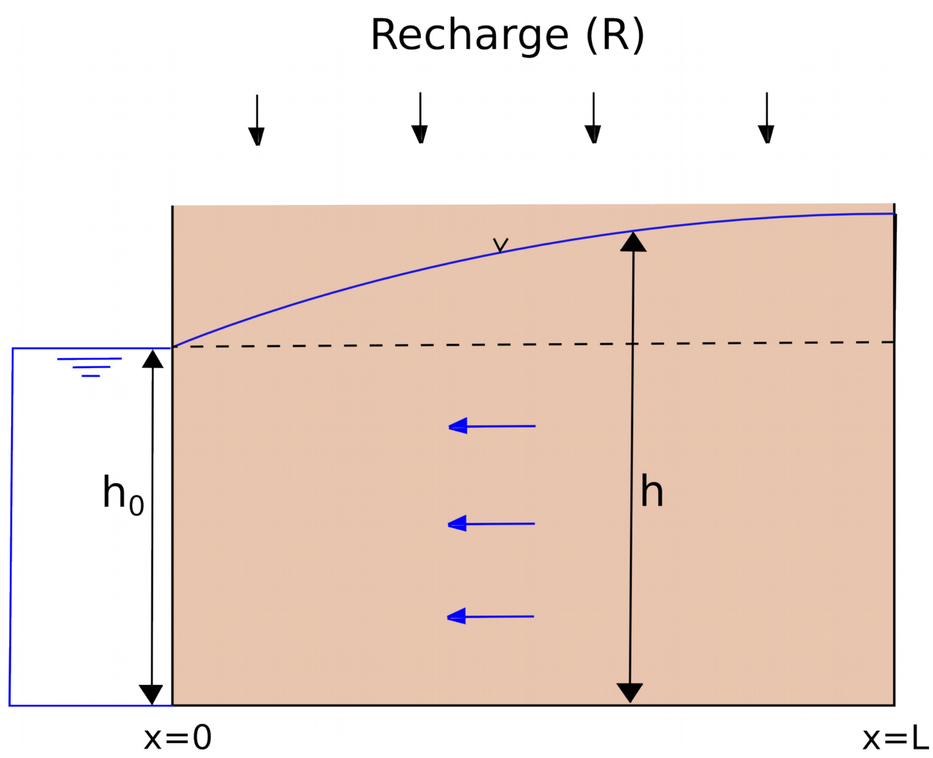
Exercise 1: Groundwater modeling in excel



M.Geo.239

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Earth science research:

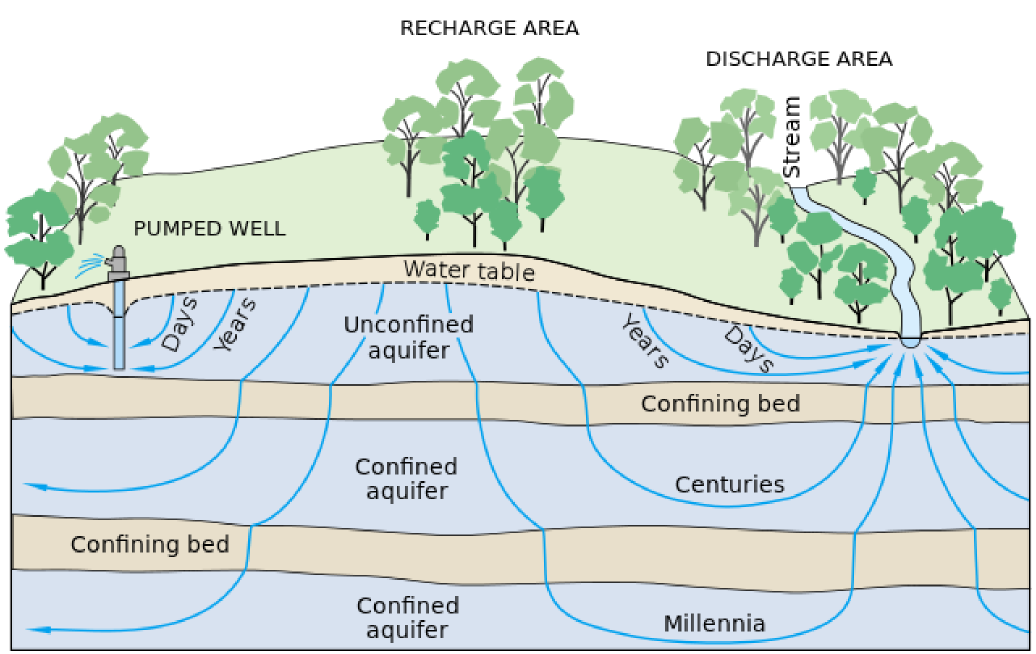
- Observations:
  - Field, lab data, etc ...
  - Use observations to formulate theory of underlying processes
  - This theory may or may not be physically realistic

- Modeling:
  - Solve physics eq. to predict earth science process
  - Outcome may or may not be realistic, input parameters often highly uncertain

- Combining models & data
  - Use observations and datasets (temperature, pressure, etc...) to constrain process-based models
  - More insight into underlying processes than obtained by just qualitative analysis of data or modeling alone

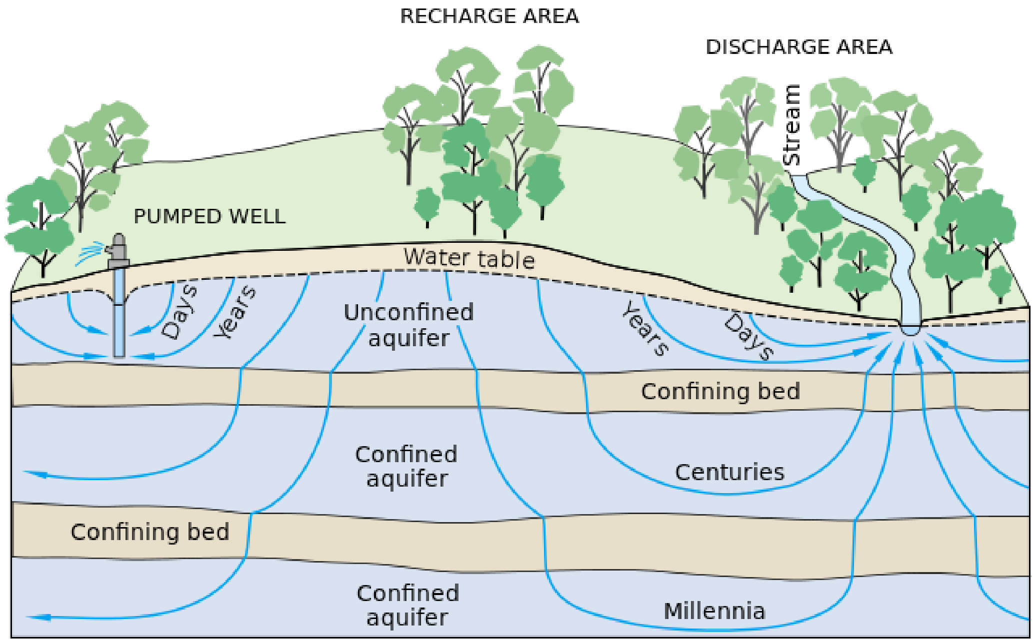
Exercise 1: your very first groundwater model

- Make your very first computer model in excel
- Objectives
  - Learn how to set up a model of an earth science process
  - Learn how to constrain models with data (model calibration)
  - Learn how groundwater systems function and respond to changes in recharge and pumping



Exercise 1: your very first groundwater model

- Simple flow system: groundwater flow to a stream
- Groundwater levels are a function of
  1) Groundwater recharge (ie., how much precipitation infiltrates to the groundwater table)
  2) Flow resistance by porous rocks (=permeability/hydraulic conductivity)
  3) Water level in stream
- Can we predict the water table?

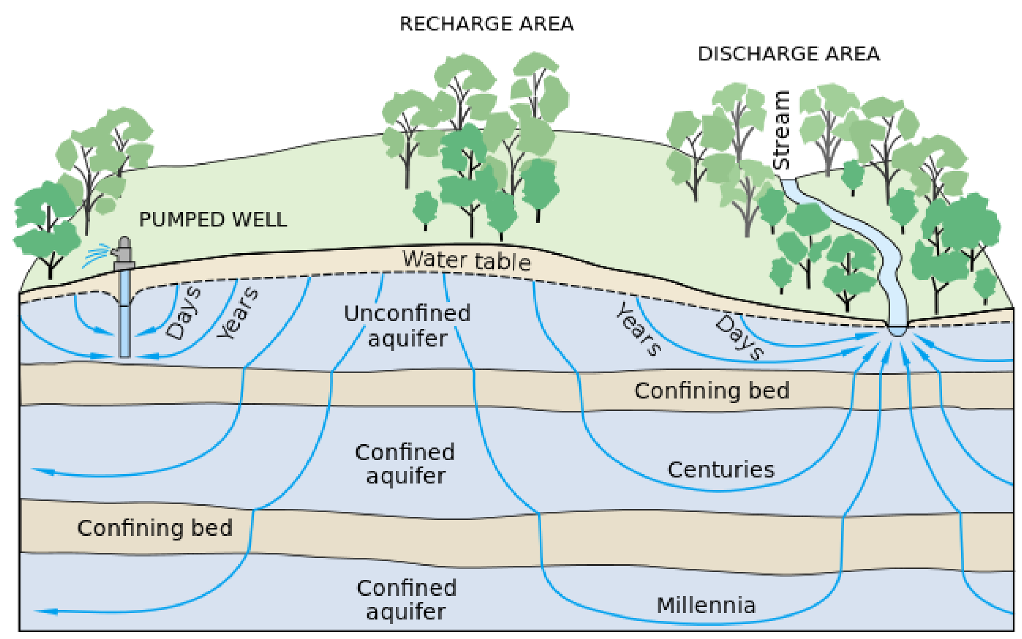


Exercise 1: your very first groundwater model

- Solution governed by groundwater flow equation
  - Darcy's law: groundwater flow is proportional to the difference in hydraulic head (h, ~watertable) and hydraulic conductivity (K)

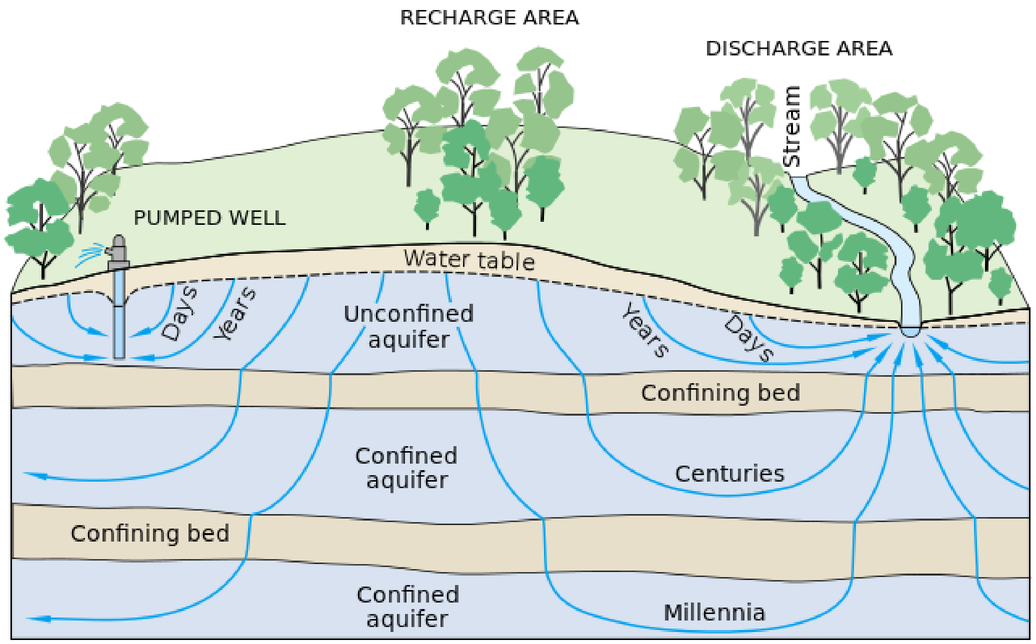
$$q = K \frac{\partial h}{\partial x}$$

- 2 ways to solve this: exact analytical solution of equation or numerical approximation



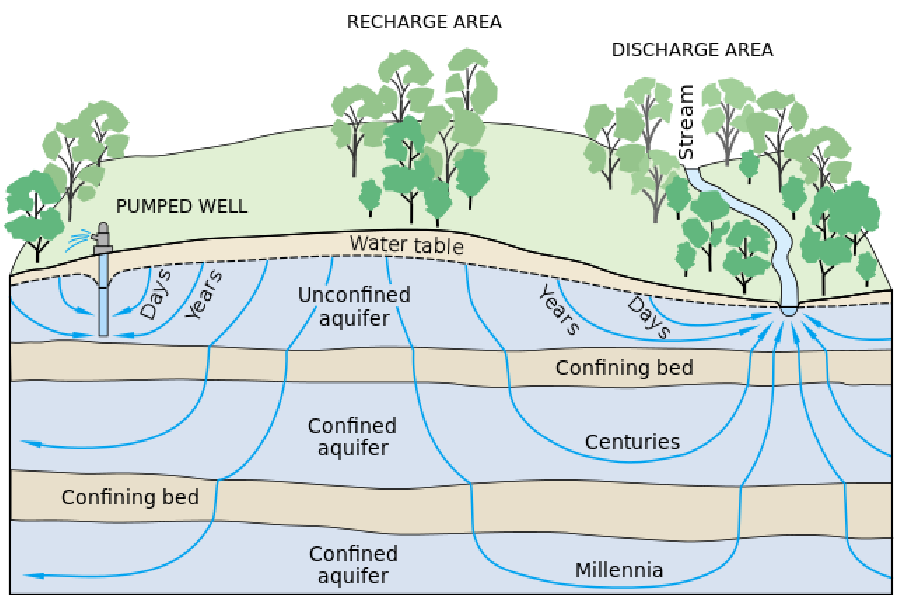
Analytical solutions of the groundwater flow equation

- Highly simplified: analytical solutions usually require assumptions of constant parameters (K, R), only horizontal or vertical flow, simplified geometries (rectangular, circular or infinite aquifers), etc...

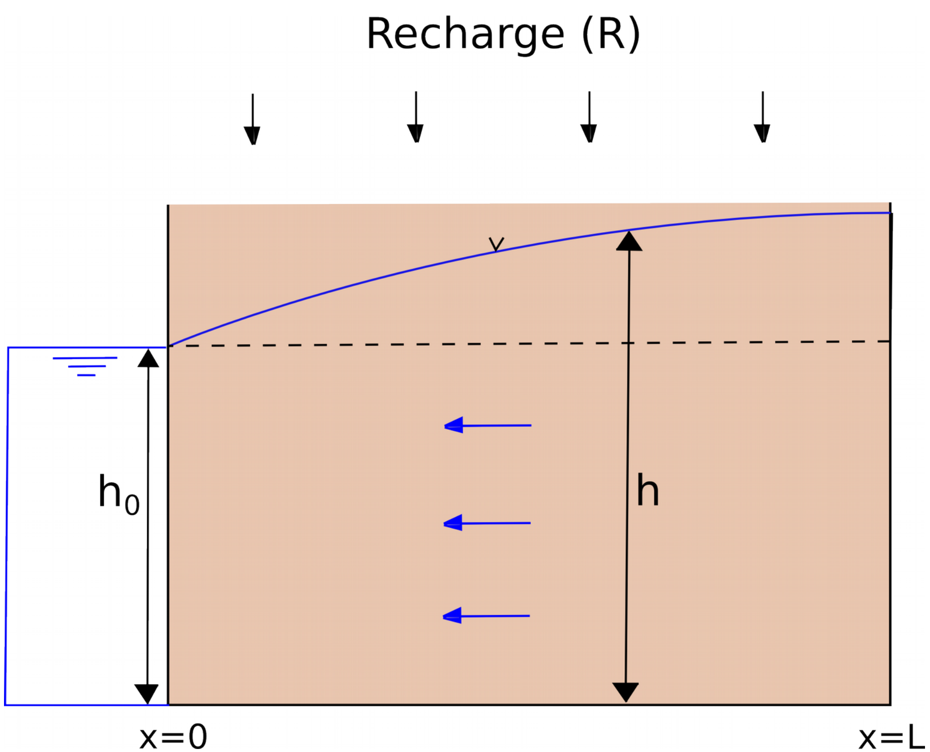


Analytical solution of groundwater flow eq.

- Simplifications in this case:
  - Rectangular aquifer (=permeable layer) with impermeable base
  - Stream reaches all the way to the bottom of the aquifer
  - Horizontal groundwater flow only, no vertical flow



Realistic groundwater system



Simplified groundwater system

Analytical solution of groundwater flow eq.

- At any point x, the flow towards the left (Q) has to be equal to the water input to the right (R * (L-x)):

$$Q = R\left(L - x\right)$$

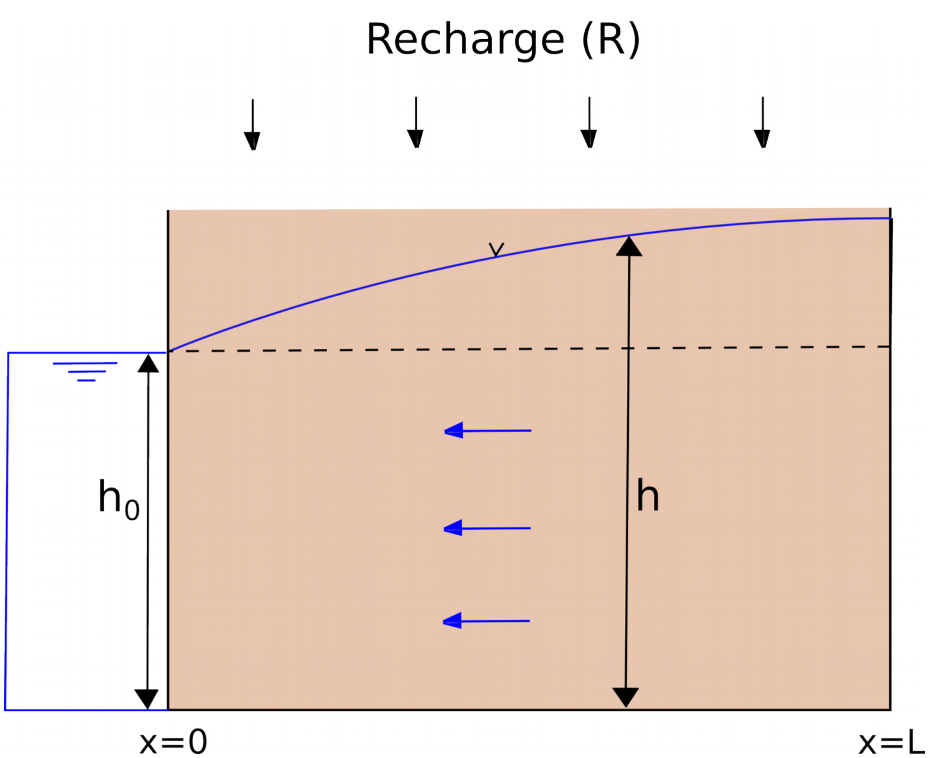
- Darcy's law for flow in an aquifer with constant thickness (b)

$$Q = Kb\frac{\partial h}{\partial x}$$

- Combine the two, some shuffling, integration and voila: an analytical solution for hydraulic head (h):

$$h = h_0 + \frac{R}{Kb}(Lx - \frac{1}{2}x^2)$$

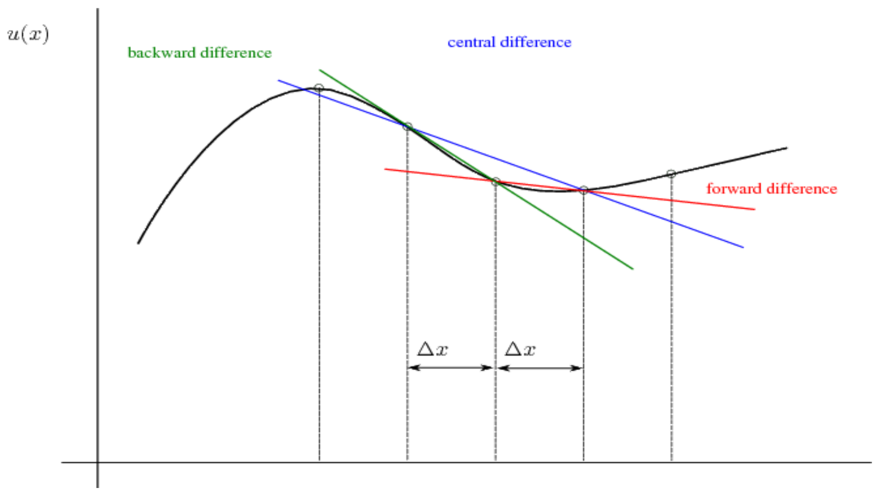
(see handout for full derivation)



Numerical solution of groundwater flow eq.

- Numerical solution: approximate derivatives in groundwater flow equation (=hydraulic gradient) with differences over discrete intervals in time or space
- In this case: the hydraulic gradient (dh/dx) at point x is approximately equal to the difference in h between two nodes divided by their distance (dx):

$$\frac{\partial y}{\partial x} \approx \frac{y(x+\Delta x) - y(x)}{\Delta x}$$

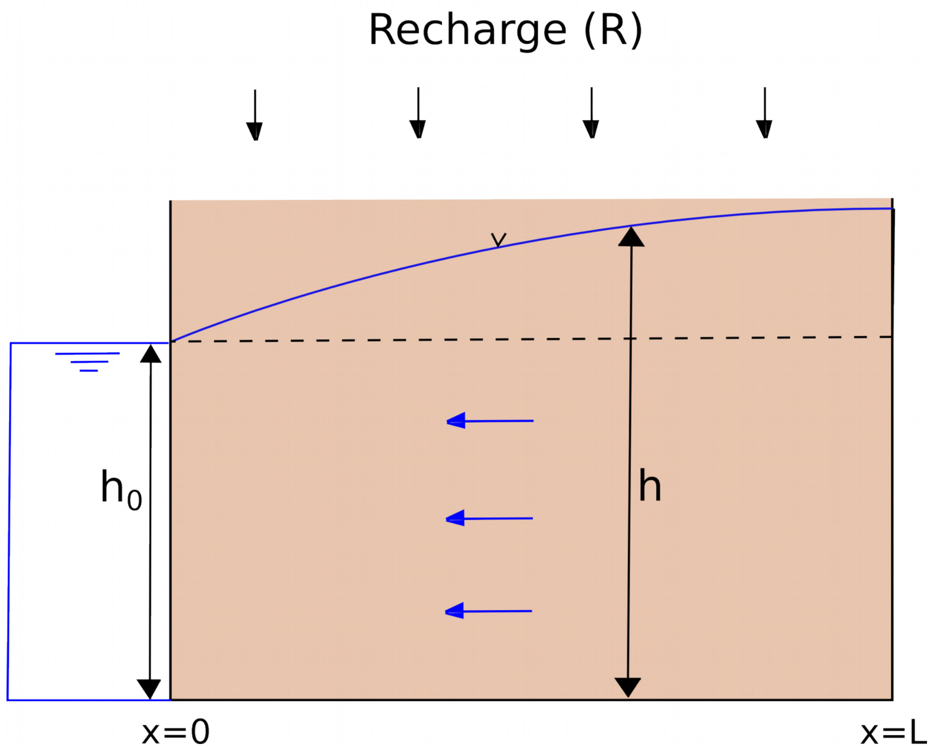


Numerical solution of groundwater flow eq.

- Applying the finite difference method to get rid of derivatives in the groundwater flow equation gives a numerical solution for the hydraulic head in our simplified groundwater system:

$$h(x) = \frac{1}{2}\left(\frac{Wb\Delta x^2}{Kb} + h(x+\Delta x) + h(x-\Delta x)\right)$$

(see handout for full derivation)



Numerical solution of groundwater flow eq.

- Numerical solution: hydraulic head as a function of h at 2 adjacent points in space → needs to be solved iteratively
- Advantages: much more flexible. All parameters can vary in space or time (variable recharge, hydraulic conductivity, etc...)
- Disadvantages:
  - Approximate solution → need to make sure error stays small
  - Computationally intensive, especially in 2D or 3D
  - Potential numerical stability issues for solutions of more complex systems, coupled fluid and heat flow, deformation, solute transport etc....

$$h(x) = \frac{1}{2}\left(\frac{Wb\Delta x^2}{Kb} + h(x+\Delta x) + h(x-\Delta x)\right)$$

Exercise 1: Design your very first groundwater model in excel

- Numerical solution of groundwater flow to a stream in excel
- Follow step by step instructions in pdf handout
- We will model the global median aquifer using recently published data on stream density, recharge and permeability
- Use analytical solution to check how well your numerical solution is doing
- Experiment with adding a pumping well (question 2) and calibrating model parameters to match observations (question 3)
- Questions are free….

Good luck!