SVM

In[∘]:=
```
ClearAll[X]; ClearAll[Y]; ClearAll[BigMatrix];
```

In[∘]:= X =
$$\begin{pmatrix} 1.0 & 1 \\ 2 & 1 \\ 2 & 2 \\ 4 & 1 \\ 4 & 4 \end{pmatrix};$$

In[∘]:= Y =
$$\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{pmatrix};$$

Calculus

In[∘]:=
```
BigMatrix = Table[Y⟦i⟧⟦1⟧ * Y⟦j⟧⟦1⟧ * (X⟦i⟧.X⟦j⟧), {i, 1, Length[X]}, {j, 1, Length[X]}];
```

In[∘]:=
```
BigMatrix // TableForm
```

Out[∘]//TableForm=
```
2.     3.     -4.    -5.    -8.
3.     5      -6     -9     -12
-4.    -6     8      10     16
-5.    -9     10     17     20
-8.    -12    16     20     32
```

In[∘]:=
```
ClearAll[alphas];
ClearAll[W];
ClearAll[TmpW];
ClearAll[rules];
ClearAll[W0];
ClearAll[svPos]
```

In[∘]:=
```
alphas = Table[Symbol["alpha" ~~ ToString[i]], {i, 1, Length[X]}];
```

In[∘]:=
```
rules = Minimize[{1 / 2 * alphas.BigMatrix.alphas +
      Table[-1, {i, 1, Length[X]}].alphas, Transpose[Y].alphas == 0 &&
      (Table[alph ≥ 0, {alph, alphas}] /. (List → And))}, alphas]⟦2⟧
```

Out[∘]= $\{\text{alpha1} \to -1.11022 \times 10^{-16}, \text{alpha2} \to 2.5, \text{alpha3} \to 2., \text{alpha4} \to 0.5, \text{alpha5} \to 1.4363 \times 10^{-24}\}$

In[∘]:=
```
TmpW = Sum[(alphas⟦n⟧ /. rules) * Y⟦n⟧⟦1⟧ * X⟦n⟧, {n, 1, Length[X]}]
```

Out[∘]=
```
{-1., -2.}
```

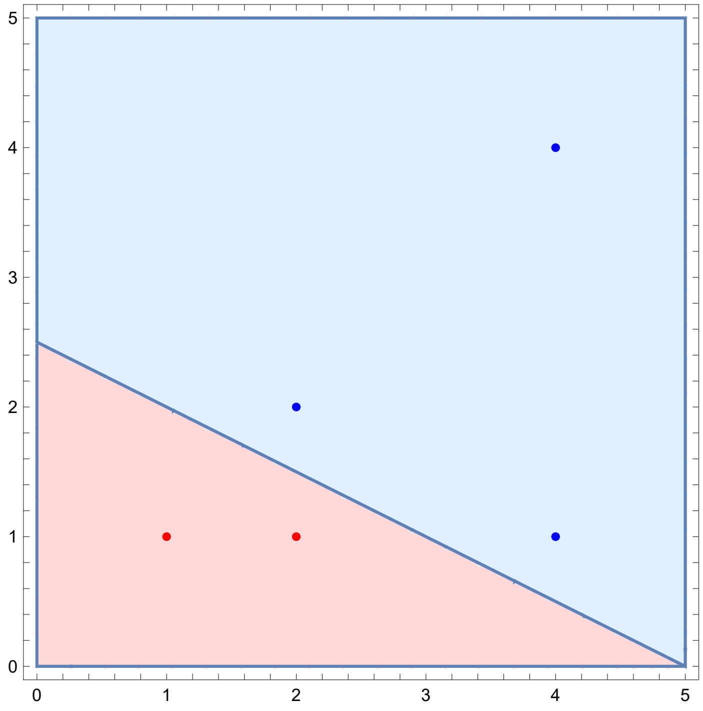
In[∘]:=
```
svPos = FirstPosition[rules, alpha_Rule /; alpha⟦2⟧ > 0.000001]⟦1⟧;
```

```
In[∘]:= W0 = 1 / Y⟦svPos⟧⟦1⟧ - Sum[TmpW⟦i⟧ * X⟦svPos⟧⟦i⟧, {i, 1, Length[X⟦svPos⟧]}]
Out[∘]= 5.

In[∘]:= W = Prepend[TmpW, W0]
Out[∘]= {5., -1., -2.}
```

Result

```
In[∘]:= Show[RegionPlot[Sign[W.{1, x, y}] > 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → LightRed],
         RegionPlot[Sign[W.{1, x, y}] ≤ 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → LightBlue],
         ListPlot[Select[X, Y⟦FirstPosition[X, #]⟦1⟧⟧⟦1⟧ == 1 &], PlotStyle → Red],
         ListPlot[Select[X, Y⟦FirstPosition[X, #]⟦1⟧⟧⟦1⟧ == -1 &], PlotStyle → Blue]]
```

Out[∘]=



Kernel Trick

```
In[∘]:= ClearAll[X]; ClearAll[Y]; ClearAll[BigMatrix]; ClearAll[radialKernel]
```

In[∘]:= X = $\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 2 & 2 \\ 2 & 0.5 \\ 4 & 4 \\ 0 & 0 \end{pmatrix}$;

```
In[∘]:=      ( 1 )
             ( 1 )
        Y =  ( 1 ) ;
             (-1 )
             (-1 )
             (-1 )
```

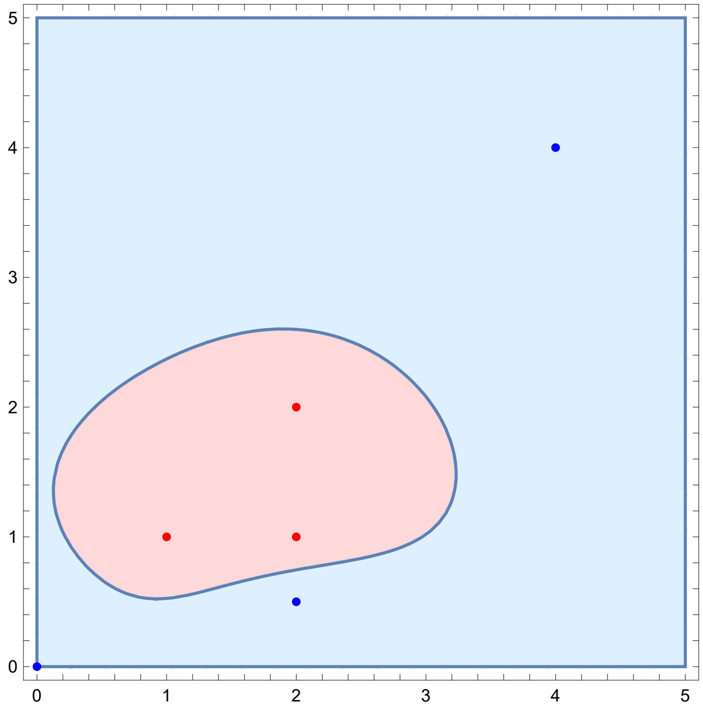
Calculus

```
In[∘]:= radialKernel[xn_, xm_] := Exp[-xn.xn] * Exp[-xm.xm] * Exp[2 xn.xm] // N

In[∘]:= BigMatrix =
          Table[Y⟦i⟧⟦1⟧ * Y⟦j⟧⟦1⟧ * radialKernel[X⟦i⟧, X⟦j⟧], {i, 1, Length[X]}, {j, 1, Length[X]}];

In[∘]:= BigMatrix // TableForm
Out[∘]//TableForm=
        1.               0.367879          0.135335          -0.286505         -1.523 × 10^-8      -0.1353
        0.367879         1.                0.367879          -0.778801         -2.26033 × 10^-6    -0.0067
        0.135335         0.367879          1.                -0.105399         -0.000335463        -0.0003
        -0.286505        -0.778801         -0.105399         1.                8.76425 × 10^-8     0.01426
        -1.523 × 10^-8   -2.26033 × 10^-6  -0.000335463      8.76425 × 10^-8   1.                  1.26642
        -0.135335        -0.00673795       -0.000335463      0.0142642         1.26642 × 10^-14    1.

In[∘]:= ClearAll[alphas];
        ClearAll[rules];
        ClearAll[W0];
        ClearAll[svPos];
        ClearAll[model];

In[∘]:= alphas = Table[Symbol["alpha" ~~ ToString[i]], {i, 1, Length[X]}];

In[∘]:= rules = Minimize[{1 / 2 * alphas.BigMatrix.alphas +
              Table[-1, {i, 1, Length[X]}].alphas, Transpose[Y].alphas == 0 &&
              (Table[alph ≥ 0, {alph, alphas}] /. (List → And))}, alphas]⟦2⟧

Out[∘]= {alpha1 → 1.14086, alpha2 → 4.21583, alpha3 → 0.149222,
         alpha4 → 4.20481, alpha5 → 0.589142, alpha6 → 0.711958}

In[∘]:= svPos = FirstPosition[rules, alpha_Rule /; alpha⟦2⟧ > 0.000001]⟦1⟧;

In[∘]:= W0 = 1 / Y⟦svPos⟧⟦1⟧ -
          Sum[(alphas⟦k⟧ /. rules) * Y⟦k⟧⟦1⟧ * radialKernel[X⟦k⟧, X⟦svPos⟧], {k, 1, Length[X]}]

Out[∘]= -0.410917

In[∘]:= model[input_] :=
         Sum[(alphas⟦k⟧ /. rules) * Y⟦k⟧⟦1⟧ * radialKernel[X⟦k⟧, input], {k, 1, Length[X]}] + W0
```

Result

```
In[∘]:= Show[RegionPlot[model[{x, y}] > 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → LightRed],
         RegionPlot[model[{x, y}] < 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → LightBlue],
         ListPlot[Select[X, Y⟦FirstPosition[X, #][[1]]⟧[[1]] == 1 &], PlotStyle → Red],
         ListPlot[Select[X, Y⟦FirstPosition[X, #][[1]]⟧[[1]] == -1 &], PlotStyle → Blue]]
```

Out[∘]=



Kernel Trick With Lots of samples

```
In[∘]:= ClearAll[X]; ClearAll[Y]; ClearAll[BigMatrix]; ClearAll[radialKernel]

In[∘]:= X = Table[RandomReal[{0.5, 4.5}], {i, 1, 20}, {j, 1, 2}];

In[∘]:= Y = Table[RandomChoice[{-1, 1}], {i, 1, 20}, {j, 1, 1}];
```

Calculus

```
In[∘]:= radialKernel[xn_, xm_] := Exp[-xn.xn] * Exp[-xm.xm] * Exp[2 xn.xm] // N

In[∘]:= BigMatrix =
          Table[Y⟦i⟧[[1]] * Y⟦j⟧[[1]] * radialKernel[X⟦i⟧, X⟦j⟧], {i, 1, Length[X]}, {j, 1, Length[X]}];
```

```
In[∘]:= BigMatrix // TableForm
Out[∘]//TableForm=
        1.                  -0.129367         0.250647          -0.00197724       -0.109743          0.9465
        -0.129367           1.                -0.0175152        0.0624299         0.00597073         -0.155
        0.250647            -0.0175152        1.                -0.00858707       -0.907575          0.1379
        -0.00197724         0.0624299         -0.00858707       1.                0.00695176         -0.001
        -0.109743           0.00597073        -0.907575         0.00695176        1.                 -0.052
        0.946505            -0.155876         0.137931          -0.00129047       -0.0522123         1.
        -6.55965 × 10^-7    0.00157998        -1.36675 × 10^-6  0.100468          8.30922 × 10^-7    -5.615
        -0.819643           0.0988668         -0.0741276        0.000357619       0.0249782          -0.938
        0.00875344          -0.498264         0.000304541       -0.0144171        -0.000069892       0.0144
        -6.69307 × 10^-6    0.0117004         -4.7169 × 10^-8   0.000248523       6.63883 × 10^-9    -0.000
        -0.00490606         0.219066          -0.0060428        0.750164          0.00350508         -0.004
        0.000491543         -0.0519502        1.5458 × 10^-6    -0.000077429      -1.88006 × 10^-7   0.0013
        0.0453244           -0.28785          0.0830941         -0.581458         -0.0557882         0.0331
        -0.000363103        0.0811418         -2.53972 × 10^-6  0.000548864       3.73489 × 10^-7    -0.000
        -0.00247816         0.0023395         -2.01291 × 10^-6  4.01885 × 10^-8   1.91675 × 10^-7    -0.007
        9.74367 × 10^-7     -0.00172175       2.71074 × 10^-6   -0.137385         -1.78815 × 10^-6   7.8229
        0.00187849          -0.126998         0.0000109149      -0.000344341      -1.57132 × 10^-6   0.0043
        -5.85153 × 10^-6    0.0133575         -8.38814 × 10^-8  0.000873046       1.40997 × 10^-8    -0.000
        -0.720194           0.0501544         -0.0581055        0.000114539       0.0192442          -0.828
        -1.30502 × 10^-6    0.00228418        -2.03428 × 10^-9  6.09065 × 10^-6   1.93863 × 10^-10   -4.317
```

```
In[∘]:= ClearAll[alphas];
        ClearAll[rules];
        ClearAll[W0];
        ClearAll[svPos];
        ClearAll[model];
```

```
In[∘]:= alphas = Table[Symbol["alpha" ~~ ToString[i]], {i, 1, Length[X]}];
```

```
In[∘]:= rules = Minimize[{1 / 2 * alphas.BigMatrix.alphas +
             Table[-1, {i, 1, Length[X]}].alphas, Transpose[Y].alphas == 0 &&
             (Table[alph ≥ 0, {alph, alphas}] /. (List → And))}, alphas]⟦2⟧
```

```
Out[∘]= {alpha1 → 1.12858 × 10^-9, alpha2 → 5.55214, alpha3 → 6.78379, alpha4 → 4.35518,
         alpha5 → 7.22967, alpha6 → 16.2192, alpha7 → 47.5887, alpha8 → 15.5059, alpha9 → 6.92404,
         alpha10 → 3.1236 × 10^-9, alpha11 → 1.19368, alpha12 → 6.85907, alpha13 → 5.46551,
         alpha14 → 9.22159, alpha15 → 0.418223, alpha16 → 48.8134, alpha17 → 5.38197 × 10^-9,
         alpha18 → 1.50079 × 10^-8, alpha19 → 7.5261 × 10^-9, alpha20 → 8.4002 × 10^-9}
```

```
In[∘]:= svPos = FirstPosition[rules, alpha_Rule /; alpha⟦2⟧ > 0.000001]⟦1⟧;
```

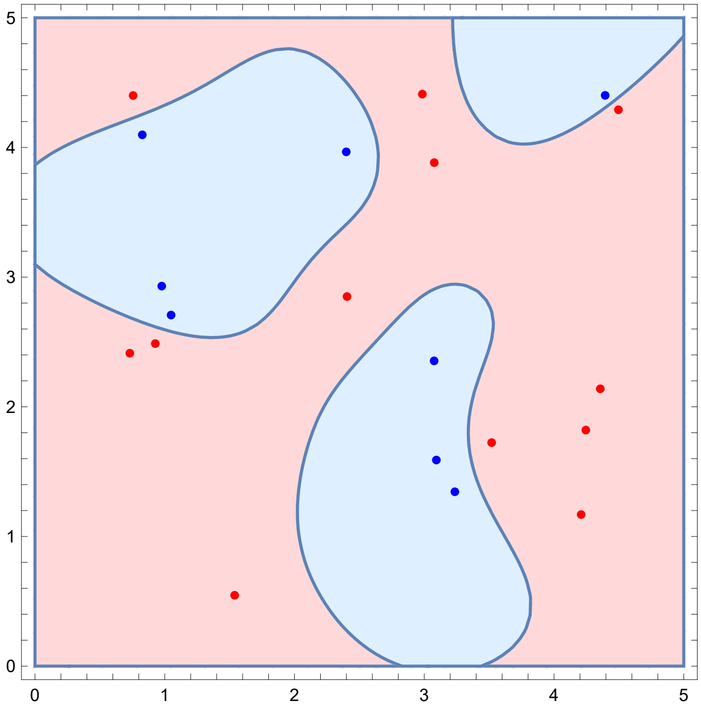
```
In[∘]:= W0 = 1 / Y⟦svPos⟧⟦1⟧ -
          Sum[(alphas⟦k⟧ /. rules) * Y⟦k⟧⟦1⟧ * radialKernel[X⟦k⟧, X⟦svPos⟧], {k, 1, Length[X]}]
Out[∘]= 0.624492
```

```
In[∘]:= model[input_] :=
         Sum[(alphas⟦k⟧ /. rules) * Y⟦k⟧⟦1⟧ * radialKernel[X⟦k⟧, input], {k, 1, Length[X]}] + W0
```

Result

```
In[∘]:= Show[RegionPlot[model[{x, y}] > 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → LightRed],
         RegionPlot[model[{x, y}] < 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → LightBlue],
         ListPlot[Select[X, Y⟦FirstPosition[X, #]⟦1⟧⟧⟦1⟧ == 1 &], PlotStyle → Red],
         ListPlot[Select[X, Y⟦FirstPosition[X, #]⟦1⟧⟧⟦1⟧ == -1 &], PlotStyle → Blue]]
```

Out[∘]=



SVM With Soft Margin

```
ClearAll[X]; ClearAll[Y]; ClearAll[BigMatrix];
```

$$X = \begin{pmatrix} 2 & 1 \\ 1.0 & 3 \\ 2 & 0 \\ 4 & 1 \\ 4 & 4 \\ 4 & 3 \\ 1 & 2 \end{pmatrix};$$

Y = $\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$;

Calculus

```
BigMatrix = Table[Y⟦i⟧⟦1⟧ * Y⟦j⟧⟦1⟧ * (X⟦i⟧.X⟦j⟧), {i, 1, Length[X]}, {j, 1, Length[X]}];
```

```
BigMatrix // TableForm
```

```
5      5.      4     -9     -12     -11     -4
5.     10.     2.    -7.    -16.    -13.    -7.
4      2.      4     -8     -8      -8      -2
-9     -7.     -8    17     20      19      6
-12    -16.    -8    20     32      28      12
-11    -13.    -8    19     28      25      10
-4     -7.     -2    6      12      10      5
```

```
ClearAll[alphas];
ClearAll[W];
ClearAll[TmpW];
ClearAll[rules];
ClearAll[W0];
ClearAll[svPos]
```

```
alphas = Table[Symbol["alpha" ~~ ToString[i]], {i, 1, Length[X]}];
```

```
rules = Maximize[{-1 / 2 * alphas.BigMatrix.alphas - Table[-1, {i, 1, Length[X]}].alphas,
    Transpose[Y].alphas == 0 && (Table[alph ≥ 0, {alph, alphas}] /. (List → And)) &&
     (Table[alph ≤ 500, {alph, alphas}] /. (List → And))}, alphas]⟦2⟧
```

```
{alpha1 → 375.875, alpha2 → 249.75, alpha3 → 0.,
 alpha4 → 125.625, alpha5 → 0., alpha6 → 0., alpha7 → 500.}
```

```
TmpW = Sum[(alphas⟦n⟧ /. rules) * Y⟦n⟧⟦1⟧ * X⟦n⟧, {n, 1, Length[X]}]
```

```
{-1.00046, -0.5002}
```

```
svPos = FirstPosition[rules, alpha_Rule /; alpha⟦2⟧ > 0.000001]⟦1⟧;
```

```
W0 = 1 / Y⟦svPos⟧⟦1⟧ - Sum[TmpW⟦i⟧ * X⟦svPos⟧⟦i⟧, {i, 1, Length[X⟦svPos⟧]}]
```

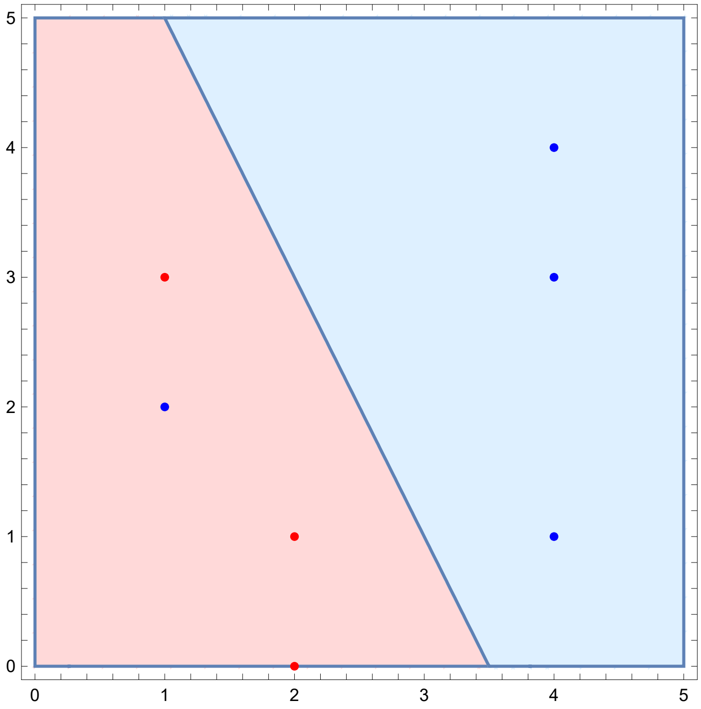
```
3.50112
```

```
W = Prepend[TmpW, W0]
```

```
{3.50112, -1.00046, -0.5002}
```

Result

```
Show[RegionPlot[Sign[W.{1, x, y}] > 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → LightRed],
 RegionPlot[Sign[W.{1, x, y}] ≤ 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → LightBlue],
 ListPlot[Select[X, Y⟦FirstPosition[X, #][[1]]⟧[[1]] == 1 &], PlotStyle → Red],
 ListPlot[Select[X, Y⟦FirstPosition[X, #][[1]]⟧[[1]] == -1 &], PlotStyle → Blue]]
```


0
1
2
3
4
5
0
1
2
3
4
5