Zürcher Hochschule
für Angewandte Wissenschaften
School of Engineering
IAMP Institut für Angewandte
Mathematik und Physik

Lecture 4: Exercise Solutions

MSE Algorithms - Metaheuristics

Solutions: Manual Tabu Search

Task 1: Manual Tabu Search

a) red: Tabu Search with t = 3

b) green: Tabu Search with t = 1

y\x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
-6	248	216	210	222	230	234	256	304	336	372	428	495	585	650	754
-5	193	175	157	166	174	181	215	249	295	329	382	454	539	597	707
-4	138	144	126	116	124	150	184	194	250	305	361	425	480	566	646
-3	123	89	85	97	105	109	129	179	209	246	302	368	458	525	627
-2	92	58	70	70	78	94	98	148	168	223	282	339	413	510	582
-1	68	34	46	46	54	70	74	124	144	199	258	315	388	486	558
0	51	17	14	25	33	38	57	107	136	174	230	296	386	454	555
1	18	25	5	-4	3	29	65	74	131	185	240	305	361	445	527
2	27	6	-10	0	8	13	46	83	126	160	213	284	371	429	539
3	33	0	-3	7	15	20	39	89	118	156	212	278	368	436	537
4	33	12	-4	6	14	19	52	89	132	166	219	290	377	435	545
5	30	37	17	7	15	41	77	86	143	197	252	317	373	457	539
6	69	35	32	43	51	56	75	125	154	192	248	314	404	472	573
7	92	58	70	70	78	94	98	148	168	223	282	339	412	510	582

Solutions: Tabu Search on Knapsack Problem

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Maximize $11x_1 + 10x_2 + 9x_3 + 12x_4 + 10x_5 + 6x_6 + 7x_7 + 5x_8 + 3x_9 + 8x_{10}$

subject to $33x_1 + 27x_2 + 16x_3 + 14x_4 + 29x_5 + 30x_6 + 31x_7 + 33x_8 + 14x_9 + 18x_{10} \leq 100$

$x_i = \{0,1\}, i = 1, \ldots, 10$

Initial solution : (0, 0, 0, 0, 0, 0, 0, 0, 0, 0) revenue : 0

Move : flip a bit $x_i \leftarrow 1 - x_i$

Tabu condition : don't flip the same bit for 3 (complete) iterations

Tabu list : iteration AFTER which a bit can be flipped again

Initial tabu list : (0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

Iteration	Variable flipped	Current solution	Reve-nue	Vol.	Tabu list
1	x_4	(0, 0, 0, 1, 0, 0, 0, 0, 0, 0)	12	14	(0, 0, 0, 4, 0, 0, 0, 0, 0, 0)
2	x_1	(1, 0, 0, 1, 0, 0, 0, 0, 0, 0)	23	47	(5, 0, 0, 4, 0, 0, 0, 0, 0, 0)
3	x_2	(1, 1, 0, 1, 0, 0, 0, 0, 0, 0)	33	74	(5, 6, 0, 4, 0, 0, 0, 0, 0, 0)
4	x_3	(1, 1, 1, 1, 0, 0, 0, 0, 0, 0)	42	90	(5, 6, 7, 4, 0, 0, 0, 0, 0, 0)
5	x_4	(1, 1, 1, 0, 0, 0, 0, 0, 0, 0)	30	76	(5, 6, 7, 8, 0, 0, 0, 0, 0, 0)
6	x_{10}	(1, 1, 1, 0, 0, 0, 0, 0, 0, 1)	38	94	(5, 6, 7, 8, 0, 0, 0, 0, 0, 9)
7	x_2	(1, 0, 1, 0, 0, 0, 0, 0, 0, 1)	28	67	(5, 10, 7, 8, 0, 0, 0, 0, 0, 9)
8	x_5	(1, 0, 1, 0, 1, 0, 0, 0, 0, 1)	38	96	(5, 10, 7, 8, 11, 0, 0, 0, 0, 9)
9	x_3	(1, 0, 0, 0, 1, 0, 0, 0, 0, 1)	29	80	(5, 10, 12, 8, 11, 0, 0, 0, 0, 9)
10	x_4	(1, 0, 0, 1, 1, 0, 0, 0, 0, 1)	41	94	(5, 10, 12, 13, 11, 0, 0, 0, 0, 9)

Solutions: Neighbourhood Analysis for CVRP

Task 3: Neighbourhood Analysis for CVRP (*n* customers, *m* tours)

a) Properties:

- *Size of Neighborhood*: $O\left(n \cdot \left(\sum_{i=1}^{m}(1 + number\ of\ cust. in\ tour\ T_i)\right)\right) = O(n^2)$.
- *Connectivity*: No, since the number of tours can not change.
- *Run Time for one move*: $O(1)$, since only randomly chosen vertices and the corresponding edges in the original and destination tour are changed.

b) Properties:

- *Size of Neighborhood*: $O(n \cdot m) = O(n^2)$
  (worst case: every customer is in a tour on his own).
- *Connectivity*: No, since the number of tours can not change.
- *Run Time for one move*: $O(numbrof\ cust. in\ destination\ tour) = O(n)$, since best insertion position has to be found in destination tour
  (worst case: all customers in a single tour).