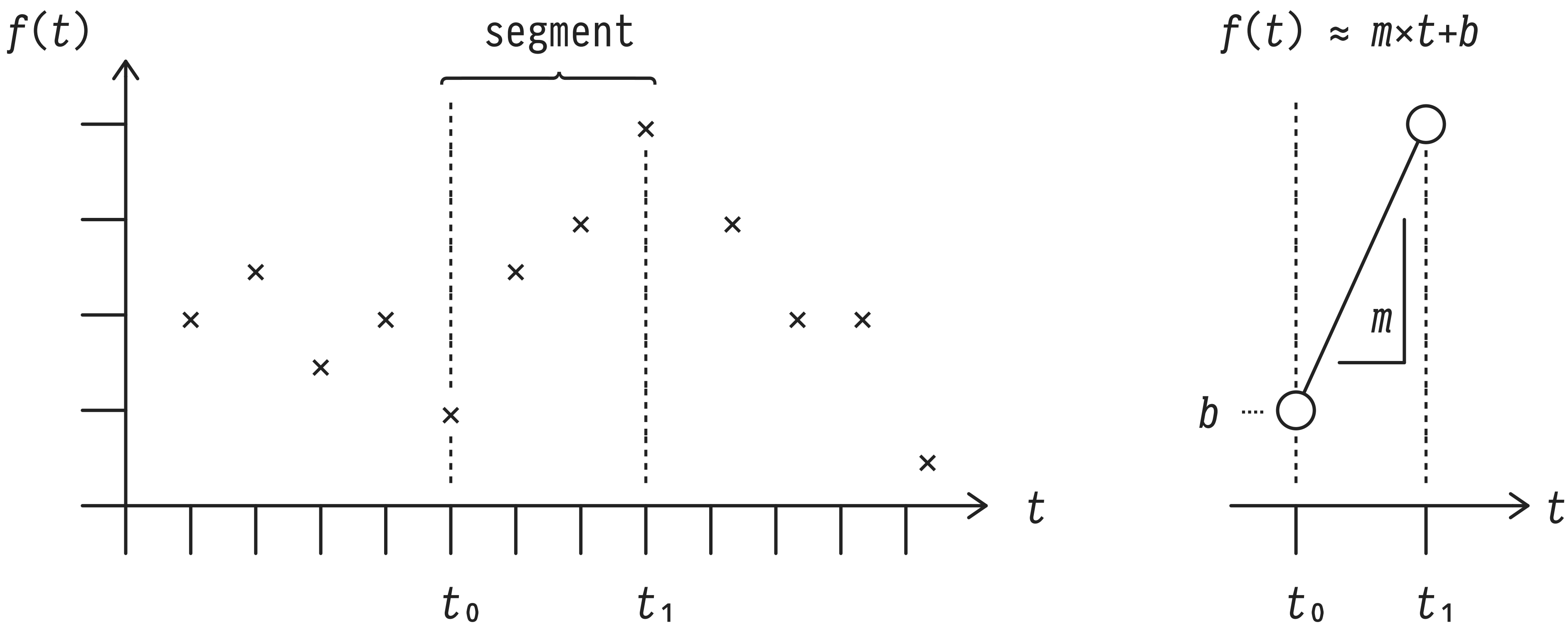
- Classify the altitudes of a mountain range into
  1. **equal-sized** vegetation **zones** and
  2. lowlands (altitude in the lowest **20%**) and highlands (between **60%–80%** of maximum altitude).



Altitudinal Mountain Zones

```sql
-- Classify altitudinal zones in table mountains(x, alt)
--
SELECT
  m.x, m.alt,
  NTILE(4) OVER altitude AS zone,
  CASE
    WHEN PERCENT_RANK() OVER altitude BETWEEN 0.6 AND 0.8
         THEN 'highlands'
    WHEN PERCENT_RANK() OVER altitude < 0.2
         THEN 'lowlands'
         ELSE '-'
  END AS region
FROM   mountains AS m
WINDOW altitude AS (ORDER BY m.alt)
ORDER BY m.x;
```

Linear Approximation of a Time Series



1. NTILE(n) **segments** time series at desired granularity.
2. Compute m, b in each **segment ≡ window frame**.

9 Summary: Window Function Semantics[2]

Scope	Computation	Function	Description
frame	aggregation	(aggregates)	SUM, AVG, MAX, array_agg, …
	row access	FIRST_VALUE(*e*)	*e* at first row in frame
		LAST_VALUE(*e*)	*e* at last row in frame
		NTH_VALUE(*e*,*n*)	*e* at n^{th} row in frame
partition	row access	LAG(*e*,*n*,*d*)	*e* at *n* rows *before* current row
		LEAD(*e*,*n*,*d*)	*e* at *n* rows *after* current row
	ranking	ROW_NUMBER()	number of current row
		RANK()	rank with gaps (“Olympics”)
		DENSE_RANK()	rank without gaps
		PERCENT_RANK()	relative rank of current row
		CUME_DIST()	ratio of rows up to –"–
		NTILE(*n*)	rank on a scale {1,2,…,*n*}

[2] FIRST_VALUE(*e*): expression *e* will be evaluated *as if* we are at the first row in the frame.
LAG(*e*,*n*,*d*): default expression *d* is returned if there is no row at offset *n* before the current row.