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$$1 + 2 + 3 + \cdots + 100 = S$$

$$100 + 99 + 98 + \cdots + 1 = S$$

$$\underbrace{101 \quad 101 \quad 101 \quad \cdots \quad 101}_{100 \text{ times}} = 2S$$

$$\frac{100 \times 101}{2} = 50 \times 101 = 5050$$

$$1 + 2 + \cdots + n = S_n$$

$$n + n-1 + \cdots + 1 = S_n$$

$$n+1 + n+1 + \cdots + n+1 = 2S_n$$

$$S_n = \frac{n(n+1)}{2} = \sum_{r=1}^{n} r$$

- $f(x) = O(g(x))$ as $x \to \infty$ if there exists a positive number M and a real number x_0 such that

$$f(x) \leq M \cdot g(x) \quad \forall x \geq x_0$$

$\|x = n$

Insertion sort $\quad f(x) = a_2x^2 + a_1x + a_0$ $\quad \leq Bx^2 + Bx + B$

with $a_2, a_1, a_0 \in \mathbb{R}$ $\quad \forall\ x \geq 1 \quad x^2 \geq x$, $x^2 \geq 1$

$$f(x) = \underline{\underline{O}}(x^2).$$

$\leq \underset{M}{\underline{3B}}x^2$

$B = \max\{a_2, a_1, a_0\}$; $\quad M = 3B$

for $x \geq 1$; $\quad f(x) \leq Mx^2$

Pragmatic part : in a Polynomial the highest exponent gives the Big-O class

$f(x) = O(x^3)$ ✓ $\qquad f(x) = O(x^{1000})$ ✓

$f(x) = O(x^3)$ ✓ $\quad f(x) = O(x^{1000})$ ✓

- $f(x) = \Omega(g(x))$ as $x \to \infty$ if there exists a positive number m and a real number x_0 such that

$$m \cdot g(x) \leq f(x) \quad \forall x \geq x_0$$

$m = \min\{a_2, a_1, a_0\}$ $\quad \left(f(x) = a_2 x^2 + a_1 x + a_0 \right)$

$f(x) \geqslant m \quad \forall x \geqslant 1$

$f(x) = \Omega(1)$.

$T(n)$ running time of insertion sort

"worst case" $\quad T(n) = a_2 n^2 + a_1 n + a_0 = \Theta(n^2)$

"best case" $\quad T(n) = b_1 n + b_0 = \Theta(n)$

$\Longrightarrow$ we say Insertion sort has running time $O(n^2)$.

$$c < \log(n) < \underset{\alpha > 0}{n^{\alpha}} < e^n < n! < n^n$$

$c = \Theta(1)$

$\cos(n) \leq 1 \Rightarrow \cos(n) = O(1)$