3.1 HW

3.1.1 a) $27 \in A$ = True
because 27 is an integer and multiple of 3.

b) $27 \in B$ = False
because 27 has no perfect square .. $x = y^2$ $27 = y^2$
$y = \sqrt{27}$

c) $100 \in B$ = True
because 100 has a perfect square. $100 = y^2$ $y = \sqrt{100}$
$y = \pm 10$

d) $E \subseteq C$ or $C \subseteq E$ = False
because neither are a subset of each other.

e) $E \subseteq \dot{A}$ = True
because $\{3, 6, 9\}$ are all multiples of three.

f) $A \subset E$ = False
because E can be a perfect subset of A but not the other way around.

g) $E \in A$ = False
because E is a set and not an element.

3.2 HW

3.2.1 a) $2 \in X \rightarrow$ True

b) $\{2\} \subseteq X \rightarrow$ True

c) $\{2\} \in X \rightarrow$ False

d) $3 \in X \rightarrow$ False

e) $\{1,2\} \in X \rightarrow$ True

f) $\{1,2\} \subseteq X \rightarrow$ True

g) $\{2,4\} \subseteq X \rightarrow$ True

h) $\{2,4\} \in X \rightarrow$ False

i) $\{2,3\} \subseteq X \rightarrow$ False

j) $\{2,3\} \in X \rightarrow$ False

k) $|X| = 7 \rightarrow$ False

3.2.2 a) $\{a\}$

$P(\{a\}) = \{\emptyset, \{a\}\}$

b) $\{1,2\}$

$P(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

3.3 HW

3.3.3 a) $\bigcap_{i=2}^{5} A_i = A_2 \cap A_3 \cap A_4 \cap A_5$

$= \{i^0, i^1, i^2\}$

$= \{2^0, 2^1, 2^2\} \cap \{3^0, 3^1, 3^2\} \cap \{4^0, 4^1, 4^2\} \cap \{5^0, 5^1, 5^2\}$

$= \{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\}$

$= \{1\}$

b) $\bigcup_{i=2}^{5} A_i = A_2 \cup A_3 \cup A_4 \cup A_5$

$= \{i^0, i^1, i^2\}$

$= \{2^0, 2^1, 2^2\} \cup \{3^0, 3^1, 3^2\} \cup \{4^0, 4^1, 4^2\} \cup \{5^0, 5^1, 5^2\}$

$= \{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\}$

$= \{1, 2, 4, 3, 9, 16, 5, 25\}$

c) $\bigcap_{i=1}^{100} B_i$

$= \{x \in R : -1 \leq x \leq 1/100\}$

d) $\bigcup_{i=1}^{100} B_i$

$= \{x \in R : -100 \leq x \leq 1\}$

e) $\bigcap_{i=1}^{100} C_i$

$\bigcap_{i=1}^{100} = \{x \in R : -\frac{1}{i} \leq x \leq \frac{1}{i}\} = [-\frac{1}{i}, \frac{1}{i}]$

$= \{-\frac{1}{100}, \frac{1}{100}\} \cap \{-\frac{1}{99}, \frac{1}{99}\} \cap \{-\frac{1}{88}, \frac{1}{88}\} \cap \ldots \cap \{-1, 1\}$

$\bigcap_{i=1}^{100} C_i = \{-\frac{1}{100}, \frac{1}{100}\}$

3.3.3 f) $\bigcup_{i=1}^{100} C_i \qquad C_i = \{x \in \mathbb{R} : -1/i \le x \le \frac{1}{i}\}$

$\bigcup_{i=1}^{100} C_i = \{x \in \mathbb{R} : \frac{-1}{i}, \frac{1}{i}\}$

$\bigcup_{i=1}^{100} C_i = \{x \in \mathbb{R} : -1 \le x \le 1\}$

3.4.4 a) $|A \cap B| = 1 \rightarrow$ False

b) $\{1, 2\} \subset P(A) \rightarrow$ True

c) $G \subseteq H \longrightarrow$ False

d) $|C - F| = 1 \longrightarrow$ False

e) $A \cup B = A \oplus B \rightarrow$ True

f) $1 \in A \cap B \cap C \rightarrow$ False

g) $\emptyset \in C \longrightarrow$ False

h) $\{\{0\}\} \subseteq P(C) \longrightarrow$ True

i) $C \cap F = C \cap G \rightarrow$ True

j) $E \cup F \subseteq \mathbb{R} \longrightarrow$ True

k) $\emptyset \in P(B) \longrightarrow$ True

3.5. HW

3.5.1 a) $(B \cap C) \cup \overline{B \cap C} = U \rightarrow$ Complement Law

b) $\overline{A \cup (A \cap B)} = \overline{A} \rightarrow$ Absorption Law

c) $A \cup (\overline{B \cap C}) = A \cup (\overline{B} \cup \overline{C}) \rightarrow$ De Morgan's Law

d) $(B \cap \overline{\overline{C}}) = B \cap C \rightarrow$

e) $(B-A) \cup (B-A) = (B-A) \rightarrow$ Idempotent

f) $((A \oplus B) - C \cap \emptyset = \emptyset \rightarrow$ Domination Law

3.5.2 a) $(\bar{A} \cap C) \cup (A \cap C) = C$

$= \{(\bar{A} \cap C) \cup A\} \cap \{(\bar{A} \cap C) \cup C\}$ distribution law

$= \{(A \cup A) \cap (C \cup A)\} \cap \{(\bar{A} \cup C) \cap (C \cup C)\}$ Identity law

$= \{1 \cap (C \cup A)\} \cap \{(\bar{A} \cup C) \cap C\}$ Idempotent law

$= \{(C \cup A) \cap C\}$ Absorption law

$= C$

3.5.2 b) $(B \cup A) \cap (\bar{B} \cup A) = A$

$= (B \cap \bar{B}) \cup A$ distributive law

$= \emptyset \cup A$ complement law

$= A$ Identity law

c) $\overline{A \cap \bar{B}} = \bar{A} \cup B$

LHS $= \bar{\bar{B}} \cup \bar{A}$ Demorgan's Law

$= B \cup \bar{A}$ Commutative law

$= \bar{A} \cup B$

d) $\bar{A} \cap (A \cup B) = \bar{A} \cap B$

$= (\bar{A} \cap A) \cup (\bar{A} \cap B)$ distributive law

$= \emptyset \cup (\bar{A} \cap B)$ complement law

$= \bar{A} \cap B$ Identity law

e) $\bar{A} \cup (A \cap B) = \bar{A} \cup B$

LHS $= (\bar{A} \cup A) \cap (\bar{A} \cup B)$ distributive law

$= U \cap (\bar{A} \cup B)$ complement law

$= \bar{A} \cup B$ identity law

f) $A \cap (B \cap \bar{B}) = \emptyset$

$(B \cap \bar{B}) = \emptyset$

$= A \cap (\emptyset)$ Complement law

$= A$ domination law

3.5.2 g) $A \cup (B \cup \bar{B}) = U$

$A \cup (U) =$ complement law

$U =$ domination law

$U = U$

$A \cup \bar{A} = U$

$A \cup U = U$

3.7.1 a) No, the sets A, B, C do not form a partition with D.

b) No, the sets B and C do not form a partition of D.

c) yes, the sets B and C form a partition of set E

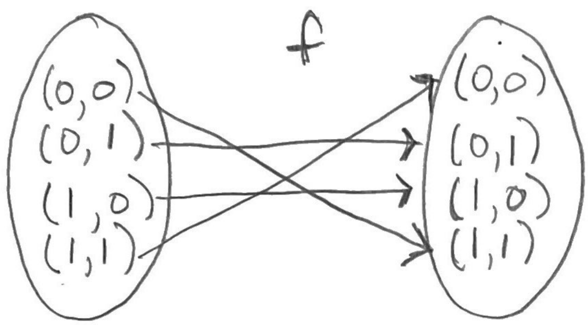
3.8 HW

3.8.4 let $B=(0,1)$, $f: B\times B \to B\times B$ as $f(x,y)=(1-y, 1-x)$

a) $B\times B = \{(0,0), (0,1), (1,0), (1,1)\}$

Domain: $\{(0,0), (0,1), (1,0), (1,1)\}$

b)



c) Range: $\{(0,0),(0,1),(1,0),(1,1)$

3.10 HW

3.10.2 a) $f: R \to R$. $f(x) = x^2$

- when $f(1) = 1^2$ and $f(-1) = -1^2$

$= 1$ and $= 1$

2 different elements of x map to 2 different elements of y. Hence its not one-to-one

- when $-1 \in R$ there doesn't exist any $x \in R$ that $f(x) = x^2 = -1$. Such that f is not onto.

Neither

b) $g: R \to R$. $g(x) = x^3$

Let $x_1, x_2 \in R$ such that $g(x_1) = g(x_2)$

$= x_1^3 = x_2^3$

$= (x_1 - x_2)(x_1^2 + x_2^2 + x_1 x_2) = 0$

$= x_1 = x_2$

g is one-to-one. Now for every $y \in R$, $\exists x \in R$ such that $g(x) = y$, so g is also onto.

Both one-to-one and onto

c) $h: Z \to Z$. $h(x) = x^3$

let $x_1, x_2 \in R$ such that $h(x_1) = h(x_2)$

$= x_1^3 = x_2^3$

$= (x_1 - x_2)(x_1^2 + x_2^2 + x_1 x_2) = 0$

$= x_1 = x_2$

h is one-to-one. Now let $y \in Z$ such that there does not exist any $x \in Z$ that $h(x) = y$. Thus h is not onto.

h is one-to-one, but not onto.

3.10.2 d) $f: Z \to Z$, $f(x) = \left\lceil \frac{x}{5} \right\rceil - 4$

Let $1, 2 \in Z$ such that

$f(1) = \left\lceil \frac{1}{5} \right\rceil - 4 \quad f(2) = \left\lceil \frac{2}{5} \right\rceil - 4$

$= 0 - 4 \quad = 0 - 4$

$f(1) = -4 \quad f(2) = -4$

Since $x_1 \neq x_2$ and $f(1) = f(2)$ then the function f is not one-to-one. For any integer $\lceil 2 \rceil = 2$ so the function is onto

f is not one-to-one but is onto

e) $f: Z \to Z$, $f(x) = 5x - 4$

let $x_1, x_2 \in Z$ $\quad f(x_1) = f(x_2)$

$5(x_1) \cancel{-4} = 5(x_2) \cancel{-4}$

$5(x_1) = 5(x_2)$

$x_1 = x_2$

f is one-to-one. Now let $f(x) = 3$

$5x - 4 = 3$

$+4 \quad +4$

$\frac{5x}{5} = \frac{7}{5} \quad x = \frac{7}{5} \notin Z$

$f(x) = 3 = \frac{7}{5} \notin Z$

f is one-to-one, but is not onto

f) $f: Z \to Z$, $f(x) = x - 4$

let $x_1, x_2 \in Z$

$f(x_1) = (x_1) - 4 \quad x_1 = x_2$

$f(x_2) = (x_2) - 4$

f is one-to-one. Now let $f(x) = y$ & $x = f^{-1}(y)$

$y = x - 4 \quad x = x + 4$

$x = f^{-1}(y) = y + 4 = f^{-1}(y)$

$f^{-1}(x) = x + 9$

f is both one-to-one and onto

3.11 HW

3.11.2 a) $f(x) = x+3$ $\quad f(y) = y+3$

$x = y$ such that f is one-to-one and therefore has a well defined inverse.

When $f(x) = x+3 = y$, then $x = y-3$, $f^{-1}(y) = y-3$ is the inverse.

b) $x, y \in \mathbb{Z}$, $f(x) = f(y) \rightarrow 2x+3 = 2y+3$

$x = y$ such that f is one-to-one.

When $f(x) = 2x+3 = y$, then $x = \frac{y-3}{2}$,

$f(y) = \frac{y-3}{2}$

$f(2) = \frac{2-3}{2}$

$f(2) = -\frac{1}{2} \notin \mathbb{Z}$ and therefor the inverse is not well defined

c) $x, y \in \mathbb{Z}$, $f(x) = f(y) \rightarrow 2x+3 = 2y+3$

$x = y$ so that f is one-to-one.

When $f(x) = 2x+3 = y$, then $x = \frac{y-3}{2}$, such that

$f^{-1}(y) = \frac{y-3}{2}$ is the inverse and is well defined

d) The function is not one-to-one, since there are different subsets of A having the same cardinality.

$C = \{1, 2, 3, 4\}$, $D = \{2, 3, 4, 5\}$

$|C| = 4$ $\quad |D| = 4$

$C = D$ such that f is not one-to-one and does not have a well defined inverse.

e) $X \subseteq A$, $f(X) = \overline{X}$ where $A = \{1,2,3,4,5,6,7,8\}$

since $\overline{X} = \overline{Y}$ which is also $X = Y$ the inverse is well defined. $C = \{1, 2, 3, 4\}$ $D = \{2, 3, 4, 5\}$

$f(C) = \overline{C} = \{5, 6, 7, 8\}$

$f(D) = \overline{D} = \{1, 6, 7, 8\}$ $\quad$ Well defined.

f) Since different sequences can be mapped to the same sequence as with $f(001) = 101$, $f(101) = 101$

Meaning the inverse function can not be well defined

3.12 HW

3.12 a) Range of $g = \{2, 3\}$

b) Domain of $h \circ g = \{a, b, c\}$

c) $h^{-1}(y) = 3$

d) domain of $h^{-1} \circ h = \{1, 2, 3, 4\}$

e) $(h \circ g)(b)$

$$= h(g(b))$$
$$= h(2)$$
$$= w$$

f) g is neither one-to-one or onto

g) h is a bijection, but g is not a bijection.