FTP_Alg_Week 2: Exercises

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Exercise 1 *Viewing a heap as a tree, we define the height of a node in a heap to be the number of edges on the longest simple downward path from the node to a leaf, and we define the height of the heap to be the height of its root.*

What are the minimum and maximum numbers of elements in a heap of height h?

Exercise 2 *Show that an n-element heap has height $\lfloor \lg n \rfloor$. (Where $\lg(\cdot)$ denotes logarithm in base 2).*

Exercise 3 *(*) Show that, with the array representation for storing an $n-$element heap, the leaves are the nodes indexed by $\lfloor n/2 \rfloor + 1, \lfloor n/2 \rfloor + 2, \ldots, n$.*

Exercise 4

1. *We consider the running time of a recursive algorithm $y(n)$. Suppose that $y(n)$ verifies the following:*

$$\begin{cases} y(1) = 0 \\ y(n) = y\left(\frac{n}{2}\right) + 1 \quad n \geq 1 \end{cases}$$

If possible calculate the running time.

2. *We consider the running time of a recursive algorithm $y(n)$. Suppose that $y(n)$ verifies the following:*

$$\begin{cases} y(1) = 0 \\ y(n) = 3y\left(\frac{n}{4}\right) + n^2 \log_2 n \quad n \geq 1 \end{cases}$$

If possible calculate the running time

3. *We consider the running time of a recursive algorithm $y(n)$. Suppose that $y(n)$ verifies the following:*

$$\begin{cases} y(1) = 0 \\ y(n) = 5y\left(\frac{n}{3}\right) + \log_2 n \quad n \geq 1 \end{cases}$$

If possible calculate the running time is