M1: Homework

Graded

Student

Jonathan C Elder

Total Points

47 / 50 pts

Question 1

1.1.2 — **5** / 5 pts

- ✔ **– 0 pts** Correct 1.1.2 (a) thourgh (f)

Question 2

1.2.7 — **4** / 5 pts

- ✔ **– 1 pt** 1.2.7 graded, (b) incorrect

Question 3

1.3.5 — **5** / 5 pts

- ✔ **– 0 pts** 1.3.5 graded: Correct

Question 4

1.4.3 — **5** / 5 pts

- ✔ **– 0 pts** 1.4.3 graded: Correct

Question 5

1.5.2 — **5** / 5 pts

- ✔ **– 0 pts** Correct

Question 6

1.6.4 — **5** / 5 pts

- ✔ **– 0 pts** 1.6.4 graded: Correct

Question 7

1.7.10 — **3.5** / 5 pts

- ✔ **– 0.5 pts** (g) incorrect
- ✔ **– 0.5 pts** (c) incorrect
- ✔ **– 0.5 pts** (h) incorrect

Question 8

1.8.1 **5** / 5 pts

✔ **− 0 pts** 1.8.1 graded: Correct

Question 9

1.9.5 **5** / 5 pts

✔ **− 0 pts** 1.9.5 graded: Correct

Question 10

1.10.4 **4.5** / 5 pts

✔ **− 0.5 pts** h incorrect

Question assigned to the following page: 1

HW 1.1

1.1.2 a) $n \wedge m$
b) $t \wedge m$
c) $n \vee m$
d) $\neg m$
e) $t \wedge n$
f) $\neg t$

1.1.3 a) true
b) false
c) true
d) true
e) false
f) true

1.1.4 a) inclusive: true
exclusive: true
b) inclusive: false
exclusive: false
c) inclusive: true
exclusive: false
d) inclusive: true
exclusive: true
e) inclusive: true
exclusive: false

No questions assigned to the following page.

HW 1.2

1.2.4 a) $\neg p \oplus q$

Truth Table

p	q	$\neg p$	$\neg p \oplus q$
T	T	F	T
T	F	F	F
F	T	T	F
F	F	T	T

b) $\neg(p \vee q)$

Truth Table

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

c) $r \vee (p \wedge \neg q)$ Truth Table

r	p	q	$\neg q$	$p \wedge \neg q$	$r \vee (p \wedge \neg q)$
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

Question assigned to the following page: 2

1.2.4 d) $(r \vee p) \wedge (\neg r \vee \neg q)$

Truth Table

r	p	q	$\neg r$	$\neg q$	$r \vee p$	$\neg r \vee \neg q$	$\boxed{(r \vee p) \wedge (\neg r \vee \neg q)}$
T	T	T	F	F	T	F	F
T	T	F	F	T	T	T	T
T	F	T	F	F	T	F	F
T	F	F	F	T	T	T	T
F	T	T	T	F	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	F	F	T	F
F	F	F	T	T	F	T	F

1.2.7 a) $B \vee D \vee M$

b) $B \wedge (D \vee M)$

c) $B \vee (D \wedge M)$

1.2.8 a) $r \wedge (p \vee q)$

$(r \wedge p) \vee q$

① $r = p = T \wedge q = F$

② $r = q = F \wedge p = T$

r	p	q	$r \wedge (p \vee q)$	$(r \wedge p) \vee q$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	F
F	F	F	F	F

No questions assigned to the following page.

1.2.8 b) $\neg p \wedge q$

$\neg(p \wedge q)$

① $p=T, q=F$

② $p=q=F$

p	q	$\neg p \wedge q$	$\neg(p \wedge q)$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	F	T

c) $p \vee q$

$(\neg p \wedge q) \vee (p \wedge \neg q)$

p	q	$\neg p \wedge q$	$p \wedge \neg q$	$p \vee q$	$(\neg p \wedge q) \vee (p \wedge \neg q)$
T	T	F	F	T	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	F	F	F	F

When $p=q=$ True $\left.\begin{array}{l} p \vee q \\ (\neg p \wedge q) \vee (p \wedge \neg q) \end{array}\right\}$ have different truth values

No questions assigned to the following page.

1.3 HW

$p \quad \rightarrow \quad q$

1.3.2 a) If she finished her homework, then she went to the party.

$\neg p \rightarrow \neg q$ Inverse: If she did not finish her homework,
then she did not go to the party.

$\neg q \rightarrow \neg p$ Contrapositive: If she did not go to the party,
then she did not finish her homework.

$q \rightarrow p$ Converse: If she went to the party,
then she finished her homework.

$p \quad \rightarrow \quad q$

b) If he trained for the race, then he finished the race.

Inverse: If he did not train for the race,
then he did not finish the race.

Contrapositive: If he did not finish the race,
then he did not train for the race.

Converse: If he finished the race, then he trained for the race.

$p \quad \rightarrow \quad q$

c) If the patient took the medicine, then she had side effects.

Inverse: If the patient did not take the medicine,
then she did not have side effects.

Contrapositive: If she did not have side effects,
then the patient did not take the medicine.

Converse: If she had side effects, then the patient took the medicine

$p \quad \rightarrow \quad q$

d) If it was sunny, then the game was held.

Inverse: If it was not sunny, then the game was not held.

Contrapositive: If the game was not held, then it was not sunny.

Converse: If the game was held, then it was sunny.

e) Inverse: If it did not snow last night,
then school will not be cancelled.

Contrapositive: If school will not be cancelled,
then it did not snow last night.

Converse: If school will be cancelled, then it snowed last night.

Question assigned to the following page: 3

1.3.5 a) $\neg j \to c$
b) $c \to \neg j$
c) $\neg j \wedge \neg c$
d) $c \to \neg j$
e) $\neg c$
f) $j \wedge c$

1.3.8 a) If the roads were wet, then there was heavy traffic.
b) The roads were wet and there was an accident.
c) There was not an accident or traffic was not heavy.
d) If traffic was heavy then there was an accident or the roads were wet.
e) The roads were wet and traffic was not heavy.

No questions assigned to the following page.

1.4 HW

1.4.1 a) $(p \vee q) \vee (q \to p)$ is a tautology

p	q	$p \vee q$	$q \to p$	$(p \vee q) \vee (q \to p)$
T	T	T	T	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

Always true

b) $(p \to q) \leftrightarrow (p \wedge \neg q)$ is a contradiction

p	q	$p \to q$	$p \wedge \neg q$	$(p \to q) \leftrightarrow (p \wedge \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

Always false

c) $(p \to q) \leftrightarrow p$ is neither a tautology or contradiction

p	q	$p \to q$	$(p \to q) \leftrightarrow p$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

differing truth values

d) $(p \to q) \vee p$ is a tautology

p	q	$p \to q$	$(p \to q) \vee p$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Always True

No questions assigned to the following page.

1.4.1 e) $(\neg p \vee q) \leftrightarrow (p \wedge \neg q)$ is a contradiction

p	q	$\neg p \vee q$	$p \wedge \neg q$	$(\neg p \vee q) \leftrightarrow (p \wedge \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

[illegible]

f) $(\neg p \vee q) \leftrightarrow (\neg p \wedge q)$ is neither a tautology or contradiction

p	q	$\neg p \vee q$	$\neg p \wedge q$	$(\neg p \vee q) \leftrightarrow (\neg p \wedge q)$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

different truth values

1.4.2 a) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

* have the same truth table

b) $\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$

p	q	$p \leftrightarrow q$	$\neg p$	$\neg(p \leftrightarrow q)$	$\neg p \leftrightarrow q$
T	T	T	F	F	F
T	F	F	F	T	T
F	T	F	T	T	T
F	F	T	T	F	F

* have the same truth table

Question assigned to the following page: 4

1.4.2 c) $\neg p \to q \equiv p \vee q$

p	q	$\neg p$	$\neg p \to q$	$p \vee q$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	F

* have the same truth table so they are logically equivalent.

1.4.3 a) $p \to q \not\equiv q \to p$

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

* Since the truth tables are not the same, they are not logically equivalent.

b) $\neg p \to q \not\equiv \neg p \vee q$

p	q	$\neg p$	$\neg p \to q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	T

* Since the truth tables are not the same, they are not logically equivalent

Question assigned to the following page: 4

1.4.3 c) $(p \to q) \land (r \to q) \not\equiv (p \land r) \to q$

p	q	r	$p \to q$	$r \to q$	$p \land r$	$(p \to q) \land (r \to q)$	$(p \land r) \to q$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	F	F	T	F	F
T	F	F	F	F	F	F	T
F	T	T	T	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	T	F	F	F	T
F	F	F	T	T	F	T	T

Truth tables do not match so they are not logically equivalent.

d) $p \land (p \to q) \not\equiv p \lor q$

p	q	$p \to q$	$p \land (p \to q)$	$p \lor q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	F

Truth tables do not match so they are not logically equivalent.

No questions assigned to the following page.

1.5.1

(a)

$(p \to q) \wedge (q \vee p)$	
$(\neg p \vee q) \wedge (q \vee p)$	Conditional Identity
$(q \vee \neg p) \wedge (q \vee p)$	Commutative law.
$q \vee (\neg p \wedge p)$	Distributive law
$q \vee (p \wedge \neg p)$	Commutative law
$q \vee F$	Complement law
q	Idenity law

(b)

$(\neg p \vee q) \to (p \wedge q)$	
$\neg(\neg p \vee q) \vee (p \wedge q)$	Conditional identity
$(\neg\neg p \wedge \neg q) \vee (p \wedge q)$	De Morgan's law
$(p \wedge \neg q) \vee (p \wedge q)$	Double Negation
$p \wedge (\neg q \vee q)$	Distributive law
$p \wedge T$	Complement law
p	Idenity law

(c)

$r \vee (\neg r \to p)$	
$r \vee (\neg\neg r \vee p)$	Conditional identity
$r \vee (r \vee p)$	Double Negation
$(r \vee r) \vee p$	Associative law
$r \vee p$	Idemotent law

Question assigned to the following page: 5

1.5.2 a) $\neg p \to \neg q \equiv q \to p$

$\neg p \to \neg q$

$\neg p \to \neg q \equiv \neg\neg p \vee \neg q$ Conditional

$\equiv p \vee \neg q$ double negation

$\equiv \neg q \vee p$ Commutative

$\equiv q \to p$ Conditional

b) $p \wedge (\neg p \to q) \equiv p$

$p \wedge (\neg p \to q) \equiv p \wedge (\neg\neg p \vee q)$ Conditional

$\equiv p \wedge (p \vee q)$ Double negation

$\equiv (p \wedge p) \vee (p \wedge q)$ Distributive

$\equiv p \vee (p \wedge q)$ Idempotent

$\equiv p$ Absorption

c) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to q) \wedge (p \to r)$

$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$ Conditional Identity

$\equiv (\neg p \vee q) \wedge (r \vee \neg p)$ Commutative

$\equiv \neg p \vee (q \wedge r) \vee \neg p$ Associative

$\equiv \neg p \vee \neg p \vee (q \wedge r)$ Commutative

$\equiv (\neg p \vee \neg p) \vee (q \wedge r)$ Associative

$\equiv \neg p \vee (q \wedge r)$ idempotent

$\equiv p \to (q \wedge r)$ Conditional Identity

Question assigned to the following page: 5

1.5.2 d) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg p \to (q \to r)$

$\equiv \neg p \to (\neg q \vee r)$ Conditional

$\equiv \neg(\neg p) \vee (\neg q \vee r)$ Conditional

$\equiv p \vee (\neg q \vee r)$ Double negation

$\equiv \neg q \vee (p \vee r)$ Commutative

$\equiv q \to (p \vee r)$ Conditional

e) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

$(p \to r) \vee (q \to r)$

$\equiv (\neg p \vee r) \vee (\neg q \vee r)$ Conditional

$\equiv (\neg p \vee \neg q) \vee (r \vee r)$ Commutative

$\equiv (\neg p \vee \neg q) \vee r$ Idempotent

$\equiv \neg(p \wedge q) \vee r$ De Morgans

$\equiv (p \wedge q) \to r$ Conditional

$\equiv$

f) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\neg(p \vee (\neg p \wedge q))$

$\equiv (\neg p \wedge \neg(\neg p \wedge q))$ De Morgans

$\equiv \neg p \wedge (p \wedge \neg q)$ Double negation

$\equiv (\neg p \wedge p) \wedge \neg q$ Associative

$\equiv (F) \vee \neg q$ Complement

$\equiv \neg p \wedge (F \vee \neg q)$ Associative

$\equiv \neg p \wedge \neg q$ Identity

Question assigned to the following page: 5

1.5.2 g) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ distributive

$(p \vee p) \wedge (q \vee \neg q) \wedge (\neg r \vee \neg r)$

$p \wedge (q \vee \neg q) \wedge \neg r$ Idempotent

$p \wedge T \wedge \neg r$ Complement

$p \wedge \neg r$ identity

h) $p \leftrightarrow (p \wedge r) \equiv \neg p \vee r$

$p \leftrightarrow (p \wedge r)$

$\equiv (p \rightarrow (p \wedge r)) \wedge ((p \wedge r) \rightarrow p$ biconditional

$\equiv (\neg p \vee (p \wedge r)) \wedge (\neg (p \wedge r) \vee p)$ conditional

$\equiv ((\neg p \vee p) \wedge (\neg p \vee r)) \wedge (\neg p \vee \neg r \vee p)$ distributive

$\equiv [T \wedge (\neg p \vee r)] \wedge (\neg p \vee \neg r \vee p)$ Complement

$\equiv (\neg p \vee r) \wedge (\neg p \vee \neg r \vee p)$ Identity

$\equiv (\neg p \vee r) \wedge (T \vee \neg r)$ Complement

$\equiv (\neg p \vee r) \wedge T$ Domination

$\equiv (\neg p \vee r)$ Idenity

i) $(p \wedge q) \rightarrow r \equiv (p \wedge \neg r) \rightarrow \neg q$

$(p \wedge q) \rightarrow r$

$\neg (p \wedge q) \vee r$ Conditional

$(\neg p \vee r) \vee \neg q$ Associative

$\neg \neg (\neg p \vee r) \vee \neg q$ Double Negation

$\neg (p \wedge \neg r) \vee \neg q$ De Morgans

$((p \wedge \neg r) \rightarrow \neg q)$ Conditional

No questions assigned to the following page.

1.6 HW

Domain = all integers

1.6.2 a) $\exists x(x+x=1)$

False, there is no integer x that equals 1.

b) $\exists(x+2=1)$

Existential statement is true because there exist an x value

$x=-1$

$(-1)+2=1$

$1=1$ - true

c) $\forall x\,(x^2-x\neq 1)$ ← True

This is true for the domain given

$x=-1$

$(-1)^2-(-1)\neq 1$

$1+1$

$x=1$ $1^2-x\neq 1$

$0=1$

$x=2$ $(2^2-2\neq 1$

$4-2\neq 1$

$2\neq 1$

No x value that makes statement not true

d) $\forall x(x^2-x\neq 0)$ False

$x=1$ $1^2-1\neq 0$

$1-1\neq 0$

$0\neq 0$ ← since $x=1$ makes the predicate not true the statement is false.

e) $\forall x(x^2>0)$ True

This is true for all integers so the universal statement is true

f) $\exists x(x^2>0)$

$x=1$ $1^2>0$

$1>0$ This is true for $x=1$ so the existential statement is true.

Question assigned to the following page: 6

1.6.4 a) $\forall x\, P(x)$

$P(a) = P(b) = P(c) = P(d) = \text{true}$

True

b) $\exists x\, P(x)$

$P(a) = \text{true}$

true

c) $\forall x\, Q(x)$

$Q(b) = Q(c) = Q(d) = \text{False}$

False

d) $\exists x\, Q(x)$

$Q(a) = \text{True}$

true

e) $\forall x\, R(x)$

$R(a) = \text{false}$

$R(b) = \text{false}$

False

f) $\exists x\, R(x)$

$R(a) = R(b) = R(c) = R(d) = \text{False}$

False

Question assigned to the following page: 7

1.7 HW

1.7.2 a) $\exists x(E(x) \wedge T(x))$

b) $\forall x(E(x) \rightarrow T(x))$

c) $\forall x(T(x) \rightarrow E(x))$

d) $\exists x(E(x) \wedge \neg T(x))$

1.7.4 a) $\exists x\, S(x)$
b) $\forall x(\neg S(x) \wedge W(x))$
c) $\forall x(S(x) \wedge \neg W(x))$
d) $\exists x(S(x) \wedge W(x))$
e) $\forall x(\neg W(x) \rightarrow S(x))$
f) $\forall x(\neg W(x) \rightarrow (S(x) \vee V(x)))$
g) $\exists x(\neg W(x) \wedge \neg S(x) \wedge \neg V(x))$
h) $\forall x(\neg W(x) \rightarrow (S(x) \vee V(x)))$
i) $S(Ingrid) \wedge W(Ingrid)$
j) $\exists x((x \neq ingrid) \wedge S(x))$
k) $\forall x((x \neq Ingrid) \rightarrow S(x))$

1.7.10 a) False. Counter example: Jeb
b) True
c)
d) True. Example: Hilary
e) True. For any x value, $m(x)$, $O(x)$ or $P(x)$ is true
f) False. Counter example: Hilary and Jeb
g)
h)
i) True. Example: Hilary
j) False. For every value of x

Question assigned to the following page: 8

1.8 HW

1.8.1 a) $\neg \exists x \, P(x)$
$= \forall x \, \neg P(x)$

b) $\neg \exists x (P(x) \vee Q(x))$
$= \forall x \neg (P(x) \vee Q(x))$
$= \forall x \, \neg P(x) \wedge \neg Q(x)$

c) $\neg \forall x (P(x) \wedge Q(x))$
$= \exists x \neg (P(x) \wedge Q(x))$
$= \exists x \, \neg P(x) \vee \neg Q(x)$

d) $\neg \forall x [P(x) \wedge (Q(x) \vee R(x))]$
$= \exists x \neg [P(x) \wedge (Q(x) \vee R(x))]$
$= \exists x \, \neg P(x) \vee \neg (Q(x) \vee R(x))$
$= \exists x \, \neg P(x) \vee \neg Q(x) \wedge \neg R(x)$

1.8.2 a) $\forall x (D(x))$
Negation: $\neg \forall x (D(x))$
Applying Demorgans law: $\exists x (\neg D(x))$
English: There exist a male patient that was not given the medication

b) $\forall x (P(x) \vee D(x))$
Negation: $\neg \forall x (P(x) \vee D(x))$
Demorgan's law: $\exists x (\neg P(x) \wedge \neg D(x))$
English: There exist a male patient that was not given the placebo and also not given medication.

c) $\exists x (M(x) \wedge D(x))$
Negation: $\neg \exists x (M(x) \wedge D(x))$
Demorgan's law: $\forall x (\neg M(x) \vee \neg D(x))$
English: Every patient either did not have migraines, or was not given the medication.

No questions assigned to the following page.

1.8.2 d) $P(x) \to M(x) \equiv \neg P(x) \vee M(x)$

Expression: $\forall x (P(x) \to M(x))$

Negation: $\neg \forall x (P(x) \to M(x))$

De morgan's law: $\exists x (P(x) \wedge \neg M(x))$

English: There exist a patient that was given the placebo and did not have migraines

e) $\exists x (M(x) \wedge P(x))$

Negation: $\neg \exists x (M(x) \wedge P(x))$

De morgan's law: $\forall x (\neg M(x) \vee \neg P(x))$

English: Every patient either did not have migraines or was not given the placebo.

1.8.4 a) $\neg \forall (P(x) \wedge \neg Q(x)) \equiv \exists x (\neg P(x) \vee Q(x))$

Demorgan's law

$\exists x (\neg P(x) \vee \neg\neg Q(x))$ Double negation

$\exists x (\neg P(x) \vee Q(x))$ $\equiv$

b) $\neg \forall x (\neg P(x) \to Q(x)) \equiv \exists x (\neg P(x) \wedge \neg Q(x))$

$\equiv \exists x \neg (\neg (P(x)) \vee Q(x))$

$\equiv \exists x \neg (P(x) \vee Q(x))$

$\equiv \exists x (\neg P(x) \wedge \neg Q(x))$ $\equiv$

c) $\neg \exists x [\neg P(x) \vee (Q(x) \wedge \neg R(x))] \equiv \forall x [P(x) \wedge (\neg Q(x) \vee R(x))]$

$\equiv \forall x \neg (\neg P(x) \vee (Q(x) \wedge \neg R(x)))$

Demorgans law

$\equiv \forall x [\neg\neg P(x) \wedge \neg (Q(x) \wedge \neg R(x))]$

$\equiv \exists x [P(x) \wedge (\neg Q(x) \vee R(x))]$

$\equiv \neg \exists x [\neg P(x) \vee (Q(x) \wedge \neg R(x))]$

Question assigned to the following page: 9

1.9 HW

1.9.1 a) $m(1,1)$ is a proposition and is true
b) $\forall y\ m(x,y)$ is not a proposition since the value of x is not defined. Truth value is not obtainable.
c) $\exists x\ m(x,3)$ is a proposition and is true.
d) $\exists x \exists y\ m(x,y)$ is a proposition and is true
e) $\exists x \forall y\ m(x,y)$ is a proposition and is false
f) $m(x,2)$ is not a proposition.
g) $\exists y \forall x\ m(x,y)$ is a proposition and is true

1.9.5 a) $\forall x \forall y\ F(x,y)$
Negation: $\neg \forall x \forall y\ F(x,y)$
Demorgan's: $\exists x \exists y \neg F(x,y)$
English: Someone is an enemy of someone.

b) $\exists x \exists y\ F(x,y)$
Negation: $\neg \exists x \exists y\ F(x,y)$
Demorgan's: $\forall x \forall y \neg F(x,y)$
English: Everyone is an enemy of everyone.

c) $\exists x \forall y\ F(x,y)$
Negation: $\neg \exists x \forall y\ F(x,y)$
Demorgan's: $\forall x \exists y \neg F(x,y)$
English: Everyone is an enemy of someone

d) $\forall x \exists y\ F(x,y)$
Negation: $\neg \forall x \exists y\ F(x,y)$
Demorgan's: $\exists x \forall y \neg F(x,y)$
English: Someone is an enemy of everyone.

Question assigned to the following page: 10

1.10 HW

1.10.1 a) False, when $x=2, y=2$ $M(x,y)$ is false so for all $M(x,y)$ is a false statement.

b) True, since $M(x,y)$ is true whenever $x=y$

c) True, since $M(2,2)$ is false which the "not" of makes a true statement

d) False, since because of the truth table. $M(x,y)$ is true whenever $x\neq y$ which disproves the statement.

e) False, take $x=1$ then its not possible to find y such that $M(x,y)$ is false.

1.10.4 a) $\exists x \exists y \left(\frac{x}{y} < 1\right)$

b) $\forall x \left(x > 0 \rightarrow \frac{1}{x} > 0\right)$

c) $\exists x \exists y \left(\frac{x}{y} < 1\right)$

d) $\exists x \exists y \left[(x > 0 \wedge y > 0) \rightarrow \frac{x}{y} > 0\right]$

e) $\forall x \left[(x > 0 \wedge x < 1) \rightarrow \frac{1}{x} > 1\right]$

f) $\neg \exists x \forall y (x \leq y)$

g) $\forall x \left[(x \neq 0) \rightarrow \exists y (xy = 1)\right]$

h) $\forall x \left[(x \neq 0) \rightarrow \exists ! y (xy = 1)\right]$