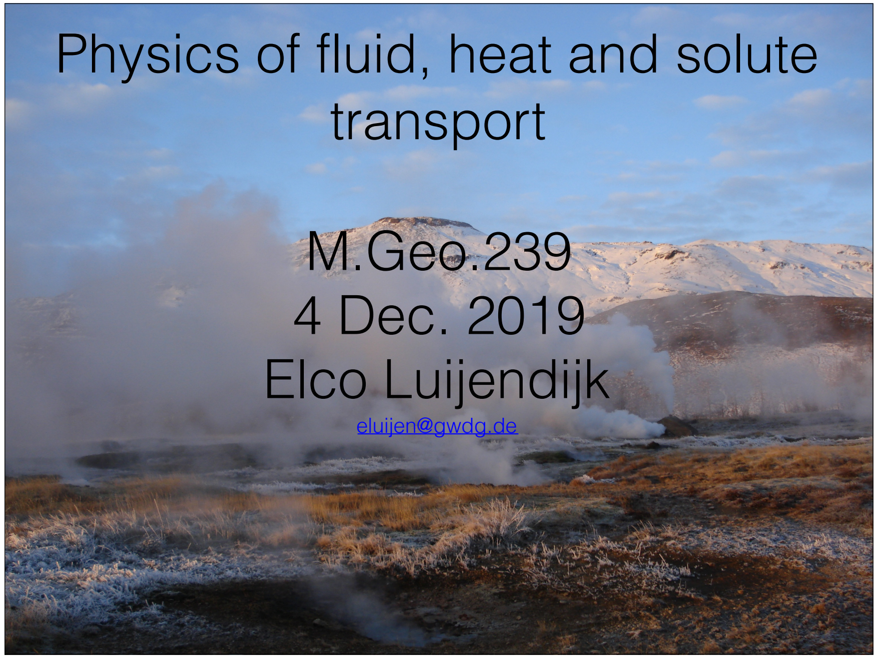

Physics of fluid, heat and solute transport

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Lectures so far:

- Fluid volumes in the crust
- Diffusion and transport of fluid, heat and solutes in the crust
- Permeability of the crust
- Temperatures and heat flow in the crust
- Terrestrial hydrothermal systems

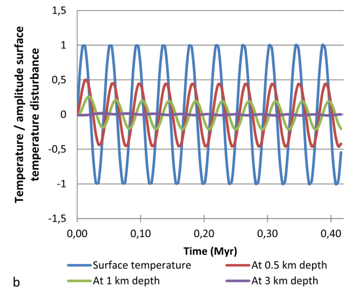
This week

- Physics of crustal fluid, heat and solute transport:
  - 1- Completing the diffusion equations: linking flux and change of fluid pressure, temperature or solute concentration over time
  - 2- Storativity, fluid pressure and compressibility of porous rocks
  - 3- Deriving the full fluid flow equation: combining Darcy's law and the mass balance equation

Part 1: Diffusion equations

- The diffusion equations discussed so far for fluid flow, heat transport and solute transport:
- These equations are not complete. They describe the flux of fluid, heat or solute, but not how these fluxes in turn change change fluid pressure, temperature or solute concentration in the subsurface…..

$$q_f = -K_f \nabla h$$
$$q_h = -K_h \nabla T$$
$$q_s = -K_s \nabla C$$



b

Changing solute concentrations

$$q_s = -K_s \nabla C$$

- Easiest example: How does solute concentration change in response to a diffusive solute flux?
- Let assume a solute flux of 1 kg / (m^2 s) towards a cube of 1 m^3 of porous rock
- How much does the solute concentration increase in this cube of porous rock?
- Question: What variables does this depend on?

Changing solute concentrations

$$q_s = -K_s \nabla C$$

- Answer: Change in solute concentration (C) only depends on the porosity (Φ)
- Low porosity: the same solute flux affects a smaller porewater volume -> stronger concentration increase

Changing temperatures

$$q_h = -K_h \nabla T$$

- Second example: changing temperatures in the subsurface
- Heat flux of 1 W/m^2 towards a cube of 1 m^3 of porous rock
- Temperature increases, but how much?
- Question: What variables does this depend on?

Changing temperatures

$$q_h = -K_h \nabla T$$

- Answer: change depends on how easy or hard it is for a porous rock to heat up
- This is determined by three parameters: volume (m^3), density (ρ, kg/m^3) and heat capacity (c_p, J / (kg K))
- Volume & Density -> more molecules -> more energy needed to heat up material
- Heat capacity -> different materials heat up a different rate

Changing temperatures

$$q_h = -K_h \nabla T$$

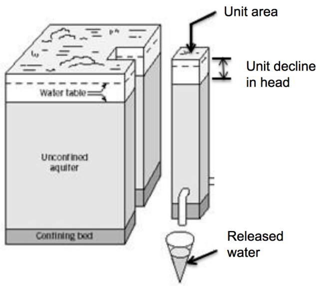
- Heat capacity of most minerals ~600 - 1100 J /(kg K))
- Water has a much higher heat capacity than most rocks: approx. 4200 J / (kg K)
- heating up 1 x 10^{-3} m^3 of a porous rocks with 25% porosity at a rate of 700 W (= average microwave)
- -> bulk density = 2000 kg/m3, c_p = 1750 J kg K
- dT/dt = 700 / (1 x 10^{-3} x 2000 x 1750) = 0.2 K/s = 12 K/min

Changing hydraulic head

- Final example: changing fluid energy potential in the subsurface (=hydraulic head)
- Fluid flux of 1 m/s towards a cube of 1 m^3 of porous rock
- Hydraulic head increases, but how much?
- Question: What variables does this depend on?

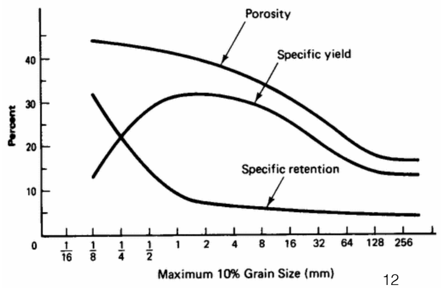
Part 2: Storativity, what happens when you change hydraulic head?

- Answer: its complicated…
- Simplest case: Adding water to an unconfined aquifer
- Unconfined aquifer: a water-bearing layer in which the water table can move freely
- Additional water increases the water table:
- Question: by how much?



Specific yield

- Increase/decrease in hydraulic head is controlled by the specific yield (Sy)
- Specific yield is often close but not equal to the porosity
- Especially for fine-grained sediments & clays: not all pore water can be drained freely due to capillary forces

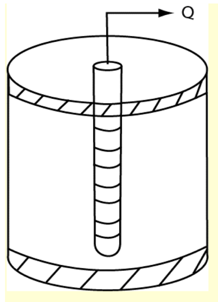


McCuen (2004)

FIGURE 3–10 Variation of specific retention, specific yield, and porosity with the grain size for which the cumulative total (starting with the coarsest material) reaches 10% of the total.

Aquifer storage

- Consider a pump in a confined aquifer
- Aquifer is saturated and bounded on bottom and top by confining (low permeability) layers. The watertable is located above the aquifer and the rocks remain saturated.
- Pumping decreases the hydraulic head in the aquifer
- However, the porous rocks remain saturated
- Question: where does the water come from?
- 1 - water from the expansion of the fluid volume (as pressure & hydraulic head decrease)
- 2 - water from the decrease in pore space (as pore pressure decreases and the rock matrix collapses)

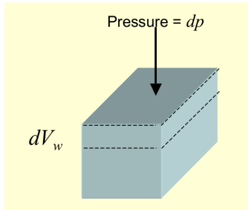


Changing hydraulic head

- More complex and subtle change: confined aquifer, porous rock at some distance below the water table
- Fluid pressure affects fluid volume (and density if we are looking at a fixed volume) due to the compressibility of the fluid (β_f)
- Fluid compressibility -> how much does the volume of a fluid change if you increase or reduce the pressure:
- compressibility equation in words: change in water volume = porosity x water compressibility x total volume x change in pressure

$$dV_f = \phi \beta_f V_t dp$$

(Fluid compressibility eq.)

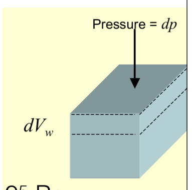


Source: J. Wilson (NMT)

Fluid compressibility

$$dV_f = \phi \beta_f V_t dp$$

(Fluid compressibility eq.)



- typical compressibility of water = 4.4 x 10^{-10} Pa^{-1}
- example of volume change: increase pressure by 1 x 10^5 Pa (= pressure of 10 m column of water): for a 1 m^3 volume of saturated porous rock with a porosity of 0.3

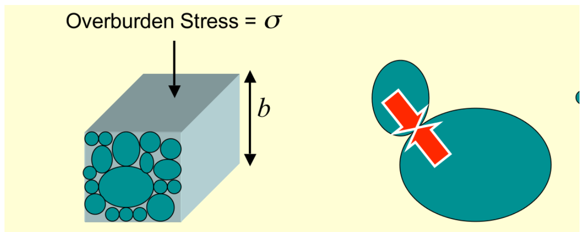
$$dV_f = \phi \beta_f V_t dp$$

$$dV_f = 0.3 \times 4.4 \times 10^{-10} \times 1.0 \times 1 \times 10^5 = 1.3 \times 10^{-5} m^3$$

- adding a column of 10 m of water on a 1 m^3 cube only decreases the fluid volume in the cube of porous rock by 0.013 liter -> water is nearly incompressible (!)

Rock matrix compressibility

- However, fluid pressure is not the only thing that changes
- Changes in fluid pressure will also compress/decompress the rock matrix.



Rock matrix compressibility

- Rock compressibility equation:
- In words: change in rock volume = rock compressibility x total volume x change in effective stress (σ_e)
- effective stress = total (overburden) stress - fluid pressure
- change in fluid pressure =~ change in hydraulic head (h)
- and change in volume / volume = change in porosity

$$dV = -\beta_m V d\sigma_e$$

$$dV = \beta_m V dP$$

$$dV = \beta_m V dh$$

$$d\phi = \beta_m dh$$

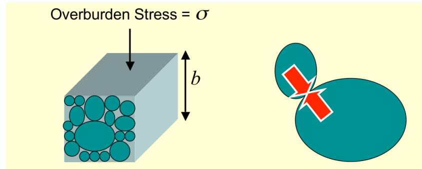
note: more on effective stress and rock compressibility in the lecture on compaction & compaction-driven flow

Rock matrix compressibility

- Typical values of rock compressibility:

material	$\beta_m\ (Pa^{-1})$
Solid granite	10^{-11} to 10^{-9}
Consolidated rock	10^{-10} to 10^{-8}
Unconsolidated sand	10^{-9} to 10^{-7}
Clay	10^{-9} to 10^{-6}

- Water: β = ~4.4×10^{-10} Pa^{-1}



Specific storage

- The total change in fluid mass (for a fixed volume) for a change in hydraulic head is called specific storage (S_s):

$$S_s = \frac{1}{\rho_f}\frac{\partial \phi \rho_f}{\partial h}$$

- definition: "The volume of water released per unit aquifer volume per unit decline in head."
- In this eq: fluid mass change = $d\Phi\rho$ = change in (porosity x density):
- Mathematical trick: chain rule expansion of derivatives:

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial z}\frac{\partial z}{\partial x}$$

- Using chain rule to rewrite the specific storage equation:

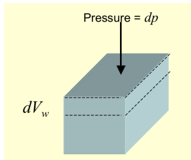
$$S_s = \frac{1}{\rho_f}\left(\rho_f \frac{\partial \phi}{\partial h} + \phi \frac{\partial \rho_f}{\partial h}\right)$$

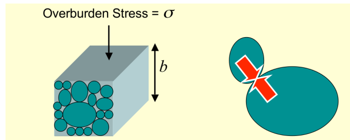
Specific storage

$$S_s = \frac{1}{\rho_f}\left(\rho_f \frac{\partial \phi}{\partial h} + \phi \frac{\partial \rho_f}{\partial h}\right)$$

- In words: specific storage = water released from the reduction of porosity due to compression of rock matrix: $d\Phi/dh$
- + water from the compression or expansion of fluid: $d\rho_f/dh$
- We can rewrite this using the compressibility equations for fluid and rock
- -> The specific storage is a function of the compressibility of the fluid (βf) and the rock matrix (βm):

$$S_s = \rho g \left(\phi \beta_f + \beta_m\right)$$





Specific storage

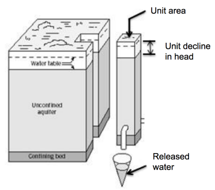
- Typical values of specific storage:

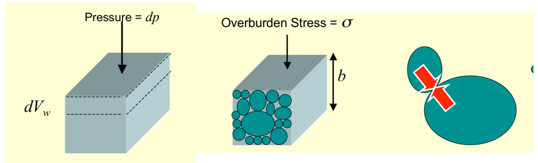
Specific storage:

Material	$\beta_m\ (Pa^{-1})$	$S_s\ (m^{-1})$
Solid granite	10^{-11} to 10^{-9}	10^{-7} to 10^{-6}
Consolidated rock	10^{-10} to 10^{-8}	$\approx 10^{-5}$
Unconsolidated sand	10^{-9} to 10^{-7}	$\approx 10^{-4}$
Clay	10^{-9} to 10^{-6}	$\approx 10^{-3}$

Storativity

- In reality both the confined and unconfined response are active at the same time
- For unconfined aquifers: specific yield dominates, because it is several orders of magnitude higher than specific storage
- For confined aquifers: specific yield = 0 (the hydraulic head does not change saturation of the rock), storativity is equal to specific storage

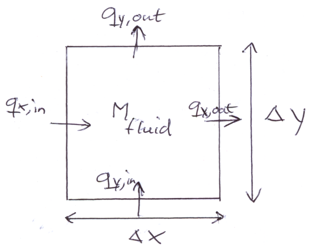




Part 3: Groundwater flow equation

- Deriving the full groundwater equation
- Basis of physical laws: any combination of mass balance + energy balance + force balance
- Fluid mass balance for a fixed volume of space:
  - change in mass over time = mass flux in - mass flux out

$$\frac{\Delta M}{\Delta t} = \frac{M_{in} - M_{out}}{\Delta t}$$



Fluid mass balance

- Fluid mass balance (kg/s)

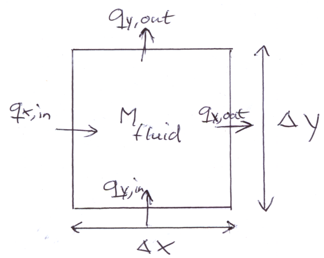
$$\frac{\Delta M}{\Delta t} = \frac{M_{in} - M_{out}}{\Delta t}$$

- Fluid mass of a cube is equal to porosity(ϕ) x density(ρ) x volume

$$M_f = \phi\rho\Delta x\Delta y$$

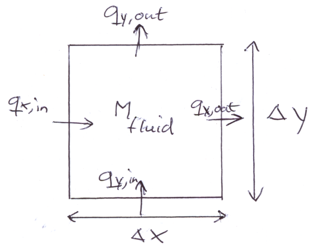
- Use this to rewrite the equation:

$$\frac{\Delta(\phi\rho\Delta x\Delta y)}{\Delta t} = \frac{M_{in} - M_{out}}{\Delta t}$$



Fluid mass balance

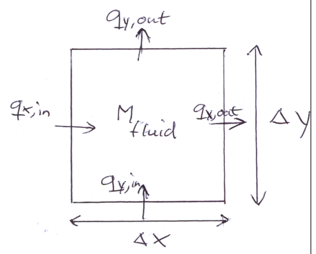
- rewrite mass change in fluid flux (q) terms:
- Change in mass= change in density x volume
  = density x flux x area



$$\frac{\Delta(\phi\rho\Delta x\Delta y)}{\Delta t} = \frac{M_{in} - M_{out}}{\Delta t}$$

$$\frac{\Delta(\phi\rho\Delta x\Delta y)}{\Delta t} = \rho q_{x,in}\Delta y - \rho q_{x,out}\Delta y + \rho q_{y,in}\Delta x - \rho q_{y,out}\Delta x$$

Fluid mass balance

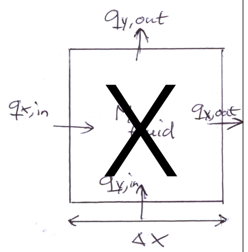


- Simplify: divide both sides of equation by the volume of the cube (Δx Δy)
- and rewrite: q_in - q_out = Δq

$$\frac{\Delta(\phi\rho\Delta x\Delta y)}{\Delta t} = \rho q_{x,in}\Delta y - \rho q_{x,out}\Delta y + \rho q_{y,in}\Delta x - \rho q_{y,out}\Delta x$$

$$\frac{\Delta(\phi\rho)}{\Delta t} = \rho\frac{\Delta q_x}{\Delta x} + \rho\frac{\Delta q_y}{\Delta y}$$

Fluid mass balance



- The world is not made of cubes -> we need an equation that is valid for any point in space
- $\lim x \to 0, \lim y \to 0, \lim t \to 0$

$$\frac{\Delta(\phi\rho)}{\Delta t} = \rho\frac{\Delta q_x}{\Delta x} + \rho\frac{\Delta q_y}{\Delta y}$$

$$\frac{\partial(\phi\rho)}{\partial t} = -\left(\rho\frac{\partial q_x}{\partial x} + \rho\frac{\partial q_y}{\partial y}\right)$$

$$\frac{\partial(\phi\rho)}{\partial t} = -\rho\nabla q$$

Adding Darcy's law

- We now have a fluid mass balance law that is valid for any point in space (ie. the subsurface):

$$\frac{\partial(\phi\rho)}{\partial t} = -\rho\nabla q$$

- The eq. contains a flux term (q) which we have seen already in Darcy's law:

$$q = -\frac{\rho g k}{\mu}\nabla h \qquad \text{(Darcy's law)}$$

Insert Darcy's law into the mass balance equation:

$$\frac{\partial(\phi\rho)}{\partial t} = -\rho\nabla\left(-\frac{\rho g k}{\mu}\nabla h\right) \qquad \text{(Fluid balance + Darcy)}$$

Adding storativity to get the final fluid flow equation:

- Ok, almost there…
- Recall that storativity is: $S_s = \frac{1}{\rho_f}\frac{\partial \phi \rho_f}{\partial h}$
- By inserting this into the fluid flow equation we finally get the full fluid flow equation, with just one unknown (h):

$$\frac{\partial(\phi\rho)}{\partial t} = -\rho\nabla\left(-\frac{\rho g k}{\mu}\nabla h\right)$$

(Fluid balance + Darcy)

insert the storativity term:

$$S_s\frac{\partial h}{\partial t} = \nabla\frac{\rho g k}{\mu}\nabla h$$

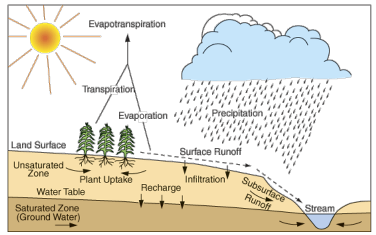
(fluid flow eq., with permeability)

Source term:

- Open instead of closed system: adding a source term (W) = external source or sink of water

$$S_s\frac{\partial h}{\partial t} = \nabla\frac{\rho g k}{\mu}\nabla h + W$$

- Sources: Recharge, fluid injection (fracking, brine disposal)
- Sinks: Groundwater discharge to rivers, lakes, oceans, evapotranspiration, pumping
- Fluid consumption or generation in chemical reactions



The fluid flow equation

- Ok, we now have one equation for fluid flow with a single unknown (h)
- Different forms of the fluid flow equation:

$$S_s\frac{\partial h}{\partial t} = \nabla\left(\frac{\rho g k}{\mu}\nabla h\right) + W$$

fluid flow eq.,

$$S_s\frac{\partial h}{\partial t} = \nabla\left(K\nabla h\right) + W$$

fluid flow eq., with hydraulic conductivity (K), assumes constant fluid viscosity and density

$$0 = \nabla\left(K\nabla h\right) + W$$

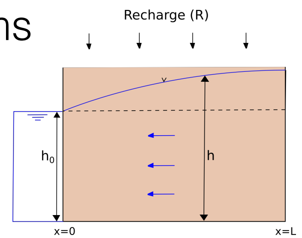
Steady state (no change over time)

$$S_s\frac{\partial h}{\partial x} = \frac{\partial K\frac{\partial h}{\partial x}}{\partial x} + W$$

one-dimensional, with hydraulic conductivity (K)

Using the fluid flow equation to study flow systems

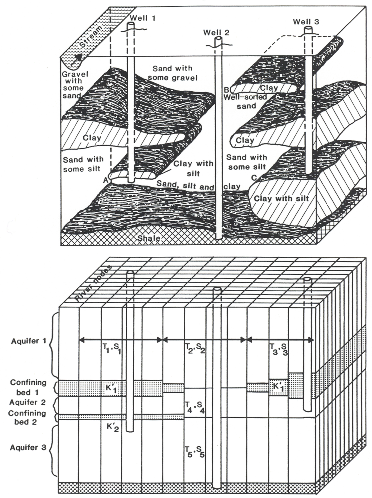
$$h = h_0 + \frac{R}{Kb}(Lx - \frac{1}{2}x^2)$$



- Analytical solutions: Exact solutions of fluid flow equations
- Fast, computationally easy
- Only for idealised conditions:
  - simple geometry only (rectangular system, circular islands, etc…), often only horizontal flow (q_z=0), homogenous K and recharge

Using the fluid flow equation to study flow systems

- Numerical models: approximate solutions of fluid flow equations
- Advantages: Can deal with complex geometries, heterogeneous properties (variable K, recharge, porosity, etc.…)
- Much better suited to simulate realistic natural flow systems
- However, computationally much more expensive and potentially inaccurate



Summary

- Change in solute concentration in response to a solute flux in the subsurface is determined by porosity
- Change in temperature in response to heat flux is a function of density and heat capacity porous rock and pore water
- Change in hydraulic head in response to flux of water determined by storativity = specific storage + specific yield
- Specific storage function of compressibility of pore water and rock matrix
- The full groundwater equation is a combination of the fluid mass balance for a fixed volume and Darcy's law
- Fluid flow equation describes the change in hydraulic head over time as a function the groundwater flux, which is a function of the derivative of the hydraulic head in space (=hydraulic gradient)
- -> we will use this equation in the exercises to model fluid flow systems over time

Next week (11 Dec.):

- The first of a series of lectures on the driving forces of fluid flow
  - What drives groundwater flow
  - Topography-driven flow
- Reading material: Garven (1995) Continental-scale groundwater flow and geologic processes.
  - Excellent introduction the importance of topography-driven flow and the other driving forces for crustal fluid & heat flow.