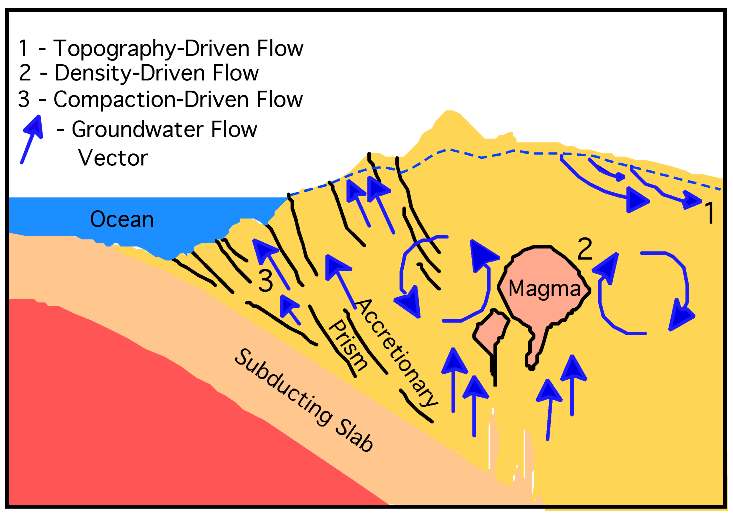
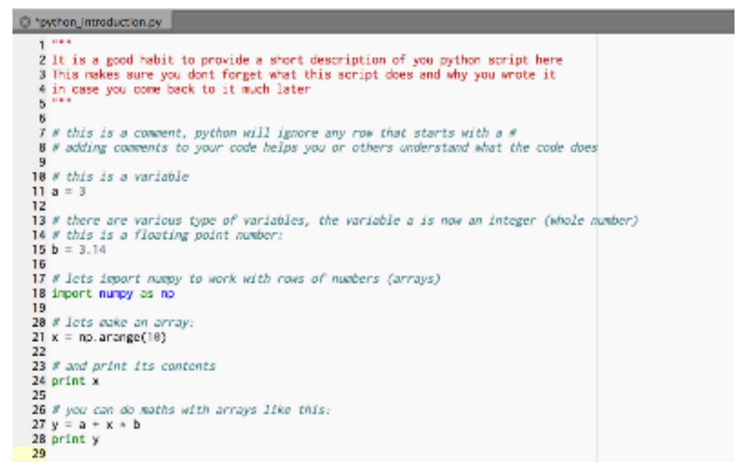
Exercise 3: Modelling transient fluid, heat and solute transport in Python





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Summary of exercises

- Exercise 1 & 2: Steady-state groundwater flow (=no change over time) in excel and Python

- Steady-state flow eq. can be used to describe average situations, such as annual average groundwater level or longer term averages

- somewhat artificial: no natural system is in steady-state, there are always changes over time, earth is a dynamic place

- Exercise 3 & 4: Transient groundwater flow (& heat transport) -> how do fluid and heat flow in the crust change over time?

Transient diffusion equations

- Transient (time-dependent) fluid flow, heat flow and solute diffusion equations:

- Can be used to explore how these systems evolve over time

- example: how does a groundwater flow system react to a decrease in recharge? on what timescales will springs dry up or streams run dry?

$$S_s \frac{\partial h}{\partial t} = \nabla K_f \nabla h + W_f$$
$$\rho_b c_b \frac{\partial T}{\partial t} = \nabla K_h \nabla T + W_h$$
$$\phi \frac{\partial C}{\partial t} = \nabla K_s \nabla C + W_s$$

from top to bottom:
fluid flow, heat flow, solute diffusion

Transient flow equations

$$S_s \frac{\partial h}{\partial t} = \nabla K_f \nabla h + W_f$$

$$\rho_b c_b \frac{\partial T}{\partial t} = \nabla K_h \nabla T + W_h$$

$$\phi \frac{\partial C}{\partial t} = \nabla K_s \nabla C + W_s$$

$$S \frac{\partial u}{\partial t} = \nabla K \nabla u + W$$

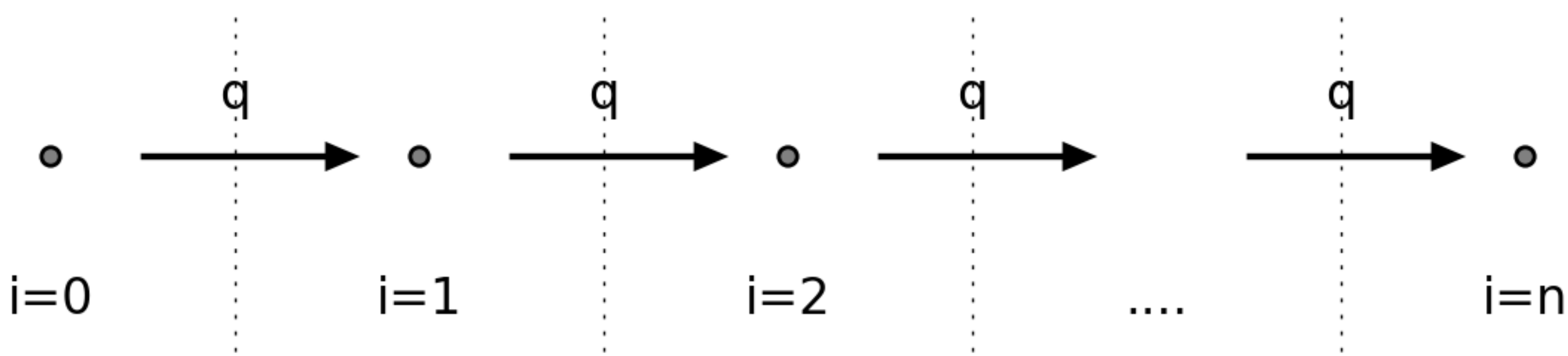
general form of transient diffusion equation

from top to bottom:
fluid flow, heat flow, solute diffusion

Solving the transient diffusion equation

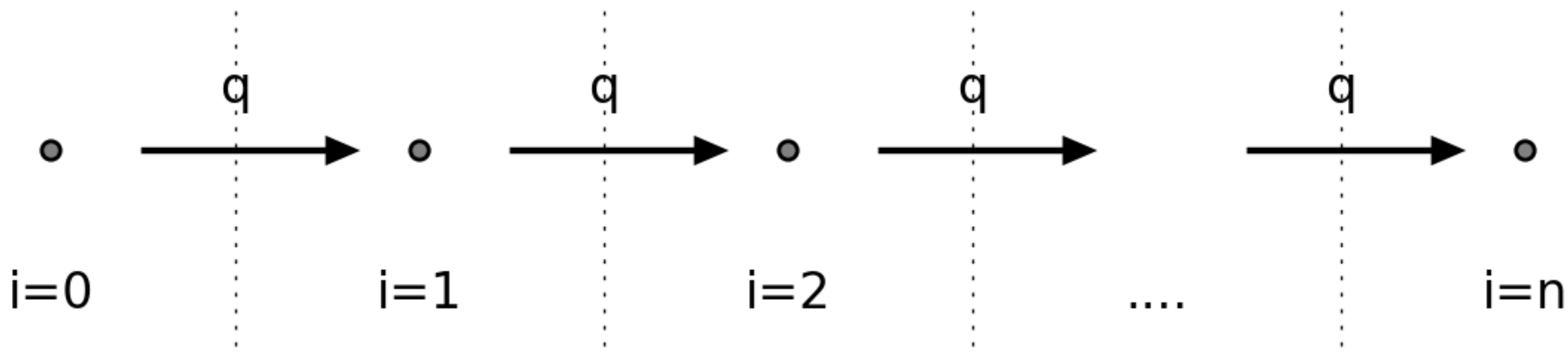
$$S\frac{\partial u}{\partial t} = \nabla K \nabla u + W$$

- Equation contains three terms:
  - change of *u* (=h/T/C) over time = flux balance + source term
  - *u* is defined on nodes, flux is defined in between nodes:



Solving the transient diffusion equation

$$S\frac{\partial u}{\partial t} = \nabla K \nabla u + W$$



- Solution:
  1. start with initial value of u (at each node i=0..n)
  2. calculate flux (*q*) between nodes
  3. use flux balance (*q* in- *q* out) to calculate new value of *u*
  4. and repeat step 2 and 3 for a number of time steps (nt)

Solving the transient diffusion equation

- Step 1: calculate and initial steady-state value of u (in exercise 2)

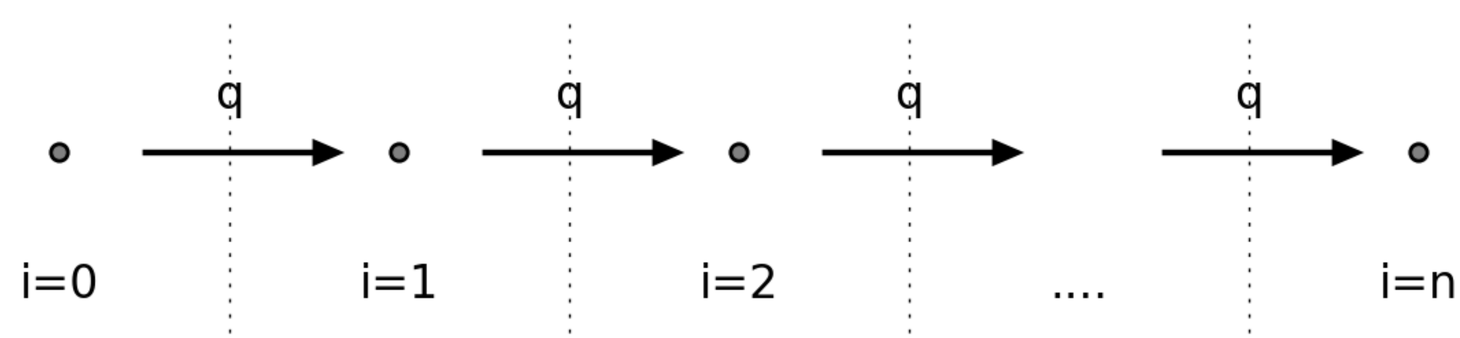
- Step 2: calculate flux between nodes:

$$q = K\frac{\partial u}{\partial x}$$

flux equation

$$q_{0-1} = K\frac{(u_0 - u_1)}{\Delta x}$$

discretized version for flux between node 0 and 1



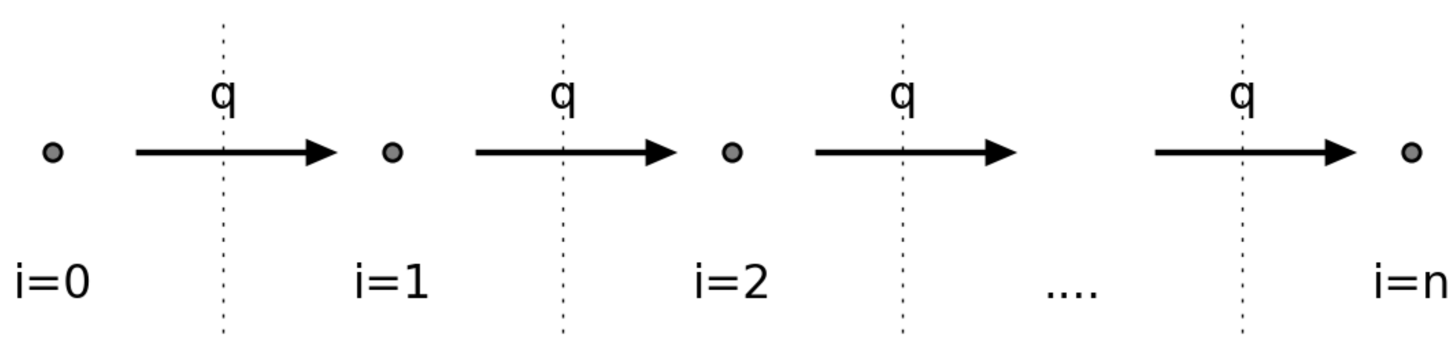
Solving the transient diffusion equation

- Step 3: use fluxes to update node values:

$$S\frac{\partial u}{\partial t} = \nabla K \nabla u + W$$ Full equation

$$S\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = -\frac{q_{i+1/2,j} - q_{i-1/2,j}}{2\Delta x} + W$$ Discretised equation

calculate the new node value of *u* at timestep j+1
as a function of flux *q* at current timestep j



Solving the transient diffusion equation

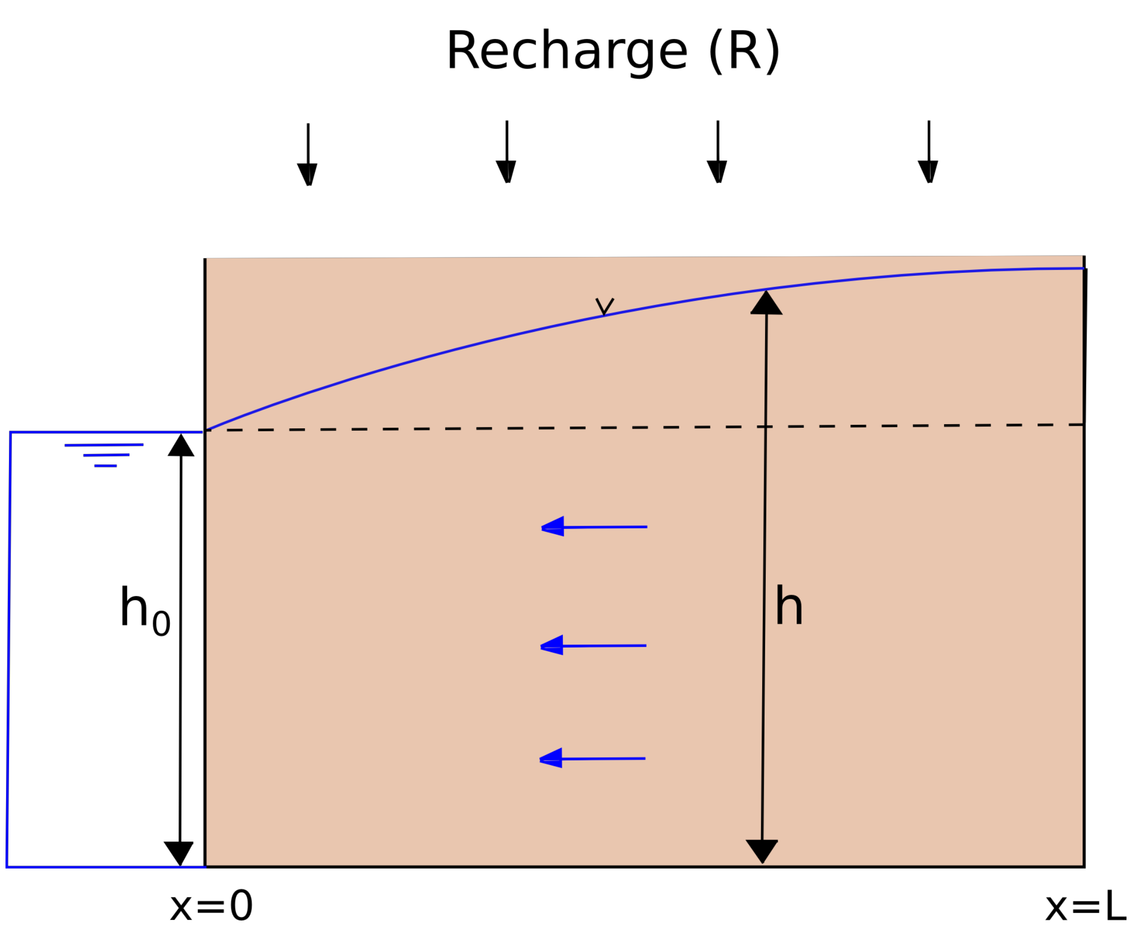
$$q_{1/2} = K \frac{(u_0 - u_1)}{\Delta x}$$

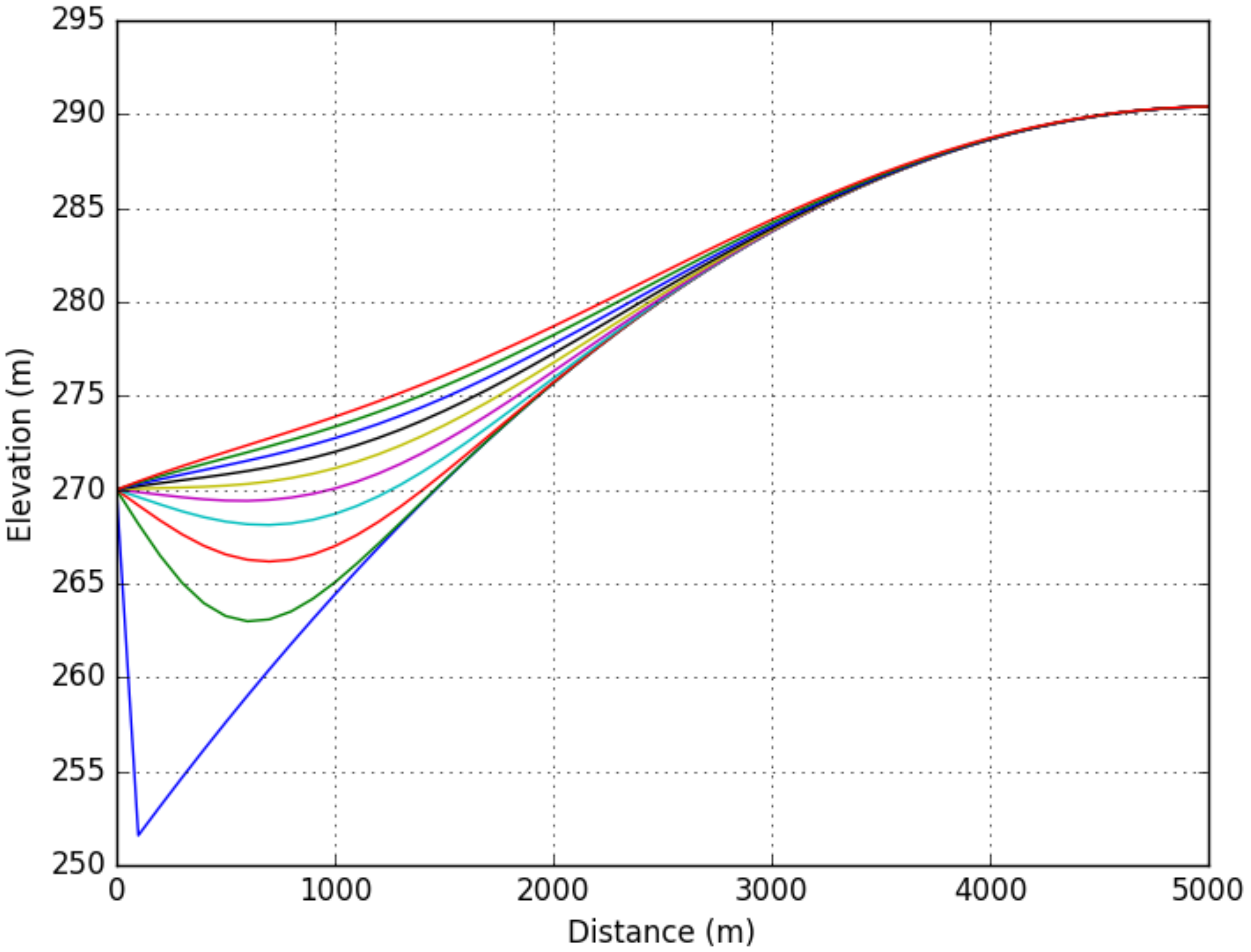
$$S \frac{u_{i,j+1} - u_{i,j}}{\Delta t} = - \frac{q_{i+1/2,j} - q_{i-1/2,j}}{2\Delta x} + W$$

- Repeat step 2 & 3 for a number of time steps
- Optional: change boundary conditions, source term or parameter values (*K*, *S*) in between and run an additional number of time steps to see what happens

Transient diffusion:

- Example: change in hydraulic head over time as a response to an increase in stream level (h_0):





Case study 1: Groundwater recharge and pore pressure below mt. Hood

- Case study by Saar and Manga (2003) who explore the reason for seasonal changes in seismicity and whether these could be driven by seasonal recharge and pore pressure change

Pore pressure & deformation

- Seismic activity fluctuates over time, and shows a seasonal trend
- link = additional fluid mass -> increase in normal stress on critically stressed faults -> increasing chance of failure and seismic activity
- The time lag is due to the diffusion of pore pressure, which is a function of permeability and storativity

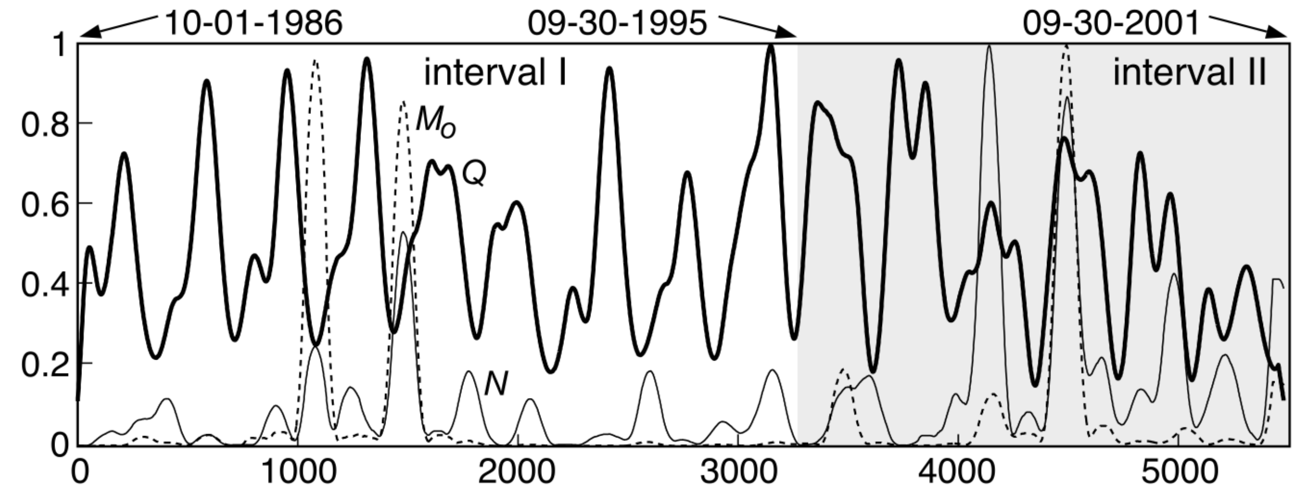
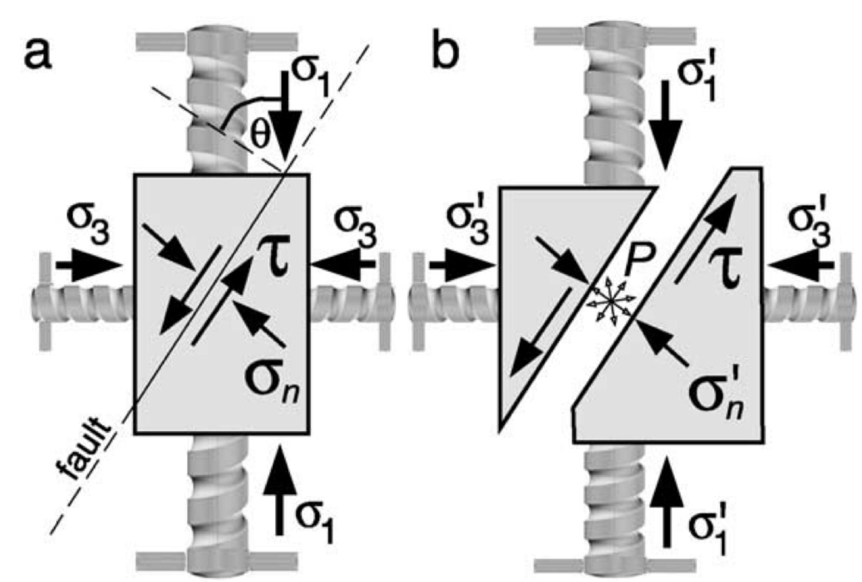


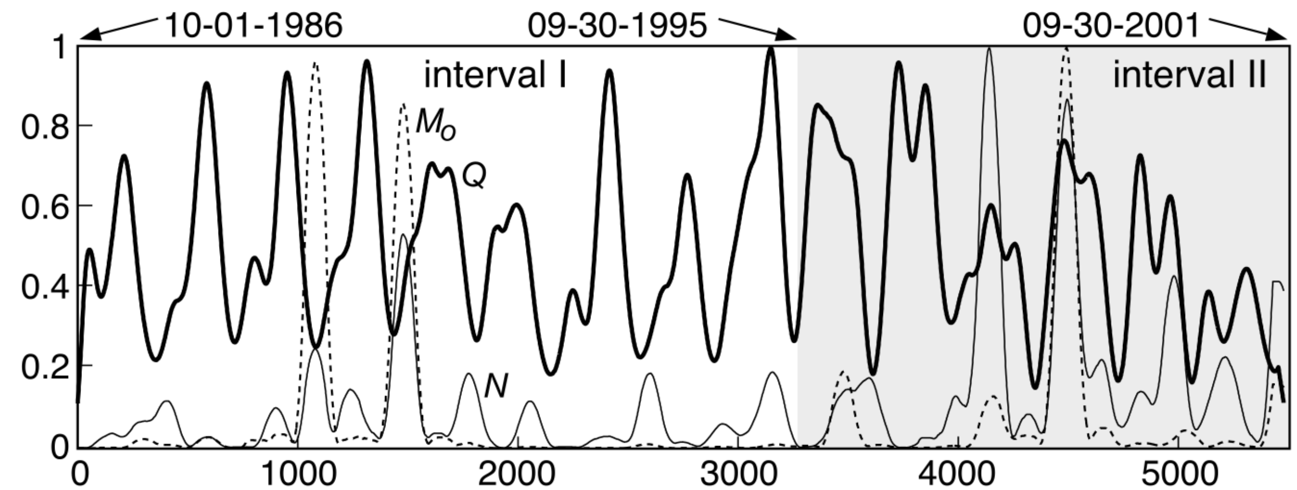
Fig. 7. Interpolated time series from Fig. 6, normalized by each series' absolute maximum value. Local maxima of number of earthquakes (thin solid line) and seismic moment (thin dashed line) typically follow local maxima of stream discharge (bold solid line) after a time lag of about 151 days.



Time series of seasonal groundwater recharge and seismic activity (top) and conceptual model of fluid pressure in faults (bottom)
Saar & Manga (2003) EPSL

Model workflow:

- Model the effect of an instantaneous increase in recharge and record how long it takes before it reaches the depth where seismic activity occurs (4500 m below the surface)
- Adjust hydraulic conductivity and storativity to find the optimal values that correspond to the phase shift between the recharge events and the spike in seismic activity (151 days)
- Implement a periodic boundary condition to simulate seasonal recharge and calculate the pore-pressure change at depth due to seismic activity



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Option 2: Modelling transient groundwater flow to a qanat

- Qanat = horizontal tunnels dug into alluvial fans or hillslope to tap the groundwater table

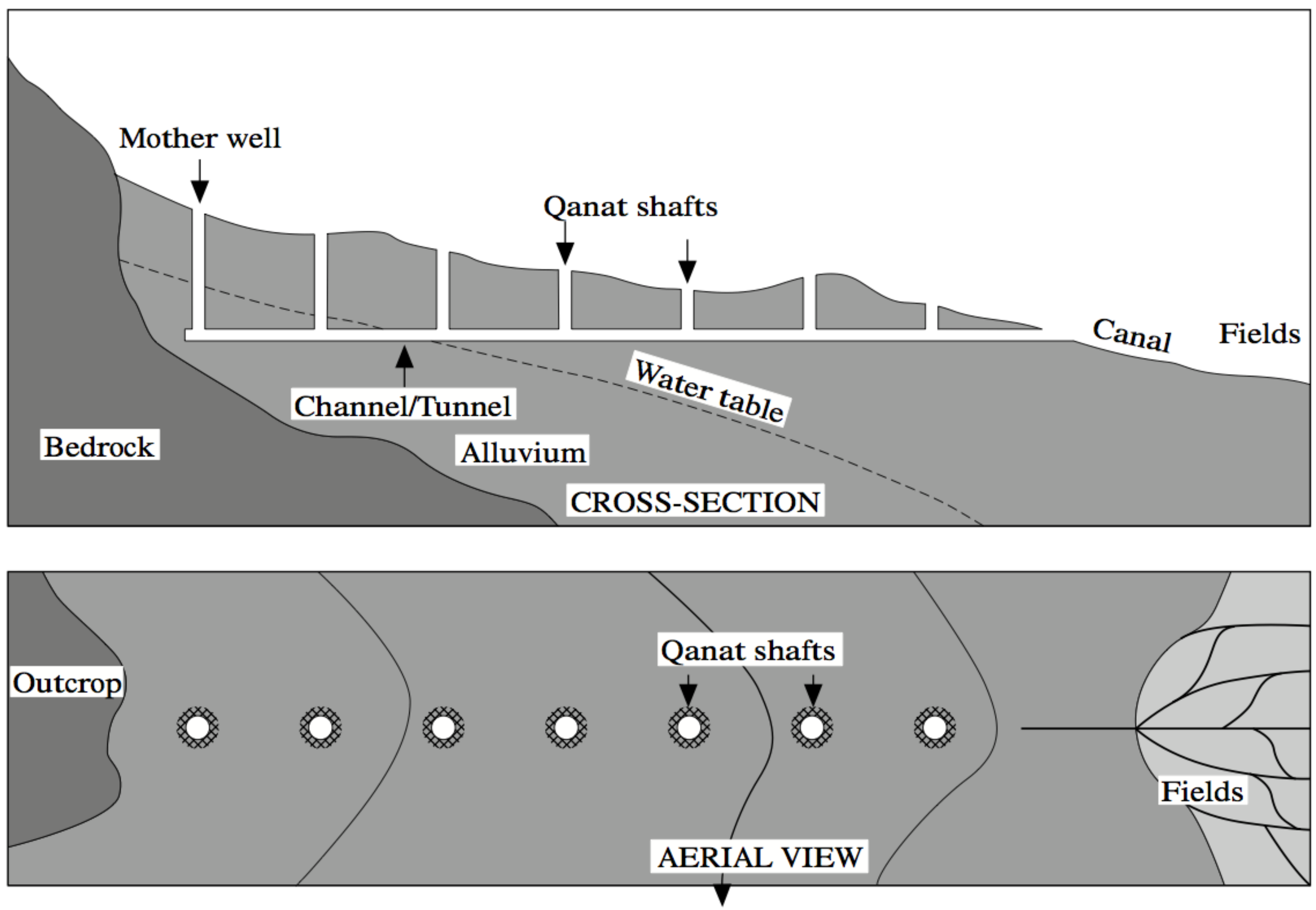
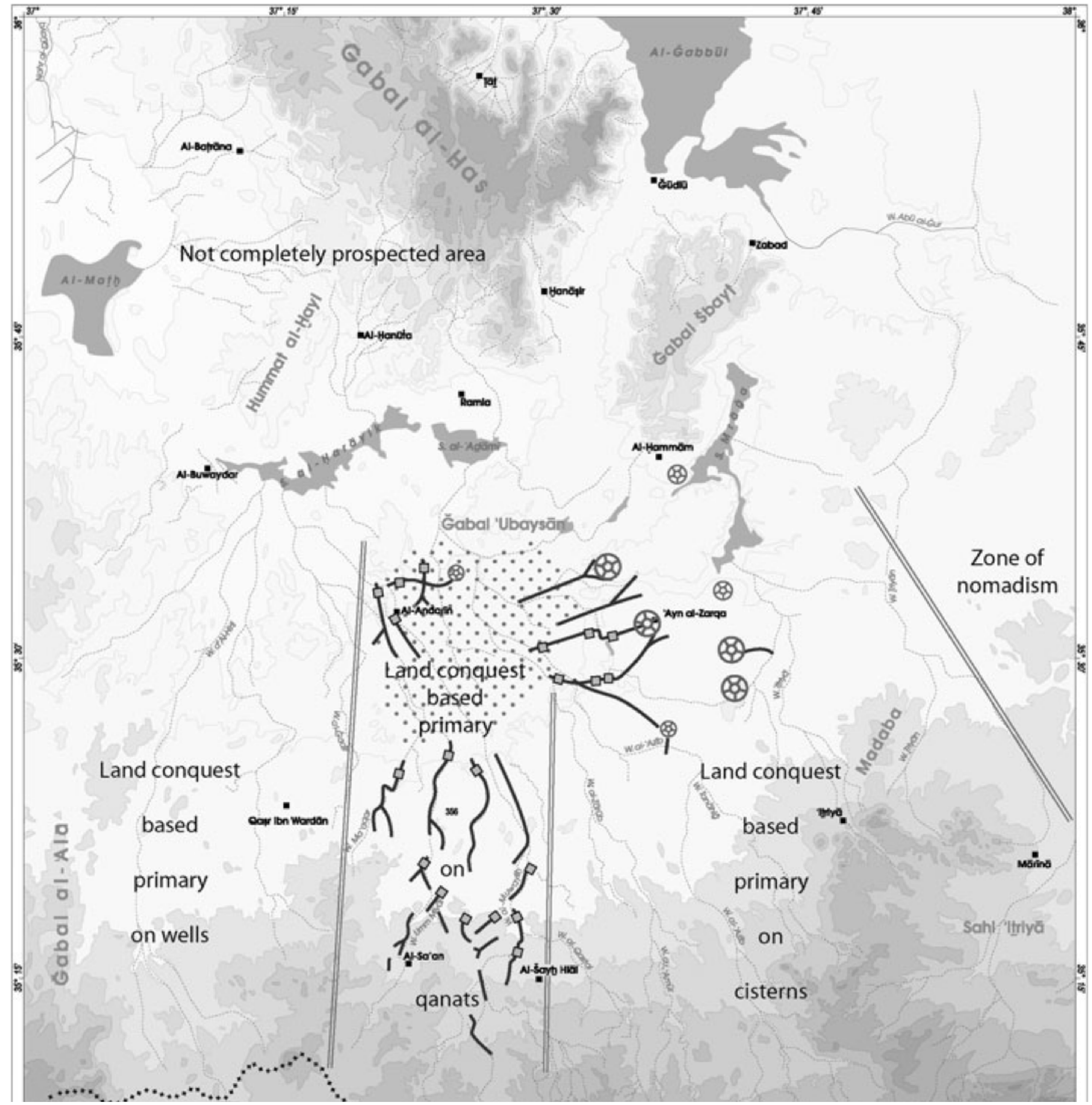


Figure 1. Plan and profile view of a typical qanat.

Lightfoot (1996)

Qanats

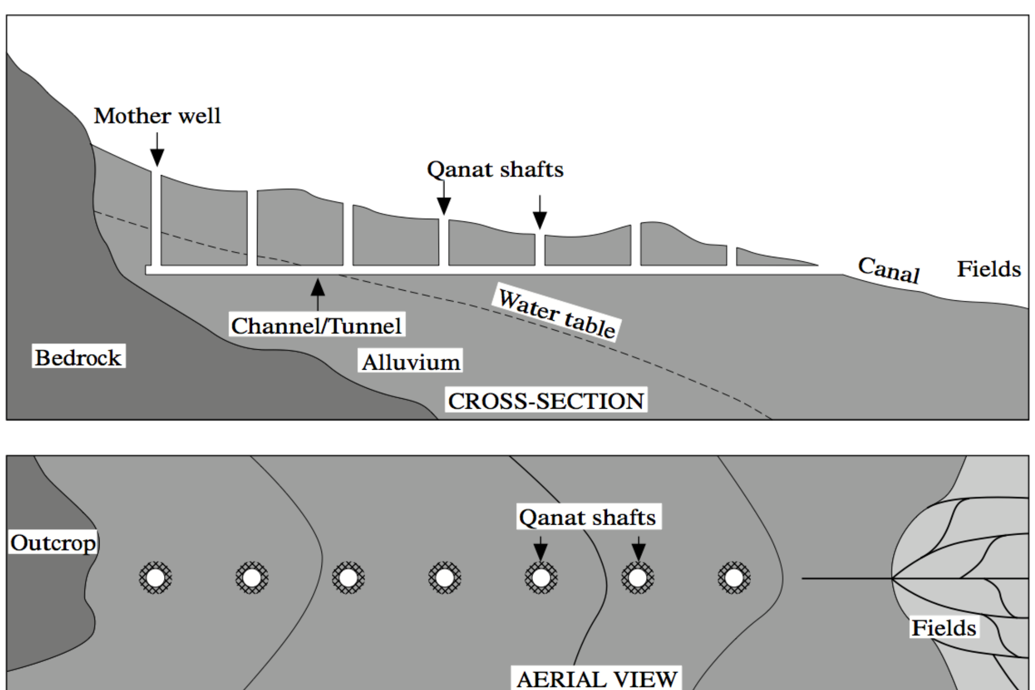
- Qanats date back 3000 years
- Played an important role in enabling settlement and agriculture in dry areas in history



Braemer et al. (2010)

Qanats

- We will try to answer the question: How sensitive are these systems to change in climate/recharge?



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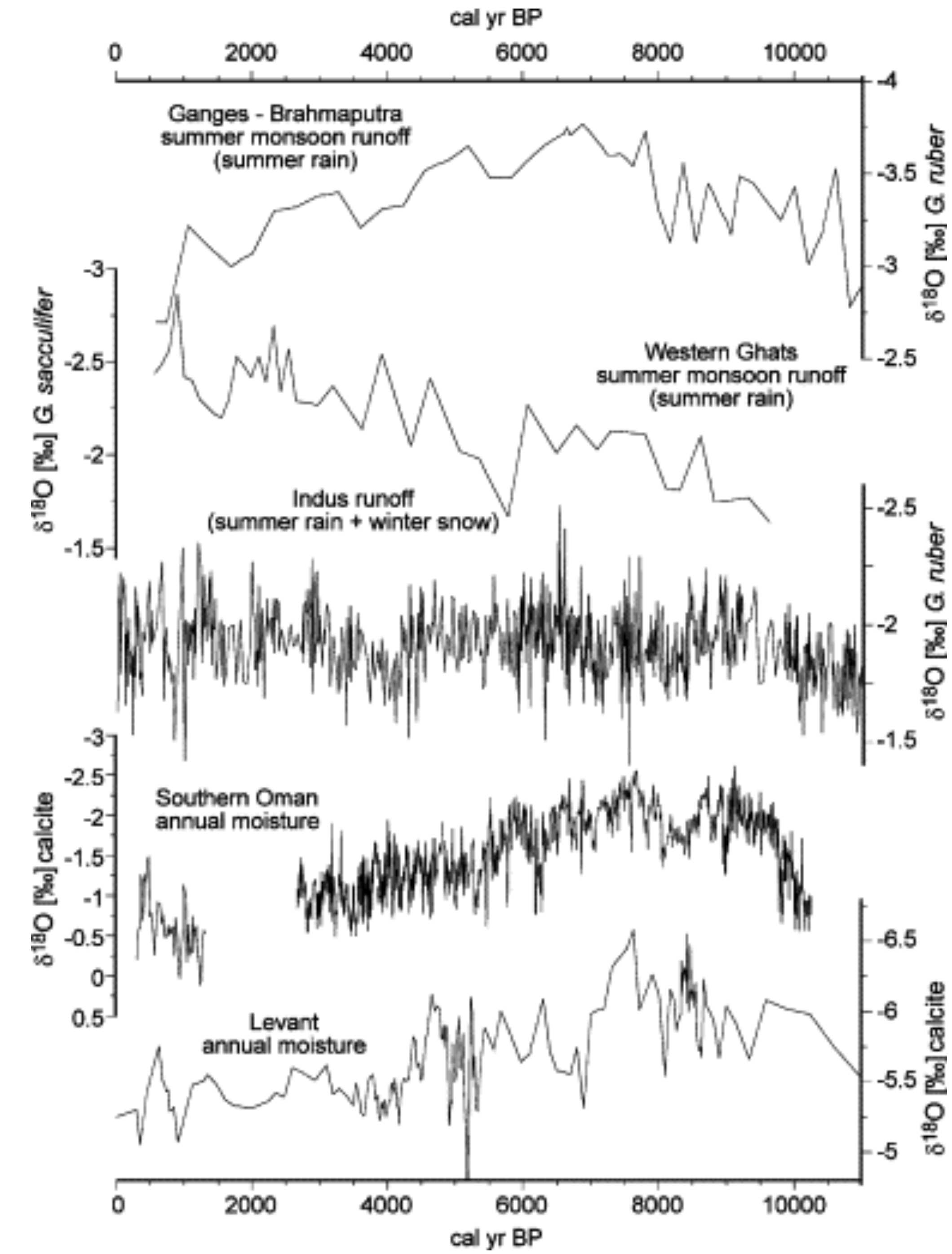


Staubwasser & Weiss (2006)

Case study: a qanat in northern Syria

- Case study: a Roman qanat in northern Syria

Case study: a qanat in northern Syria

- Recently restored:

Source: Wessels (2000)

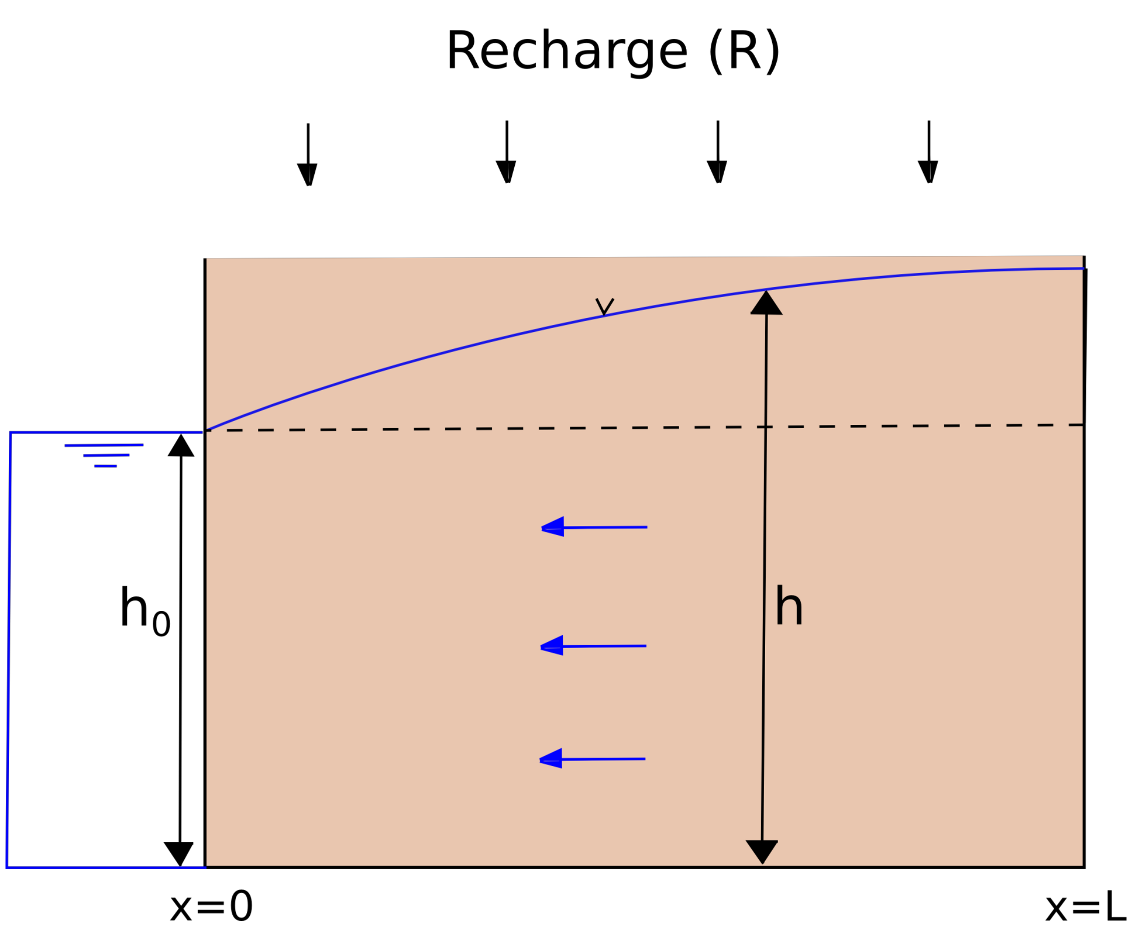
Case study: a qanat in northern Syria

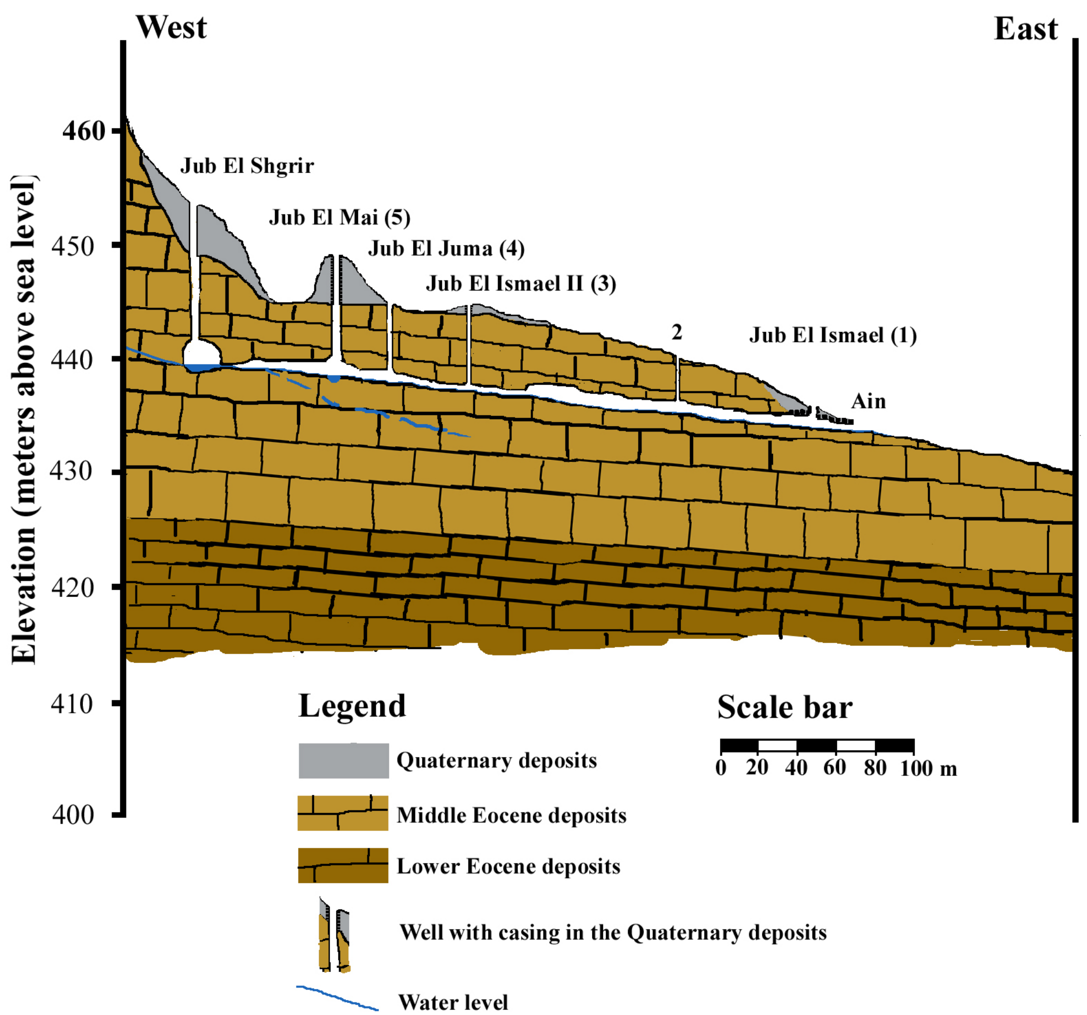
Air shaft

These hills provide recharge for the qanat

Case study: flow towards a qanat

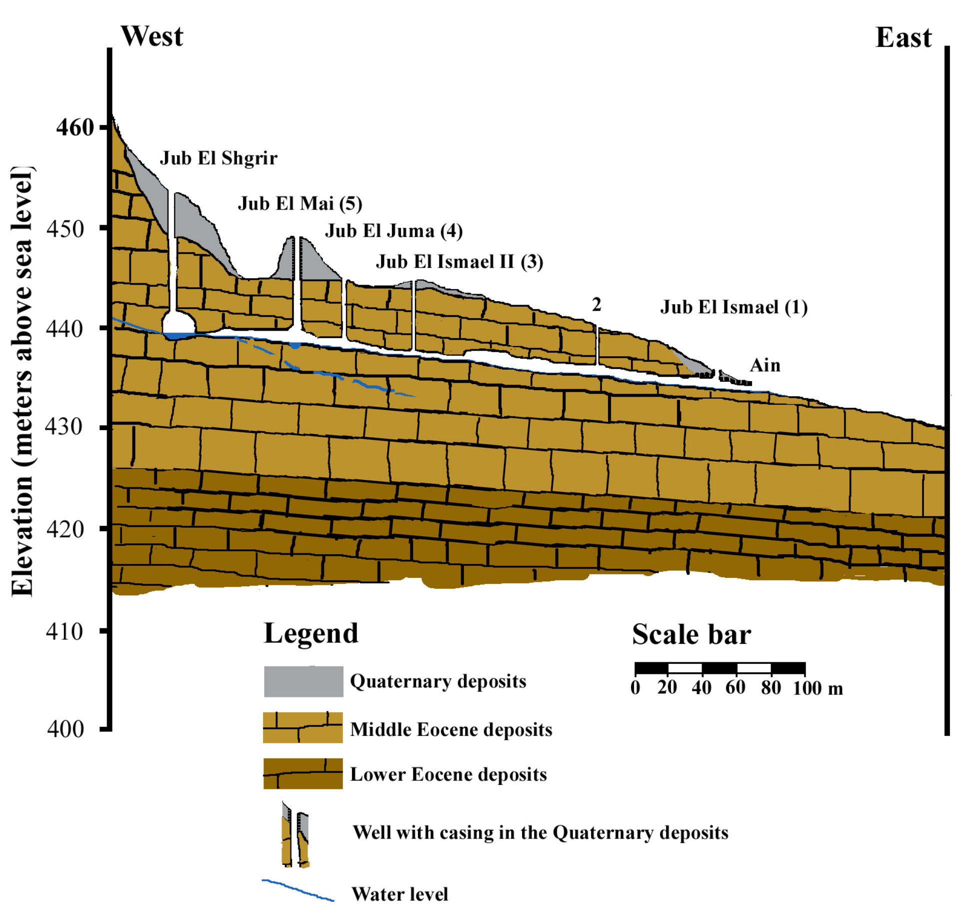
- Question: how long does the flow to the qanat last in case of a multi-year drought, when recharge=0?

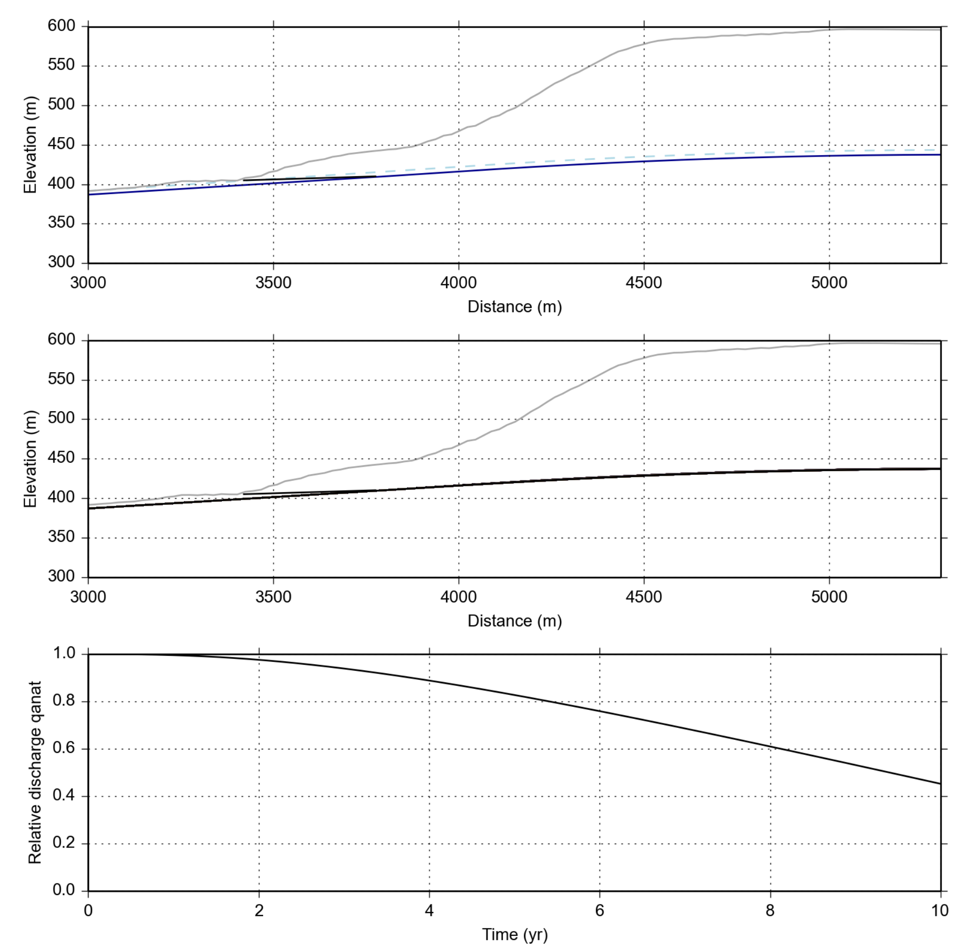




Case study: flow towards a qanat

- Qanat outflow simulated with a second fixed hydraulic head node at the source of the qanat
- We will make this a conditional fixed head: if the simulated water table starts to fall below the qanat we will remove this condition





Transient modelling workflow

1. Calculate steady-state hydraulic head

2. Apply a fixed hydraulic head condition at the source of the qanat and recalculate steady-state h

3. Use steady-state as initial condition, set recharge to 0 and run the transient calculations