

Parcial 1- Señales y sistemas

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Punto 1:

$$x(t) = |6 \sin(3t + \frac{\pi}{4})|^2$$

Resolvemos el cuadrado de la función para obtener una expresión más fácil de trabajar.

Usamos la propiedad $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

$$|6(\underbrace{\sin(3t)}_{\frac{\sqrt{2}}{2}} \underbrace{\cos(\frac{\pi}{4})}_{\frac{\sqrt{2}}{2}} + \cos(3t) \underbrace{\sin(\frac{\pi}{4})}_{\frac{\sqrt{2}}{2}})|^2$$

$$|6[\sqrt{2} \sin(3t) + \sqrt{2} \cos(3t)]|^2$$

$$|3\sqrt{2} \sin(3t) + 3\sqrt{2} \cos(3t)|^2$$

$$18 \sin^2(3t) + 36 \sin(3t)\cos(3t) + 18 \cos^2(3t)$$
$$18 + 18 \sin(6t)$$

Por serie trigonométrica tenemos que:

$$x(t) = a_0 + \sum_{n=-N}^N a_n \cos(n\omega_0 t) + a_n \sin(n\omega_0 t)$$

$$a_0 = c_0 = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} x(t) dt \quad y$$

$$a_n = \frac{2}{t_f - t_i} \int_{t_i}^{t_f} x(t) \cos(n\omega_0 t) dt$$

$$b_n = \frac{2}{t_f - t_i} \int_{t_i}^{t_f} x(t) \sin(n\omega_0 t) dt$$

Determinamos si es una función par o impar, para así poder evitar cálculos innecesarios de ser posible

Si $f(x) = f(-x)$ es par y si $f(-x) = -f(x)$ es impar en caso contrario no es: ∴ ninguno de los 2 y debemos hacer el proceso completo

$$f(x) = 18 + 18 \sin(6t)$$

$$f(-x) = 18 + 18 \sin(-6t) \rightarrow 18 - 18 \sin(6t)$$

$$-f(x) = -(18 + 18 \sin(6t)) \rightarrow -18 - 18 \sin(6t)$$

$$f(x) \neq f(-x) \text{ y } f(-x) \neq -f(x)$$

por ende no es par ni impar.

Procedemos a calcular los valores para la función

$$\left| 6 \sin\left(3t + \frac{\pi}{4}\right) \right|^2 = 18 + 18 \sin(6t)$$

$$= a_0 + \sum_{n=1}^N a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} 18 + 18 \sin(6t) dt$$

$$a_0 = \frac{18}{2\pi} \int_{-\pi}^{\pi} dt - \frac{18}{2\pi} \int_{-\pi}^{\pi} \sin(6t) dt$$

$$= \frac{18}{2\pi} t \Big|_{-\pi}^{\pi} + \frac{18 \cos(6t)}{12\pi} \Big|_{-\pi}^{\pi}$$

$$\left[\frac{18\pi}{2\pi} + \frac{18\pi}{2\pi} \right] + \left[\frac{18 \cos(6\pi)}{12\pi} - \frac{18 \cos(6(-\pi))}{12\pi} \right]$$

$$a_0 = 18$$

Procedemos a encontrar a_n

$$a_n = \frac{2}{\pi - (-\pi)} \int_{-\pi}^{\pi} [18 + 18 \sin(6t)] [\cos n\omega t] dt$$

$$\frac{2}{2\pi} \int_{-\pi}^{\pi} 18 \cos(n\omega t) + 18 \sin(6t) \cos(n\omega t) dt$$

$$18\pi \int_{-\pi}^{\pi} \cos(n\omega t) + 18\pi \int_{-\pi}^{\pi} \sin(6t) \cos(n\omega t) dt$$

teniendo en cuenta que $\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$, reemplazamos

$$18\pi \int_{-\pi}^{\pi} \cos(nt) + 18\pi \int_{-\pi}^{\pi} \sin(6t) \cos(nt) dt$$

Aplicamos $\sin(a) \cdot \cos(b) = \frac{1}{2} \sin(a+b) + \sin(a-b)$

$$\textcircled{1} 18\pi \int_{-\pi}^{\pi} \cos(nt) + \frac{18\pi}{2} \int_{-\pi}^{\pi} \sin((6+n)t) + \frac{18\pi}{2} \int_{-\pi}^{\pi} \sin((6-n)t)$$

Evaluamos cada trozo de la integral a parte para mayor facilidad:

$$1) \frac{18 \sin(nt)}{n} \Big|_{-\pi}^{\pi} \rightarrow \frac{36 \pi \sin(n\pi)}{n} \quad ①$$

$$② \frac{18 \pi \cos((6+n)t)}{12+2n} \Big|_{-\pi}^{\pi} = 0$$

$$③ \frac{18 \cos((6-n)t)}{2(6-n)} \Big|_{-\pi}^{\pi} = 0$$

Entonces tenemos que

$$a_0 = \frac{36 \pi \sin(n\pi)}{n}$$

Encontramos el b_n

$$b_n = \frac{2}{T} \int_{-\pi}^{\pi} [18 + 18 \sin(6t)] [\sin(n\omega_0 t)] dt$$

$$\frac{2}{2\pi} \int_{-\pi}^{\pi} 18 \sin(n\omega_0 t) + 18 \sin(6t) \sin(n\omega_0 t) dt$$

Con $\omega_0 = 1$ tenemos

$$\frac{2}{2\pi} \int_{-\pi}^{\pi} 18 \sin(nt) + 18 \sin(6t) \sin(nt) dt$$

Resolviendo la integral anterior nos queda lo

$$b_n = \frac{18(6+n) \sin(\pi n) + 18(6-n) \sin(\pi n)}{\pi(36-n^2)}$$

de acuerdo a la expresion anterior, al tener $n \neq 6$ obtenemos una indeterminacion, por ello aplicamos a L'Hopital.

$$\lim_{n \rightarrow 6} \left(\frac{\frac{d}{dn} (18(6+n) \sin(\pi n) + 18(6-n) \sin(\pi n))}{\frac{d}{dn} \pi(36-n^2)} \right)$$

$$\lim_{n \rightarrow 6} \frac{18 \cos(\pi n)(6+n) + 18\pi \cos(\pi n)(6-n)}{-2\pi n} = +18$$

$$\lim_{n \rightarrow -6} = -18$$

Entonces, para la serie exponencial compleja tenemos que:

$$C_0 = a_0 = 18 \quad y \quad C_n = \frac{a_n - j b_n}{2}$$

$$C_6 = \frac{0 - (-18)}{2} = 9$$

entonces

$$C_n = \begin{cases} 18 & n=0 \\ 9j & n=6 \\ -9j & n=-6 \end{cases}$$

$0 \quad \forall n \in \{0, 6, -6\}$

$$x(t) = C_6 e^{-j6t} + C_0 e^0 + C_6 e^{j6t}$$

$$= 9j(\cos(6t) - j \sin(6t)) + 18 - 9j(\cos(6t) - j \sin(6t))$$

$$= 18 + 18 \sin(6t)$$

Procedemos a calcular el error relativo

$$E_r = \left[1 - \frac{1}{P_x} \sum_{n=-N}^N |C_n|^2 \right] 100\%$$

Calculamos la potencia de la señal.

$$P_x = \frac{1}{2\pi} \int_{-\pi}^{\pi} |18 + 18 \sin(6t)|^2 dt$$

$$P_x = 486$$

$$E_r[\%] = \left[1 - \frac{|C_{-6}|^2 + |C_0|^2 + |C_6|^2}{P_x} \right] 100\%$$

$$\left[1 - \frac{81 + 324 + 81}{486} \right] 100\% \rightarrow \left[1 - \frac{486}{486} \right] 100\%$$

$$\boxed{E_r[\%] = 0\%}$$