Binding by Oscillatory Dynamics in Neural Architectures for Relational Reasoning

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Introduction

Relational reasoning tasks (e.g., same/different judgments or analogies) challenge neural networks due to the lack of explicit binding mechanisms. Inspired by neuroscience, we address the *binding problem* using **oscillatory dynamics**, leveraging neural synchrony to dynamically group features. We introduce a **relational bottleneck** to enforce abstract, generalizable relational representations.

Model Architecture

Our model binds object features through Kuramoto oscillator synchronization:

- **Feature Extraction:** Object images z_i encoded by CNN: $E_i = f_\theta(z_i) \in \mathbb{R}^D$.
- **Oscillatory Binding (Kuramoto Dynamics):** Features drive oscillators $\mathbf{x}_{i,d}(t) \in S^{N-1}$ on a unit sphere, evolving as:

$$\dot{\mathbf{x}}_{i,d}(t) = \mathbf{\Omega}_d \mathbf{x}_{i,d}(t) + \mathrm{Proj}_{\mathbf{x}_{i,d}(t)}\left(\mathbf{c}_{i,d} + \sum_{d'} \mathbf{J}^{\mathrm{IN}}_{d,d'} \mathbf{x}_{i,d'}(t) + \sum_{d'} \mathbf{J}^{\mathrm{OUT}}_{d,d'} \mathbf{x}_{3-i,d'}(t)\right)$$

  Dynamics include:
  - *Natural frequency* ($\mathbf{\Omega}_d$): Skew-symmetric matrices generating intrinsic rotations, ensuring diverse oscillator frequencies and preventing trivial synchronization.
  - *Conditional input* ($\mathbf{c}_{i,d} = W_d \cdot E_i[d] + \mathbf{b}_d$): Encodes object-specific features into oscillator dynamics, guiding task-relevant synchronization patterns. W_d, $\mathbf{b}_d$ are learnable parameters.
  - *Within-object coupling* ($\mathbf{J}^{\mathrm{IN}}_{d,d'}$): Synchronizes features within the same object.
  - *Between-object coupling* ($\mathbf{J}^{\mathrm{OUT}}_{d,d'}$): Synchronizes features across different objects.
- **Relational Bottleneck:** Coherence measures synchronization strength:

$$\rho_d = \|\mathbf{x}_{1,d}(T) + \mathbf{x}_{2,d}(T)\|_2$$

- **Classification:** Classifier predicts relationship based on coherence signals.

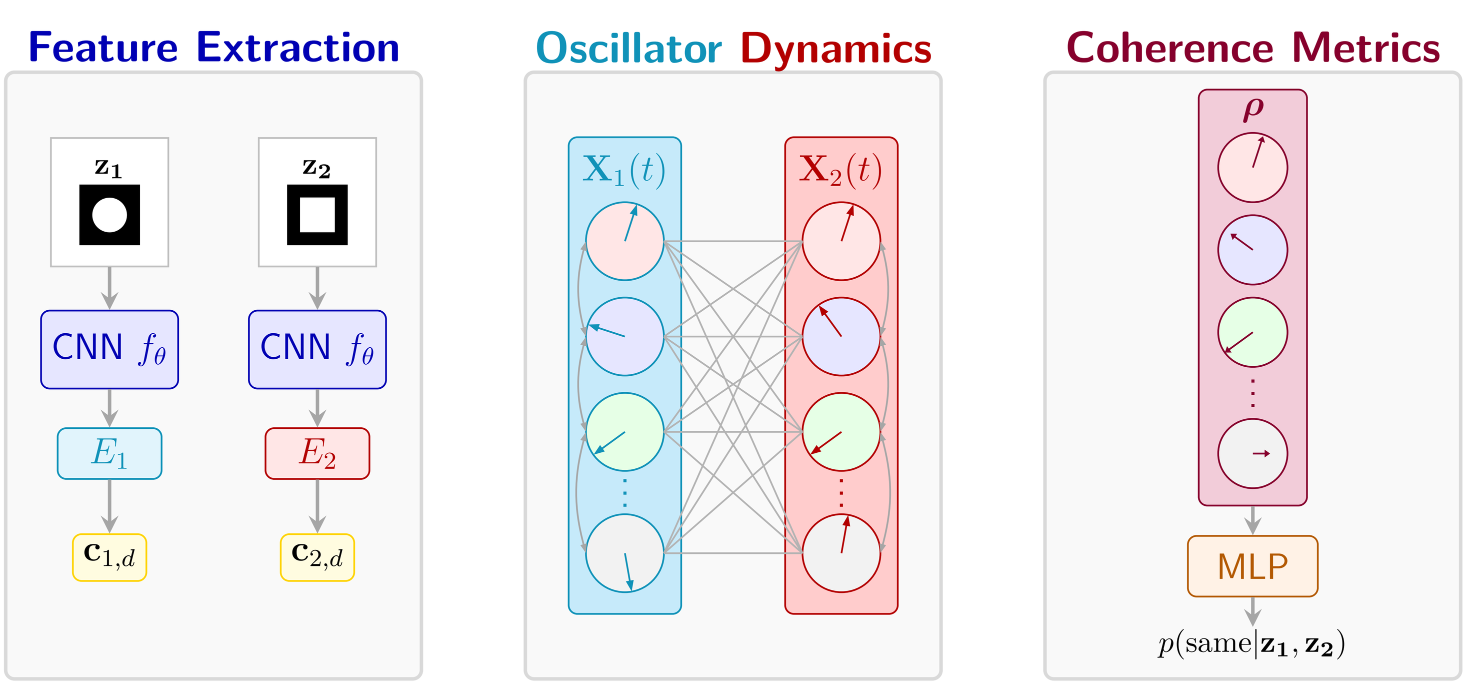


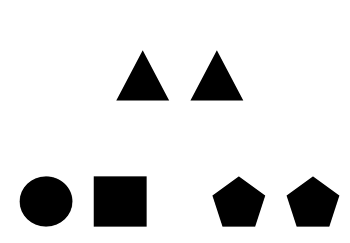
Figure 1. Model architecture overview.

Tasks

We evaluate on two visual reasoning tasks, using 100 Unicode characters with random scaling/positioning:

- **Same/Different:** Decide if two objects belong to the *same* category.
- **Relational Match-to-Sample (RMTS):** Given a source pair (defining "same" or "different"), choose which of two target pairs matches that relation. This is reasoning about *relations of relations*.
- **Generalization Regimes:** Train on $m \in \{95, 50, 15\}$ icons, test on the remaining $\{5, 50, 85\}$.

(a) Same/Different



(b) RMTS

Figure 2. Task Illustrations.

Transfer Learning Approach

- **Stage 1:** Train the full CNN + Oscillator network on the Same/Different task.
- **Stage 2:** Freeze CNN and oscillator parameters; extract coherence vectors for each pair and train a new MLP on the RMTS task.

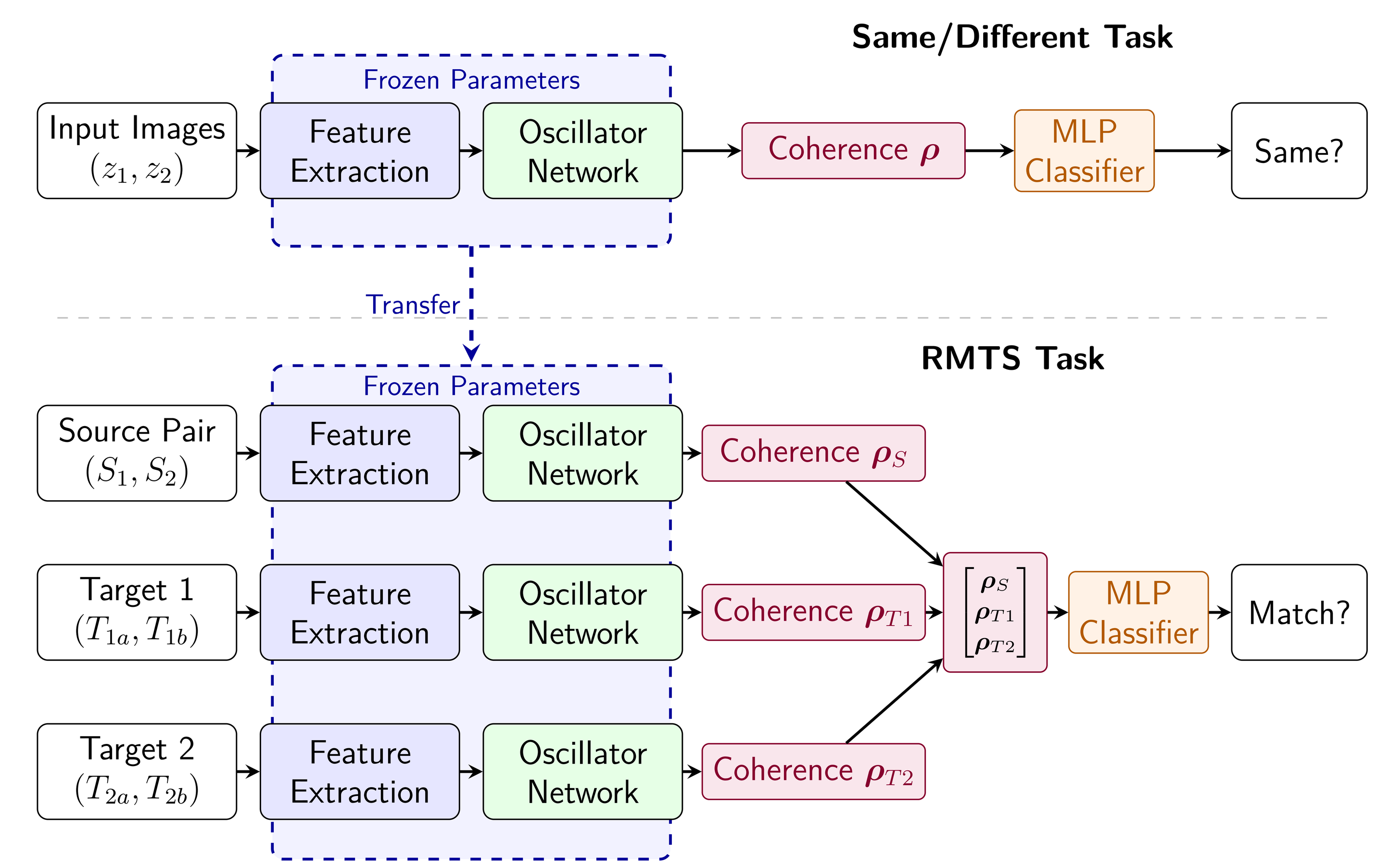
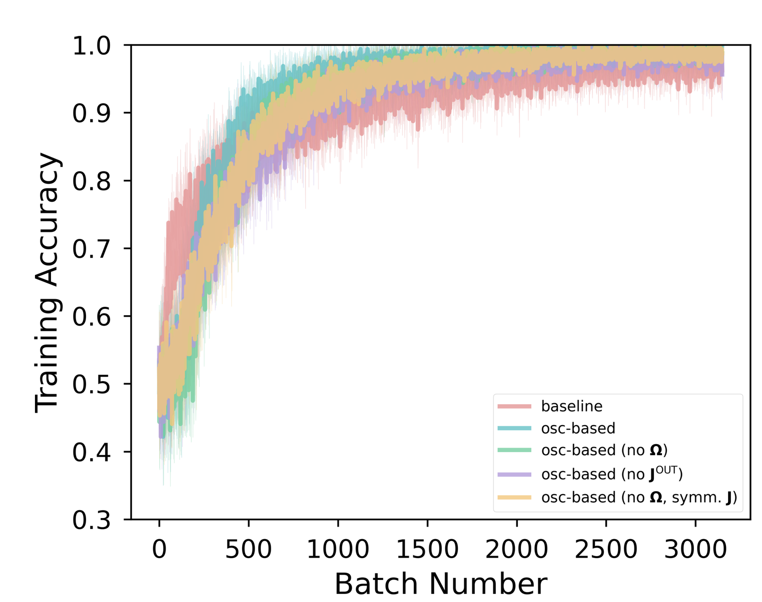
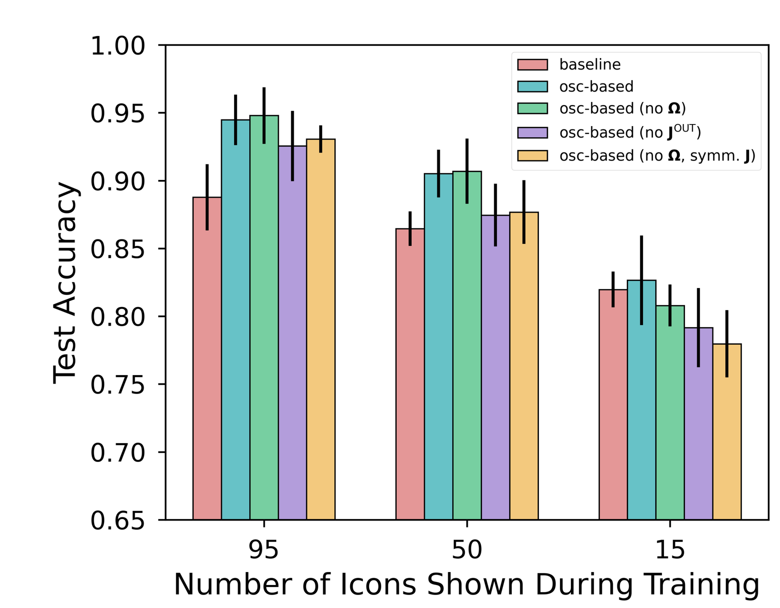


Figure 3. Two-stage transfer learning paradigm.

Same/Different Performance

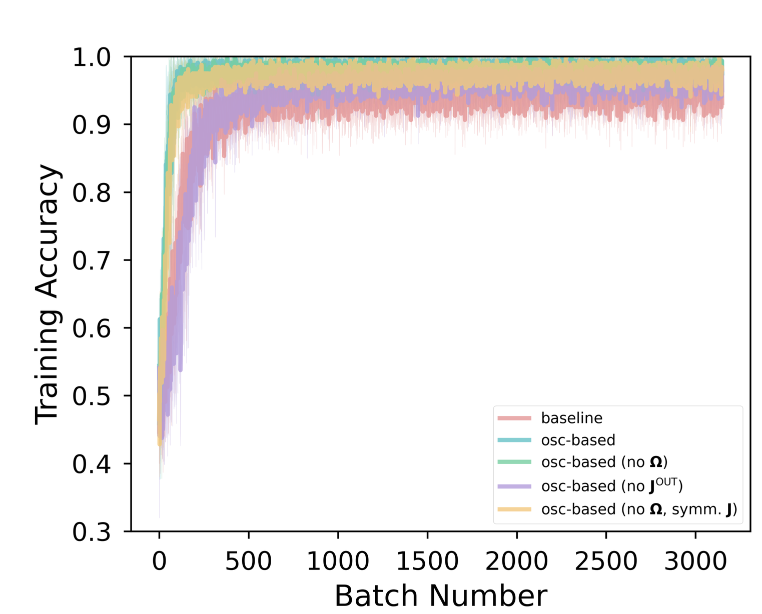


(a) Training accuracy over batches ($m = 95$)

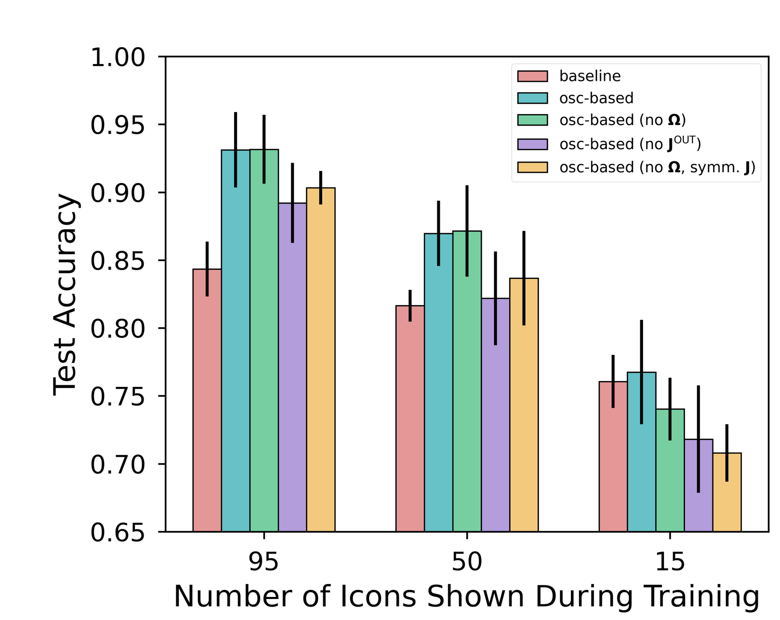


(b) Test accuracy across generalization regimes

RMTS Performance



(a) RMTS training accuracy ($m = 95$)



(b) RMTS test accuracy across regimes

Energy Minimization

Under symmetric coupling ($\mathbf{J} = \mathbf{J}^\top$) and zero natural frequencies ($\mathbf{\Omega} = 0$), the Kuramoto network minimizes the Lyapunov energy

$$\begin{aligned} E &= -\tfrac{1}{2}\sum_{i,j=1}^{2} \mathbf{x}_i^\top \mathbf{J}_{ij} \mathbf{x}_j - \sum_{i=1}^{2} \mathbf{c}_i^\top \mathbf{x}_i \\ &= -\tfrac{1}{2}\Big(\mathbf{x}_1^\top \mathbf{J}^{\mathrm{IN}} \mathbf{x}_1 + \mathbf{x}_2^\top \mathbf{J}^{\mathrm{IN}} \mathbf{x}_2 + \mathbf{x}_1^\top \mathbf{J}^{\mathrm{OUT}} \mathbf{x}_2 + \mathbf{x}_2^\top \mathbf{J}^{\mathrm{OUT}} \mathbf{x}_1\Big) - \left(\mathbf{c}_1^\top \mathbf{x}_1 + \mathbf{c}_2^\top \mathbf{x}_2\right). \end{aligned}$$

During each integration step, E decreases monotonically. Empirically, "same" input pairs converge to *lower* energy minima than "different" pairs.

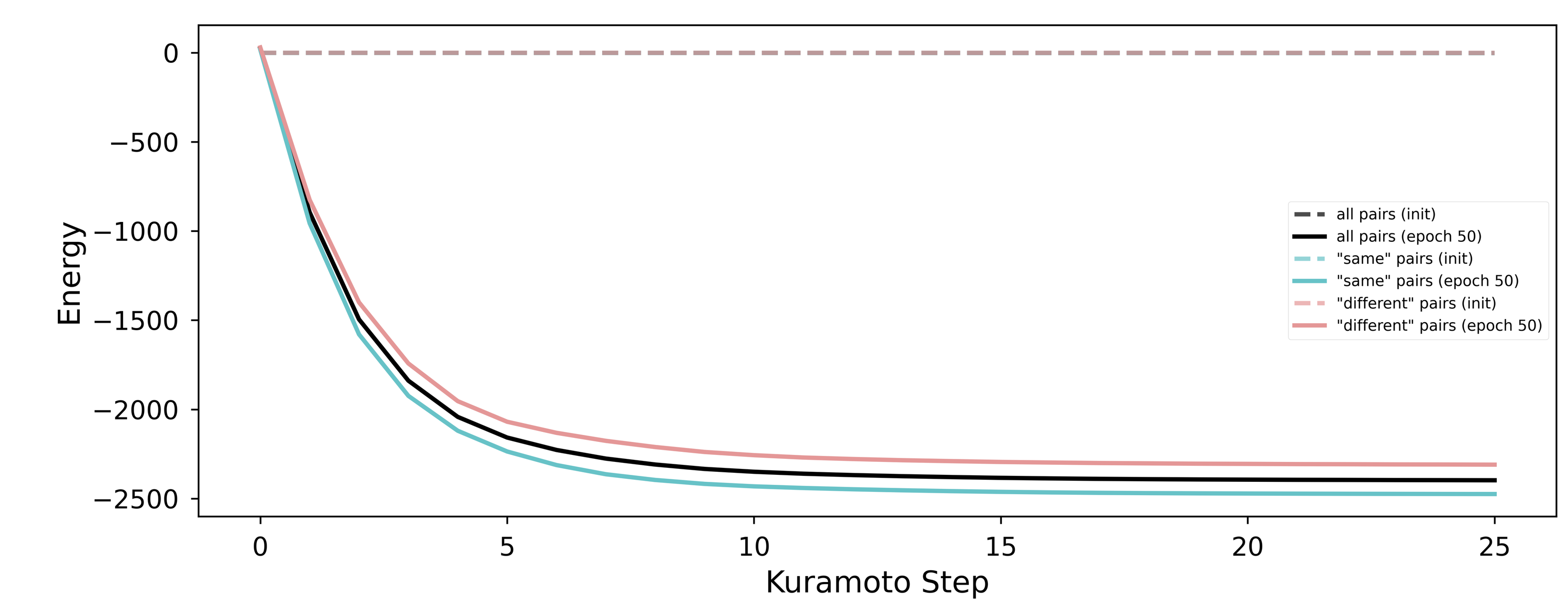
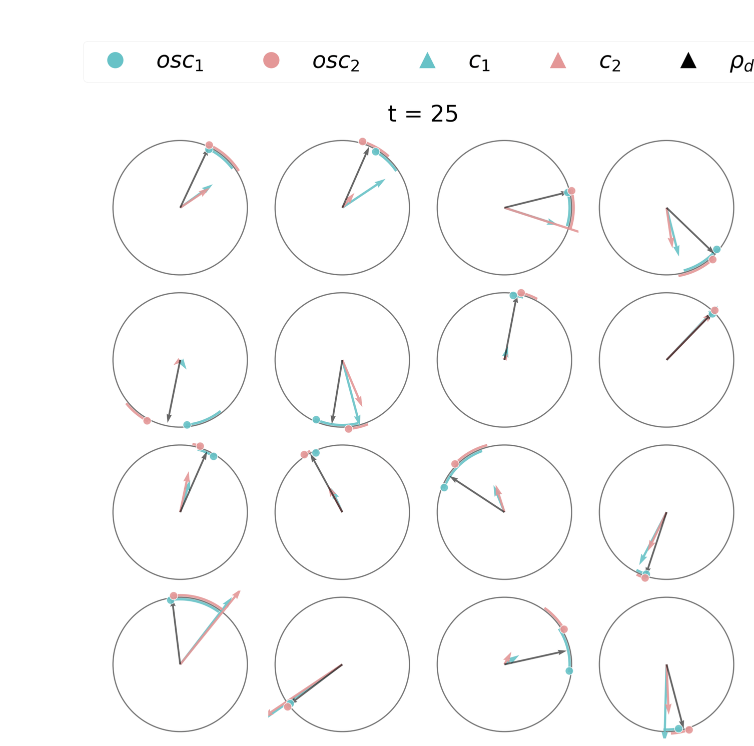


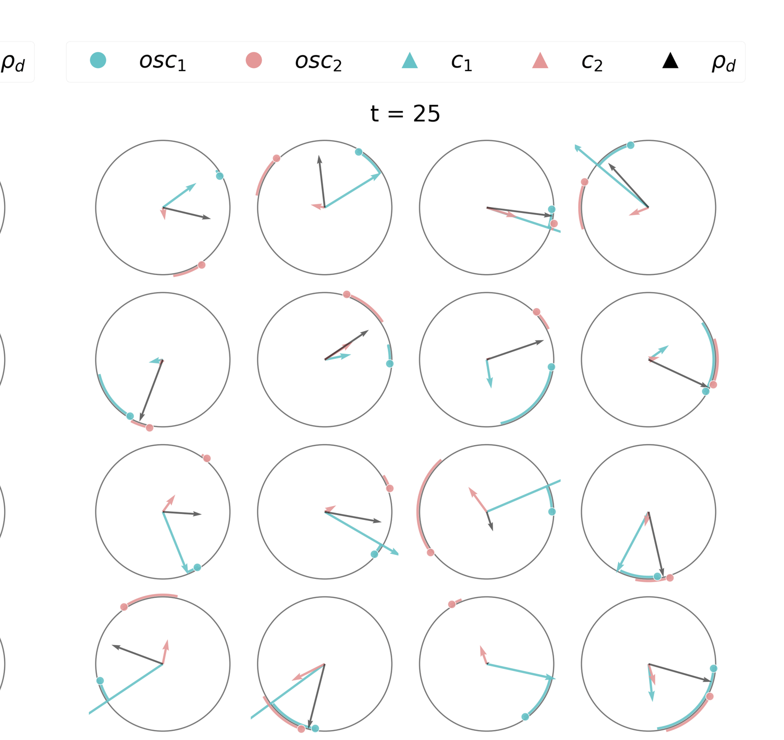
Figure 6. Average energy trajectory for "same" vs "different" pairs over $T = 25$ steps.

Oscillator State Dynamics

- **Initialization:** All D oscillators begin with random phases on the circle.
- **Trained "Same" Pair:** For every dimension d, the two corresponding oscillators lock in phase—i.e. they overlap as tight pairs on the circle.
- **Trained "Different" Pair:** Only shared–feature dimensions phase–lock; others remain asynchronous, producing a mix of tight and dispersed pairs.



(a) "Same" pair oscillators (epoch 50)



(b) "Different" pair oscillators (epoch 50)

Figure 7. All D oscillator phases at $t = T$ for sample "same" vs. "different" pairs.

References

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Poster Session

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