MASTER OF SCIENCE
IN ENGINEERING

# Final Exam Autumn Semester 2021/2022

Modul / *Module*: **FTP_Alg**
Datum / *Date*: **08.02.2022**
Dozierende/r / *Lecturer/s*: **J.K. Canci / S. Beer**
Teil / *Part*: 2 von 2 */ 2 of 2*

Name / Last name: ---

Vorname / First name: ---

E-Mail-Adresse / Email address: ---

FH / UAS: ---

Teilschule / Part of UAS: ---

Max. Punktzahl/max. points: 60 (for part 2)
Erreichte Punktzahl/ Schlussnote/
Reached points / final mark: ---

**Allgemeine Hinweise**

- Beachten Sie die allgemeinen Hinweise im Abschnitt des Moduls im Moodle-Kurs.

**Viel Erfolg!**

***General information:***

- *Please follow the general instructions in the section of the Moodle course.*

***Good luck!***

## Frage/Question 1: (3 Punkte/points)

For a Knapsack Problem instance with $n$ items and weight limit $W$, a neighborhood is defined as follows: Remove the item with the worst value-to-weight-ratio from the knapsack and then randomly insert an item that fits into (i.e. such that the weight limit is not exceeded). What is the *best upper bound* for the size of this neighborhood?

**Size: $O(n^2)$.**

## Frage/Question 2: (4 Punkte/points)

We apply Partially Mapped Crossover to the following permutations with swapping positions 4 to 7 (in blue):

**P1:** 9 3 1 **7 4 6 2** 5 8
**P2:** 6 4 2 **9 8 7 5** 3 1

Provide the resulting two offsprings **O1** and **O2**.

**O1: 6 3 1 9 8 7 5 2 4**
**O2: 9 8 5 7 4 6 2 3 1**

## Frage/Question 3: (5.5 Punkte/points)

(***Note:*** for every correct answer you will get 0.5 point, for every incorrect answer 0.5 points will be subtracted. For every "Not sure" answer you will neither get nor lose any points. Maximum amount of points for this task is 5.5 points, minimum is 0 points, i.e. a negative total will be set to 0).

a) Travelling Salesperson Problem (TSP) instances with up to 50 cities are most easily solved by an Exhaustive Search.
- True
- **False**
- Not sure

b) If a problem is NP-hard, then it is in NP.
- True
- **False**
- Not sure

c) The Steiner Tree Problem is similar to the Minimum Spanning Tree Problem and can hence be solved in polynomial time.
- True
- **False**
- Not sure

d) In the Capacitated Vehicle Routing Problem (CVRP), the location of the depot is usually fixed.
- **True**
- False
- Not sure

e) Tabu Search with a tabu duration $t=0$ is just like a Local Search.
- **True**
- False
- Not sure

f) The 3-*opt* neighborhood for a TSP instance with $n$ cities is of size $O(n^3)$.
- **True**
- False
- Not sure

g) The memory used to represent the adjacency matrix of a graph with $n$ vertices and $e$ edges is of size $O(n^2)$.
- **True**
- False
- Not sure

h) For maximization problems the temperature in Simulated Annealing is increased over time.
- True
- **False**
- Not sure

i) If a problem is NP-complete, then it is proven that there exist no algorithms, which solve the problem in polynomial time.
- True
- **False**
- Not sure

j) A Genetic Algorithm with a higher mutation rate usually converges slower but towards better solutions in turn.
- **True**
- False
- Not sure

k) Best Insertion is a family of methods for improving existing solutions.
- True
- **False**
- Not sure

## Frage/Question 4: (6 Punkte/points)

Given is a TSP instance by its asymmetric distance matrix below (e.g. the distance from c3 to c4 equals 7, whereas the distance from c4 to c3 equals 3.

| To / From | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $c_1$ | - | 2 | 9 | 4 |
| $c_2$ | 3 | - | 1 | 5 |
| $c_3$ | 3 | 5 | - | 7 |
| $c_4$ | 4 | 2 | 3 | - |

a) (1 point) What is the cost of tour c1-c2-c3-c4-c1? **Cost: 14.**

b) (5 points) We apply the *Pilot Method*, and use the Nearest Neighbor heuristic as the pilot strategy. A tour always starts and ends in city c1. What is the cost of the resulting tour?
**Tour: c1-c4-c2-c3-c1.**
**Cost: 10.**

## Frage/Question 5: (15 Punkte/points)

The values of a function $f(x, y)$ of two integer variables defined on the domain $[-7, 7] \times [-7, 7]$ are given by the following table:

| x \ y | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -7 | 175 | 153 | 125 | 104 | 88 | 60 | 39 | 24 | 15 | 11 | 24 | 49 | 59 | 81 | 109 |
| -6 | 133 | 116 | 92 | 79 | 62 | 52 | 31 | 18 | 9 | 8 | 24 | 30 | 44 | 67 | 81 |
| -5 | 85 | 72 | 63 | 53 | 42 | 34 | 26 | 13 | 4 | 9 | 13 | 29 | 30 | 42 | 54 |
| -4 | 46 | 41 | 36 | 34 | 22 | 21 | 12 | 7 | 6 | 10 | 11 | 15 | 25 | 30 | 28 |
| -3 | 18 | 13 | 14 | 14 | 10 | 7 | 13 | 10 | 3 | 3 | 3 | 11 | 12 | 10 | 15 |
| -2 | 21 | 19 | 10 | 10 | 9 | 9 | 10 | 6 | 4 | 9 | 3 | 7 | 6 | 11 | 13 |
| -1 | 52 | 44 | 39 | 29 | 28 | 19 | 14 | 13 | 10 | 4 | 7 | 14 | 18 | 31 | 33 |
| 0 | 88 | 72 | 64 | 52 | 40 | 35 | 20 | 13 | 6 | 4 | 12 | 24 | 35 | 46 | 52 |
| 1 | 127 | 116 | 96 | 81 | 61 | 48 | 30 | 20 | 6 | 5 | 25 | 39 | 49 | 68 | 74 |
| 2 | 176 | 146 | 126 | 101 | 81 | 68 | 42 | 25 | 12 | 15 | 26 | 46 | 63 | 82 | 101 |
| 3 | 218 | 189 | 156 | 129 | 100 | 82 | 53 | 33 | 11 | 11 | 37 | 53 | 78 | 109 | 132 |
| 4 | 264 | 227 | 196 | 155 | 125 | 94 | 65 | 39 | 12 | 13 | 38 | 65 | 99 | 131 | 163 |
| 5 | 318 | 273 | 227 | 191 | 146 | 116 | 74 | 39 | 13 | 17 | 47 | 78 | 108 | 150 | 184 |
| 6 | 365 | 309 | 261 | 214 | 171 | 131 | 92 | 50 | 17 | 13 | 51 | 88 | 132 | 169 | 213 |
| 7 | 416 | 357 | 298 | 253 | 195 | 145 | 100 | 57 | 21 | 17 | 61 | 98 | 142 | 197 | 249 |

We define a neighborhood that allows adding or subtracting 1 to/from one of the two variables (i.e. horizontal or vertical moves, but no diagonal moves).

a) (5 points) The **minimum** of $f$ is determined using a **Local Search** parametrized as follows:
   - Initialization: $x = -7$, $y = -7$, $f(x, y) = 175$.
   - Selection Criterion: **First Improving Move**.
   - Ordering of the moves: Right, Up, Left, Down.

   Provide the resulting path of this search as the sequence of the values of $f$.
   **Sequence: 175-153-125-104-88-60-39-24-15-11-8-stop.**

b) (10 points) Use the same function table and neighborhood as given above. The **minimum** of $f$ is determined using a **Tabu Search** parameterized as follows:
   - Initialization: $x = -7$, $y = -7$, $f(x, y) = 175$.
   - Tabu Condition: If *1* is added to a variable, there is no subtractions from this variable allowed for $t$ iterations. Same if *1* is subtracted, then no addition is allowed for $t$ iterations. [Note: $t=0$ would be no tabu condition, and $t=1$ prohibits to reverse the last step].
   - The Tabu Condition applies unless a step leads to an improvement of the best solution so far. In that case, the step is taken anyway.
   - Selection Criterion: Always take the best possible move, even if it increases the function value.
   - Stopping criteria: Maximum of 14 iterations reached, or no more moves allowed.

   Provide the path resulting for a **Tabu Value of $t = 4$** as the sequence of the values of $f$.
   **Sequence: 175-133-85-46-18-13-14-10-10-9-9-7-13-10-3-stop.**

## Frage/Question 6: (3 Punkte/points)

We consider the Santa's Sleigh Challenge. Please answer ONE of the following two questions with at most 500 characters.

Briefly describe a strategy to find an initial solution.
**OR**
Briefly describe a strategy to improve an existing solution.

## Frage/Question 7: (3 Punkte/points)

Consider the Traveling Salesperson Problem instance *sw24978*. Give a good lower bound estimate of the number of possible tours, as a power of 10.
**Number of cities: 24978.**
**E.g. number of tours > $12489^{12489} > 10^{51161}$.**

## Frage/Question 8: (3 Punkte/points)

For a Travelling Salesperson Problem instance with seven cities, consider the tour:

$c_1$-$c_2$-$c_3$-$c_4$-$c_5$-$c_6$-$c_7$-$c_1$.

Give the resulting tour of applying the *2-opt* move, where the edges $c_2$-$c_3$ and $c_5$-$c_6$ are replaced.
**Tour: $c_1$-$c_2$-$c_5$-$c_4$-$c_3$-$c_6$-$c_7$-$c_1$.**

## Frage/Question 9: (7 Punkte/points)

Consider an instance of the Capacitated Vehicle Routing Problem (CVRP) with $n$ customers and vehicle capacity $Q$. We define a neighborhood of a given solution $S$ with $m$ non-empty tours as follows:

- Select a random tour $t_s$ (source tour).
- Select a random customer $X$ from $t_s$.
- Traverse all other tours $t_d$ in a random ordering:
  - Find a customer $Y$ in $t_d$ such that the tours of $X$ and $Y$ can be swapped without exceeding the vehicle capacity $Q$ for both tours.
  - If such $Y$ is found, then insert $X$ in $t_d$ and $Y$ in $t_s$ at random positions and then EXIT the traversing loop.

a) (4 points) Give a good estimate of the *running time* (in O-Notation) to compute a new solution from this neighborhood. Hereby, assume that computing the total weight of a tour in the current solution takes $O(1)$.
**Running time: O(n).**

b) (3 points) Give a good *upper bound estimate* for the size of the neighborhood of $S$.
**Size: $O(n^4)$.**

## Frage/Question 10 (5.5 Punkte/points)

(***Note:*** for every correct answer you will get 0.5 point, for every incorrect answer 0.5 points will be subtracted. For every "Not sure" answer you will neither get nor lose any points. Maximum amount of points for this task is 5.5 points, minimum is 0 points, i.e. a negative total will be set to 0).

a) Vertex Coloring for graphs in general is NP-hard.
- **True**
- False
- Not sure

b) Although the Steiner Tree Problem is theoretically interesting, it is of no great practical use.
- True
- **False**
- Not sure

c) Genetic Algorithms are constructive methods.
- True
- **False**
- Not sure

d) Genetic Algorithms are suitable for solving Knapsack Problems, since Crossover und Mutation are easy to define for bit vectors.
- **True**
- False
- Not sure

e) In the Roulette Wheel Method solutions are chosen with uniformly distributed probability.
- True
- **False**
- Not sure

f) Minimum Shortest Path Problem is in NP.
- **True**
- False
- Not sure

g) Choosing an appropriate neighborhood is crucial for solving a problem with Simulated Annealing.
- **True**
- False
- Not sure

h) Random Sampling converges fast towards good solutions if the parameters of the algorithm are set appropriately.
- True
- **False**
- Not sure

i) In Simulated Annealing, the temperature schedule must be chosen depending on the size of the problem to be solved.
- **True**
- False
- Not sure

j) Capacitated Vehicle Routing Problems (CVRP) can always be solved optimally with a greedy algorithm.
- True
- **False**
- Not sure

k) Tabu Search is an improving method.
- **True**
- False
- Not sure

## Frage/Question 11: (3 Punkte/points)

(***Note:*** for every correct answer you will get 0.5 point, for every incorrect answer 0.5 points will be subtracted. For every "Not sure" answer you will neither get nor lose any points. Maximum amount of points for this task is 5.5 points, minimum is 0 points, i.e. a negative total will be set to 0).

We consider the Santa's Sleigh Challenge. Which of the following statements is correct?

a) Distances between locations are computed using Euclidean coordinates.
- True
- **False**
- Not sure

b) The goal of the challenge is to minimize the total amount of time used to deliver all the presents.
- True
- **False**
- Not sure

c) Presents may be temporarily stored at any location.
- True
- **False**
- Not sure

d) The cost of a tour depends on the weight of the sleigh.
- **True**
- False
- Not sure

e) The weight of the sleigh is higher than its carrying capacity.
- True
- **False**
- Not sure

f) The winning team of the official challenge used an exclusively greedy method to find their final solution.
- True
- **False**
- Not sure

## Frage/Question 12: (2 Punkte/points)

(***Note:*** for every correct answer you will get 0.5 point, for every incorrect answer 0.5 points will be subtracted. For every "Not sure" answer you will neither get nor lose any points. Maximum amount of points for this task is 5.5 points, minimum is 0 points, i.e. a negative total will be set to 0).

A basic Ant System algorithm (i.e. without any local search application to the intermediate solutions) is applied to the Traveling Sales Person problem (TSP). Which of the following statements are correct?

a) Choosing an appropriate neighborhood is crucial for solving the problem with the Ant System algorithm.
- True
- **False**
- Not sure

b) When building a path, ants choose an edge with a probability proportional to the amount of pheromones accumulated on that edge.
- **True**
- False
- Not sure

c) Inside every iteration of the algorithm, artificial ants cooperate in building a new path.
   - True
   - **False**
   - Not sure

d) To get good solutions, the number of ants must be chosen depending on the number of cities of the TSP instance to be solved.
   - **True**
   - False
   - Not sure