Parcial 1- Señales y sistemas

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Punto 1:

$$x(t) = |6\,\text{sen}(3t + \frac{\pi}{4})|^2$$

Resolvemos el cuadrado de la funcion para obtener una expresion mas facil de trabajar.

Usamos la propiedad $\text{sen}(a+b) = \text{sen}(a)\cos(b) + \cos(a)\,\text{sen}(b)$

$$|6(\text{sen}(3t)\underbrace{\cos(\tfrac{\pi}{4})}_{\frac{\sqrt{2}}{2}} + \cos(3t)\underbrace{\text{sen}(\tfrac{\pi}{4})}_{\frac{\sqrt{2}}{2}})|^2$$

$$|\cancel{6}\left[\frac{\sqrt{2}\,\text{sen}(3t) + \sqrt{2}\cos(3t)}{\cancel{2}}\right]|^2$$

$$|3\sqrt{2}\,\text{sen}(3t) + 3\sqrt{2}\cos(3t)|^2$$

$$18\,\text{sen}(3t)^2 + 36\,\text{sen}(3t)\cos(3t) + 18\cos(3t)^2$$

$$18 + 18\,\text{sen}(6t)$$

Por serie trigonometrica tenemos que

$$x(t) = a_0 + \sum_{n=-N}^{N} a_n \cos(n w_0 t) + a_n \sin(n w_0 t)$$

$$a_0 = C_0 = \frac{1}{t_f - t_i}\int_{t_i}^{t_f} x(t)\,dt \quad y$$

$$a_n = \frac{2}{t_f - t_i} \int_{t_i}^{t_f} x(t) \cos(n w_0 t)\, dt$$

$$b_n = \frac{2}{t_f - t_i} \int_{t_i}^{t_f} x(t) \operatorname{sen}(n w_0 t)\, dt$$

Determinamos si es una funcion par o impar, para asi poder evitar calculos innesesarios de ser posible

si $f(-x) = f(x)$ es par y si $f(-x) = -f(x)$ es impar en caso contrario no es ninguno de los 2 y debemos hacer el proceso completo

$f(x) = 18 + 18 \operatorname{sen}(6t)$

$f(-x) = 18 + 18 \operatorname{sen}(-6t) \Rightarrow 18 - 18 \operatorname{sen}(6t)$

$-f(x) = -(18 + 18 \operatorname{sen}(6t) \Rightarrow -18 - 18 \operatorname{sen}(6t)$

$f(x) \neq f(-x)$ y $f(-x) \neq -f(x)$

por ende no es par ni impar.

Procedemos a calcular los valores para la funcion

$$\left|6 \operatorname{sen}(3t + \frac{\pi}{4})\right|^2 = 18 + 18 \operatorname{sen}(6t)$$

$$= a_0 + \sum_{n=N}^{N} a_n \cos(n w_0 t) + b_n \operatorname{sen}(n w_0 t)$$

$$a_0 = \frac{1}{\pi - (-\pi)} \int_{-\pi}^{\pi} 18 + 18 \operatorname{sen}(6t)\, dt$$

$$a_0 = \frac{18}{2\pi}\int_{-\pi}^{\pi} dt - \frac{18}{2\pi}\int_{-\pi}^{\pi} sen(6t)\, dt$$

$$= \frac{18}{2\pi} t \Big|_{-\pi}^{\pi} + \frac{18 \cos(6t)}{12\pi}\Big|_{-\pi}^{\pi}$$

$$\left[\frac{18\pi}{2\pi} + \frac{18\pi}{2\pi}\right] + \left[\cancel{\frac{18\cos(6\pi)}{12\pi}} - \cancel{\frac{18\cos(6\pi)}{12}}\right]^{0}$$

$$a_0 = 18$$

Prodecedos a encontrar a_n

$$a_n = \frac{2}{\pi-(-\pi)}\int_{-\pi}^{\pi}[18 + 18\, sen(6t)][\cos n w_0 t\, dt$$

$$\frac{2}{2\pi}\int_{-\pi}^{\pi} 18\cos(nw_0t) + 18\, sen(6t)\cos(nw_0t)\, dt$$

$$18\pi\int_{-\pi}^{\pi}\cos(nw_0t) + 18\pi\int_{-\pi}^{\pi} sen(6t)\cos(nw_0t)\, dt$$

teniendo en cuenta que $w_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$, reemplazamos

$$18\pi\int_{-\pi}^{\pi}\cos(nt) + 18\pi\int_{-\pi}^{\pi} sen(6t)\cos(nt)\, dt$$

Aplicamos $sen(a)\cdot\cos(B) = \frac{1}{2}\, sen(a+b) + sen(a-B)$

① $18\pi\int_{-\pi}^{\pi}\cos(nt) +$ ② $\frac{18\pi}{2}\int_{-\pi}^{\pi} sen((6+n)t) +$ ③ $\frac{18\pi}{2}\int_{-\pi}^{\pi} sen((6-n)t)$

Evaluamos cada trozo de la integral a parte para mayor facilidad:

1) $\frac{18 \operatorname{sen}(nt)}{n}\Big|_{-\pi}^{\pi} \rightarrow \frac{36\pi \operatorname{sen}(\pi n)}{\pi n}$ ①

② $\frac{18\pi \cos((6+n)t)}{12+2n}\Big|_{-\pi}^{\pi} = 0$

③ $\frac{18 \cos((6-n)t)}{2(6-n)}\Big|_{-\pi}^{\pi} = 0$

Entonces tenemos que

$$\boxed{a_0 = \frac{36\pi \operatorname{sen}(\pi n)}{\pi n}}$$

Encontramos el b_n

$$b_n = \frac{2}{T}\int_{-\pi}^{\pi} [18 + 18 \operatorname{sen}(6t)][\operatorname{sen}(n w_0 t)\, dt$$

$$\frac{2}{2\pi}\int_{-\pi}^{\pi} 18 \operatorname{sen}(n w_0 t) + 18 \operatorname{sen}(6t) \operatorname{sen}(n w_0 t)\, dt$$

Con $w_0 = 1$ tenemos

$$\frac{2}{2\pi}\int_{-\pi}^{\pi} 18 \operatorname{sen}(nt) + 18 \operatorname{sen}(6t) \operatorname{sen}(nt)\, dt$$

Resolviendo la integral anterior nos queda so

$$b_n = \frac{18(6+n)\,\text{sen}(\pi n) + 18(6-n)\,\text{sen}(\pi n)}{\pi(36-n^2)}$$

deacuerdo a la expresion anterior, al tener $n=6$ obtenemos una indeterminacion, por ello aplicamos L'Hopital.

$$\lim_{n\to 6} \left(\frac{\frac{d}{dn}\left(18(6+n)\,\text{sen}(\pi n) + 18(6-n)\,\text{sen}(\pi n)\right.}{\frac{d}{dn}\quad \pi(36-n^2)} \right.$$

$$\lim_{n\to 6} \frac{18\cos(\pi n)(6+n) + 18\pi\cos(\pi n)(6-n)}{-2\pi n} = -18$$

Entonces, para la serie exponencial compleja tenemos que:

$$C_0 = a_0 = 18 \quad y \quad C_n = \frac{a_n - j b_n}{2}$$

$$C_6 = \frac{0-(-18)}{2} = 9$$

entonces

$$C_n = \begin{cases} 18 & n=0 \\ 9 & n=\{6,-6\} \\ 0 & \forall n \in \{0,6,-6\} \end{cases}$$

$$x(t) = C_6 e^{-j6t} + C_0 e^{0} + C_6 e^{j6t}$$

$$= 9\left(\cos(6t) - j\,\text{sen}(6t)\right) + 18 - 9\left(\cos(6t) - j\,\text{sen}(6t)\right)$$

$$= 18 + 18\,\text{sen}(6t)$$

Procedemos a calcular el error relativo

$$E_r = \left[1 - \frac{1}{P_x} \sum_{n=-N}^{N} |C_n|^2 \right] 100\%$$

Calculamos la potencia de la señal.

$$P_x = \frac{1}{2\pi} \int_{-\pi}^{\pi} |18 + 18\,\mathrm{sen}(6t)|^2 \, dt$$

$$P_x = 486$$

$$E_r[\%] = \left[1 - \frac{|C_{-6}|^2 + |C_0|^2 + |C_6|^2}{P_x} \right] 100\%$$

$$\left[1 - \frac{81 + 324 + 81}{486} \right] 100\% \rightarrow \left[1 - \frac{486}{486} \right] 100\%$$

$$\boxed{E_r[\%] = 0\%}$$