2022秋 概率论期末

一、填空题

1. $1-(1-p)^n$　　2. 0.7　　3. 3　　4. $1-\sqrt[3]{\frac{1}{2}}$　　5. 7

6. $\frac{1}{2}$　　~~8~~ 7. 1.71　　8. $\chi^2(n)$

二、选择题

1. C

$$\left.\begin{aligned} P(AB)&=P(A)+P(B)-P(A\cup B)\\ P(\overline{A}\,\overline{B})&=1-P(A\cup B)\end{aligned}\right\}\Rightarrow \begin{aligned} &P(A)+P(B)=1\\ &P(A)=1-P(B)\end{aligned}$$

2. D　　3. A　　4. B　　5. C　　6. C　　7. D

8. C

$$\begin{aligned} &X\sim N(0,1)\\ &Y\sim\chi^2(9)\end{aligned}\Rightarrow \frac{X}{\sqrt{\frac{Y}{9}}}\sim t(9)\rightarrow\frac{3X}{\sqrt{Y}}\sim t(9)$$

三、应用题

1. (1)

解：记乘坐火车、轮船、汽车为事件 A_1、A_2、A_3，迟到为事件 B

$$P(B)=P(A_1)P(B|A_1)+P(A_2)P(B|A_2)+P(A_3)P(B|A_3)=0.3\times\frac{1}{4}+0.2\times\frac{1}{3}+0.5\times\frac{1}{12}=\frac{11}{60}$$

(2) $$P(A_2|B)=\frac{P(A_2)P(B|A_2)}{P(B)}=\frac{0.2\times\frac{1}{3}}{\frac{11}{60}}=\frac{4}{11}$$

2.

解：待求 $P_Y(y)$.

$$P_Y(y)=F_Y'(y)$$

$$F_Y(y)=P(Y\leq y)=P(X^2+1\leq y)=P(X^2\leq y-1)=P(-\sqrt{y-1}\leq X\leq +\sqrt{y-1})$$

$$=\int_{-\sqrt{y-1}}^{\sqrt{y-1}}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx$$ ，根据对称性： $$F_Y(y)=2\int_{-\infty}^{\sqrt{y-1}}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx-1$$

$$P_Y(y)=F_Y'(y)=\frac{d\left(2\int_{-\infty}^{\sqrt{y-1}}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx-1\right)}{dy}$$

变上限积分后求导：

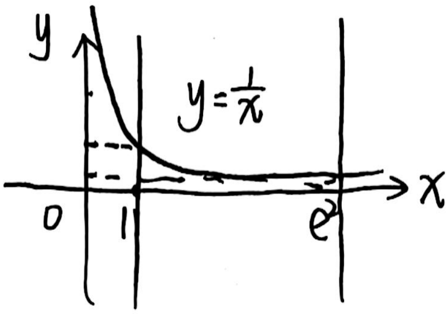
$$P_Y(y)=F_Y'(y)=\frac{1}{\sqrt{2\pi(y-1)}}e^{-\frac{(y-1)}{2}}$$

注意别漏掉

3.

解:

(1) 先画出 x,y 围成的区域 D.



求区域 D 面积 S:

$$S=\int_1^{e^2}\frac{1}{x}dx$$

$$=\ln x\Big|_1^{e^2}=2$$

$$\therefore\ f(x,y)=\begin{cases}\frac{1}{2}, & x,y\in D\\ 0, & \text{其他}\end{cases}$$

$$f_X(x)=\int_{-\infty}^{+\infty}f(x,y)\,dy=\int_0^{\frac{1}{x}}\frac{1}{2}dy=\frac{1}{2x}$$

$$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)\,dx.$$

当 $0<y<\frac{1}{e^2}$ 时, $1<x<e^2$, $f_Y(y)=\int_1^{e^2}\frac{1}{2}dx=\frac{e^2-1}{2}$

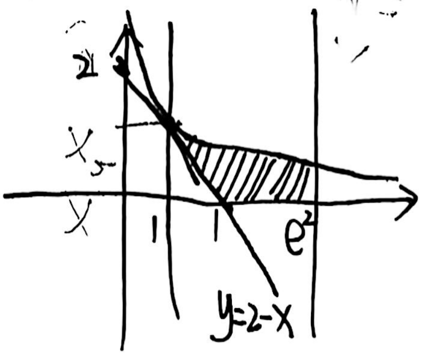
当 $\frac{1}{e^2}<y<1$ 时, $1<x<\frac{1}{y}$, $f_Y(y)=\int_1^{\frac{1}{y}}\frac{1}{2}dx=\frac{1}{2}(\frac{1}{y}-1)$

$$\therefore\ f_Y(y)=\begin{cases}\frac{e^2-1}{2}, & 0<y<\frac{1}{e^2}\\ \frac{1}{2}(\frac{1}{y}-1), & \frac{1}{e^2}<y<1\\ 0, & \text{其他}\end{cases}$$

(2) $f(x,y)\neq f_X(x)\cdot f_Y(y)$

$\therefore$ 不独立.

(3) $P(X+Y\geq 2)=P(Y\geq 2-X)=1-P(Y\leq 2-X)$



阴影面积 $S_2=S_D-\frac{1}{2}\times 1\times 1$

$=\frac{3}{4}$

区域 D 服从均匀分布

$\therefore\ P(X+Y\geq 2)=\frac{3}{4}$

5.

解:

① $\int_0^{+\infty} ke^{-5x}dx = \frac{k}{-5}e^{-5x}\Big|_0^{+\infty} = 0-\left(\frac{k}{-5}\right)=1 \Rightarrow k=5$

② $P(X>0.2) = \int_{0.2}^{+\infty}5e^{-5x}dx = -e^{-5x}\Big|_{0.2}^{+\infty} = 0-(-e^{-1}) = \frac{1}{e}$

③ 当 $x>0$:

$F(x) = \int_0^x 5e^{-5x}dx = -e^{-5x}\Big|_0^x = -e^{-5x}-(-1) = -e^{-5x}+1$

$\therefore F(x) = \begin{cases} 1-e^{-5x}, & x>0 \\ 0, & \text{其他} \end{cases}$