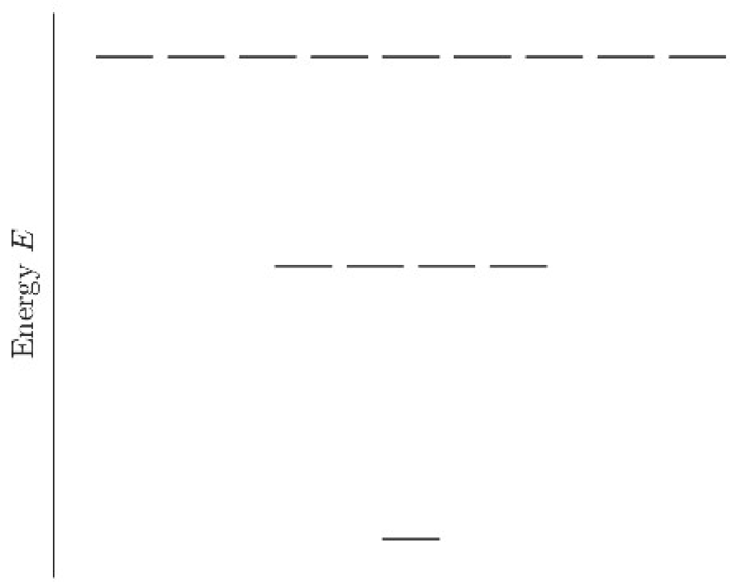
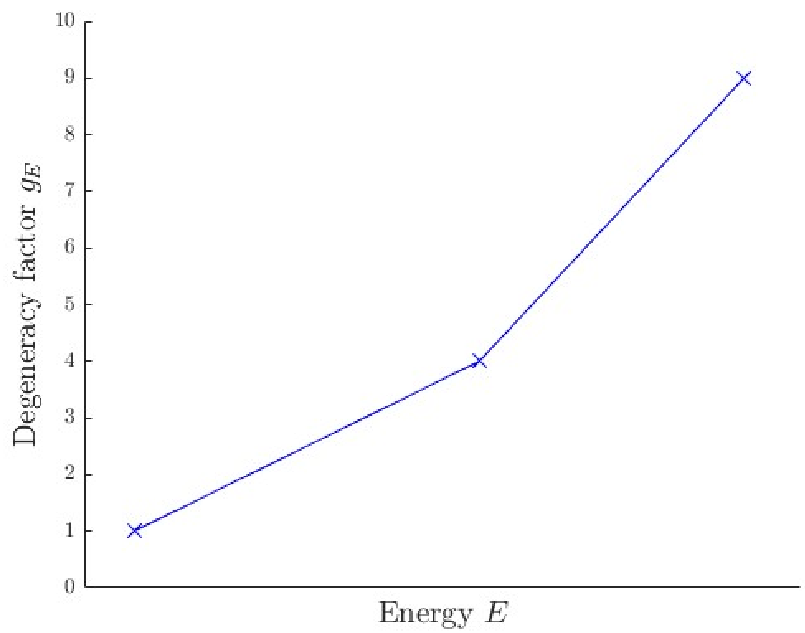
Reminder: density of states

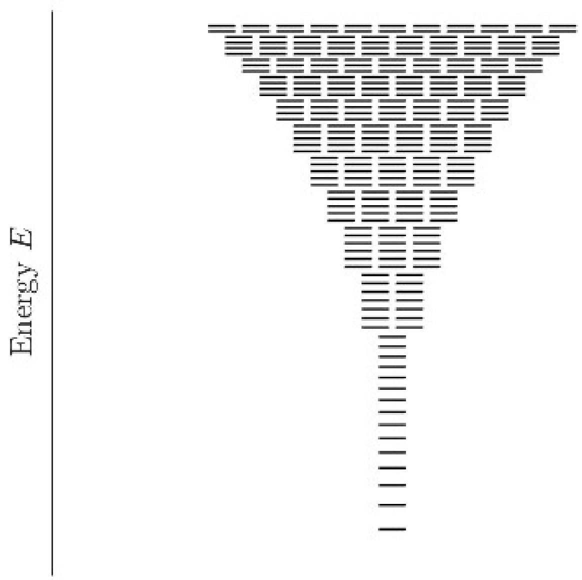
In the final tutorial, we recalled the *degeneracy factor* g_E:

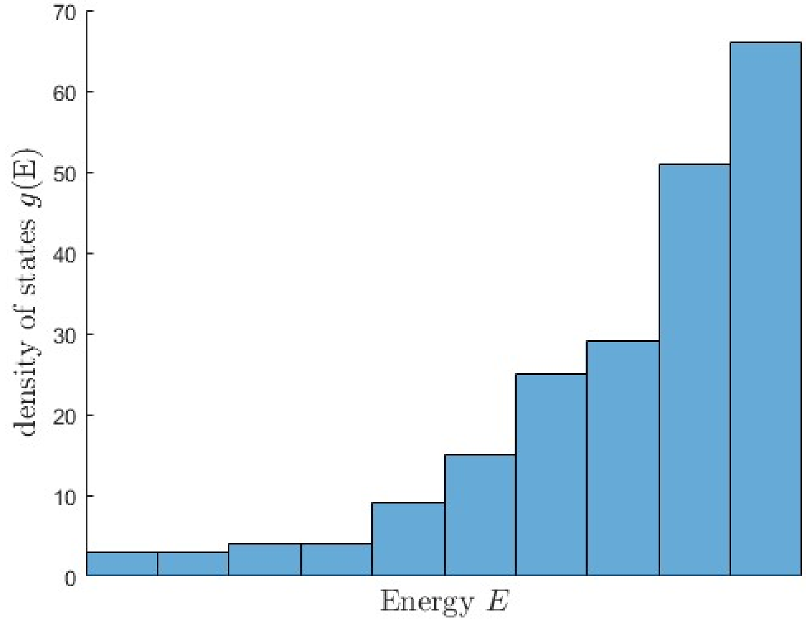




Reminder: density of states

When there are many states (thermodynamic limit) we think of this as a *density*:





Q: Given a state of energy *E*, what is it's probability of occupation?

(by a fermion)

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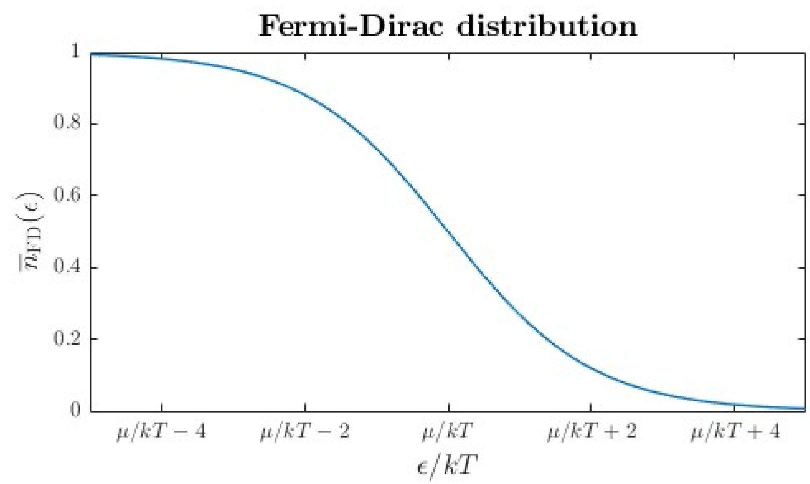
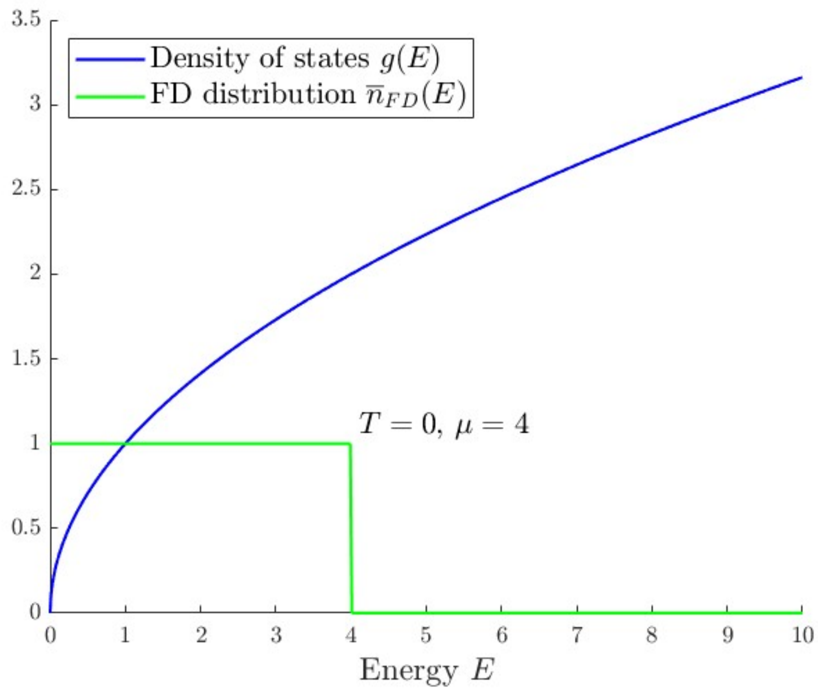
(by a fermion)

A: The Fermi-Dirac distribution:

$$\overline{n}_{\text{FD}}(E) = \frac{1}{e^{+\beta(E-\mu)} + 1}$$



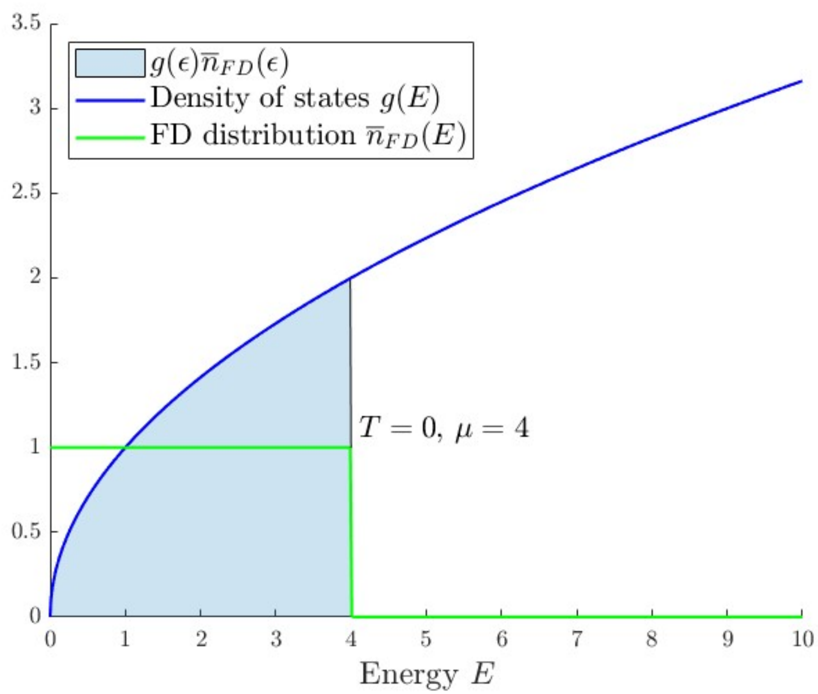
How many states are occupied around E?



At $T = 0$, what does the FD distribution look like?

How are the states occupied?

How many states are occupied around E?



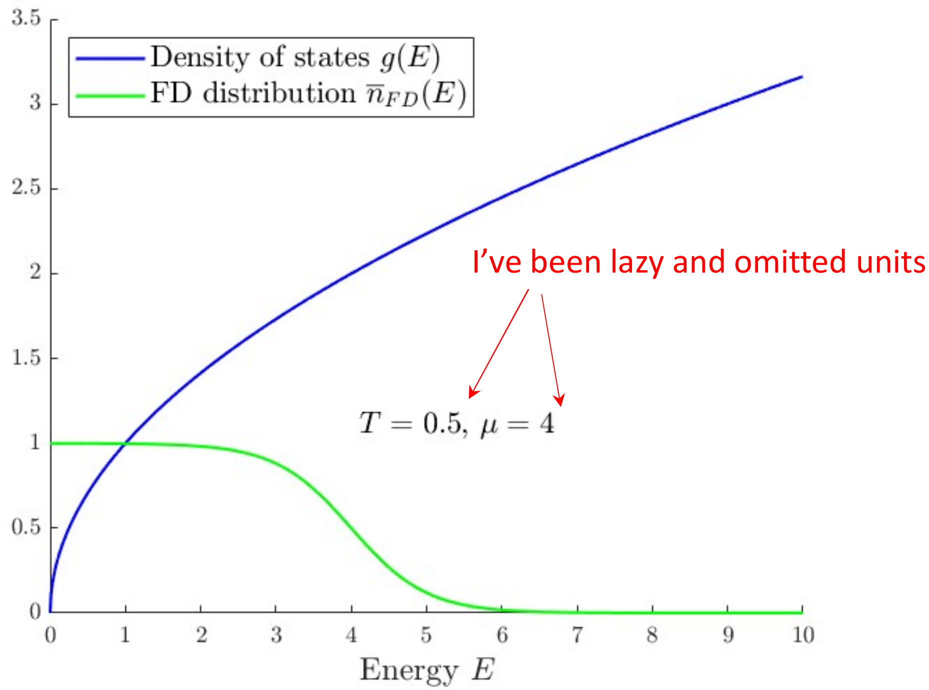
At $T = 0$, what does the FD distribution look like?

How are the states occupied?

What does the *area* mean?

$$N = \int_0^\infty g(E)\overline{n}_{\mathrm{FD}}(E)\,\mathrm{d}E$$

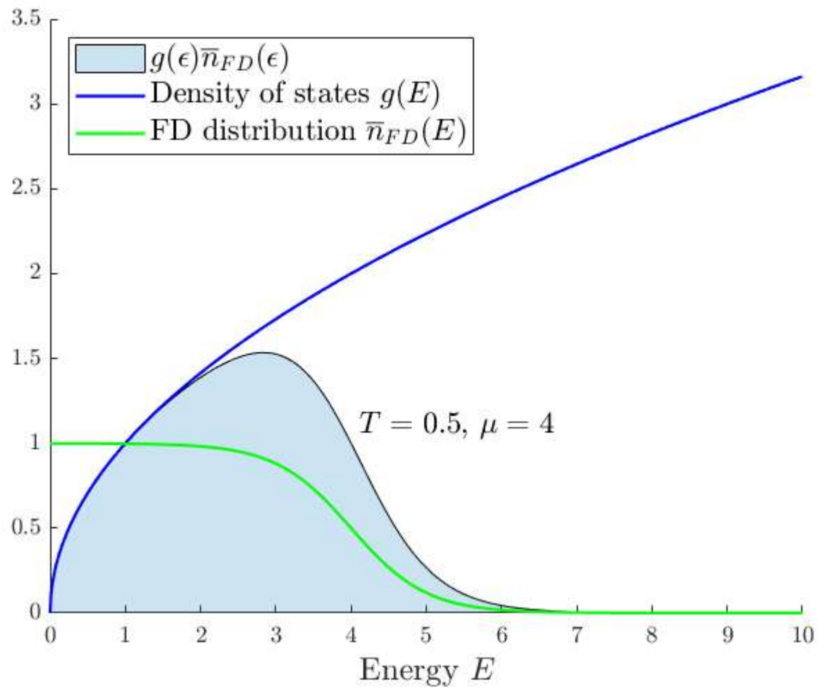
Hotter



Increase T, fix μ

How are the states occupied?

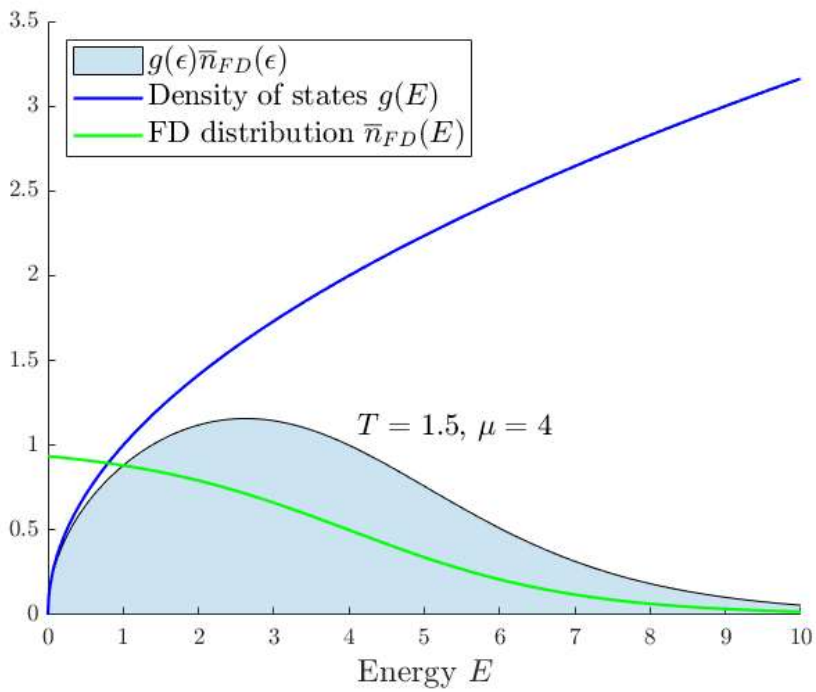
Hotter



Increase T, fix μ

How are the states occupied?

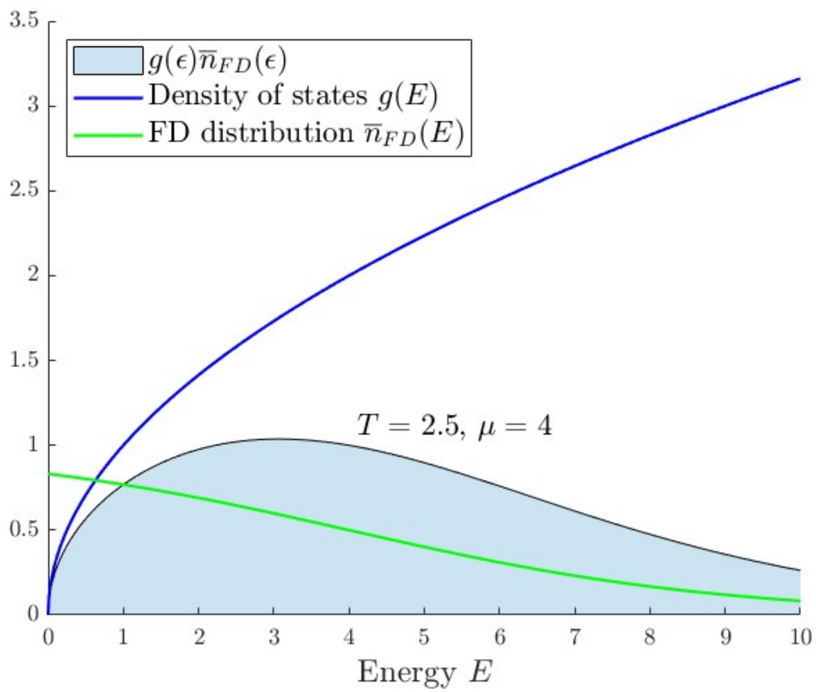
Even hotter?



Even hotter T, fix μ.

What is happening to the area?

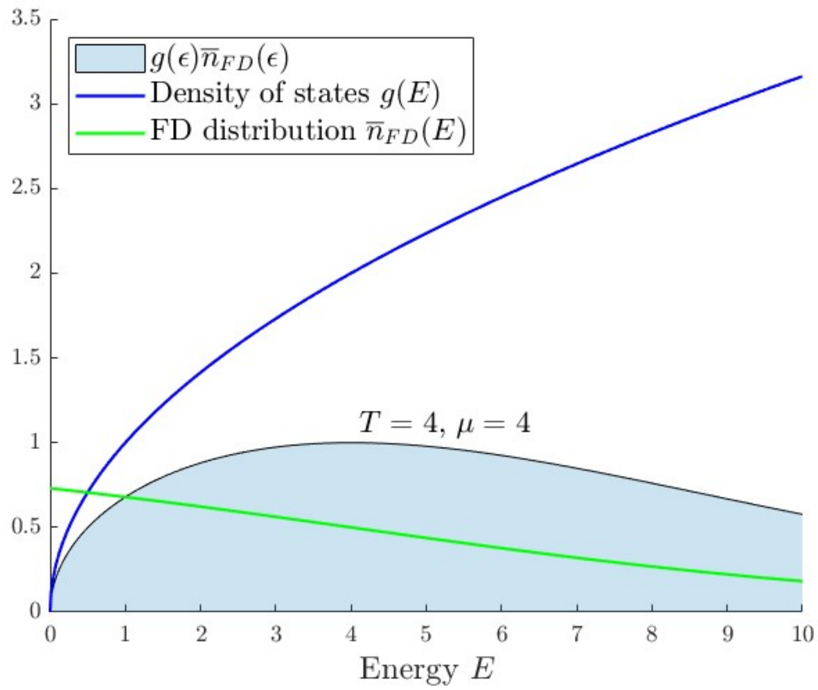
Even hotter??



Even hotter T, fix μ.

What is happening to the area?

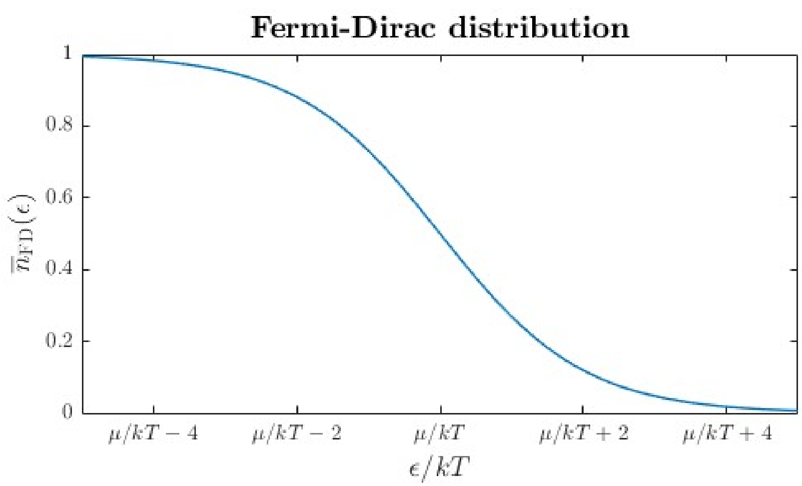
Even hotter???



Even hotter T, fix μ.

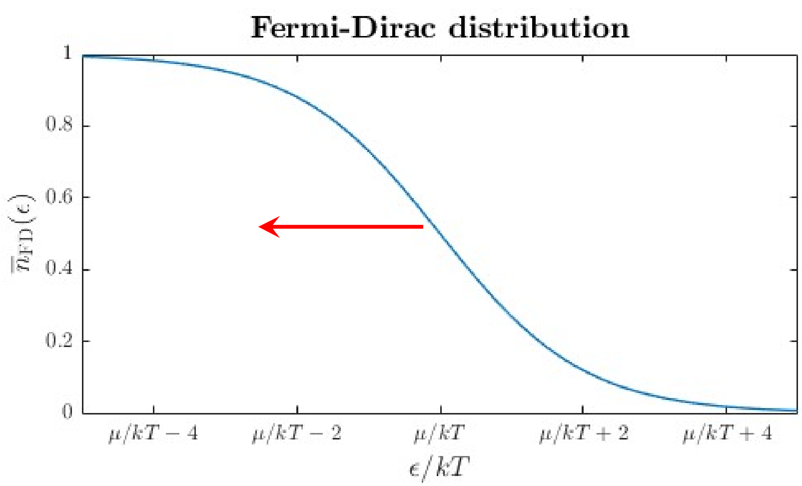
What is happening to the area?

Q: What should happen if *particle number* is conserved?



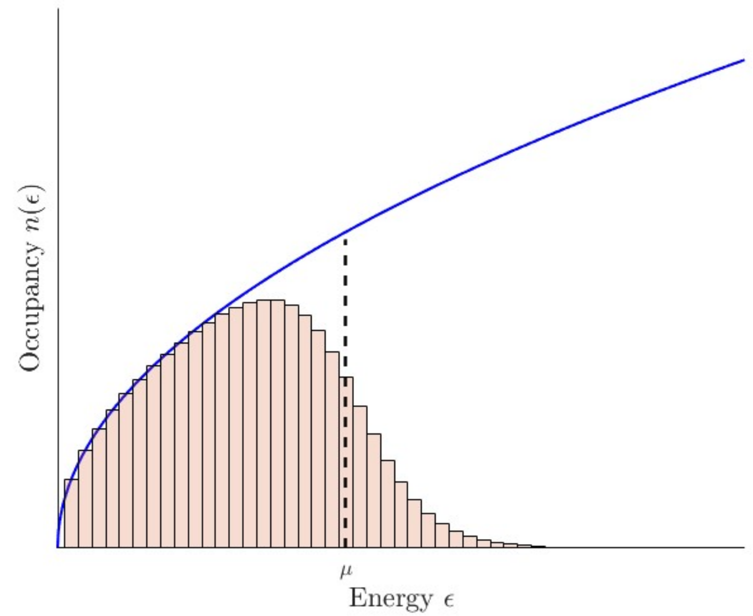
Q: What should happen if *particle number* is conserved?

A: μ should decrease

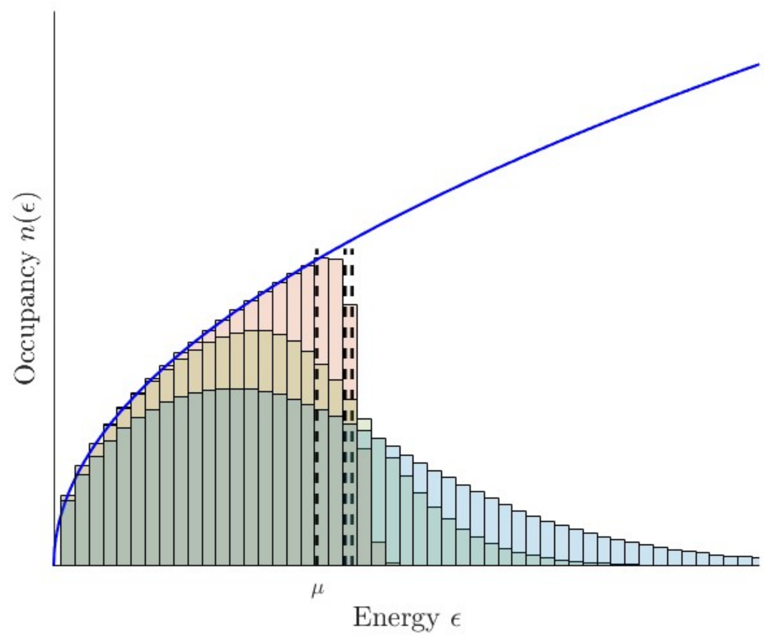


Finding μ with Matlab

```
mu = 2 % Pick a starting mu

for s = several_steps
    occupancies = g(E).*n(E,T,mu)

    Current_N = sum(occupancies)

    if (abs(Current_N) < tolerance)
        break % break out and use the current distribution
    end

    % Otherwise, correct mu to make Current_N closer to target
    % Making mu bigger makes this sum bigger, so we step appropriately
    if (Current_N > N)
        mu = mu - small_step;
    else
        mu = mu + small_step;
    end
end


% [Other code above and below]
```



μ should decrease with increasing temperature!



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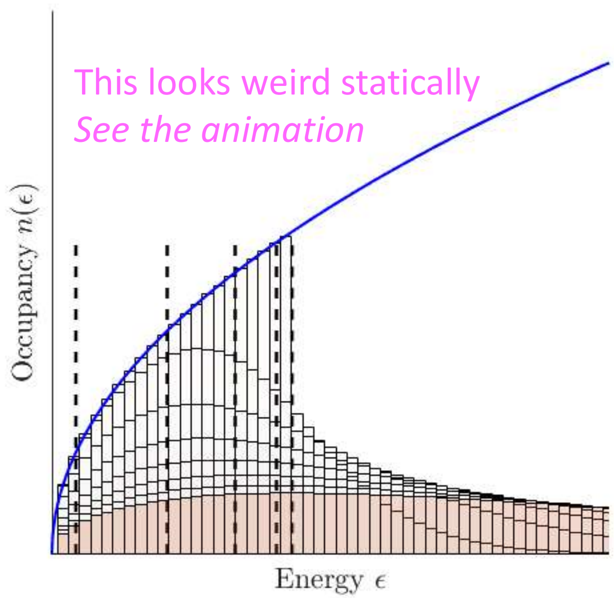

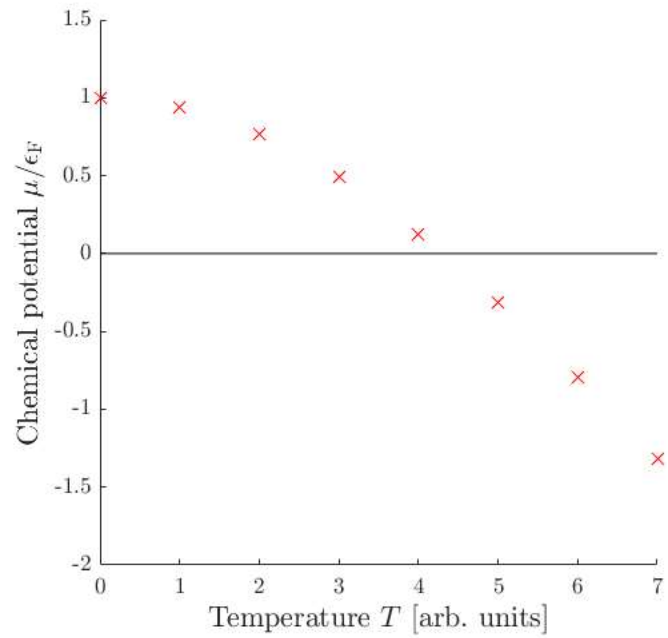

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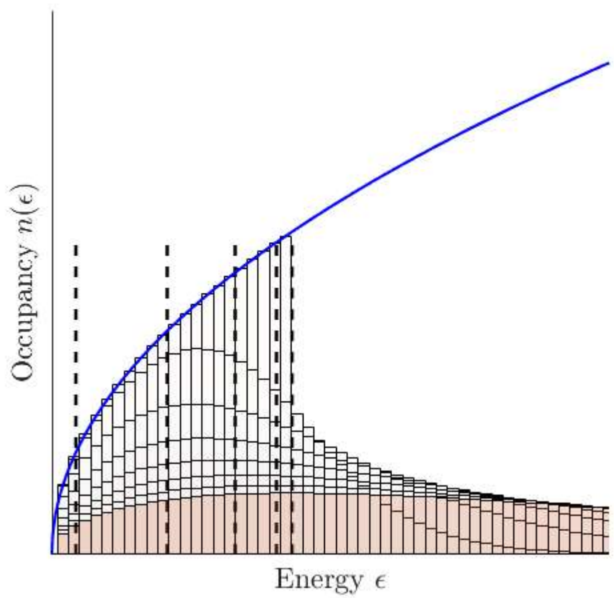
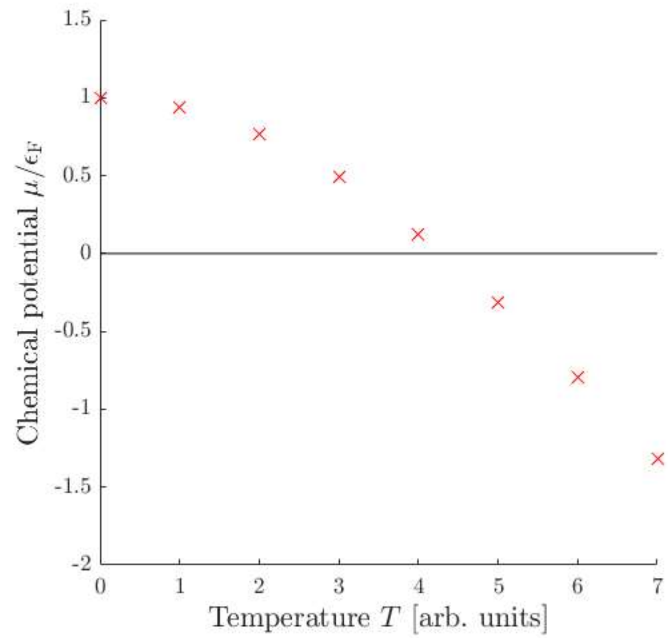




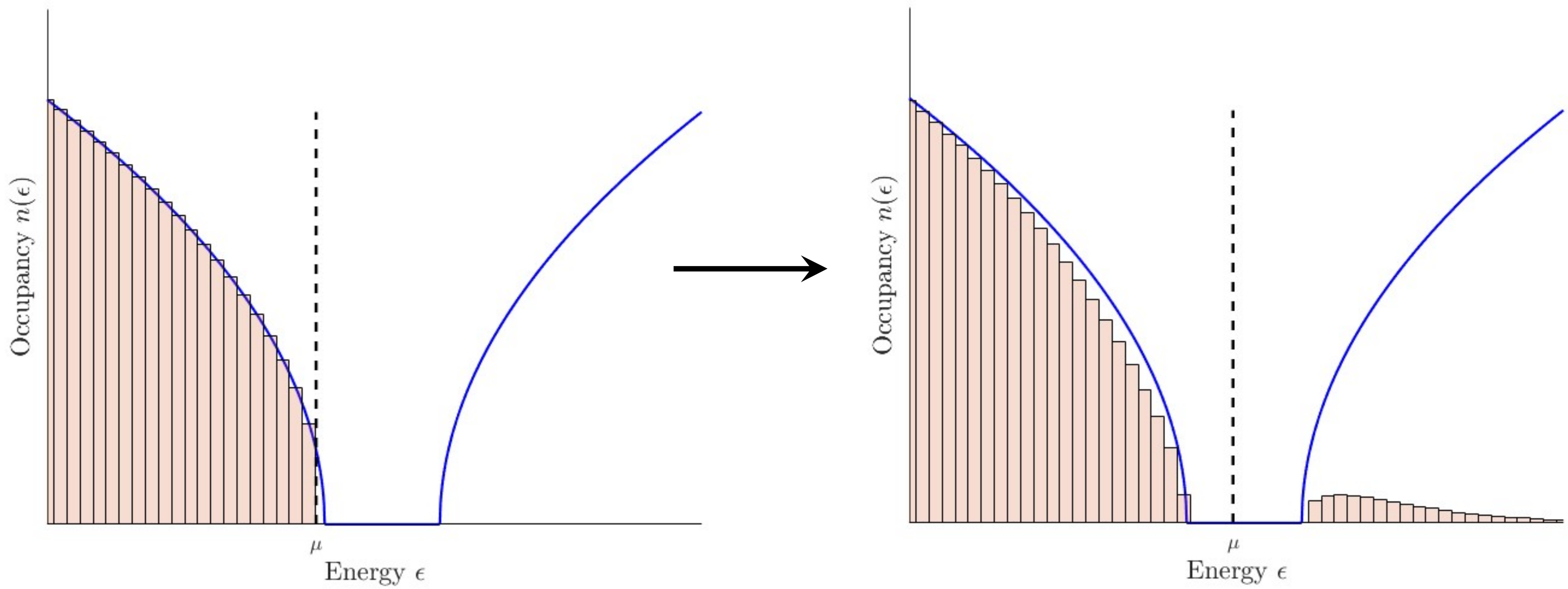
This is the *Sommerfeld expansion*

$$U = \frac{3}{4}N\epsilon_{\mathrm{F}} + \frac{\pi^2}{4}N\frac{(kT)^2}{\epsilon_{\mathrm{F}}} + \cdots$$

$$\frac{\mu}{\epsilon_{\mathrm{F}}} = 1 - \frac{\pi^2}{12}\left(\frac{kT}{\epsilon_{\mathrm{F}}}\right)^2 + \cdots$$

Condensed Matter Physics

We can also look at more exotic density-o'-state functions…



$T = 0$ nowhere for electrons to go = no conductivity

$T = 1$ electrons can move around! Conduction.