

# Lecture 1: Exercises

MSE Algorithms - Metaheuristics



## \* Relevant for Exam

### **Task 1: Drawing 5 Lines (5 min)**

Proof that you cannot draw 5 lines on the Euclidean plane in such a way that each line cuts exactly 3 other lines.

### **\* Task 2: Mathematical TSP Formulation (3 min)**

Give an exact formulation of the travelling salesperson problem: Input data, feasible tours, and objective.

### **\* Task 3: Objective Function for Cards Problem (10 min)**

50 cards are numbered from 1 to 50. The cards must be separated into 2 stacks such that the sum of the cards of the first stack is 1170 and the product of the cards of the second stack is 36000. Formulate this problem as an optimization problem.

## Task 4: Timetable for Exams (15 min)

Assume that  $k$  students are inscribed for the written examinations of various modules of a university (an example with students A..M is given in the following table). Each student can do at most one exam per day, and all students in a module have to take the exam at the same day. Goal is to find the minimum number of days for the examinations.

Show how this problem can be modelled as Vertex Colouring Problem (see e.g. <http://mathworld.wolfram.com/MinimumVertexColoring.html>).

	A	B	C	D	E	F	G	H	I	J	K	L	M
ARO2				X		X	X		X	X			
ASD	X					X				X	X		
GRE		X						X			X		X
INF3	X			X									
MAD2			X		X					X			X
MAI2			X					X					
POO	X	X							X		X		
PST			X				X					X	

## \* Task 5: Number of Solutions for Permutation Flow Shop Problem (10 min)

Given  $n$  jobs and  $m$  machines, how many different solutions does the Permutation Flow Shop Problem have?

## \* Task 6: Asymptotic Runtime (5 min)

a) What is the asymptotic runtime of the following code?

```
sum = 0
for (int i = 1; i < n; i = i + 2) {
    for(int j = 0; j < i; j = j + 1) {
        sum = sum + 1;
    }
}
```

b) Put the following functions into ascending order according to their asymptotic order:

$n^2$ ,  $2^n$ ,  $n$ ,  $\log n$ ,  $n^3$ ,  $(3/2)^n$ ,  $1$

- \* Task 7: Complete the example on Dijkstra's Algorithm from the slides (p. 42) (15 min)**
- \* Task 8: Complete the example on Prim's Algorithm from the slides (p. 45) (15 min)**