JHARKHAND RAKSHA SHAKTI UNIVERSITY 2019

Subject : Mathematics (GE-1) F.M. : 70

Class : **BCACS Sem-I** Time : 2½ hrs. P.M. : 32

GROUP-A

1. Answer all the following questions :- 15x1=15

a) If $y = e^{ax}\sin(bx+c)$, then $y_n =$ ________.

b) If $y = \dfrac{1}{x^2+a^2}$, then $y_n =$ ________.

c) Give conditions of failure for Taylor's series and Maclaurin's series.

d) Write the expansion of e^x.

e) State Euler's Theorem on homogeneous function of two variable.

f) $\int x^n dx$ is equal to ________.

g) $\int e^x \left\{ f(x) + f^1(x) \right\}$ is equal to ________.

h) $\lim\limits_{n\to\infty} \sum\limits_{r=0}^{n-1} \left[\dfrac{b-a}{n} . f\left(a + r.\dfrac{b-a}{n} \right) \right]$ is equal to ____.

i) Write the rule of integration for integration by parts.

j) Write the Limit of the integral

$$\int_{-a}^{a} \int_{0}^{\sqrt{a^2-y^2}} f(x,y)\,dx\,dy$$

k) Write the equation of a straight line passing through two given points (x, y) and (x_2, y_2).

l) Write the co-ordinates of a point $p(x, y)$ if the direction of ones is changed by an angle but the origin is fixed.

m) Write the length of lotus-rectum for parabola $y^2 = \Delta ax$. 4

n) The directorcircle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

o) Write down the co-ordinate of foci for hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

GROUP-B

2. Answer any five questions :- 5x5=25

a) Expand $\frac{x}{e^x - 1}$ as far as x^4.

b) $u = \log(x^3 + y^3 + z^3 - 3xyz)$. Show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}.$$

c) Integrate $\int \frac{d\theta}{(2+\cos\theta)}$

d) Derive equation of tangent at the point (α, β) on the parabola $y^2 = 4ax$. Also write the Co-ordinates of the point of intersection of tangents $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$.

3.

e) Write properties of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

f) Find the equation of the normal at the point (x_1, y_1) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

g) If ϕ and ψ be differentiable scalar functions of position (x,y,z). Then prove that grad $(\phi \pm \psi)=$ grade $\phi \pm$ grad ψ.

h) Prove that a monotonic decreasing sequence tends to its lower bound.

GROUP-C

3. Answer any **TWO** questions :- 2x15=30

a) State and prove Leibnitz's Theorem.

b) Change the order of integration in

$$\int_0^{a\cos\alpha}\int_{x\tan\alpha}^{\sqrt{(a^2-x^2)}} f(x,y)dxdy$$ and verify the result when $f(x,y)=1$

c) Derive the equation of hyperbola and focal distances of a point $p(x,y)$ on the hyperbola.

d) Prove that the necessary and sufficient condition for the vector function $\vec{V}(t)$ to have constant direction is $\vec{u}\times\frac{d\vec{u}}{dt}=\vec{0}$.

e) State and prove Bolzano-weierstrass Theorem.

Jharkhand Raksha Shakti University-2017

Subject : BCACS-GE-I
Class : **Semester-I**
Pass Marks : 32

F.M. : 70
Time : 2½ Hrs.

GROUP-B

1. Answer all the following questions :- 15x1=15

a) Define homogeneous function with an example.

b) Find $\frac{\partial u}{\partial x}$ if $u = x^3 + y^3 + 3axy$

c) State Euler's theorem for the functions of too variable.

d) Can $f(x) = \sqrt{x}$ be expanded in ascending powers of x ?

e) For the curve $y = f(x)$, the length of subnormal is..................? $y\sqrt{1+y_1^2}$

f) Write down the equation of tangent at any point (α, β) of the curve $y = f(x)$. $(Y-\beta) = \frac{dy}{dx}(X-\alpha)$

g) $\int e^x \{f(x) + f^1(x)\}dx$ is equal to ....$e^x \cdot fx + C$

h) If $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx$ is equal to.......0.......

i) The volume of the solid generated by the revolution of the area bounded by the curve $y = f(x)$ y axis and $y = c,\ y = d$ about y axis is ....

j) Write down the formula for finding the area bounded by the curve $x = f(y)$, between $y = c, y = d$ and Y-axis, when the axes of coordinates are rectangular.

k) If $\vec{r} = \cos 2\pi t\vec{i} + 3\sin 2\pi t\vec{j}$, find $\frac{d\vec{r}}{dt}$ at $t = 0$.

l) Define divergence of a vector point function.

m) Evaluate $\nabla . \vec{r}$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

a) of
a funct^n f(x, y) in x + y is said to be homogeneous
funct^n of degree n, if the degree of each

n) Write down the parametric equation of the parabola $y^2 = 4ax$.

o) Define eccentricity of the conic section.

GROUP-A (5x5=25)

2. If $u = \sin^{-1} \dfrac{x^2+y^2}{x+y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$.

3. Expand $\cos x$ upto three terms using Maclaurin's theorem.

4. Show that in the curve $y = b\,.\,e^{-a/x}$, the subtangent varies as the square of the abscissa.

5. Evaluate $\displaystyle\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx$.

6. Evaluate $\displaystyle\int_0^3 \int_1^2 xy(1+x+y)\,dy\,dx$.

7. A particle moves along the curve $x = 3t^2$, $y = t^2 - 2t$, $z = t^3$, where t is the time. Find the magnitudes of velocity and acceleration at time $t = 1$.

8. Find the constants a,b,c so that the vector $\vec{v} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$ is irrotational.

9. Find the equation of the tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, at point (x_1, y_1).

GROUP-C (2x15=30)

10. State and prove Leibnitz theorem.

11. Show that the intercepts of the tangent to $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the coordinate axes is constant.

12. Find the volume of this sphere with radius 'a'.

13. Show that $r^m.\vec{r}$ is solenoidal if $m = -3$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

JHARKHAND RAKSHA SHAKTI UNIVERSITY

Subject : Mathematics (BCACS-102) F.M. : 70
Class : **Sem-I** Time : 2½ P.M. : 32

Group-A

1. Answer all the following questions :- $15\times1=15$

a) State Leibnitz Theorem

b) For the curve $y=f(x)$, the length of subtangent is......?

c) Give one condition when the expansion of any function $f(x)$ by Taylor's series will not be valid.

d) $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$, if..............?

e) Let $\vec{f}$ be a vector valued function, then geometrical meaning of div $\vec{f}$ is................

f) Let P be a point on the curve $\vec{r}=f(t)$ then geometrically $\frac{d\vec{r}}{dt}$ at P is ....

g) A sufficient condition for the vector function $\vec{u}$ of a scalar variable t to have constant magnitude is..........

h) The condition that the general equation of the second degree $ax^2+2hxy+by^2+2gx+2fy+c=0$ may represent two straight lines is.............

i) If the line $y=mx+c$ is a tangent to the parabola $y^2=4ax$ then the point of contact is............

j) The equation of normal to the parabola $y^2 = 4ax$ is.....

k) State Archimedian property of real numbers.

l) Let $A = \left\{2 + \frac{1}{n}, n \in N\right\}$ then the set of limit points of A is ....

m) Let $a_n = (-1)^n\left(1 + \frac{1}{n}\right)$ then the sequence ossillates finitely between.............

n) Limit of $\left(1 + \frac{x}{n}\right)^n$ is...............

o) Consider the double integral $\iint_R f(x,y)\,dxdy$ defined over the region R. Let the independent variable x, y be changed to u,v by transformations given by $x = \phi(u,v),\ y = \Psi(u,v)$ then the double integral is transformed into...............

Group-B

2. Answer any five questions :- 5×5=25

a) If $u = e^{xyz}$, show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$

b) If $y = e^{\tan^{-1}x}$ show that

$$(1+x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n = 0$$

c) Change the order of integration in the double integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dxdy$ and hence find the value.

d) Prove that curlgrad $\phi = \vec{0}$

3.

e) Find the directional derivatives of $f(x, y, z) = x^2yz + 4xz^2$ at the point (1, −2, −1) in the direction of the vector $2\vec{i} - \vec{j} - 2\vec{k}$.

f) Find the equation of a tangent to the conic $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ at the point (x, y)

g) Prove that the intersection of a finite number of open sets is an open set.

h) Prove that a monotonic increasing sequence bounded above tends to a limit which is its least upper bound.

Group-C

3. Answer any two questions :- 2×15=30

a) i) State and prove Euler's Theorem.

ii) If $u = tan^{-1}\dfrac{x^3 + y^3}{x - y}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin\,2u$$

b) Prove that $dir\left(\vec{u} \times \vec{v}\right) = \vec{v}.curl\,\vec{u} - \vec{u}.curl\,\vec{v}$

c) Find the volume a tetrahedron by the use of triple integral.

d) If $a_n = \dfrac{1.3.5.........(2n-1)}{2.4.6.............(2n)}$ and $b_n = \dfrac{3.5.7.........(2n+1)}{2.4.6.............(2n)}$

show that $\{a_n\}$ and $\{b_n\}$ are monotonic sequences and

$\{a_nb_n\}$ bounded within $\dfrac{1}{2}$ and 1.

JHARKHAND RAKSHA SHAKTI UNIVERSITY

END SEMESTER EXAMINATION 2021 DATE:25.03.2021

SUBJECT: BCACS-GE-1 MATHEMATICS F.M.: 70

CLASS: BCACS-I TIME: $2^1/2$ hrs P.M.: 32

SECTION A

Q. 1 Select the most appropriate answers from the following Multiple Choice Questions.
All questions are compulsory. 1x15 = 15 Marks

1. $\frac{d}{dx}\sec x$ *is*
   a) sec x sin x
   b) sec x tan x
   c) cosec x cot x
   d) sec x cot x

2. According to Euler's Theorem,
   a) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n^2u$
   b) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$
   c) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$
   d) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

3. $\int \cot x\, dx$
   a) log |sin x|
   b) log | tan x|
   c) log |sec x|
   d) None of these

4. $\int \frac{1}{|x|\sqrt{x^2-1}}dx =$
   a) $\sec x$
   b) $\sec^{-1} x$
   c) $\cos^{-1} x$
   d) $\sin^{-1} x$

5. Sum of the series $1^2 + 2^2 + 3^2 + 4^2 + ---- + n^2$ *is*
   a) $\frac{n(n+1)(2n+1)}{6}$
   b) $\frac{n(n+1)(2n+1)}{2}$
   c) $\frac{n(n+1)(2n-1)}{6}$
   d) $\frac{n(n-1)(2n+1)}{6}$

6. $\int e^x(\sin x + \cos x)dx =$
   a) $e^x \sin x$
   b) $e^x \tan x$
   c) $e^x \cos x$
   d) e^x

7. *If* $A = x\hat{i} + y\hat{j} + z\hat{k}$ then $\nabla . A$
   a) 3
   b) 1
   c) 0
   d) 2

8. *curl curl F is equal to*
   a) $grad\ div\ F - \nabla^2 F$
   b) $grad\ div\ F - \nabla F$
   c) $curl\ div\ F - \nabla^2 F$
   d) $grad\ curl\ F - \nabla^2 F$

9. The value of curl (grad f), where $f = 2x^2 - 3y^2 + 4z^2$
   a) 4x – 6y + 8z
   b) $4x\hat{i} - 6y\hat{j} + 8z\hat{k}$
   c) 0
   d) 3

10. If A(2, 0) is the vertex and the y – axis is the directrix of a parabola, then its focus is

a) F(2, 0)
b) F (-2, 0)
c) F(4, 0)
d) F (-4, 0)

11. A line passes through the point (2, 2) and is perpendicular to the line 3x + y = 3. Its y – intercept is

a) 1/3
b) 2/3
c) 1
d) 4/3

12. The eccentricity of the ellipse $4x^2 + 9y^2 + 8x + 36y + 4 = 0$ *is*

a) 5/6
b) $\sqrt{2}/3$
c) 3/5
d) $\sqrt{5}/3$

13. *If* $y = e^{ax}$ *then* $y_n =$

a) e^{ax}
b) $a^n e^{ax}$
c) ae^{ax}
d) $n^a e^{ax}$

14. According to Nested Interval Property of the real line, which of the following is true

a) $\cap_{m=1}^{\infty} I_m \neq \emptyset$
b) $\cap_{m=1}^{\infty} I_m = \emptyset$
c) $\cap_{m=1}^{\infty} I_m = 1$
d) $\cup_{m=1}^{\infty} I_m \neq \emptyset$

15. "Between any two distinct real numbers there lie infinity of rationals and infinity of irrationals. This theorem is known as

a) Density Theorem
b) *Bolzano Weierstrass theorem*
c) Sandwich Theorem
d) Schroeder–Bernstein Theorem

SECTION B

Answer any FIVE questions. **5x5 = 25**

1. *If* $u = \sin^{-1}\frac{x^2+y^2}{x+y}$, *show that* $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$
2. State and prove Archimedean property of real numbers.
3. *Show that* $Div(curl\,\vec{v}) = 0$ *i.e.* $\nabla.(\nabla \times \vec{v}) = 0$
4. *Evaluate* $\int \sqrt{tan\theta}\, d\theta$.
5. *Apply Maclaurin's Theorem to obtain the expansion of* $\log(1 + \tan x)$
6. *Find the equation of the parabola whose vertex is at* $(2,1)$ *and the directrix is* $x = y - 1$
7. *If* $A = 5t^2\hat{\imath} + t\hat{\jmath} - t^3\hat{k}, B = \sin t\,\hat{\imath} - \cos t\,\hat{\jmath}$, *find i)* $\frac{d}{dt}(A.B)$ *ii)* $\frac{d}{dt}(A \times B)$
8. *If* $y = (\sin^{-1} x)^2$, *prove that*

   i) $(1 - x^2)y_2 - xy_1 = 2$ *ii)* $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$

SECTION C

Answer any TWO questions. **15x2 = 30**

1. *Prove that* $\int_0^{\frac{\pi}{2}} \log sinx\, dx = -\frac{\pi}{2}\log 2$
2. *State and prove Bolzano Weierstrass theorem.*
3. Find equation of tangent and normal of 2nd degree general equation of conic at any arbitrary point using calculus method.
4. a) Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point (-1, -1, 2).

   b) If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = yz + zx + xy$, prove that grad u, grad v and grad w are coplanar.