Zürcher Hochschule
für Angewandte Wissenschaften

School of
Engineering
IAMP Institut für Angewandte
Mathematik und Physik

Lecture 1: Exercises Solutions

MSE Algorithms - Metaheuristics

Solutions

Task 1: Drawing 5 Lines

Proof that you cannot draw 5 lines on the Euclidean plane in such a way that each line cuts exactly 3 other lines.

Graph model : one line = one node ; one intersection = one edge

Problem in terms of graphs : draw a graph with 5 nodes, each of degree 3. Sum of the degrees : 15.

In any graph, the sum of the degrees must be even since every edge has 2 extremities

Conclusion : infeasible !

Solutions

Task 2: Mathematical TSP Formulation

Give an exact formulation of the travelling salesperson problem: Input data, feasible tours, and objective.

The travelling salesman problem data are given by a distances matrix $D = (d_{ij})$ between cities i and j. The objective is to find a permutation p of the n cities minimizing : $\sum_{i=1}^{n-1} d_{p_i p_{i+1}} + d_{p_n p_1}$

Solutions

Task 3: Objective Function for Cards Problem

50 cards are numbered from 1 to 50. The cards must be separated into 2 stacks such that the sum of the cards of the first stack is 1170 and the product of the cards of the second stack is 36000. Formulate this problem as an optimization problem.

Encoding: Zero-one vector s where $s_i = 0$ means that card i is in the first stack.

Objective Function f(s) to be minimized:

$$f(s) = \left| \frac{1170 - \sum_{i=1}^{50} i \cdot (1 - s_i)}{1170} \right| + \left| \frac{36000 - \prod_{i=1}^{50} i^{s_i}}{36000} \right|$$

This function becomes 0 if and only if the product and the sum both reach the target value. Also note the weighting of the summands of the function.

Solutions

Task 4: Timetable for Exams

This problem can be formulated as Vertex Colouring Problem: Each examination is a node, and two nodes are connected if at least one student exists who has to take both exams.

Solutions

Task 5: Number of Solutions for Permutation Flow Shop Problem

Given n jobs and m machines, how many different solutions does the Permutation Flow Shop Problem have?

For Permutation Flow Shop Problems, only the ordering of the n jobs on the first machine need to be selected, since the ordering of tasks on the other machines is then fixed.

So there are n! permutations of n jobs.

Solutions

Task 6: Asymptotic Runtime

a) The asymptotic runtime of the given code is $O(n^2)$

b) The ascending ordering is: 1, log n, n, n^2, n^3, $(3/2)^n$, 2^n.

Task 7: Example on Dijkstra's Algorithm

A, B, C, D, E

lambda = (0, 5, 2, 10, 12)

p = (-, C, A, B, D)

Task 8: Example on Prim's Algorithm

A, B, C, D, E, F, G

lambda = (0, 7, 5, 5, 7, 6, 9)

p = (-, A, E, A, B, D, E)

E_T = {(A,D), (D,F), (A, B), (B,E), (E,C), (E,G)}