32021017 7
Mohamed Eldagla

Section: 9
Sheet 1 Q 3

① $T(n) = \overset{a}{3}\, T(n/\overset{b}{2}) + n^{2^{d}}$

$a = 3$ $b = 2$ $d = 2$

$n^{\log_2 3} < n^2$

$O(n^2)$ Case 3

② $T(n) = 4T(n/2) + n^2$

$n^{\log_2 4} = n^2$ Case 2

$O(n^2 . \log_2 n)$

③ $T(n) = T(n/2) + 2^n$ $n^{0} < 2^n$

$f(n)$ is not a Polynomial

Can't be solved be Master Method

Using Master generalization $O(2^n)$

④ $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$, it must be a Constant and > 1

Can't be solved

⑤ $T(n) = 16T(n/4) + n$

$a = 16$ $b = 4$ $d = 1$

$n^{\log_4 16}$ Case 1

$n^2 > n$

$O(n^2)$

⑥ $T(n) = 2T(n/2) + n \log n$

$a = 2$, $b = 2$ $d = 1$, $k = 1$ Case 2

$O(n \log_2^2 n)$

$n^{\log_2 2} = n^1 = n$

(12)

(7) $T(n) = 2T(n/2) + n/\log n$

$F(n) = n \log^{-1} n$ $\qquad$ K must be > 0

Can't be solved by master

(8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2$, $b = 4$ $\qquad$ $d = 0.51$

$n^{\log_4 2}$ $\qquad$ Case 3

$n^{0.5} < n^{0.51}$ $\qquad$ $O(n^{0.51})$

(9) $T(n) = 0.5\,T(n/2) + \frac{1}{n}$

$a = 0.5$, $b = 2$

$T(n) = 0.5\,T(n/2) + n^{-1}$

$a = 0.5$, $b = 2$, $d = -1$

$n^{\log_2 0.5}$

$n^{-1} = n^{-1}$

Can't be solved directly by Master as $a = 0.5 < 1$ but I solved it with other method

$O(n^{-1} \cdot \log_2 n) = O(\frac{1}{n} \cdot \log_2 n)$

(10) $T(n) = 16T(n/4) + n!$ $\qquad$ Master generalization Case 3

Can't be solved by Master as $f(n)$ not polynomial but if we solved it will be $O(n!)$

(11) $T(n) = \sqrt{2}\,T(n/2) + \log n$

Can't be solved by master

We can solve it by Master theorem generalization

as $n^{\log_2 \sqrt{2}} = n^{\frac{1}{2}} = \sqrt{n} > \log n$

$O(\sqrt{n})$

(12) $T(n) = 3T(n/2) + n$

$n^{\log_2 3}$ $a = 3,\ b = 2$

$n^{1.58} > n$ <u>Case 1</u>

$O(n^{\log_2 3})$ which is also $\simeq O(n^{\log_2 3})$

(13) $T(n) = 3T(n/3) + \sqrt{n}$

$3T(n/3) + n^{\frac{1}{2}}$

$a = 3$ $d = \frac{1}{2}$

$b = 3$

$n^{\log_3 3}$ $O(n)$

$n > n^{\frac{1}{2}}$

(14) $T(n) = 4T(n/2) + Cn$

$a = 4,\ b = 2$ $d = 1$

$n^{\log_2 4}$

$n^2 > Cn$ $O(n^2)$

(15) $T(n) = 3T(n/4) + n\ \underline{\log n}$

$a = 3,\ b = 4$

Can't be solved by Master

$n^{\log_4 3}$

$n^{0.7924} \neq n$

$n^{0<c<1} = n^{1<c<2}$

$n^{0.7924} < n^{1<c<2}$

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$O(n \log n)$

(16) $T(n) = 3T(n/3) + \frac{n}{2}$ not a Polynomial

$a=3$, $b=3$

$n^{\log_3 3} = \boxed{n}$

Using master generlization

$O(n \log_3 n)$ Case 2

We can't write $\frac{n}{2}$ or the ford n^d

So, we can not Solve it with master directly.

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(17) $T(n) = 6T(n/3) + n^2 \log n$

$a=6$ $b=3$ $d=2$

$n^{\log_3 6} = n^{1.63} \neq n^2$

Can't be Solved by Master

Using master generlization for non Polynomial

$O(n^2 \log n)$ Case 3

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(18) $T(n) = 4T(n/2) + n/\log n$ Case 1 $O(n^2)$

Can't be Solved by master

Using master generlizatio $O(n^2) > n/\log n$

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(19) $T(n) = 64T(n/8) - n^2 \log n$

$a=64$ $b=8$ $d=2$, $k=1$

$n^{\log_8 64}$

$n^2 = n^2$

$O(n^2 \log_8^2 n)$

? other technique

Can't be Solved by Master as f(n) is negative

(20) $T(n) = 7T(n/3) + n^2$

$a = 7 \quad b = 3 \quad d = 1$

$n^{\log_3 7}$

$n^{1.77} < n^2$

$O(n^2)$ Case 3

(21) $T(n) = 4T(n/2) + \log n$

$a = 4 \quad b = 2$ $O(n^2)$

Can't be solved by Master theorem

as $f(n)$ cannot be $\log n$ and we can't write it on the form of n^d Case 1

Using Master generalization for $f(n)$ not Polynomial $O(n^2) > \log n$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

Also, we can not write the $f(n) \rightarrow n(2 - \cos n)$ on the form of n^d

Can't be solved by Master theorem

Mohamed Eldagla

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