# Heat transport equations in escript

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## 1 Escript

Escript is a generic finite element model that solves the following partial differential equation:

$$-\nabla(A\nabla u + Bu) + C\nabla u + Du = -\nabla X + Y \tag{1}$$

# 2 Heat transport

$$\rho_b c_b \frac{\partial T}{\partial t} = \nabla \kappa \nabla T - \rho_f c_{p,f} \vec{q} \nabla T \tag{2}$$

In which $T$ is temperature (K), $t$ is time (sec), $c$ is heat capacity, $\rho$ is bulk density ($kgm^{-3}$), $\kappa$ is thermal conductivity ($Wm^{-1}s^{-1}$), $\phi$ is porosity, $\rho$ is density (kg m$^{-3}$), $c$ the heat capacity ($Jkg^{-1}K^{-1}$) and $\vec{v}$ is the flow velocity vector (m s$^{-1}$). Subscripts $_b$, $_f$ and $_s$ denote properties of the bulk material, the fluid and the solid phase, respectively.

## 2.1 Implicit form

Time discretization of derivative of $T$:

$$\frac{\partial T}{\partial t} = \frac{T^{t+1} - T^t}{\delta t} \tag{3}$$

Insert into heat transport equation:

$$\rho_b c_b \frac{T^{t+1} - T^t}{\partial t} = \nabla \kappa \nabla T^{t+1} - \rho_f c_{p,f} \vec{q} \nabla T^{t+1} \tag{4}$$

$$-\nabla dt\ \kappa \nabla T^{t+1} + dt\ \phi \rho_f c_{p,f} \vec{q} \nabla T^{t+1} + \rho_b c_b T^{t+1} = \rho_b c_b T^t \tag{5}$$

Escript equation:

$$-\nabla(A\nabla u + Bu) + C\nabla u + Du = -\nabla X + Y \tag{6}$$

Cast implicit solute transport equation into escript form:

$$-\nabla \underbrace{dt\ \kappa}_{A} \nabla T^{t+1} + (\underbrace{dt \rho_f c_f \vec{q}}_{C}) \nabla T^{t+1} + \underbrace{\rho_b c_b}_{D} T^{t+1} = \underbrace{\rho_b c_b T^t}_{Y} \tag{7}$$

Dependent variable:
$u = T^{t+1}$
Escript constants:
$A = dt\ \kappa$
$C = dt \rho_f c_f \vec{q}$
$D = \rho_b c_b$
$Y = \rho_b c_b T^t$

## 2.2 Steady-state

$$0 = \nabla \kappa \nabla T - \rho_f c_f \vec{q} \nabla T \tag{8}$$

$$-\nabla \kappa \nabla T + \rho_f c_f \vec{q} \nabla T = 0 \tag{9}$$

Escript equation:

$$-\nabla(A\nabla u + Bu) + C\nabla u + Du = -\nabla X + Y \tag{10}$$

Cast implicit solute transport equation into escript form:

$$-\nabla \underbrace{\kappa}_{A} \nabla T + (\underbrace{\rho_f c_f \vec{q}}_{C}) \nabla T = 0 \tag{11}$$

Dependent variable:
$u = T$
Escript constants:
$A = \kappa$
$C = \rho_f c_f \vec{q}$