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MSE Algorithms

L05: Ant Colony Optimization, Work on Santas Challenge

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Course Overview

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1. Introduction: Basic problems and algorithms
2. Constructive methods: Random building, Greedy
3. Local searches
4. Randomized methods
5. Threshold accepting, Simulated Annealing
6. Decomposition methods: Large neighborhood search
7. Learning methods for solution improvement: Tabu search
8. Application: Santa's Challenge
9. **Learning methods for solution building: Artificial ant systems**
10. Methods with a population of solutions: Genetic algorithms
11. Work on Santa's Challenge
12. Final Lecture

Ant Colony Optimization (ACO)

Ant Systems are used to **CONSTRUCT** solutions

Ants can find Shortest Paths

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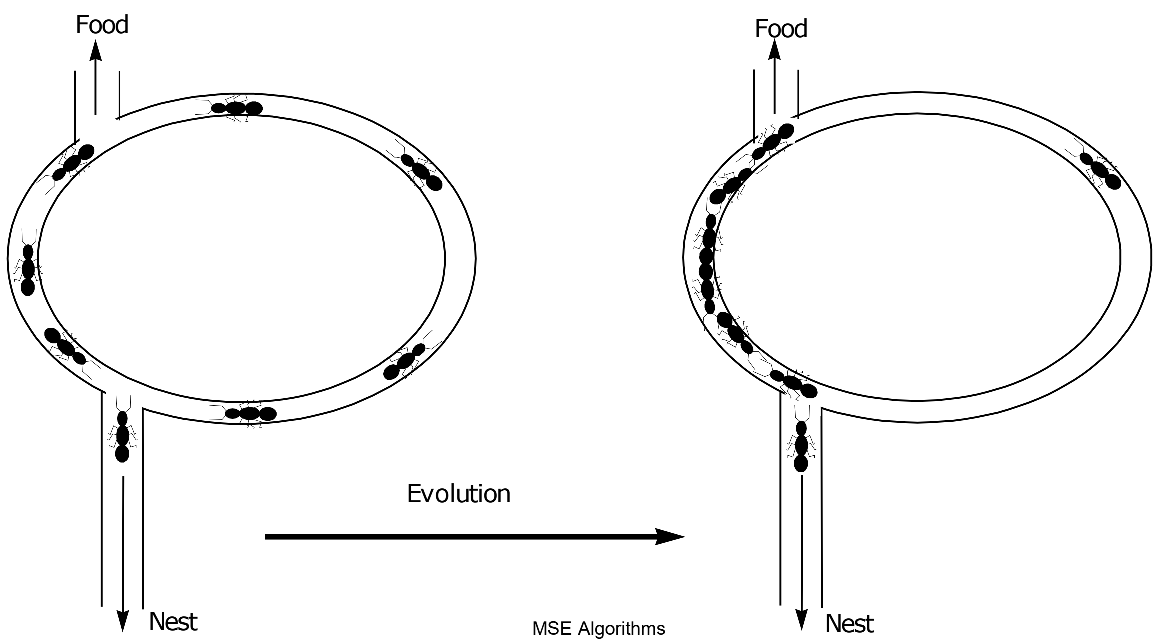
- Ants leave a "pheromone trail" on their way from the nest to a target (e.g. food)
- The more pheromone on a segment, the more likely a next ant is to use this segment
- Experiments have shown that ants can find shortest path between source and target (2-bridge-experiment)
- Ant Colony Optimization simulates behaviour of ants to solve combinatorial problems



Meta-Heuristic: Ant Colony Optimization (ACO)

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Set parameters

Initialize pheromone trails

Do

Let artificial Ants construct solutions

Apply Local Search (optional)

Update Pheromones

Keep best solution so far

While termination condition not met

ACO is a Constructive Method, i.e. it generates new solutions

Components of an ACO System

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Memory

- Associate a value τ_e (pheromone trail) to each element *e* constituting a solution
- τ_e depends on the number of times *e* has been used ***and*** on the quality of solutions using *e*

Construct Ant Solutions (building phase)

- Let *m* artificial ants build solutions in parallel, starting from empty solutions
- For each partial solution, ants choose a feasible next element *e* with a probability proportional to $\tau_e^{\alpha} \cdot c(s,e)^{\beta}$, where *c*(*s*, *e*) is an incremental cost function that measures the quality of adding element *e* to partial solution *s*
- Parameters $\alpha \geq 0$ and $\beta \leq 0$ balance the importance of learning versus greedy choice

Update Pheromones

- **Evaporate:** Multiply all τ_e by $1 - \rho$, where $0 \leq \rho \leq 1$ is a parameter to simulate pheromone evaporation
- **Reinforcement:** For each element *e* belonging to a solution *s* at a given iteration, add Q / f(s) to τ_e, where Q is a parameter and *f(s)* the cost of solution *s*. This simulates the leaving of pheromone marks on trails by ants

Ant System for TSP

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Data
Distance matrix $D = (d_{ij})$ between cities
Trace matrix $\mathrm{T} = (\tau_{ij})$
Parameters α, β, ρ, τ_0, Q, m, *max_iter*

Initialisation
$\tau_{ij} = \tau_0$ **for all** i, j

Repeat for *max_iter* iterations
$R = (r_{ij}) = 0$ // Edge reinforcement
For each $k = 1, \ldots, m$ // Ant k build a new solution
$L = 0$ // Tour length
Choose a city i at random // Current city : i
Repeat, while all cities have not been visited
Choose a city j not yet visited with probability proportional to $\tau_{ij}^{\alpha} \cdot d_{ij}^{\beta}$
$L = L + d_{ij}$
$i = j$

Note: $\beta \leq 0$

For all arc (i, j) of the tour just built, set
$r_{ij} = r_{ij} + Q/L$

The shorter a tour of ant k, the higher the reinforcement value r_{ij} for the edges on this tour.

$\mathrm{T} = (1-\rho)\mathrm{T} + R$ // Update pheromone trail after having built m solutions

Return The best tour found

Question

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1. What is the effect if we set $\alpha = 0$.

2. Same for $\beta = 0$.

Ant Algorithm Variants

Algorithm	*Authors*	*Year*
Ant System (AS)	Dorigo et al.	1991
Elitist AS	Dorigo et al.	1992
Ant-Q	Gambardella & Dorigo	1995
Ant Colony System	Dorigo & Gambardella	1996
$\mathcal{MAX}$–$\mathcal{MIN}$ AS	Stützle & Hoos	1996
Rank-based AS	Bullnheimer et al.	1997
ANTS	Maniezzo	1999
BWAS	Cordón et al.	2000
Hyper-cube AS	Blum et al.	2001

Source: Marco Dorigo et al, Ant Colony Optimization, 2006

Resources:

- *Ant Colony Optimization – Artificial Ants as a Computational Intelligence Technique* by Marco Dorigo, Mauro Birattari, and Thomas Stützle, 2006.

- https://www.researchgate.net/publication/230660829_Ant_Colony_Optimization_Artificial_Ants_as_a_Computational_Intelligence_Technique

META-HEURISTIC: MaxMin Ant System (MMAS)

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MMAS is one of the most successful ACO variants

Changes in comparison to Classical ACO:

- Only the best ant is used to update the pheromone trails
- Maintains lower and upper bounds τ_{min} and τ_{max} for pheromone values τ_e. Updates to τ_e cannot exceed or fall below these boundaries.
- Pheromone is updated only for elements of the "best" solution found. "Best" solution could be smallest of solutions from current iteration, or "best-so-far", the best solution ever found.
- Update depends on the cost of the best solution (e.g. proportional to 1/f(s))

MMAS for TSP with Nearest Neighbors

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```
Standard parameter settings
 α = 1, β = -2, ρ = 0.98, Q = 1, b = 20,  m = problem size
 τ_min = …, τ_max = …, max_iter = …                      // Problem dependent
Initialisation
   τ_ij = τ_max for all arc (i, j)

Repeat for max_iter iterations

   For each ant k = 1, …, m
      s_k = ∅                                           // Solution built by ant k
      Choose a city i at random                         // Current city : i
      Repeat, while all cities have not been visited
         If there is a city j not yet visited among b nearest neighbours of i
            Choose a city j among them with probability proportional to  τ_ij^α · d_ij^β

         Else Choose a city j not yet visited with maximum  τ_ij^α · d_ij^β
         s_k = s_k + (i, j)
         i = j
      Improve solution s_k with a local search

   For all arc (i, j) of the best tour s* among s_1, s_2, …, s_m
         τ_ij = max(τ_min , min(τ_max , (1 − ρ)τ_ij + Q/Length(s*) ) )

Return The best tour found
```

MMAS for TSP: Pherome Update

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For all arc (i, j) of the best tour s^* among $s_1, s_2, \ldots, s_m$

$$\tau_{ij} = \max(\tau_{min}\ ,\ \min(\tau_{max}\ ,\ (1-\rho)\tau_{ij} + Q/Length(s^*)\ )\ )$$

Steiner Tree Problem

Definition: Steiner Tree Problem

Problem Definition:

Given a network $G = (V, E, C)$ with

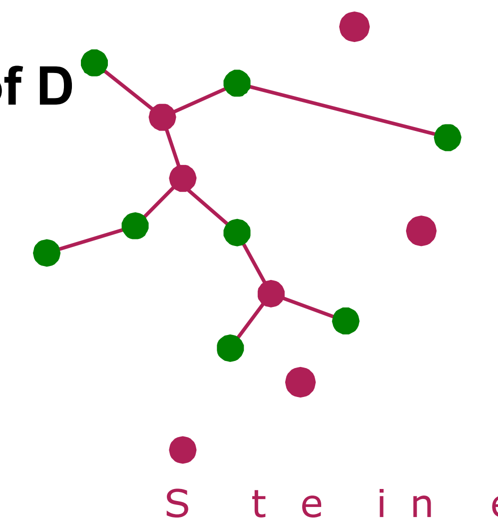
V set of vertices

E set of (undirected) edges

C a function that specifies the weights of the edges

and a subset $D \subseteq V$,

find a tree of minimum weight that contains **all vertices of D** (and may include additional vertices of V not in D).



The optional nodes (which are not in D) are called Steiner nodes.

The nodes in D are called terminals.

Question

Which problem is "easier" to solve?

a) Minimum Spanning Tree

b) Steiner Tree

c) Both are equal

Steiner Tree vs. MST

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Mentimeter

Which problem is "easier" to solve?

SOLUTION: Steiner Tree vs. MST

- If D = V in the Steiner Tree Problem, then we have an MST-Problem. Thus, in general if D ≠ V the Steiner Tree Problem is harder.
- In fact, the Steiner Tree Problem is NP-hard, while MST is solvable in linear time (Prim’s algorithm, with respect to the edges).
- Remark: If |D|=2, then the Steiner Tree Problem is equivalent to the shortest path problem and thus solvable in linear time (Dijkstra’s algorithm, with respect to the edges).

Steiner Tree Problem: Real World Applications

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- Layout of electrical networks, water distribution networks, etc.: The edges of the input graph typically correspond to segments in the local street map, with vertices representing street intersections and the location of potential customers (terminals). The cost c associated with an edge is the cost of laying the pipe or cable on the corresponding street segment.

- Efficient point to multipoint transfers across multiple datacenters in cloud services (see e.g. https://www.usenix.org/system/files/conference/hotcloud17/hotcloud17-paper-noormohammadpour.pdf)

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ACO for Steiner Tree Problem

Propose an ACO for the Minimum Steiner Tree problem. Describe how to choose the pheromone trails and how they are exploited.

SOLUTION: ACO for Steiner Tree

IMPORTANT: there is no need in ACO that the "ants" really follow a "consecutive" path

ACO for Steiner Tree Problem

- Element *e* of a partial solution = edge of a tree
- An edge can only be selected as next element if it does not create a cycle
- Probability to chose edge *e* depends on lengths of *e* (in the graph) and on pheromone trail on *e*
- Stop building as soon as all mandatory nodes are in the solution and solution is a tree

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ACO for Numerical Problems

In graph-based problems such as TSP or Steiner Tree Problem, it is usually obvious what a "step" of an ant is: Choosing an edge to the next city or node.

What about problems that are **not** graph-based, such as the Knapsack Problem? What would a "step" of an ant be here?

SOLUTION: ACO for Knapsack

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ACO for Numerical Problems

For Knapsack Problem with items a_1, .. a_n:

- Start with empty solution.
- Partial solution is a selection of k items.
- Potential next steps are taking any of the items that are not yet in the knapsack and which still fit in.
- Options: Either use value or ratio of value/weight to select the next item.
- Probability to chose item *a* depends on value (or value/weight) of *a* and on pheromone trail on *a.*

Santa's Challenge