Practical problems

Problem 1:

1. Define some line ($y = ax + b$) in $\mathbb{R}^2$.
2. Generate N random points in $\mathbb{R}^2$ which approximates the line defined previously.
3. Plot this data.
4. Now write a program that solves the best approximation problem for the above, that is:

$$\min_{a,b\in\mathbb{R}} \frac{1}{2}\left(\sum_{i=1}^{N} \left(y_i - (ax_i - b)\right)^2 \right)$$

You can also try to plot the correlation of the effort in time in the course and the corresponding final grade.

Problem 2:

Given n points $\{\vec{x}_i\}_{i=1}^n$ (for example images of dogs and cats) and a corresponding labels $\{y_i\}_{i=1}^n$ (for example 1 is for a dog image and -1 for cat). Read about classifiers, support vector machine and implement a code for separating between dogs and cats.
For mathematical background see:
https://medium.com/@ankitnitjsr13/math-behind-support-vector-machine-svm-5e7376d0ee4d
and
https://en.wikipedia.org/wiki/Support-vector_machine#Linear_SVM

Problem 3:

A *Chebyshev center* (https://en.wikipedia.org/wiki/Chebyshev_center) is a geometric problem in which for a given bounded set C, the aim is to find the center of a minimal-radius ball enclosing C, or alternatively the center of largest inscribed ball.

So, given random n points $\{\vec{x}_i\}_{i=1}^n$, plot them and find their Chebyshev center.

Problem 4:

Given n data points $\{\vec{x}_i\}_{i=1}^n$ representing the locations of customers and given are weights $\{w_i > 0\}_{i=1}^n$ that stan for their demands.

The goal is to locate $1 \le K < n$ facilities and assign each customer to one facility, so as to solve:

$$\min_{c_1,\dots,c_k} \sum_{k=1}^{K} \sum_{x_i \in C_k} w_i \left\| x_i - c_k \right\|^2$$

where $\{c_k\}$ are the locations (or centers) of the facilities, and C_k is the cluster of customers that are assigned to the k-th–facility.
Solve this problem for random data and a general $K \in \mathbb{N}$
or for $K = 1$ the Fermat–Weber location problem (http://benisrael.net/IYIGUN-BENISRAEL-GEN-W.pdf), find a point c that minimizes:

$$\min_{c} \sum_{i=1}^{n} w_i \left\| x_i - c \right\|^2$$

Problem 4:
Read the file “IMAGE INTERPOLATION” and study the work:

S. Sajikumar and A. K. Anilkumar, Image compression using Chebyshev polynomial surface fit, International Journal of Pure and Applied Mathematical Sciences 10(1) (2017), 15-27.

Implement the proposed algorithm (in the above paper) and test it for the given images in the paper. Try other polynoms (Lagrange, Newton) and compare their performances.