Question:

点$P(x,y)$是椭圆$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b>0)$上的任意一点，$F_1,F_2$是椭圆的两个焦点，且$\angle F_1PF_2\le 90^\circ$，则该椭圆的离心率的取值范围是?

(Let $P(x,y)$ be an arbitrary point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b>0)$. F_1 and F_2 are the two foci of the ellipse, and $\angle F_1PF_2\le 90^\circ$. What is the range of values for the eccentricity of the ellipse?)

Rationale:

由题意可知，当点P位于$(0,b)$或$(0,-b)$处时，$\angle F_1PF_2=90^\circ$最大，此时$\cos\angle F_1PF_2=\frac{a^2+a^2-4c^2}{2a^2}=\frac{a^2-2c^2}{a^2}\ge 0, a\ge\sqrt{2}c$。因为$e=c/a$，所以$e\le\frac{\sqrt{2}}{2}$。因为$e$是椭圆离心率，$0<e<1$，所以$0<e\le\frac{\sqrt{2}}{2}$。

(When the point P is located at $(0,b)$ or $(0,-b)$, the angle $\angle F_1PF_2\le 90^\circ$ is at its maximum. In this case, $\cos\angle F_1PF_2=\frac{a^2+a^2-4c^2}{2a^2}=\frac{a^2-2c^2}{a^2}\ge 0, a\ge\sqrt{2}c$. Since $e=\frac{c}{a}$, we have $e\le\frac{\sqrt{2}}{2}$. As e represents the eccentricity of the ellipse, and it lies within the range $0<e<1$, we can conclude that $0<e\le\frac{\sqrt{2}}{2}$.)

Answer:

$(0,\frac{\sqrt{2}}{2}]$

Formal Representation:	Span:
`P: Point`	点$P(x,y)$
`PointOnCurve(P, G)=True`	点$P(x,y)$是椭圆…上的任意一点
`Coordinate(P)=(x1, y1)`	$P(x,y)$
`x1,y1: Number`	$P(x,y)$
`G: Ellipse`	椭圆$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b>0)$
`Expression(G)=(y^2/b^2+x^2/a^2=1)`	椭圆$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b>0)$
`a, b: Number`	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b>0)$
`a > b`	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b>0)$
`b > 0`	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b>0)$
`F1, F2: Point`	F_1,F_2
`Focus(G)={F1, F2}`	F_1,F_2是椭圆两个焦点
`AngleOf(F1,P,F2)<=Unit(90,degree)`	$\angle F_1PF_2\le 90^\circ$
`Range(Eccentricity(G))=?`	该椭圆的离心率的取值范围是?