HW 1.1

1.1.2 a) $n \wedge m$
b) $t \wedge m$
c) $n \vee m$
d) $\neg m$
e) $t \wedge n$
f) $\neg t$

1.1.3 a) true
b) false
c) true
d) true
e) false
f) true

1.1.4 a) inclusive: true
exclusive: true
b) inclusive: false
exclusive: false
c) inclusive: true
exclusive: false
d) inclusive: true
exclusive: true
e) inclusive: true
exclusive: false

HW 1.2

1.2.4 a) $\neg p \oplus q$

Truth Table

p	q	$\neg p$	$\neg p \oplus q$
T	T	F	T
T	F	F	F
F	T	T	F
F	F	T	T

b) $\neg(p \vee q)$

Truth Table

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

c) $r \vee (p \wedge \neg q)$ Truth Table

r	p	q	$\neg q$	$p \wedge \neg q$	$r \vee (p \wedge \neg q)$
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

1.2.4 d) $(r \vee p) \wedge (\neg r \vee \neg q)$

Truth Table

r	p	q	$\neg r$	$\neg q$	$r \vee p$	$\neg r \vee \neg q$	$(r \vee p) \wedge (\neg r \vee \neg q)$
T	T	T	F	F	T	F	F
T	T	F	F	T	T	T	T
T	F	T	F	F	T	F	F
T	F	F	F	T	T	T	T
F	T	T	T	F	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	F	F	T	F
F	F	F	T	T	F	T	F

1.2.7 a) $B \vee D \vee M$
b) $B \wedge (D \vee M)$
c) $B \vee (D \wedge M)$

1.2.8 a) $r \wedge (p \vee q)$
$(r \wedge p) \vee q$

(1) $r = p = T \wedge q = F$

(2) $r = q = F \wedge p = T$

r	p	q	$r \wedge (p \vee q)$	$(r \wedge p) \vee q$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	F
F	F	F	F	F

1.2.8 b) $\neg p \wedge q$

$\neg(p \wedge q)$

① $p = T, q = F$

② $p = q = F$

p	q	$\neg p \wedge q$	$\neg(p \wedge q)$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	F	T

c) $p \vee q$

$(\neg p \wedge q) \vee (p \wedge \neg q)$

p	q	$\neg p \wedge q$	$p \wedge \neg q$	$p \vee q$	$(\neg p \wedge q) \vee (p \wedge \neg q)$
T	T	F	F	T	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	F	F	F	F

When $p = q =$ True, $p \vee q$ and $(\neg p \wedge q) \vee (p \wedge \neg q)$ have different truth values

1.3 HW

1.3.2 a) If she finished her homework, then she went to the party. ($p \to q$)

$\neg p \to \neg q$ Inverse: If she did not finish her homework, then she did not go to the party.

$\neg q \to \neg p$ Contrapositive: If she did not go to the party, then she did not finish her homework.

$q \to p$ Converse: If she went to the party, then she finished her homework.

b) If he trained for the race, then he finished the race. ($p \to q$)

Inverse: If he did not train for the race, then he did not finish the race.

Contrapositive: If he did not finish the race, then he did not train for the race.

Converse: If he finished the race, then he trained for the race.

c) If the patient took the medicine, then she had side effects. ($p \to q$)

Inverse: If the patient did not take the medicine, then she did not have side effects.

Contrapositive: If she did not have side effects, then the patient did not take the medicine.

Converse: If she had side effects, then the patient took the medicine

d) If it was sunny, then the game was held. ($p \to q$)

Inverse: If it was not sunny, then the game was not held.

Contrapositive: If the game was not held, then it was not sunny.

Converse: If the game was held, then it was sunny.

e) Inverse: If it did not snow last night, then school will not be cancelled.

Contrapositive: If school will not be cancelled, then it did not snow last night.

Converse: If school will be cancelled, then it snowed last night.

1.3.5 a) $\neg j \rightarrow c$
b) $c \rightarrow \neg j$
c) $\neg j \wedge \neg c$
d) $c \rightarrow \neg j$
e) $\neg c$
f) $j \wedge c$

1.3.8 a) If the roads were wet, then there was heavy traffic.
b) The roads were wet and there was an accident.
c) There was not an accident or traffic was not heavy.
d) If traffic was heavy then there was an accident or the roads were wet.
e) The roads were wet and traffic was not heavy.

1.4 HW

1.4.1 a) $(p \vee q) \vee (q \to p)$ is a tautology

p	q	$p \vee q$	$q \to p$	$(p \vee q) \vee (q \to p)$
T	T	T	T	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

Always true

b) $(p \to q) \leftrightarrow (p \wedge \neg q)$ is a contradiction

p	q	$p \to q$	$p \wedge \neg q$	$(p \to q) \leftrightarrow (p \wedge \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

Always false

c) $(p \to q) \leftrightarrow p$ is neither a tautology or contradiction

p	q	$p \to q$	$(p \to q) \leftrightarrow p$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

differing truth values

d) $(p \to q) \vee p$ is a tautology

p	q	$p \to q$	$(p \to q) \vee p$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Always True

1.4.1 e) $(\neg p \vee q) \leftrightarrow (p \wedge \neg q)$ is a contradiction

p	q	$\neg p \vee q$	$p \wedge \neg q$	$(\neg p \vee q) \leftrightarrow (p \wedge \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

[illegible]

f) $(\neg p \vee q) \leftrightarrow (\neg p \wedge q)$ is neither a tautology or contradiction

p	q	$\neg p \vee q$	$\neg p \wedge q$	$(\neg p \vee q) \leftrightarrow (\neg p \wedge q)$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

different truth values

1.4.2 a) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

* have the same truth table

b) $\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$

p	q	$p \leftrightarrow q$	$\neg p$	$\neg(p \leftrightarrow q)$	$\neg p \leftrightarrow q$
T	T	T	F	F	F
T	F	F	F	T	T
F	T	F	T	T	T
F	F	T	T	F	F

* have the same truth table

1.4.2 c) $\neg p \to q \equiv p \vee q$

p	q	$\neg p$	$\neg p \to q$	$p \vee q$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	F

* have the same truth table so they are logically equivalent.

1.4.3 a) $p \to q \not\equiv q \to p$

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

* Since the truth tables are not the same, they are not logically equivalent.

b) $\neg p \to q \not\equiv \neg p \vee q$

p	q	$\neg p$	$\neg p \to q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	T

* Since the truth tables are not the same, they are not logically equivalent

1.4.3 c) $(p \to q) \wedge (r \to q) \not\equiv (p \wedge r) \to q$

p	q	r	$p \to q$	$r \to q$	$p \wedge r$	$(p \to q) \wedge (r \to q)$	$(p \wedge r) \to q$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	F	F	T	F	F
T	F	F	F	F	F	F	T
F	T	T	T	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	T	F	F	F	T
F	F	F	T	T	F	T	T

Truth tables do not match so they are not logically equivalent.

d) $p \wedge (p \to q) \not\equiv p \vee q$

p	q	$p \to q$	$p \wedge (p \to q)$	$p \vee q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	F

Truth tables do not match so they are not logically equivalent.

1.5.1

(a)

$(p \to q) \land (q \lor p)$	
$(\neg p \lor q) \land (q \lor p)$	← Conditional Identity
$(q \lor \neg p) \land (q \lor p)$	← Commutative law.
$q \lor (\neg p \land p)$	← Distributive law
$q \lor (p \land \neg p)$	← Commutative law
$q \lor F$	← Complement law
q	← Idenity law

(b)

$(\neg p \lor q) \to (p \land q)$	
$\neg(\neg p \lor q) \lor (p \land q)$	← Conditional identity
$(\neg\neg p \land \neg q) \lor (p \land q)$	← De Morgan's law
$(p \land \neg q) \lor (p \land q)$	← Double Negation
$p \land (\neg q \lor q)$	← Distributive law
$p \land T$	← Complement law
p	← Idenity law

(c)

$r \lor (\neg r \to p)$	
$r \lor (\neg\neg r \lor p)$	← Conditional identity
$r \lor (r \lor p)$	← Double Negation
$(r \lor r) \lor p$	← Associative law
$r \lor p$	← Idemotent law

1.5.2 a) $\neg p \to \neg q \equiv q \to p$

$\neg p \to \neg q$

$\neg p \to \neg q \equiv \neg\neg p \vee \neg q$ Conditional

$\equiv p \vee \neg q$ double negation

$\equiv \neg q \vee p$ Commutative

$\equiv q \to p$ Conditional

b) $p \wedge (\neg p \to q) \equiv p$

$p \wedge (\neg p \to q) \equiv p \wedge (\neg\neg p \vee q)$ Conditional

$\equiv p \wedge (p \vee q)$ Double negation

$\equiv (p \wedge p) \vee (p \wedge q)$ Distributive

$\equiv p \vee (p \wedge q)$ Idempotent

$\equiv p$ Absorption

c) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to q) \wedge (p \to r)$

$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$ Conditional Identity

$\equiv (\neg p \vee q) \wedge (r \vee \neg p)$ Commutative

$\equiv \neg p \vee (q \wedge r) \vee \neg p$ Associative

$\equiv \neg p \vee \neg p \vee (q \wedge r)$ Commutative

$\equiv (\neg p \vee \neg p) \vee (q \wedge r)$ Associative

$\equiv \neg p \vee (q \wedge r)$ idempotent

$\equiv p \to (q \wedge r)$ Conditional Identity

1.5.2 d) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg p \to (q \to r)$

$\equiv \neg p \to (\neg q \vee r)$ Conditional

$\equiv \neg(\neg p) \vee (\neg q \vee r)$ Conditional

$\equiv p \vee (\neg q \vee r)$ Double negation

$\equiv \neg q \vee (p \vee r)$ Commutative

$\equiv q \to (p \vee r)$ Conditional

e) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

$(p \to r) \vee (q \to r)$

$\equiv (\neg p \vee r) \vee (\neg q \vee r)$ Conditional

$\equiv (\neg p \vee \neg q) \vee (r \vee r)$ Commutative

$\equiv (\neg p \vee \neg q) \vee r$ Idempotent

$\equiv \neg(p \wedge q) \vee r$ De Morgans

$\equiv (p \wedge q) \to r$ Conditional

$\equiv$

f) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\neg(p \vee (\neg p \wedge q))$

$\equiv (\neg p \wedge \neg(\neg p \wedge q))$ De Morgans

$\equiv \neg p \wedge (p \wedge \neg q)$ Double negation

$\equiv (\neg p \wedge p) \wedge \neg q$ Associative

$\equiv (F) \vee \neg q$ Complement

$\equiv \neg p \wedge (F \vee \neg q)$ Associative

$\equiv \neg p \wedge \neg q$ Identity

1.5.2 g) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ distributive

$(p \vee p) \wedge (q \vee \neg q) \wedge (\neg r \vee \neg r)$

$p \wedge (q \vee \neg q) \wedge \neg r$ Idempotent

$p \wedge T \wedge \neg r$ Complement

$p \wedge \neg r$ identity

h) $p \leftrightarrow (p \wedge r) \equiv \neg p \vee r$

$p \leftrightarrow (p \wedge r)$

$\equiv (p \to (p \wedge r)) \wedge ((p \wedge r) \to p$ biconditional

$\equiv (\neg p \vee (p \wedge r)) \wedge (\neg (p \wedge r) \vee p)$ conditional

$\equiv (\neg p \vee p) \wedge (\neg p \vee r) \wedge (\neg p \vee \neg r \vee p)$ distributive

$\equiv [T \wedge (\neg p \vee r)] \wedge (\neg p \vee \neg r \vee p)$ Complement

$\equiv (\neg p \vee r) \wedge (\neg p \vee \neg r \vee p)$ Identity

$\equiv (\neg p \vee r) \wedge (T \vee \neg r)$ Complement

$\equiv (\neg p \vee r) \wedge T$ Domination

$\equiv (\neg p \vee r)$ Idenity

i) $(p \wedge q) \to r \equiv (p \wedge \neg r) \to \neg q$

$(p \wedge q) \to r$

$\neg (p \wedge q) \vee r$ Conditional

$(\neg p \vee r) \vee \neg q$ Associative

$\neg\neg(\neg p \vee r) \vee \neg q$ Double Negation

$\neg (p \wedge \neg r) \vee \neg q$ De Morgans

$((p \wedge \neg r) \to \neg q)$ Conditional

1.6 HW

Domain = all integers

1.6.2 a) $\exists x(x+x=1)$

False, there is no integer x that equals 1.

b) $\exists(x+2=1)$

Existential statement is true because there exist an x value

$x=-1$

$(-1)+2=1$

$1=1$ - true

c) $\forall x\,(x^2-x\neq 1)$ ← True

This is true for the domain given

$x=-1$

$(-1)^2-(-1)\neq 1$

$1+1$

$x=2\ (2^2-2\neq 1$

$4-2\neq 1$

$2\neq 1$

No x value that makes statement not true

$x=1\quad 1^2-x\neq 1$

$0=1$

d) $\forall x(x^2-x\neq 0)$ False

$x=1\quad 1^2-1\neq 0$

$1-1\neq 0$

$0\neq 0$ ← since x=1 makes the predicate not true the statement is false.

e) $\forall x(x^2>0)$ True

This is true for all integers so the universal statement is true

f) $\exists x(x^2>0)$

$x=1\quad 1^2>0$

$1>0$ This is true for x=1 so the existential statement is true.

1.6.4 a) $\forall x\, P(x)$

$P(a) = P(b) = P(c) = P(d) =$ true

True

b) $\exists x\, P(x)$

$P(a) =$ true

true

c) $\forall x\, Q(x)$

$Q(b) = Q(c) = Q(d) =$ False

False

d) $\exists x\, Q(x)$

$Q(a) =$ True

true

e) $\forall x\, R(x)$

$R(a) =$ false

$R(b) =$ false

False

f) $\exists x\, R(x)$

$R(a) = R(b) = R(c) = R(d) =$ False

False

1.7 HW

1.7.2 a) $\exists x(E(x) \wedge T(x))$

b) $\forall x(E(x) \rightarrow T(x))$

c) $\forall x(T(x) \rightarrow E(x))$

d) $\exists x(E(x) \wedge \neg T(x))$

1.7.4 a) $\exists x\, S(x)$
b) $\forall x(\neg S(x) \wedge W(x))$
c) $\forall x(S(x) \wedge \neg W(x))$
d) $\exists x(S(x) \wedge W(x))$
e) $\forall x(\neg W(x) \rightarrow S(x))$
f) $\forall x(\neg W(x) \rightarrow (S(x) \vee V(x)))$
g) $\exists x(\neg W(x) \wedge \neg S(x) \wedge \neg V(x))$
h) $\forall x(\neg W(x) \rightarrow (S(x) \vee V(x)))$
i) $S(\text{Ingrid}) \wedge W(\text{Ingrid})$
j) $\exists x((x \neq \text{ingrid}) \wedge S(x))$
k) $\forall x((x \neq \text{Ingrid}) \rightarrow S(x))$

1.7.10 a) False. Counter example: Jeb
b) True
c)
d) True. Example: Hilary
e) True. For any x value, m(x), O(x) or P(x) is true
f) False. Counter example: Hilary and Jeb
g)
h)
i) True. Example: Hilary
j) False. For every value of x

1.8 HW

1.8.1 a) $\neg\exists x\, P(x)$

$= \forall x\, \neg P(x)$

b) $\neg\exists x (P(x) \lor Q(x))$

$= \forall x \neg(P(x) \lor Q(x))$

$= \forall x\, \neg P(x) \land \neg Q(x)$

c) $\neg\forall x (P(x) \land Q(x))$

$= \exists x \neg(P(x) \land Q(x))$

$= \exists x\, \neg P(x) \lor \neg Q(x)$

d) $\neg\forall x [P(x) \land (Q(x) \lor R(x))]$

$= \exists x \neg[P(x) \land (Q(x) \lor R(x))]$

$= \exists x \neg P(x) \lor \neg(Q(x) \lor R(x))$

$= \exists x \neg P(x) \lor \neg Q(x) \land \neg R(x)$

1.8.2 a) $\forall x (D(x))$

Negation: $\neg\forall x (D(x))$

Applying Demorgans law: $\exists x (\neg D(x))$

English: There exist a male patient that was not given the medication

b) $\forall x (P(x) \lor D(x))$

Negation: $\neg\forall x (P(x) \lor D(x))$

Demorgan's law: $\exists x (\neg P(x) \land \neg D(x))$

English: There exist a male patient that was not given the placebo and also not given medication.

c) $\exists x (M(x) \land D(x))$

Negation: $\neg\exists x (M(x) \land D(x))$

Demorgan's law: $\forall x (\neg M(x) \lor \neg D(x))$

English: Every patient either did not have migraines, or was not given the medication.

1.8.2 d) $P(x) \to M(x) \equiv \neg P(x) \vee M(x)$

Expression: $\forall x (P(x) \to M(x))$

Negation: $\neg \forall x (P(x) \to M(x))$

De morgan's law: $\exists x (P(x) \wedge \neg M(x))$

English: There exist a patient that was given the placebo and did not have migraines

e) $\exists x (M(x) \wedge P(x))$

Negation: $\neg \exists x (M(x) \wedge P(x))$

De morgan's law: $\forall x (\neg M(x) \vee \neg P(x))$

English: Every patient either did not have migraines or was not given the placebo.

1.8.4 a) $\neg \forall (P(x) \wedge \neg Q(x)) \equiv \exists x (\neg P(x) \vee Q(x))$

Demorgan's law

$\exists x (\neg P(x) \vee \neg\neg Q(x))$ Double negation

$\exists x (\neg P(x) \vee Q(x))$

$\equiv$

b) $\neg \forall x (\neg P(x) \to Q(x)) \equiv \exists x (\neg P(x) \wedge \neg Q(x))$

$\equiv \exists x \neg (\neg (P(x)) \vee Q(x))$

$\equiv \exists x \neg (P(x) \vee Q(x))$

$\equiv \exists x (\neg P(x) \wedge \neg Q(x))$

$\equiv$

c) $\neg \exists x [\neg P(x) \vee (Q(x) \wedge \neg R(x))] \equiv \forall x [P(x) \wedge (\neg Q(x) \vee R(x))]$

$\equiv \forall x \neg (\neg P(x) \vee (Q(x) \wedge \neg R(x))$

Demorgans law

$\equiv \forall x [\neg\neg P(x) \wedge \neg (Q(x) \wedge \neg R(x))]$

$\equiv \exists x [P(x) \wedge (\neg Q(x) \vee R(x))]$

$\equiv \exists x [\neg P(x) \vee (Q(x) \wedge \neg R(x))]$

1.9 HW

1.9.1 a) $m(1,1)$ is a proposition and is true

b) $\forall y\ m(x,y)$ is not a proposition since the value of x is not defined. Truth value is not obtainable.

c) $\exists x\ m(x,3)$ is a proposition and is true.

d) $\exists x \exists y\ m(x,y)$ is a proposition and is true

e) $\exists x \forall y\ m(x,y)$ is a proposition and is false

f) $m(x,2)$ is not a proposition.

g) $\exists y \forall x\ m(x,y)$ is a proposition and is true

1.9.5 a) $\forall x \forall y\ F(x,y)$

Negation: $\neg \forall x \forall y\ F(x,y)$

Demorgan's: $\exists x \exists y \neg F(x,y)$

English: Someone is an enemy of someone.

b) $\exists x \exists y\ F(x,y)$

Negation: $\neg \exists x \exists y\ F(x,y)$

De morgan's: $\forall x \forall y \neg F(x,y)$

English: Everyone is an enemy of everyone.

c) $\exists x \forall y\ F(x,y)$

Negation: $\neg \exists x \forall y\ F(x,y)$

Demorgan's: $\forall x \exists y \neg F(x,y)$

English: Everyone is an enemy of someone

d) $\forall x \exists y\ F(x,y)$

Negation: $\neg \forall x \exists y\ F(x,y)$

Demorgan's: $\exists x \forall y \neg F(x,y)$

English: Someone is an enemy of everyone.

1.10 HW

1.10.1 a) False, when $x=2$, $y=2$ $M(x,y)$ is false so for all $M(x,y)$ is a false statement.

b) True, since $M(x,y)$ is true whenever $x=y$

c) True, since $M(2,2)$ is false which the "not" of makes a true statement

d) False, since because of the truth table. $M(x,y)$ is true whenever $x \neq y$ which disproves the statement.

e) False, take $x=1$ then its not possible to find y such that $M(x,y)$ is false.

1.10.4 a) $\exists x \exists y (\frac{x}{y} < 1)$

b) $\forall x (x > 0 \rightarrow \frac{1}{x} > 0)$

c) $\exists x \exists y (\frac{x}{y} < 1)$

d) $\exists x \exists y [(x > 0 \land y > 0) \rightarrow \frac{x}{y} > 0]$

e) $\forall x [(x > 0 \land x < 1) \rightarrow \frac{1}{x} > 1]$

f) $\neg \exists x \forall y (x \leq y)$

g) $\forall x [(x \neq 0) \rightarrow \exists y (xy = 1)]$

h) $\forall x [(x \neq 0) \rightarrow \exists! y (xy = 1)]$