Zürcher Hochschule
für Angewandte Wissenschaften
School of
Engineering
IAMP Institut für Angewandte
Mathematik und Physik

Lecture 2: Exercise Solutions

MSE Algorithms - Metaheuristics

Solutions: Time Comlexity of TSP Heuristics

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Task 1: Time Complexity of TSP Heuristics

a) *Nearest Neighbor* has time complexity $O(n^2)$, since there are *n-1* cities that need to be added to the tour, and at each step the nearest neighbor (to the last city added) out of the *j* remaining cities has to be computed, which takes O(j) (with j = n-1, n-2, …, 2, 1) for each city. So it's
$O((n-1) + (n-2) + \dots + 2 + 1) = O(n^2)$.

b) The *Pilot Method* with Nearest Neighbor has time complexity $O(n^4)$, since there are *n-1* cities that need to be added to the tour, and at each step, the best partial tour so far has to be extended in *j* different ways by one of the *j* remaining citites each, and then using the Nearest Neighbor strategy. According to Subtask a) (with j = n-1, n-2, …, 2) we get
$O((n-1)*(n-2)^2) + O((n-2)*(n-3)^2) + \dots + O(2*1^2) = O(n^4)$.

c) *Beam Search* has time complexity $O(B*n^{k+1})$, since there are *n-1* cities that need to be added to the tour. In each step, the *B* best partial tours so far are extended by *k* cities each, out of all the *j* remaining cities (with j = n-2, n-3, …, k). So it's
$O(B*(n-2)^k) + O(B*(n-3)^k) + \dots + O(B*k^k) = O(B*n^{k+1})$.
Note, that this is an upper bound which is the more pessimistic, the larger *k* is chosen (in comparison to *n*).

Solutions: Performance of heuristics for TSP

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Will be updated periodically until end of semester…								