

FTP_Alg_Week 4: Exercises

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Exercise 1 (Here it is enough to give an intuitive idea of the solution)
 Prove that the worst case in *Partitioning Algorithm* (for Quicksort) has running time $\Theta(n^2)$, where n is the cardinality of the set of element of the partitioning.

Exercise 2 We apply *COUNTING-SORT* (Lecture 7) with the input the vector

$$A = (5, 6, 5, 3, 3, 7, 4, 4, 4, 5, 3, 8, 8)$$

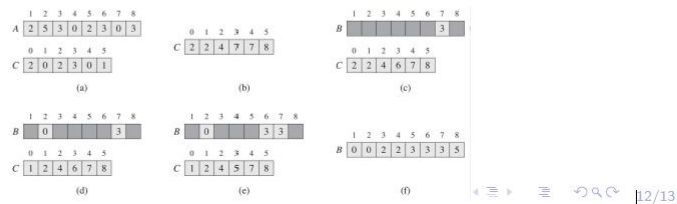
Let C and B be the arrays mentioned in the pseudocode of *COUNTING-SORT* (below is the original algorithm from our lecture slides).

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COUNTING-SORT( $A, B, k$ )
1  let  $C[0..k]$  be a new array
2  for  $i = 0$  to  $k$ 
3     $C[i] = 0$ 
4  for  $j = 1$  to  $A.length$ 
5     $C[A[j]] = C[A[j]] + 1$ 
6  //  $C[i]$  now contains the number of elements equal to  $i$ .
7  for  $i = 1$  to  $k$ 
8     $C[i] = C[i] + C[i - 1]$ 
9  //  $C[i]$  now contains the number of elements less than or equal to  $i$ .
10 for  $j = A.length$  downto 1
11    $B[C[A[j]]] = A[j]$ 
12    $C[A[j]] = C[A[j]] - 1$ 
    
```

The procedure starts by creating an array C that tell us the number of elements having key i for each $0 \leq i \leq k$. The step after is to change C , which now tells us the number of elements having key $\leq i$ for each $0 \leq i \leq k$.

Then the procedure creates the array B containing the sorted list by using the info in C .



1. Let C be the array obtained after the for loop at line 7 and 8 of the pseudocode (so the array at the step (b) of the figure at slide 12 of Lecture 7). The entry $C[7]$ is the number ...

2. We consider the first cycle **for** at line 10 of the pseudocode (see (c) of the figure at slide 12 of Lecture 7) in the figure. The number $B[13]$ is ...
3. We consider the first cycle **for** at line 10 of the pseudocode (see (c) of the figure at slide 12 of Lecture 7). At the end of the first cycle, the number $C[8]$ is ...

Exercise 3 (*) Let n be a non negative integer and $P = \{p_1, p_2, \dots, p_n\}$ n real numbers. Prove that there is a complete binary search tree having the points in P as leaves.