FTP_Alg_Week 6: Exercises

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Exercise 1 *The transpose of a directed graph* $G = (V, E)$ *is the graph* $G^T = (V, E^T)$*, where* $E^T = \{(v, u) \mid (u, v) \in E\}$*. Describe efficient algorithms for computing* G^T *from* G*, for both the adjacency–list and adjacency-matrix representations of* G*. Analyze the running times of your algorithms.*

A possible answer. Let $|V| = n$. If M_G is the adjacency–matrix of G, then the transpose M_G^T is the adjacency–matrix of G^T.

Recall that if

$$M = \begin{pmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ m_{21} & m_{22} & \dots & m_{2n} \\ \vdots & \vdots & & \vdots \\ m_{n1} & m_{n2} & \dots & m_{nn} \end{pmatrix}$$

then the transpose matrix is the following one:

$$M^T = \begin{pmatrix} m_{11} & m_{21} & \dots & m_{n1} \\ m_{12} & m_{22} & \dots & m_{n2} \\ \vdots & \vdots & & \vdots \\ m_{1n} & m_{2n} & \dots & m_{nn} \end{pmatrix}$$

E.g.

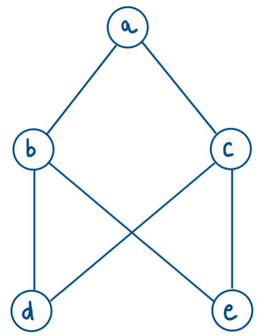
$$M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \ , \ M^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

So we have to swap all pairs of entries outside the main diagonal and symmetric with respect it, which are $n^2 - n$, thus it costs $\Theta(n^2)$.

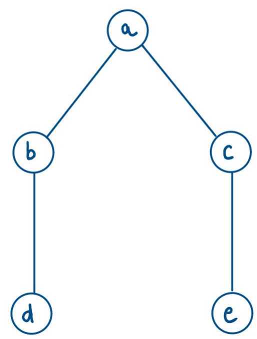
For the adjacency list of G^T, create a "vertical list" with all vertices V of G^T (that is the same of G). This costs $\Theta(n)$. Now consider the adjacency list of G. If a vertex w is in the adjacency list of v with respect G, put v in the adjacency of w with respect G^T. This operation costs $\Theta(|E|)$. Therefore we have that the running time of the whole operation is $O(|V| + |E|)$.

Exercise 2 *(Optional) Give an example of a directed graph* $G = (V, E)$*, a source vertex* $s \in V$ *, and a set of tree edges* $E_\pi \subset E$ *such that for each vertex* $v \in V$*, the unique simple path in the graph* (V, E_π) *from* s *to* v *is a shortest path in* G*, yet the set of edges* E_π *cannot be produced by running BFS on* G*, no matter how the vertices are ordered in each adjacency list.*

A possible answer. Consider the undirected graph G given by



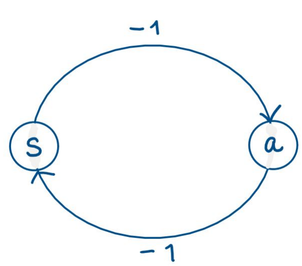
Consider the subgraph G' given by



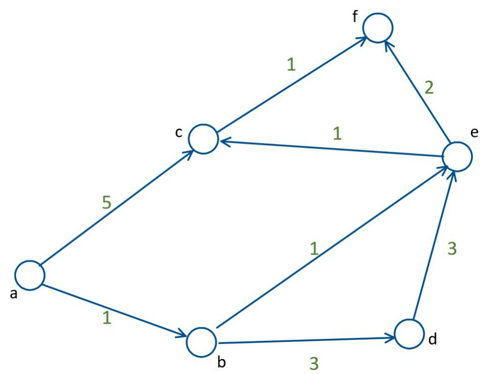
G' is a shortest path from the vertex a to any other node. One can see that BFS does never produce G' as output.

Exercise 3 *Give a simple example of a directed graph with some negative-weight edges for which Dijkstra's algorithm produces incorrect answers.*

A possible answer Any directed graph containing a cycle with a (total) negative weight produce incorrect answer. For example consider the graph:



Exercise 4 *We apply DIJKSTRA (Lecture 13 and Lecture 14) to the graph* (G, V) *represented in the following picture:*



Dijkstra is an iterative procedure, which update at each step the value v.d, that is the distance of the node v to the root a. After INITIAL-SINGLE-SOURCE(G,a) we have $a.d = 0$ *and* $v.d = \infty$ *for each vertex* $v \neq a$*. We consider the situation after two iterations (line 4 to line 8) of Dijkstra with starting node a(Look out! We consider only two iterations and not the whole Dijkstra's procedure).*

What is c.d? 5

What is f.d? ∞

What is d.d? 4