Contents

1   Funciones C++

#include <algorithm> #include <numeric>

Algo	Params	Funcion
sort, stable_sort	f, l	ordena el intervalo
nth_element	f, nth, l	*void* ordena el n-esimo, y particiona el resto
fill, fill_n	f, l / n, elem	*void* llena [f, l) o [f, f+n) con elem
lower_bound, upper_bound	f, l, elem	*it* al primer / ultimo donde se puede insertar elem para que quede ordenada
binary_search	f, l, elem	*bool* esta elem en [f, l)
copy	f, l, resul	hace resul+i=f+i $\forall i$
find, find_if, find_first_of	f, l, elem / pred / f2, l2	*it* encuentra i ∈[f,l) tq. i=elem, pred(i), i∈[f2,l2)
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca [f2,l2) ∈ [f,l)
replace, replace_if	f, l, old / pred, new	cambia old / pred(i) por new
reverse	f, l	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	*it* min, max de [f,l]
lexicographical_compare	f1,l1,f2,l2	*bool* con [f1,l1]¡[f2,l2)
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection, set_difference, set_union, set_symmetric_difference,	f1, l1, f2, l2, res	[res, ...) la op. de conj
push_heap, pop_heap, make_heap	f, l, e / e /	mete/saca e en heap [f,l), hace un heap de [f,l)
is_heap	f,l	*bool* es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum$/oper de [f,l)
inner_product	f1, l1, f2, i	$T = i + [f1, l1) . [f2, ...)$
partial_sum	f, l, r, [op]	r+i = $\sum$/oper de [f,f+i] $\forall i$ ∈[f,l)
__builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
__builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
__builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
__builtin_popcount	unsigned int	Cant. de 1's en x.
__builtin_parity	unsigned int	1 si x es par, 0 si es impar.
__builtin_XXXXXXll	unsigned ll	= pero para long long's.

Specifier	Output	Example
d or i	Signed decimal integer	392
u	Unsigned decimal integer	7235
o	Unsigned octal	610
x	Unsigned hexadecimal integer	7fa
X	Unsigned hexadecimal integer (uppercase)	7FA
f	Decimal floating point, lowercase	392.65
F	Decimal floating point, uppercase	392.65
e	Scientific notation (mantissa/exponent), lowercase	3.9265e+2
E	Scientific notation (mantissa/exponent), uppercase	3.9265E+2
g	Use the shortest representation: %e or %f	392.65
G	Use the shortest representation: %E or %F	392.65
a	Hexadecimal floating point, lowercase	-0xc.90fep-2
A	Hexadecimal floating point, uppercase	-0XC.90FEP-2
c	Character	a
s	String of characters	sample
p	Pointer address	b8000000
%	A % followed by another % will write a single %.	%

2   Compile

2.1   Compile

```
g++-13 nombre.cpp -o nombre (compilar)
./nombre (ejecutar)
g++ -std=c++23 -Wall -Wshadow -g -fsanitize=undefined -fsanitize=address
    -D_GLIBCXX_DEBUG nombre.cpp -o nombre
```

2.2   Template

```cpp
#include <bits/stdc++.h>
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
using namespace std;
#define pb push_back
#define ll long long
#define s second
#define f first
#define MOD 1000000007
#define INF 100000000000000

void solve(){

```

```
14  }
15
16  int main() {
17    ios_base::sync_with_stdio(false);  cin.tie(0);  cout.tie(0);
18    int t;cin>>t;for(int T=0;T<t;T++)
19    solve();
20  }
```

3   Data Structures

3.1   BIT

```
1  /*
2    Binary Indexed Tree (Fenwick Tree) Fast Implementation
3    --------------------------------
4    Indexing: 1-based
5    Bounds: [1, MAXN]
6    Time Complexity:
7      - update(x, val): O(log n) -> adds val to index x
8      - get(x): O(log n) -> prefix sum from 1 to x
9    Space Complexity: O(n)
10 */
11
12 const int MAXN = 10000;
13 int bit[MAXN];
14
15 // Add 'val' to index 'x'
16 void update(int x, int val) {
17   for (; x < MAXN; x += x & -x) {
18     bit[x] += val;
19   }
20 }
21
22 // Get prefix sum from 1 to 'x'
23 int get(int x) {
24   int ans = 0;
25   for (; x > 0; x -= x & -x) {
26     ans += bit[x];
27   }
28   return ans;
29 }
```

3.2   Bitset

```
1  bitset<3001> b[3001];
2
3  //set() Set the bit value at the given index to 1.
4  //reset() Set the bit value at the given index to 0.
5  //flip() Toggle the bit value at the given index.
6  //test() Check if the bit value at the given index is 1 or 0.
7  //count() Count the number of set bits.
8  //any() Checks if any bit is set
9  //all() Check if all bit is set.
10 //none() Check if no bit is set.
11 //to_string() Convert the bitset to a string representation.
12
13 #pragma GCC target("popcnt")
14 (int) __builtin_popcount(x);
15 (int) __builtin_popcountll(x);
16 __builtin_clz(x); // count leading zeros
17
18 // declare bitset
19 bitset<64> b;
```

3.3   Bit Trie

```
1  /*
2    Bit Trie (Binary Trie for Integers)
3    -----------------------------------
4    Indexing: 0-based
5    Bit Width: [0, MAX_BIT] inclusive (e.g., 31 for 32-bit integers)
6    Time Complexity:
7      - Insert: O(MAX_BIT)
8      - Query: O(MAX_BIT)
9    Space Complexity: O(N * MAX_BIT) nodes (in worst case, 1 per bit per
        number)
10 */
11
12 const int K = 2; // Each node has 2 branches (bit 0 or 1)
13 const int MAX_BIT = 30; // Max bit position (for 32-bit integers)
14
15 struct Vertex {
16   int next[K]; // next[0] = child for bit 0, next[1] = child for bit 1
17
18   Vertex() {
19     fill(begin(next), end(next), -1); // -1 means no child
20   }
```

```
21  };
22
23  vector<Vertex> trie; // Trie nodes
24
25  // Inserts a number into the binary trie
26  void insert(int num) {
27    int v = 0; // Start from root
28    for (int j = MAX_BIT; j >= 0; --j) {
29      int c = (num >> j) & 1; // Extract j-th bit (0 or 1)
30      if (trie[v].next[c] == -1) {
31        trie[v].next[c] = trie.size();
32        trie.emplace_back(); // Add new node
33      }
34      v = trie[v].next[c]; // Move to next node
35    }
36  }
```

3.4   CHMIN/CHMAX Rangesum GCD Queries

```
1  // Segment tree for chmin, chmax, add and query sum.
2  // Complexity is O (n (log n) ^ 2)
3
4  // Range chmin (l, r, v) -> a[l], ...., a[r] = min(a[i], v) call update
       with 1
5  // Range chmax (l, r, v) -> a[l], ...., a[r] = max(a[i], v) call update
       with 2
6  // Range set (l, r, v) -> a[l], ...., a[r] = v call update with 3 (this
       calls chmax and chmin on the interval)
7  // Range add (l, r, v) -> a[l], ...., a[r] += v call update with 4
8  // You can obtain sum, max, min, gcd in segment
9
10  typedef long long ll;
11
12  const ll inf = 1e17;
13
14  ll gcd(ll a, ll b){
15    a = abs(a);
16    b = abs(b);
17    if(a == 0) return b;
18    return gcd(b % a, a);
19  }
20
21  struct sum_t {
```

```
22    ll sum, mx, mx2, cnt_mx, mn, mn2, cnt_mn, gcd_no_max_no_min;
23    static const ll null_v = 0;
24    sum_t(): sum(null_v), mx(-inf), mx2(-inf), cnt_mx(0), mn(inf), mn2(inf
         ), cnt_mn(0), gcd_no_max_no_min(0) {}
25    sum_t(ll v): sum(v), mx(v), mx2(-inf), cnt_mx(1), mn(v), mn2(inf),
         cnt_mn(1), gcd_no_max_no_min(0) {}
26    sum_t(ll _sum, ll _mx, ll _mx2, ll _cnt_mx, ll _mn, ll _mn2, ll
         _cnt_mn, ll _gcd_no_max_no_min):
27      sum(_sum), mx(_mx), mx2(_mx2), cnt_mx(_cnt_mx), mn(_mn), mn2(_mn2),
           cnt_mn(_cnt_mn), gcd_no_max_no_min(_gcd_no_max_no_min) {}
28    sum_t op(sum_t& other) {
29      sum_t res;
30      res.sum = sum + other.sum;
31      res.gcd_no_max_no_min = gcd(gcd_no_max_no_min, other.
           gcd_no_max_no_min);
32
33      if(mx == other.mx){
34        res.mx = mx;
35        res.cnt_mx = cnt_mx + other.cnt_mx;
36        res.mx2 = max(mx2, other.mx2);
37      }
38      else if(mx > other.mx){
39        res.mx = mx;
40        res.cnt_mx = cnt_mx;
41        res.mx2 = max(mx2, other.mx);
42      }
43      else{
44        res.mx = other.mx;
45        res.cnt_mx = other.cnt_mx;
46        res.mx2 = max(mx, other.mx2);
47      }
48
49      if(mn == other.mn){
50        res.mn = mn;
51        res.cnt_mn = cnt_mn + other.cnt_mn;
52        res.mn2 = min(mn2, other.mn2);
53      }
54      else if(mn < other.mn){
55        res.mn = mn;
56        res.cnt_mn = cnt_mn;
57        res.mn2 = min(mn2, other.mn);
58      }
59      else{
```

```
60        res.mn = other.mn;
61        res.cnt_mn = other.cnt_mn;
62        res.mn2 = min(mn, other.mn2);
63      }
64
65      if(mx2 != -inf && mx2 != mn && other.mx2 != -inf && other.mx2 !=
            other.mn){
66        res.gcd_no_max_no_min = gcd(res.gcd_no_max_no_min, mx2 - other.mx2
            );
67      }
68
69      ll any = -1;
70
71      if(mx2 != -inf && mx2 != mn) any = mx2;
72      if(other.mx2 != -inf && other.mx2 != other.mn) any = other.mx2;
73
74      vector<ll> diff_max_min = {mx, mn, other.mx, other.mn};
75      for(ll val: diff_max_min){
76        if(val != res.mn && val != res.mx){
77          if(any != -1) res.gcd_no_max_no_min = gcd(res.gcd_no_max_no_min,
              val - any);
78          else any = val;
79        }
80      }
81
82      return res;
83    }
84  };
85
86  template <typename num_t>
87  struct segtree {
88    int n;
89    vector<num_t> tree, lazy;
90
91    void init(int s, const vector<ll> &A) {
92      n = s;
93      tree.assign(4 * n, num_t());
94      lazy.assign(4 * n, num_t());
95      init(0, 0, n - 1, A);
96    }
97
98    num_t init(int i, int l, int r, const vector<ll> &A) {
99      if (l == r) return tree[i] = num_t(A[l]);
100     int mid = (l + r) / 2;
101     num_t left = init(2 * i + 1, l, mid, A);
102     num_t right = init(2 * i + 2, mid + 1, r, A);
103     return tree[i] = left.op(right);
104   }
105
106   // update 3 is setting values to v
107   void update(int l, int r, ll v, int op) {
108     if (l > r) return;
109     if(op == 1){
110       update_chmin(0, 0, n - 1, l, r, v);
111     }
112     if(op == 2){
113       update_chmax(0, 0, n - 1, l, r, v);
114     }
115     if(op == 3){
116       update_chmin(0, 0, n - 1, l, r, v);
117       update_chmax(0, 0, n - 1, l, r, v);
118     }
119     if(op == 4){
120       update_add(0, 0, n - 1, l, r, v);
121     }
122   }
123
124   num_t update_chmin(int i, int tl, int tr, int ql, int qr, ll v) {
125     eval_lazy(i, tl, tr);
126
127     if (tr < ql || qr < tl || tree[i].mx <= v) return tree[i];
128     if (ql <= tl && tr <= qr && tree[i].mx2 < v) {
129       push_min(i, v);
130       eval_lazy(i, tl, tr);
131       return tree[i];
132     }
133
134     int mid = (tl + tr) / 2;
135     num_t a = update_chmin(2 * i + 1, tl, mid, ql, qr, v);
136     num_t b = update_chmin(2 * i + 2, mid + 1, tr, ql, qr, v);
137     return tree[i] = a.op(b);
138   }
139
140   num_t update_chmax(int i, int tl, int tr, int ql, int qr, ll v) {
141     eval_lazy(i, tl, tr);
```

```
143        if (tr < ql || qr < tl || tree[i].mn >= v) return tree[i];
144        if (ql <= tl && tr <= qr && tree[i].mn2 > v) {
145          push_max(i, v);
146          eval_lazy(i, tl, tr);
147          return tree[i];
148        }
149
150
151        int mid = (tl + tr) / 2;
152        num_t a = update_chmax(2 * i + 1, tl, mid, ql, qr, v);
153        num_t b = update_chmax(2 * i + 2, mid + 1, tr, ql, qr, v);
154        return tree[i] = a.op(b);
155      }
156
157      num_t update_add(int i, int tl, int tr, int ql, int qr, ll v) {
158        eval_lazy(i, tl, tr);
159
160        if (tr < ql || qr < tl) return tree[i];
161        if (ql <= tl && tr <= qr) {
162          push_add(i, tl, tr, v);
163          eval_lazy(i, tl, tr);
164          return tree[i];
165        }
166
167        int mid = (tl + tr) / 2;
168        num_t a = update_add(2 * i + 1, tl, mid, ql, qr, v);
169        num_t b = update_add(2 * i + 2, mid + 1, tr, ql, qr, v);
170        return tree[i] = a.op(b);
171      }
172
173      num_t query(int l, int r){
174        if (l > r) return num_t();
175        return query(0, 0, n - 1, l , r);
176      }
177
178      num_t query(int i, int tl, int tr, int ql, int qr) {
179        eval_lazy(i, tl, tr);
180
181        if (ql <= tl && tr <= qr) return tree[i];
182        if (tr < ql || qr < tl) return num_t();
183
184        int mid = (tl + tr) / 2;
185        num_t a = query(2 * i + 1, tl, mid, ql, qr);
186        num_t b = query(2 * i + 2, mid + 1, tr, ql, qr);
187        return a.op(b);
188      }
189
190      ll query_gcd(int l, int r){
191        if(l > r) return 0;
192        return query_gcd(0, 0, n - 1, l, r);
193      }
194
195      ll query_gcd(int i, int tl, int tr, int ql, int qr){
196        eval_lazy(i, tl, tr);
197
198        if (ql <= tl && tr <= qr){
199          ll res = gcd(tree[i].mx, tree[i].gcd_no_max_no_min);
200          if(tree[i].mn2 != inf) res = gcd(res, tree[i].mn2 - tree[i].mn);
201          if(tree[i].mx2 != -inf) res = gcd(res, tree[i].mx - tree[i].mx2);
202          return res;
203        }
204        if (tr < ql || qr < tl) return 0;
205
206        int mid = (tl + tr) / 2;
207        ll a = query_gcd(2 * i + 1, tl, mid, ql, qr);
208        ll b = query_gcd(2 * i + 2, mid + 1, tr, ql, qr);
209        return gcd(a, b);
210      }
211
212      void eval_lazy(int i, int l, int r) {
213        if (l != r) {
214          int mid = (l + r) / 2;
215
216          push_add(2 * i + 1, l, mid, lazy[i].sum);
217          push_add(2 * i + 2, mid + 1, r, lazy[i].sum);
218
219          push_min(2 * i + 1, tree[i].mx);
220          push_min(2 * i + 2, tree[i].mx);
221
222          push_max(2 * i + 1, tree[i].mn);
223          push_max(2 * i + 2, tree[i].mn);
224
225        }
226        lazy[i] = num_t();
227      }
228
```

```
229    void push_add(int i, int l, int r, ll v){
230      if(v == 0) return;
231      tree[i].mx += v;
232      tree[i].sum += v * (r - l + 1);
233      tree[i].mx2 += (tree[i].mx2 == -inf ? 0 : v);
234      tree[i].mn += v;
235      tree[i].mn2 += (tree[i].mn2 == inf ? 0 : v);
236      lazy[i].sum += v;
237    }
238
239    void push_min(int i, ll v){
240      if (tree[i].mx <= v) return;
241      if(tree[i].mn == tree[i].mx){
242        tree[i].mn = v;
243      }
244      else if(tree[i].mn2 == tree[i].mx){
245        tree[i].mn2 = v;
246      }
247      tree[i].sum -= tree[i].mx * tree[i].cnt_mx;
248      tree[i].mx = v;
249      tree[i].sum += tree[i].mx * tree[i].cnt_mx;
250    }
251
252    void push_max(int i, ll v){
253      if (tree[i].mn >= v) return;
254      if(tree[i].mx == tree[i].mn){
255        tree[i].mx = v;
256      }
257      else if(tree[i].mx2 == tree[i].mn){
258        tree[i].mx2 = v;
259      }
260      tree[i].sum -= tree[i].mn * tree[i].cnt_mn;
261      tree[i].mn = v;
262      tree[i].sum += tree[i].mn * tree[i].cnt_mn;
263    }
264 };
265
266 int main(){
267    ios_base::sync_with_stdio(false); cin.tie(NULL);
268    int n; cin >> n;
269    vector<ll> A(n);
270    for(int i = 0; i < n; i++) cin >> A[i];
271    segtree <sum_t> st;
272    st.init(n, A);
273    int q; cin >> q;
274    while(q--){
275      int t, l, r; cin >> t >> l >> r;
276      l--; r--;
277      if(t < 5){
278        ll x; cin >> x;
279        st.update(l, r, x, t);
280      }
281      else{
282        if(t == 8){
283          cout << st.query_gcd(l, r) << endl;
284        }
285        else{
286          auto ans = st.query(l, r);
287          if(t == 5) cout << ans.sum << endl;
288          else if (t == 6) cout << ans.mn << endl;
289          else if(t == 7) cout << ans.mx << endl;
290        }
291      }
292    }
293 }
```

3.5   Disjoint Sparse Table

```
1  // queries are given weird because it is a lot of them but it is segment
       multiplication
2  #include<bits/stdc++.h>
3  using namespace std;
4
5  #define pii pair<int,int>
6  #define pll pair<ll,ll>
7  #define eb emplace_back
8  #define ll long long
9  #define nl '\n'
10 #define deb(x) cerr<<#x" = "<<x<<nl
11 #define in() ( { int a ; scanf("%d",&a); a; } )
12
13 const int N = 3e5 + 9;
14 int mod = 1e9 + 7;
15
16 struct DST {
17    vector<vector<int>> left, right;
```

```
18    int k, n;
19    DST(vector<int> & a) {
20      n = (int)a.size();
21      k = log2(n) + 2;
22      left.assign(k + 1, vector<int>(n));
23      right.assign(k + 1, vector<int>(n));
24      for(int j = 0; (1 << j) <= n; ++j) {
25        int mask = (1 << j) - 1;
26        int nw = 1; //neutral
27        for(int i = 0; i < n; ++i) {
28          nw = 1LL * nw * a[i] % mod;//prefix value
29          left[j][i] = nw;
30          if((i & mask) == mask) nw = 1; //neutral
31        }
32        nw = 1; //neutral
33        for(int i = n - 1; i >= 0; --i) {
34          nw = 1LL * nw * a[i] % mod;//prefix value
35          right[j][i] = nw;
36          if((i & mask) == 0) nw = 1; //neutral
37        }
38      }
39    }
40    int query(int l, int r) {
41      if(l == r) return left[0][l];
42      int i = 31 - __builtin_clz(l ^ r);
43      int uno = left[i][r];
44      int dos = right[i][l];
45      return 1LL * uno * dos % mod;
46    }
47  };
48
49  int32_t main() {
50    int tc = in();
51    while(tc--) {
52      int n = in(), p = in(), q = in();
53      mod = p;
54      vector<int> a(n);
55      for(int i = 0; i < n; i++) a[i] = in();
56      DST t(a);
57      vector<int> b((q >> 6) + 2);
58      for(int i = 0; i < (int)b.size(); i++) b[i] = in();
59      int x = 0, l = 1, r = 1;
60      for(int i = 0; i < q; i++) {
61        if(i % 64 == 0) {
62          l = (b[i / 64] + x) % n;
63          r = (b[(i / 64) + 1] + x) % n;
64        } else {
65          l = (l + x) % n;
66          r = (r + x) % n;
67        }
68        if(l > r) swap(l, r);
69        int ans = t.query(l, r);
70        x = ans;
71        x++;
72        x %= mod;
73      }
74      printf("%d\n", x);
75    }
76    return 0;
77  }
```

3.6   Disjoint Set Union Bipartite

```
1   /*
2     DSU with Parity - Bipartite Checker
3     -----------------------------------
4     Indexing: 0-based
5     Node Bounds: [0, n-1] inclusive
6
7     Features:
8       - Tracks parity (even/odd length) of paths in each component
9       - Can be used to detect odd-length cycles (non-bipartite components)
10      - Supports dynamic edge additions
11
12    Functions:
13      - make_set(v): initializes singleton component
14      - find_set(v): returns root of v and its parity relative to root
15      - add_edge(a, b): merges two components and checks for bipartite
                violation
16      - is_bipartite(v): returns whether component containing v is
                bipartite
17  */
18
19  const int MAXN = 100005; // Set according to constraints
20
21  pair<int, int> parent[MAXN]; // parent[v] = {root, parity_from_root_to_v
```

```
21          }
22   int rank[MAXN];              // Union by rank
23   bool bipartite[MAXN];        // bipartite[root] = true if component is
         bipartite
24
25   // Create a new set for node v
26   void make_set(int v) {
27     parent[v] = {v, 0};   // Self-rooted, even parity to self
28     rank[v] = 0;
29     bipartite[v] = true;
30   }
31
32   // Find the root of v and track parity along the path (0 = even, 1 = odd
         )
33   pair<int, int> find_set(int v) {
34     if (v != parent[v].first) {
35       auto [par, parity] = parent[v];
36       auto root = find_set(par);
37       parent[v] = {root.first, parity ^ root.second}; // Path compression
           with parity update
38     }
39     return parent[v];
40   }
41
42   // Adds an edge between a and b, merges components and checks for odd
         cycles
43   void add_edge(int a, int b) {
44     auto [ra, pa] = find_set(a); // ra = root of a, pa = parity from root
           to a
45     auto [rb, pb] = find_set(b); // rb = root of b, pb = parity from root
           to b
46
47     if (ra == rb) {
48       // Same component: edge (a, b) adds a cycle -> check parity
49       if ((pa ^ pb) == 0) {
50         bipartite[ra] = false; // Found odd-length cycle
51       }
52     } else {
53       // Merge smaller rank under larger
54       if (rank[ra] < rank[rb]) swap(ra, rb), swap(pa, pb);
55
56       // Make rb child of ra; update parity of root
57       parent[rb] = {ra, pa ^ pb ^ 1};
```

```
58
59       bipartite[ra] &= bipartite[rb]; // Component is only bipartite if
           both were
60       if (rank[ra] == rank[rb]) rank[ra]++;
61     }
62   }
63
64   // Check if the component containing v is bipartite
65   bool is_bipartite(int v) {
66     return bipartite[find_set(v).first];
67   }
```

3.7   Disjoint Set Union

```
1    /*
2      Disjoint Set Union (Union-Find) with Rollback
3      ---------------------------------------------
4      Indexing: 0-based
5      Node Bounds: [0, N-1]
6
7      Features:
8        - Path compression + union by size
9        - Optional rollback to previous state
10       - Supports dynamic connectivity in offline algorithms (e.g., divide
             & conquer)
11
12     Functions:
13       - get(x): find root of x
14       - connected(a, b): check if a and b are in same component
15       - size(x): size of component containing x
16       - unite(x, y): union x and y, returns true if merged
17       - time(): current rollback timestamp
18       - rollback(t): revert to state at timestamp t
19   */
20
21   struct DSU {
22     vector<int> e;                  // e[x] < 0 -> root; size = -e[x]; e
           [x] >= 0 -> parent
23     vector<pair<int, int>> st;      // rollback stack: stores (index, old
           value)
24
25     DSU(int N) : e(N, -1) {}
26
```

```
27    // Find with path compression
28    int get(int x) {
29      return e[x] < 0 ? x : get(e[x]);
30    }
31
32    // Check if x and y belong to the same component
33    bool connected(int a, int b) {
34      return get(a) == get(b);
35    }
36
37    // Return size of component containing x
38    int size(int x) {
39      return -e[get(x)];
40    }
41
42    // Union by size, returns true if union happened
43    bool unite(int x, int y) {
44      x = get(x), y = get(y);
45      if (x == y) return false;
46      if (e[x] > e[y]) swap(x, y); // Ensure x has larger size (more
              negative)
47      st.push_back({x, e[x]});
48      st.push_back({y, e[y]});
49      e[x] += e[y];
50      e[y] = x;
51      return true;
52    }
53
54    // Return current rollback timestamp
55    int time() {
56      return (int)st.size();
57    }
58
59    // Roll back to previous state at time t
60    void rollback(int t) {
61      for (int i = time(); i-- > t;) {
62        e[st[i].first] = st[i].second;
63      }
64      st.resize(t);
65    }
66  };
```

3.8   Disjoint Set Union Queue Like Rollback

```
1  #include <bits/stdc++.h>
2      #define finish(x) return cout << x << endl, 0
3      typedef long long ll;
4      typedef long double ldb;
5      const int md = 1e9 + 7;
6      const ll inf = 2e18;
7      const int OO = 1;
8      const int OOO = 1;
9      using namespace std;
10
11     // used in the original data structure to represent a change in
            memory cell
12     struct change {
13       int t, value;
14       int *cell;
15       change() {}
16       change(int _time, int _value, int *_cell) {
17         t = _time;
18         value = _value;
19         cell = _cell;
20       }
21       void undo() {
22         *cell = value;
23       }
24     };
25
26     // dsu with normal undo, allowes to query bipartite-ness.
27     struct dsu {
28       int n;
29       vector<int> p, r;
30       vector<int> up;
31       int nxtupd;
32       vector<change> stack;
33       int first_violate;
34       dsu() {}
35       dsu(int sz) {
36         n = sz;
37         p.resize(n);
38         r.resize(n, 0);
39         up.resize(n, 0);
40         nxtupd = 0;
```

```
41       first_violate = -1;
42       for (int i = 0; i < n; i++) p[i] = i;
43     }
44     void upd(int *cell, int value) {
45       stack.push_back(change(nxtupd, *cell, cell));
46       *cell = value;
47     }
48     pair<int, int> find(int x) {
49       if (x == p[x]) return{ x, 0 };
50       pair<int, int> tmp = find(p[x]);
51       tmp.second ^= up[x];
52       return tmp;
53     }
54     bool is_bipartite() {
55       return first_violate == -1; // no violation
56     }
57     void unite(int x, int y) {
58       nxtupd++;
59       int col = 1;
60       pair<int, int> tmp = find(x);
61       x = tmp.first, col ^= tmp.second;
62       tmp = find(y);
63       y = tmp.first, col ^= tmp.second;
64       if (x == y) {
65         if (col == 1 && first_violate == -1) first_violate = nxtupd;
66         upd(&p[0], p[0]); // mark this update's existence.
67         return;
68       }
69       if (r[x] < r[y]) {
70         upd(&p[x], y);
71         upd(&up[x], col);
72       }
73       else {
74         upd(&p[y], x);
75         upd(&up[y], col);
76         if (r[x] == r[y]) {
77           upd(&r[x], r[x] + 1);
78         }
79       }
80       return;
81     }
82     void undo() {
83       if (!stack.size()) return;
84       int t = stack.back().t;
85       if (first_violate == t) first_violate = -1;
86       while (stack.size() && stack.back().t == t) {
87         stack.back().undo();
88         stack.pop_back();
89       }
90     }
91   };
92
93   // I'd want this 'struct update' to be more generic, and support
94   //     maintaining different kinds of updates...
     // but for now I'm not sure how. So this is a specific impl for this
     //     problem ('update' only relates to 'unite').
95   struct update {
96     char type; // 'A' or 'B'
97     int x, y;
98     update() {}
99     update(int xx, int yy) {
100      x = xx;
101      y = yy;
102      type = 'B';
103    }
104  };
105
106  struct dsuqueue {
107    dsu D;
108    vector<update> S;
109    int bottom; // bottom of the stack: S[0..bottom-1] is entirely B's
                 .
110    dsuqueue() {}
111    dsuqueue(int sz) {
112      D = dsu(sz);
113      bottom = 0;
114    }
115    // utility:
116    void advance_bottom() {
117      while (bottom < S.size() && S[bottom].type == 'B') bottom++;
118    }
119    void fix() {
120      if (!S.size() || S.back().type == 'A') return;
121      advance_bottom();
122      vector<update> saveB, saveA;
123      saveB.push_back(S.back());
```

```cpp
        S.pop_back(), D.undo();
        while (saveA.size() != saveB.size() && S.size() > bottom) {
          if (S.back().type == 'A')
            saveA.push_back(S.back());
          else
            saveB.push_back(S.back());
          S.pop_back(), D.undo();
        }
        // reverse saveA and saveB so their relative order is maintained
        reverse(saveA.begin(), saveA.end());
        reverse(saveB.begin(), saveB.end());
        for (const update &u : saveB) {
          S.push_back(u);
          D.unite(u.x, u.y);
        }
        for (const update &u : saveA) {
          S.push_back(u);
          D.unite(u.x, u.y);
        }
        advance_bottom();
      }
    void reverse_updates() {
      for (int i = 0; i < S.size(); i++)
        D.undo();
      for (int i = (int)S.size() - 1; i >= 0; i--) {
        D.unite(S[i].x, S[i].y);
        S[i].type = 'A';
      }
      reverse(S.begin(), S.end());
      bottom = 0;
    }
    void undo() {
      advance_bottom();
      if (bottom == S.size()) {
        // no more A's, let's reverse and begin again.
        reverse_updates();
      }
      fix();
      D.undo();
      S.pop_back();
    }

    // begin copying all functions from the original dsu.
```

```cpp
    bool is_bipartite() {
      return D.is_bipartite();
    }
    void unite(int x, int y) {
      D.unite(x, y);
      S.push_back(update(x, y));
    }
};

int n, m, q;
vector<pair<int, int>> e;
vector<int> R; // for each i, R[i] would be the minimum such that
               // query (i, R[i]) is answered with NO (the graph is bipartite)

int main() {
  ios::sync_with_stdio(0), cin.tie(0);
  cin >> n >> m >> q;
  dsuqueue D(n);
  e.resize(m);
  for (auto &i : e) {
    cin >> i.first >> i.second;
    --i.first, --i.second;
  }

  R.resize(m);
  for (int i = 0; i < m; i++)
    D.unite(e[i].first, e[i].second);
  int nxt = 0;
  for (int i = 0; i < m; i++) {
    while (!D.is_bipartite() && nxt < m) {
      D.undo();
      nxt++;
    }
    if (D.is_bipartite())
      R[i] = nxt - 1;
    else
      R[i] = md;

    D.unite(e[i].first, e[i].second);
  }

  while (q--) {
    int l, r;
```

```
209        cin >> l >> r;
210        --l, --r;
211        if (R[l] <= r) cout << "NO\n";
212        else cout << "YES\n";
213      }
214    }
```

3.9   Dynamic Conectivity

```
1   /*
2     Offline Dynamic Connectivity - Segment Tree + Rollback DSU
3     ------------------------------------------------------------
4     Indexing: 0-based
5     Node Bounds: [0, n-1]
6
7     Features:
8       - Handles dynamic edge insertions and deletions over time
9       - Answers queries of type: "how many connected components at time t
            ?"
10      - All operations are processed offline
11
12    Components:
13      - DSU with rollback (stores a stack of previous states)
14      - Segment tree over time to store active edge intervals
15   */
16
17   // Rollbackable Disjoint Set Union (Union-Find)
18   struct DSU {
19     vector<int> e; // e[x] < 0 means x is a root, and size is -e[x]
20     vector<pair<int, int>> st; // Stack for rollback (stores changed
          values)
21     int cnt; // Current number of connected components
22
23     DSU() {}
24     DSU(int N) : e(N, -1), cnt(N) {}
25
26     // Find root of x with path compression
27     int get(int x) {
28       return e[x] < 0 ? x : get(e[x]);
29     }
30
31     // Check if x and y are connected
32     bool connected(int a, int b) {
```

```
33       return get(a) == get(b);
34     }
35
36     // Size of component containing x
37     int size(int x) {
38       return -e[get(x)];
39     }
40
41     // Union two components; record state for rollback
42     bool unite(int x, int y) {
43       x = get(x), y = get(y);
44       if (x == y) return false; // Already connected
45
46       if (e[x] > e[y]) swap(x, y); // Union by size: ensure x is larger
47
48       // Save state for rollback
49       st.push_back({x, e[x]});
50       st.push_back({y, e[y]});
51
52       e[x] += e[y]; // Merge y into x
53       e[y] = x;
54       cnt--; // One fewer component
55       return true;
56     }
57
58     // Undo last union
59     void rollback() {
60       auto [x1, y1] = st.back(); st.pop_back();
61       e[x1] = y1;
62       auto [x2, y2] = st.back(); st.pop_back();
63       e[x2] = y2;
64       cnt++;
65     }
66   };
67
68   // Represents a single union operation (on edge u-v)
69   struct query {
70     int v, u;
71     bool united;
72     query(int _v, int _u) : v(_v), u(_u), united(false) {}
73   };
74
75   // Segment Tree for storing edge intervals [l, r]
```

```cpp
struct QueryTree {
  vector<vector<query>> t; // Each node stores queries that are active
      in that time segment
  DSU dsu;
  int T; // Number of total operations (time steps)

  QueryTree() {}
  QueryTree(int _T, int n) : T(_T) {
    dsu = DSU(n);
    t.resize(4 * T + 4);
  }

  // Internal segment tree add function
  void add(int v, int l, int r, int ul, int ur, query& q) {
    if (ul > ur) return;
    if (l == ul && r == ur) {
      t[v].push_back(q);
      return;
    }
    int mid = (l + r) / 2;
    add(2 * v, l, mid, ul, min(ur, mid), q);
    add(2 * v + 1, mid + 1, r, max(ul, mid + 1), ur, q);
  }

  // Public wrapper: add a query in interval [l, r]
  void add_query(query q, int l, int r) {
    add(1, 0, T - 1, l, r, q);
  }

  // Traverse the segment tree and simulate unions with rollback
  void dfs(int v, int l, int r, vector<int>& ans) {
    // Apply all union operations in this segment node
    for (query& q : t[v]) {
      q.united = dsu.unite(q.v, q.u);
    }

    if (l == r) {
      ans[l] = dsu.cnt; // Save answer for time l
    } else {
      int mid = (l + r) / 2;
      dfs(2 * v, l, mid, ans);
      dfs(2 * v + 1, mid + 1, r, ans);
    }

    // Rollback all operations applied in this node
    for (query& q : t[v]) {
      if (q.united)
        dsu.rollback();
    }
  }
};

int main() {
  int n, k;
  cin >> n >> k; // n nodes, k operations
  if (k == 0) return 0;
  QueryTree st(k, n);
  map<pair<int, int>, int> mp; // Edge -> start time
  vector<int> ans(k);          // Answers for '?' queries
  vector<int> qmarks;          // Indices of '?' queries
  // Parse all k operations
  for (int i = 0; i < k; i++) {
    char c;
    cin >> c;
    if (c == '?') {
      qmarks.push_back(i); // Save query index
      continue;
    }
    int u, v;
    cin >> u >> v;
    u--; v--;
    if (u > v) swap(u, v); // Normalize edge direction

    if (c == '+') {
      mp[{u, v}] = i; // Mark time edge is added
    } else {
      // Edge removed: store active interval
      st.add_query(query(u, v), mp[{u, v}], i);
      mp[{u, v}] = -1;
    }
  }
  // Any edge still active is added until end of timeline
  for (auto [edge, start] : mp) {
    if (start != -1) {
      st.add_query(query(edge.first, edge.second), start, k - 1);
    }
  }
```

```
161      }
162      // Process the tree to compute all '?'-query answers
163      st.dfs(1, 0, k - 1, ans);
164      // Output results of all '?'
165      for (int x : qmarks) {
166        cout << ans[x] << '\n';
167      }
168    }
```

3.10   Fenwick Tree

```
1    /*
2      Fenwick Tree (Binary Indexed Tree)
3      ----------------------------------
4      Indexing: 0-based
5      Bounds: [0, n-1] inclusive
6      Time Complexity:
7        - add(x, v): O(log n)
8        - sum(x): O(log n) -> prefix sum over [0, x)
9        - rangeSum(l, r): O(log n) -> sum over [l, r)
10       - select(k): O(log n) -> smallest x such that prefix sum >= k (works
                for monotonic cumulative sums)
11
12     Space Complexity: O(n)
13   */
14
15   template <typename T>
16   struct Fenwick {
17     int n;
18     std::vector<T> a;
19
20     Fenwick(int n_ = 0) {
21       init(n_);
22     }
23
24     // Initialize BIT of size n
25     void init(int n_) {
26       n = n_;
27       a.assign(n, T{});
28     }
29
30     // Add value 'v' to position 'x'
31     void add(int x, const T &v) {
32       for (int i = x + 1; i <= n; i += i & -i) {
33         a[i - 1] = a[i - 1] + v;
34       }
35     }
36
37     // Compute prefix sum on range [0, x)
38     T sum(int x) const {
39       T ans{};
40       for (int i = x; i > 0; i -= i & -i) {
41         ans = ans + a[i - 1];
42       }
43       return ans;
44     }
45
46     // Compute sum over range [l, r)
47     T rangeSum(int l, int r) const {
48       return sum(r) - sum(l);
49     }
50
51     // Find the smallest x such that sum[0, x) > k (if exists), or returns
            n
52     int select(const T &k) const {
53       int x = 0;
54       T cur{};
55       for (int i = 1 << std::__lg(n); i; i >>= 1) {
56         if (x + i <= n && cur + a[x + i - 1] <= k) {
57           cur = cur + a[x + i - 1];
58           x += i;
59         }
60       }
61       return x;
62     }
63   };
64   // Fenwick<int> bit(n);
```

3.11   Fenwick Tree 2D

```
1    /*
2      2D Fenwick Tree (Binary Indexed Tree)
3      -------------------------------------
4      Indexing: 1-based
5      Bounds: [1, n] inclusive
6      Time Complexity:
```

```
7      -update(x, y, v): O(log^2 n)
8      -get(x, y): sum of rectangle [1,1] to [x,y]
9      -get1(x1, y1, x2, y2): sum over rectangle [x1,y1] to [x2,y2]
10   Space Complexity: O(n^2)
11
12   -Can be adapted for rectangular grids by using n, m separately
13  */
14
15  struct Fenwick2D {
16    vector<vector<ll>> b; // 2D BIT array
17    int n;                 // Grid size (1-based)
18
19    Fenwick2D(int _n) : b(_n + 5, vector<ll>(_n + 5, 0)), n(_n) {}
20
21    // Add 'val' to cell (x, y)
22    void update(int x, int y, int val) {
23      for (; x <= n; x += (x & -x)) {
24        for (int j = y; j <= n; j += (j & -j)) {
25          b[x][j] += val;
26        }
27      }
28    }
29
30    // Get sum of rectangle [(1,1) to (x,y)]
31    ll get(int x, int y) {
32      ll ans = 0;
33      for (; x > 0; x -= x & -x) {
34        for (int j = y; j > 0; j -= j & -j) {
35          ans += b[x][j];
36        }
37      }
38      return ans;
39    }
40
41    // Get sum over subrectangle [(x1,y1) to (x2,y2)]
42    ll get1(int x1, int y1, int x2, int y2) {
43      return get(x2, y2)-get(x1-1, y2)-get(x2, y1-1) + get(x1-1, y1-1);
44    }
45  };
46  // Usage example:
47  Fenwick2D fw(n);
48  fw.update(3, 4, 5);                    // add 5 to (3, 4)
49  ll sum = fw.get(3, 4);                 // sum from (1,1) to (3,4)
```

```
50  ll range = fw.get1(2, 2, 5, 5);    // sum in rectangle [(2,2)-(5,5)]
```

3.12   Li Chao Tree

```
1  // min of a * x^3 + b * x ^2 + c * x + d
2
3  #include <bits/stdc++.h>
4  using namespace std;
5  typedef long long ll;
6
7  const int N = (int)1e5 + 5;
8  const int C = (int)1e5 + 5;
9  const int T = 10;
10  const int M = 350; // sqrt(10^5)
11  const ll inf = (ll)1e19;
12
13  namespace segtree {
14    struct Poly {
15      ll a, b, c, d = inf;
16      ll operator()(ll x) { return a * x * x * x + b * x * x + c * x + d;
17          }
18    } a[T][C * 4];
18    int t = -1;
19
20    void insert(int l, int r, Poly poly, int o=0) {
21      if(l + 1 == r) {
22        if(poly(l) < a[t][o](l)) a[t][o] = poly;
23        return;
24      }
25      int mid = (l + r) >> 1, lson = o * 2 + 1, rson = o * 2 + 2;
26      bool b1 = poly(mid) < a[t][o](mid), b2 = poly(l) < a[t][o](l);
27      if(b1) swap(poly, a[t][o]);
28      if(b1 != b2) insert(l, mid, poly, lson);
29      else insert(mid, r, poly, rson);
30    }
31    ll query(int l, int r, int x, int o=0) {
32      if(l + 1 == r) return a[t][o](x);
33      int mid = (l + r) >> 1, lson = o * 2 + 1, rson = o * 2 + 2;
34      if(x < mid) return min(a[t][o](x), query(l, mid, x, lson));
35      else return min(a[t][o](x), query(mid, r, x, rson));
36    }
37  }
38
```

```
39   ll ans[M];
40
41   void init() {
42     int n; cin >> n;
43     fill(ans, ans + M, inf);
44     segtree::t++;
45     while(n--) {
46       int a, b, c, d; cin >> d >> c >> b >> a;
47       segtree::Poly p = {a, b, c, d};
48       segtree::insert(M, C, p);
49       for(int i = 0 ; i < M ; ++i) ans[i] = min(ans[i], p(i));
50     }
51   }
52   void solve() {
53     int q; cin >> q;
54     while(q--) {
55       int x; cin >> x;
56       if(x < M) cout << ans[x] << '\n';
57       else cout << segtree::query(M, C, x) << '\n';
58     }
59   }
60
61   int main() {
62     ios_base::sync_with_stdio(0), cin.tie(0);
63     int t; cin >> t;
64     while(t--) {
65       init();
66       solve();
67     }
68   }
```

3.13   Link Cut Tree

```
1    #include<bits/stdc++.h>
2    using namespace std;
3
4    const int N = 1e5 + 9;
5
6    struct node {
7      int p = 0, c[2] = {0, 0}, pp = 0;
8      bool flip = 0;
9      int sz = 0, ssz = 0, vsz = 0; // sz -> aux tree size, ssz = subtree
           size in rep tree, vsz = virtual tree size
```

```
10     long long val = 0, sum = 0, lazy = 0, subsum = 0, vsum = 0;
11     node() {}
12     node(int x) {
13       val = x; sum = x;
14       sz = 1; lazy = 0;
15       ssz = 1; vsz = 0;
16       subsum = x; vsum = 0;
17     }
18   };
19   struct LCT {
20     vector<node> t;
21     LCT() {}
22     LCT(int n) : t(n + 1) {}
23
24     // <independant splay tree code>
25     int dir(int x, int y) { return t[x].c[1] == y; }
26     void set(int x, int d, int y) {
27       if (x) t[x].c[d] = y, pull(x);
28       if (y) t[y].p = x;
29     }
30     void pull(int x) {
31       if (!x) return;
32       int &l = t[x].c[0], &r = t[x].c[1];
33       push(l); push(r);
34       t[x].sum = t[l].sum + t[r].sum + t[x].val;
35       t[x].sz = t[l].sz + t[r].sz + 1;
36       t[x].ssz = t[l].ssz + t[r].ssz + t[x].vsz + 1;
37       t[x].subsum = t[l].subsum + t[r].subsum + t[x].vsum + t[x].val;
38     }
39     void push(int x) {
40       if (!x) return;
41       int &l = t[x].c[0], &r = t[x].c[1];
42       if (t[x].flip) {
43         swap(l, r);
44         if (l) t[l].flip ^= 1;
45         if (r) t[r].flip ^= 1;
46         t[x].flip = 0;
47       }
48       if (t[x].lazy) {
49         t[x].val += t[x].lazy;
50         t[x].sum += t[x].lazy * t[x].sz;
51         t[x].subsum += t[x].lazy * t[x].ssz;
52         t[x].vsum += t[x].lazy * t[x].vsz;
```

```
53        if (l) t[l].lazy += t[x].lazy;
54        if (r) t[r].lazy += t[x].lazy;
55        t[x].lazy = 0;
56      }
57    }
58    void rotate(int x, int d) {
59      int y = t[x].p, z = t[y].p, w = t[x].c[d];
60      swap(t[x].pp, t[y].pp);
61      set(y, !d, w);
62      set(x, d, y);
63      set(z, dir(z, y), x);
64    }
65    void splay(int x) {
66      for (push(x); t[x].p;) {
67        int y = t[x].p, z = t[y].p;
68        push(z); push(y); push(x);
69        int dx = dir(y, x), dy = dir(z, y);
70        if (!z) rotate(x, !dx);
71        else if (dx == dy) rotate(y, !dx), rotate(x, !dx);
72        else rotate(x, dy), rotate(x, dx);
73      }
74    }
75    // </independant splay tree code>
76
77    // making it a root in the rep. tree
78    void make_root(int u) {
79      access(u);
80      int l = t[u].c[0];
81      t[l].flip ^= 1;
82      swap(t[l].p, t[l].pp);
83      t[u].vsz += t[l].ssz;
84      t[u].vsum += t[l].subsum;
85      set(u, 0, 0);
86    }
87    // make the path from root to u a preferred path
88    // returns last path-parent of a node as it moves up the tree
89    int access(int _u) {
90      int last = _u;
91      for (int v = 0, u = _u; u; u = t[v = u].pp) {
92        splay(u); splay(v);
93        t[u].vsz -= t[v].ssz;
94        t[u].vsum -= t[v].subsum;
95        int r = t[u].c[1];
96        t[u].vsz += t[r].ssz;
97        t[u].vsum += t[r].subsum;
98        t[v].pp = 0;
99        swap(t[r].p, t[r].pp);
100       set(u, 1, v);
101       last = u;
102     }
103     splay(_u);
104     return last;
105   }
106   void link(int u, int v) { // u -> v
107     // assert(!connected(u, v));
108     make_root(v);
109     access(u); splay(u);
110     t[v].pp = u;
111     t[u].vsz += t[v].ssz;
112     t[u].vsum += t[v].subsum;
113   }
114   void cut(int u) { // cut par[u] -> u, u is non root vertex
115     access(u);
116     assert(t[u].c[0] != 0);
117     t[t[u].c[0]].p = 0;
118     t[u].c[0] = 0;
119     pull(u);
120   }
121   // parent of u in the rep. tree
122   int get_parent(int u) {
123     access(u); splay(u); push(u);
124     u = t[u].c[0]; push(u);
125     while (t[u].c[1]) {
126       u = t[u].c[1]; push(u);
127     }
128     splay(u);
129     return u;
130   }
131   // root of the rep. tree containing this node
132   int find_root(int u) {
133     access(u); splay(u); push(u);
134     while (t[u].c[0]) {
135       u = t[u].c[0]; push(u);
136     }
137     splay(u);
138     return u;
```

```
139        }
140        bool connected(int u, int v) {
141          return find_root(u) == find_root(v);
142        }
143        // depth in the rep. tree
144        int depth(int u) {
145          access(u); splay(u);
146          return t[u].sz;
147        }
148        int lca(int u, int v) {
149          // assert(connected(u, v));
150          if (u == v) return u;
151          if (depth(u) > depth(v)) swap(u, v);
152          access(v);
153          return access(u);
154        }
155        int is_root(int u) {
156          return get_parent(u) == 0;
157        }
158        int component_size(int u) {
159          return t[find_root(u)].ssz;
160        }
161        int subtree_size(int u) {
162          int p = get_parent(u);
163          if (p == 0) {
164            return component_size(u);
165          }
166          cut(u);
167          int ans = component_size(u);
168          link(p, u);
169          return ans;
170        }
171        long long component_sum(int u) {
172          return t[find_root(u)].subsum;
173        }
174        long long subtree_sum(int u) {
175          int p = get_parent(u);
176          if (p == 0) {
177            return component_sum(u);
178          }
179          cut(u);
180          long long ans = component_sum(u);
181          link(p, u);
182          return ans;
183        }
184        // sum of the subtree of u when root is specified
185        long long subtree_query(int u, int root) {
186          int cur = find_root(u);
187          make_root(root);
188          long long ans = subtree_sum(u);
189          make_root(cur);
190          return ans;
191        }
192        // path sum
193        long long query(int u, int v) {
194          int cur = find_root(u);
195          make_root(u); access(v);
196          long long ans = t[v].sum;
197          make_root(cur);
198          return ans;
199        }
200        void upd(int u, int x) {
201          access(u); splay(u);
202          t[u].val += x;
203        }
204        // add x to the nodes on the path from u to v
205        void upd(int u, int v, int x) {
206          int cur = find_root(u);
207          make_root(u); access(v);
208          t[v].lazy += x;
209          make_root(cur);
210        }
211    }t[2];
212
213    vector<int> g[N];
214    int par[N];
215    void dfs(int u, int p = 0) {
216      par[u] = p;
217      for (auto v: g[u]) {
218        if (v ^ p) {
219          dfs(v, u);
220        }
221      }
222    }
223    int col[N];
224    int32_t main() {
```

```
225    ios_base::sync_with_stdio(0);
226    cin.tie(0);
227    int n; cin >> n;
228    for (int i = 1; i < n; i++) {
229        int u, v; cin >> u >> v;
230        g[u].push_back(v);
231        g[v].push_back(u);
232    }
233    dfs(1, n + 1);
234    t[0] = LCT(n + 1);
235    t[1] = LCT(n + 1);
236    for (int i = 1; i <= n + 1; i++) {
237      t[0].t[i] = node(i);
238      t[1].t[i] = node(i);
239    }
240    for (int i = 1; i <= n; i++) {
241      t[0].link(par[i], i);
242    }
243    int q; cin >> q;
244    while (q--) {
245      int ty, u; cin >> ty >> u;
246      if (ty == 0) {
247        int z = t[col[u]].find_root(u);
248        int c = t[col[u]].t[z].c[1];
249        cout << t[col[u]].t[c].ssz << '\n';
250      }
251      else {
252        t[col[u]].cut(u);
253        col[u] ^= 1;
254        t[col[u]].link(par[u], u);
255      }
256    }
257    return 0;
258 }
```

3.14   Linked List

```
1  /*
2    Linked list
3    ------------------------------------
4    Time Complexity:
5      -insert: O(1)
6    Space Complexity: O(n)
```

```
7
8    -Can be adapted for rectangular grids by using n, m separately
9  */
10 // === Constructors and Destructor ===
11 // list()                // Default constructor, creates an empty list.
12 // list(n, val)          // Creates a list with 'n' elements, all
         initialized to 'val'.
13 // list(first, last)     // Creates a list from another range (e.g.,
         another list or array).
14 // list(const list& other) // Copy constructor.
15 // list(list&& other)    // Move constructor.
16 // ~list()               // Destructor, destroys the list.
17
18 // === Iterators ===
19 // begin()               // Returns an iterator to the first element of
         the list.
20 // end()                 // Returns an iterator to the position just
         past the last element.
21 // rbegin()              // Returns a reverse iterator to the last
         element.
22 // rend()                // Returns a reverse iterator to the position
         just before the first element.
23 // insert(it, val)       // Inserts 'val' before the position of the
         iterator 'it'.
24 // erase(it)             // Removes the element at the position of the
         iterator 'it'.
25 // erase(first, last)    // Removes a range of elements from 'first' to
         'last'.
26 // push_back(val)        // Adds 'val' to the end of the list.
27 // push_front(val)       // Adds 'val' to the front of the list.
28 // pop_back()            // Removes the last element of the list.
29 // pop_front()           // Removes the first element of the list.
30 // clear()               // Removes all elements from the list.
31
32 // === Capacity ===
33 // empty()               // Returns true if the list is empty, otherwise
         false.
34 // size()                // Returns the number of elements in the list.
35 // max_size()            // Returns the maximum number of elements the
         list can hold.
36
37 // === Modifiers ===
38 // insert(it, n, val)    // Inserts 'n' copies of 'val' before the
```

```
         position of 'it'.
39  // insert(it, first, last) // Inserts a range of elements before the
         position of 'it'.
40  // remove(val)            // Removes all occurrences of 'val' from the
         list.
41  // remove_if(pred)        // Removes all elements that satisfy the
         predicate 'pred'.
42  // unique()               // Removes consecutive duplicate elements from
          the list.
43  // merge(other)           // Merges two sorted lists into one sorted
         list.
44  // merge(other, pred)     // Merges two sorted lists into one sorted
         list using custom comparison 'pred'.
45  // splice(it, other)      // Transfers elements from another list 'other
         ' to the list at position 'it'.
46  // splice(it, other, it2) // Transfers an element from another list '
         other' at position 'it2' to the list at position 'it'.
47  // splice(it, other, first, last) // Transfers a range from another list
          'other' from 'first' to 'last' to the list at position 'it'.
48  // reverse()              // Reverses the order of elements in the list.
49  // sort()                 // Sorts the elements in the list in ascending
         order.
50  // sort(comp)             // Sorts the elements in the list using custom
         comparison 'comp'.
51  // swap(other)            // Swaps the content of the list with another
        list 'other'.
52
53  // === Element Access ===
54  // front()                // Returns a reference to the first element in
         the list.
55  // back()                 // Returns a reference to the last element in
         the list.
56
57  // === Search ===
58  // find(val)              // Returns an iterator to the first occurrence
         of 'val'.
59  // find_if(pred)          // Returns an iterator to the first element
         satisfying the predicate 'pred'.
60  // count(val)             // Returns the number of occurrences of 'val'
         in the list.
61  // count_if(pred)         // Returns the number of elements satisfying
         the predicate 'pred'.
62  // equal_range(val)       // Returns a pair of iterators representing the
```

```
         range of elements equal to 'val'.
63
64  // === Other Functions ===
65  // swap(other)            // Swaps the contents of the list with another
        list 'other'.
66  // emplace(it, args...)   // Constructs and inserts an element at the
         position 'it' using 'args...' (perfect forwarding).
67  // emplace_front(args...) // Constructs and inserts an element at the
         front using 'args...' (perfect forwarding).
68  // emplace_back(args...)  // Constructs and inserts an element at the
         back using 'args...' (perfect forwarding).
```

3.15   Merge Sort Tree

```
1   /*
2     Merge Sort Tree (Segment Tree of Sorted Arrays)
3     -------------------------------------------------
4     Indexing: 0-based
5     Node Bounds: [0, n-1] inclusive
6     Time Complexity:
7        - build(): O(n log n)
8        - q(l, r, x): O(log^2 n) -> number of elements <= x in [l, r)
9     Space Complexity: O(n log n)
10
11    Features:
12       - Supports frequency/count queries: "how many values <= x in range [
            l, r)?"
13       - Static array (no point updates unless rebuilt)
14  */
15
16  const int MAXN = 100005;         // size of original array
17  const int MAXT = 2 * MAXN;       // size of segment tree (2n)
18
19  vector<int> t[MAXT];     // Segment tree: each node holds sorted vector
20  int a[MAXN];             // Original array
21  int n;                  // Size of array
22
23  // Build merge sort tree (bottom-up)
24  void build() {
25    // Fill leaves
26    for (int i = 0; i < n; i++) {
27      t[i + n].push_back(a[i]);
28    }
```

```
29
30    // Merge children into parent
31    for (int i = n - 1; i > 0; i--) {
32      auto &left = t[2 * i], &right = t[2 * i + 1];
33      merge(left.begin(), left.end(), right.begin(), right.end(),
            back_inserter(t[i]));
34    }
35 }
36
37 // Query how many elements <= 'x' in range [l, r)
38 int q(int l, int r, int x) {
39    int res = 0;
40    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
41      if (l & 1) {
42        res += upper_bound(t[l].begin(), t[l].end(), x) - t[l].begin();
43        l++;
44      }
45      if (r & 1) {
46        r--;
47        res += upper_bound(t[r].begin(), t[r].end(), x) - t[r].begin();
48      }
49    }
50    return res;
51 }
52 // Read n and array a, then call build()
```

3.16   Minimum Cartesian Tree

```
1  /*
2    Min Cartesian Tree
3    ------------------
4    Indexing: 0-based
5    Time Complexity: O(n)
6    Space Complexity: O(n)
7    Tree Properties:
8      - Binary tree where in-order traversal = original array
9      - Tree satisfies min-heap property: parent <= children
10     - 'par[i]': parent of node i
11     - 'sons[i][0]': left child, 'sons[i][1]': right child
12     - 'root': index of root node
13
14   Use cases:
15     - RMQ construction
16     - LCA over RMQ via Cartesian Tree
17 */
18
19 struct min_cartesian_tree {
20   vector<int> par;             // parent for each node
21   vector<vector<int>> sons;    // left and right children
22   int root;
23
24   void init(int n, const vector<int> &arr) {
25     par.assign(n, -1);
26     sons.assign(n, vector<int>(2, -1)); // 0 = left, 1 = right
27     stack<int> st;
28
29     for (int i = 0; i < n; i++) {
30       int last = -1;
31
32       // Maintain increasing stack -> build min Cartesian Tree
33       // Change > to < for max Cartesian Tree
34       while (!st.empty() && arr[st.top()] > arr[i]) {
35         last = st.top();
36         st.pop();
37       }
38
39       if (!st.empty()) {
40         par[i] = st.top();
41         sons[st.top()][1] = i;
42       }
43       if (last != -1) {
44         par[last] = i;
45         sons[i][0] = last;
46       }
47
48       st.push(i);
49     }
50
51     for (int i = 0; i < n; i++) {
52       if (par[i] == -1) {
53         root = i;
54       }
55     }
56   }
57 };
58 // Example usage:
```

```
59  vector<int> a = {4, 2, 6, 1, 3};
60  min_cartesian_tree ct;
61  ct.init(a.size(), a);
62  cout << "Root␣index:␣" << ct.root << '\n';
```

3.17   Multi Ordered Set

```
1  #include <ext/pb_ds/assoc_container.hpp>
2  #include <ext/pb_ds/tree_policy.hpp>
3  using namespace __gnu_pbds;
4  template <typename T> using oset = __gnu_pbds::tree<
5      T, __gnu_pbds::null_type, less<T>, __gnu_pbds::rb_tree_tag,
6      __gnu_pbds::tree_order_statistics_node_update
7  >;
8
9  //en main
10
11 oset<pair<int,int>> name;
12     map<int,int> cuenta;
13     function<void(int)> meter = [&] (int val) {
14         name.insert({val,++cuenta[val]});
15     };
16     auto quitar = [&] (int val) {
17         name.erase({val,cuenta[val]--});
18     };
19
20 meter(x);
21 quitar(y);
22 multioset.order_of_key({y+1,-1})-multioset.order_of_key({x,0})
```

3.18   Ordered Set

```
1  #include <ext/pb_ds/assoc_container.hpp>
2  #include <ext/pb_ds/tree_policy.hpp>
3  using namespace __gnu_pbds;
4  template <typename T> using oset = __gnu_pbds::tree<
5      T, __gnu_pbds::null_type, less<T>, __gnu_pbds::rb_tree_tag,
6      __gnu_pbds::tree_order_statistics_node_update
7  >;
8  // order_of_key() primero mayor o igual;
9  // find_by_order() apuntador al elemento k;
10 // oset<pair<int,int>> os;
11 // os.insert({1,2});
12 // os.insert({2,3});
13 // os.insert({5,6});
14 // ll k=os.order_of_key({2,0});
15 // cout<<k<<endl; // 1
16 // pair<int,int> p=*os.find_by_order(k);
17 // cout<<p.f<<" "<<p.s<<endl; // 2 3
18 // os.erase(p);
19 // p=*os.find_by_order(k);
20 // cout<<p.f<<" "<<p.s<<endl; // 5 6
21
22
23 // check if upperbound or lowerbound does what you want
24 // because they give better time.
25
26 // to allow repetitions
27 #define ordered_set tree<int, null_type,less_equal<int>, rb_tree_tag,
       tree_order_statistics_node_update>
28
29 // to not allow repetitions
30 #define ordered_set tree<int, null_type,less<int>, rb_tree_tag,
       tree_order_statistics_node_update>
31
32 //order_of_key(x): number of items are strictly smaller than x
33
34 //find_by_order(k) iterator to the kth element
```

3.19   Palindromic Tree

```
1  /*
2    Palindromic Tree (Eertree)
3    --------------------------
4    Indexing: 0-based
5    Time Complexity:
6      - extend(i): O(1) amortized
7      - calc_occurrences(): O(n)
8    Space Complexity: O(n)
9
10   Features:
11     - Each node represents a unique palindromic substring
12     - Efficient online construction
13     - `oc`: how many times this palindrome occurs as suffix
14     - `cnt`: number of palindromic suffixes in its subtree
15     - `link`: suffix link to longest proper palindromic suffix
16  */
```

```cpp
const int N = 3e5 + 9;

struct PalindromicTree {
  struct node {
    int nxt[26];       // transitions by character
    int len;           // length of palindrome
    int st, en;        // start and end indices in string
    int link;          // suffix link
    int cnt = 0;       // number of palindromic suffixes
    int oc = 0;        // occurrences of the palindrome
  };

  string s;
  vector<node> t;
  int sz, last;

  PalindromicTree() {}
  PalindromicTree(const string &_s) {
    s = _s;
    int n = s.size();
    t.clear();
    t.resize(n + 5); // up to n + 2 distinct palindromes
    sz = 2;
    last = 2;
    // Root 1: imaginary (-1 length), simplifies links
    t[1].len = -1;
    t[1].link = 1;
    // Root 2: length 0, link to root 1
    t[2].len = 0;
    t[2].link = 1;
  }

  // Extend tree with s[pos], returns 1 if a new node is created
  int extend(int pos) {
    int cur = last;
    int ch = s[pos] - 'a';
    // Find longest suffix palindrome that can be extended
    while (true) {
      int curlen = t[cur].len;
      if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos]) break;
      cur = t[cur].link;
    }
    if (t[cur].nxt[ch]) {
      // Already exists
      last = t[cur].nxt[ch];
      t[last].oc++;
      return 0;
    }
    // Create new node
    sz++;
    last = sz;
    t[sz].oc = 1;
    t[sz].len = t[cur].len + 2;
    t[cur].nxt[ch] = sz;
    t[sz].en = pos;
    t[sz].st = pos - t[sz].len + 1;

    if (t[sz].len == 1) {
      t[sz].link = 2;
      t[sz].cnt = 1;
      return 1;
    }
    // Compute suffix link
    while (true) {
      cur = t[cur].link;
      int curlen = t[cur].len;
      if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos]) {
        t[sz].link = t[cur].nxt[ch];
        break;
      }
    }
    t[sz].cnt = 1 + t[t[sz].link].cnt;
    return 1;
  }
  // Accumulate total occurrences for each palindrome node
  void calc_occurrences() {
    for (int i = sz; i >= 3; i--) {
      t[t[i].link].oc += t[i].oc;
    }
  }
};

int main() {
  string s;
```

```cpp
103    cin >> s;
104    PalindromicTree t(s);
105    for (int i = 0; i < s.size(); i++) {
106      t.extend(i);
107    }
108    t.calc_occurrences();
109    long long total = 0;
110    for (int i = 3; i <= t.sz; i++) {
111      total += t.t[i].oc;
112    }
113    cout << total << '\n'; // Total palindromic substrings
114    return 0;
115 }
```

3.20   Persistent Array

```cpp
1  /*
2   Persistent Array (via Persistent Segment Tree)
3   ----------------------------------------------
4   Indexing: 0-based
5   Bounds: [0, n-1]
6   Time Complexity:
7     - point update: O(log n)
8     - point query: O(log n)
9   Space Complexity: O(log n) per update/version
10
11   Features:
12     - Supports point updates with full version history
13     - Allows querying any version at any index
14  */
15
16  struct Node {
17    int val;
18    Node *l, *r;
19
20    // Leaf node with value
21    Node(int x) : val(x), l(nullptr), r(nullptr) {}
22
23    // Internal node with children (value is not used)
24    Node(Node *ll, Node *rr) : val(0), l(ll), r(rr) {}
25  };
26
27  int n;
```

```cpp
28  int a[100001];        // Initial array
29  Node *roots[100001];  // Roots of all versions (0-based)
30
31  // Build version 0 from initial array
32  Node *build(int l = 0, int r = n - 1) {
33    if (l == r) return new Node(a[l]);
34    int mid = (l + r) / 2;
35    return new Node(build(l, mid), build(mid + 1, r));
36  }
37
38  // Create new version with a[pos] = val
39  Node *update(Node *node, int pos, int val, int l = 0, int r = n - 1) {
40    if (l == r) return new Node(val);
41    int mid = (l + r) / 2;
42    if (pos <= mid)
43      return new Node(update(node->l, pos, val, l, mid), node->r);
44    else
45      return new Node(node->l, update(node->r, pos, val, mid + 1, r));
46  }
47
48  // Query value at position 'pos' in a given version (node)
49  int query(Node *node, int pos, int l = 0, int r = n - 1) {
50    if (l == r) return node->val;
51    int mid = (l + r) / 2;
52    if (pos <= mid) return query(node->l, pos, l, mid);
53    else return query(node->r, pos, mid + 1, r);
54  }
55
56  // External helper: get value at index in version
57  int get_item(int index, int version) {
58    return query(roots[version], index);
59  }
60
61  // External helper: make new version based on 'prev_version', updating
62        one index
   void update_item(int index, int value, int prev_version, int new_version
        ) {
63    roots[new_version] = update(roots[prev_version], index, value);
64  }
65
66  // Initializes version 0 from given array
67  void init_arr(int nn, int *init) {
68    n = nn;
```

```
69    for (int i = 0; i < n; i++) a[i] = init[i];
70    roots[0] = build();
71  }
```

3.21   Persistent Segment Tree

```
1  /*
2    Persistent Segment Tree (Point Updates, Range Queries)
3    ------------------------------------------------------
4    Indexing: 1-based
5    Bounds: [1, n]
6    Time Complexity:
7      - Build: O(n)
8      - Point update: O(log n) -> returns new version
9      - Range query: O(log n)
10      - Copy version: O(1)
11
12    Features:
13      - Each update creates a new tree version with shared unchanged nodes
14      - Supports querying over any version
15      - Useful in rollback problems, version history, and functional
           programming
16  */
17
18  struct Node {
19    ll val;      // segment sum
20    Node *l, *r;
21
22    // Leaf node
23    Node(ll x) : val(x), l(nullptr), r(nullptr) {}
24
25    // Internal node with children
26    Node(Node *_l, Node *_r) {
27      l = _l;
28      r = _r;
29      val = 0;
30      if (l) val += l->val;
31      if (r) val += r->val;
32    }
33
34    // Version clone (used when copying tree version directly)
35    Node(Node *cp) : val(cp->val), l(cp->l), r(cp->r) {}
36  };
```

```
37
38  int n, sz = 1;
39  ll a[200001];         // Input array (1-indexed)
40  Node *t[200001];      // Roots of different versions (t[version_id])
41
42  // Build initial segment tree from array a[1..n]
43  Node *build(int l = 1, int r = n) {
44    if (l == r) return new Node(a[l]);
45    int mid = (l + r) / 2;
46    return new Node(build(l, mid), build(mid + 1, r));
47  }
48
49  // Update position 'pos' with 'val' in given 'node' version
50  Node *update(Node *node, int pos, int val, int l = 1, int r = n) {
51    if (l == r) return new Node(val); // replace leaf
52    int mid = (l + r) / 2;
53    if (pos <= mid)
54      return new Node(update(node->l, pos, val, l, mid), node->r);
55    else
56      return new Node(node->l, update(node->r, pos, val, mid + 1, r));
57  }
58
59  // Query sum over range [a, b] in given version
60  ll query(Node *node, int a, int b, int l = 1, int r = n) {
61    if (r < a || l > b) return 0;          // No overlap
62    if (l >= a && r <= b) return node->val; // Total overlap
63    int mid = (l + r) / 2;
64    return query(node->l, a, b, l, mid) + query(node->r, a, b, mid + 1, r)
         ;
65  }
66
67  int main() {
68    ios_base::sync_with_stdio(false); cin.tie(NULL);
69
70    int q;
71    cin >> n >> q;
72    for (int i = 1; i <= n; i++) {
73      cin >> a[i];
74    }
75
76    // Build version 0
77    t[0] = build();
78    sz = 1;
```

```
79
80    while (q--) {
81      int ty;
82      cin >> ty;
83
84      if (ty == 1) {
85        // Point update: create new version from t[k] with a[pos] = x
86        int k, pos, x;
87        cin >> k >> pos >> x;
88        t[k] = update(t[k], pos, x);
89
90      } else if (ty == 2) {
91        // Range query on version k over [l, r]
92        int k, l, r;
93        cin >> k >> l >> r;
94        cout << query(t[k], l, r) << '\n';
95
96      } else if (ty == 3) {
97        // Clone version k into new version
98        int k;
99        cin >> k;
100       t[sz++] = new Node(t[k]);
101     }
102   }
103   return 0;
104 }
```

3.22   Polynomial Queries

```
1  // adds d * (i - l + 1) + a to range [l, r]
2  // d * (i - l + 1) = d*(1-l) + d*i
3  // Sum = A x segment_length + B x sum_of_indices_in_segment
4
5  struct sum_t {
6    ll val;
7    static const ll null_v = 0;
8    sum_t(): val(null_v) {}
9    sum_t(ll v): val(v) {}
10   sum_t op(sum_t& other) {
11     return sum_t(val + other.val);
12   }
13   sum_t lazy_op(sum_t& v, int size) {
14     return val + v.val * size;
```

```
15   }
16   sum_t lazy_op(sum_t& v, ll l, ll r) {
17     return val + v.val * ((l + r) * (r - l + 1)) / 2;
18   }
19 };
20
21 template <typename num_t>
22 struct segtree {
23   int n;
24   vector<num_t> tree, lazy, lazy1;
25
26   void init(int s) {
27     n = s;
28     tree.assign(4 * n, num_t(0));
29     lazy.assign(4 * n, num_t(0));
30     lazy1.assign(4 * n, num_t(0));
31     init(0, 0, n - 1);
32   }
33
34   num_t init(int i, int l, int r) {
35     if (l == r) return tree[i] = num_t(0);
36
37     int mid = (l + r) / 2;
38     num_t left = init(2 * i + 1, l, mid);
39     num_t right = init(2 * i + 2, mid + 1, r);
40     return tree[i] = left.op(right);
41   }
42
43   // do -l if you want it to add 0 to l. adds d * (i - l + 1) + a to
        range [l, r]
44   void update(int l, int r, ll d, ll a) {
45     if (l > r) return;
46     ll dd = d;
47     d = (1 - l) * d + a;
48     a = dd;
49     update(0, 0, n - 1, l, r, d, a);
50   }
51
52   num_t update(int i, int tl, int tr, int ql, int qr, ll D, ll A) {
53     eval_lazy(i, tl, tr);
54
55     if (tr < ql || qr < tl) return tree[i];
56     if (ql <= tl && tr <= qr) {
```

```
57        lazy[i].val += D;
58        lazy1[i].val += A;
59        eval_lazy(i, tl, tr);
60        return tree[i];
61      }
62
63      int mid = (tl + tr) / 2;
64      num_t a = update(2 * i + 1, tl, mid, ql, qr, D, A);
65      num_t b = update(2 * i + 2, mid + 1, tr, ql, qr, D, A);
66      return tree[i] = a.op(b);
67    }
68
69    num_t query(int l, int r) {
70      if (l > r) return num_t::null_v;
71      return query(0, 0, n - 1, l, r);
72    }
73
74    num_t query(int i, int tl, int tr, int ql, int qr) {
75      eval_lazy(i, tl, tr);
76
77      if (ql <= tl && tr <= qr) return tree[i];
78      if (tr < ql || qr < tl) return num_t::null_v;
79
80      int mid = (tl + tr) / 2;
81      num_t a = query(2 * i + 1, tl, mid, ql, qr);
82      num_t b = query(2 * i + 2, mid + 1, tr, ql, qr);
83      return a.op(b);
84    }
85
86    void eval_lazy(int i, int l, int r) {
87      tree[i] = tree[i].lazy_op(lazy1[i], l, r);
88      tree[i] = tree[i].lazy_op(lazy[i], r - l + 1);
89      if (l != r) {
90        lazy[2 * i + 1].val += lazy[i].val;
91        lazy[2 * i + 2].val += lazy[i].val;
92        lazy1[2 * i + 1].val += lazy1[i].val;
93        lazy1[2 * i + 2].val += lazy1[i].val;
94      }
95      lazy[i] = lazy1[i] = num_t(0);
96    }
97 };
98
99 int main(){
```

```
100   ios_base::sync_with_stdio(false); cin.tie(NULL);
101   int n, q; cin >> n >> q;
102   segtree <sum_t> st;
103   st.init(n);
104   while(q--){
105     int t; cin >> t;
106     if(t == 1){
107       int l, r, a, d; cin >> l >> r >> a >> d;
108       l--; r--;
109       st.update(l, r, d, a);
110     }
111     else{
112       int l; cin >> l;
113       l--;
114       cout << st.query(l, l).val << endl;
115     }
116   }
117 }
```

3.23   Segment Tree

```
1  /*
2     Segment Tree (Iterative, Range Minimum Query)
3     ---------------------------------------------
4     Indexing: 0-based
5     Bounds: [0, n-1] inclusive
6     Time Complexity:
7       - update(pos, val): O(log n)
8       - get(l, r): O(log n) -> query min in range [l, r]
9     Space Complexity: O(2n)
10 */
11 struct SegmentTree {
12   vector<ll> a; // segment tree array
13   int n;        // number of elements in original array
14
15   SegmentTree(int _n) : a(2 * _n, 1e18), n(_n) {}
16   // Update position `pos` to value `val`
17   void update(int pos, ll val) {
18     pos += n;      // move to leaf
19     a[pos] = val;  // set value
20     for (pos /= 2; pos > 0; pos /= 2) {
21       a[pos] = min(a[2 * pos], a[2 * pos + 1]); // update parent
22     }
```

```
23       }
24       // Get minimum value in range [l, r)
25       ll get(int l, int r) {
26         ll res = 1e18;
27         for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
28           if (l & 1) res = min(res, a[l++]); // if l is right child
29           if (r & 1) res = min(res, a[--r]); // if r is left child
30         }
31         return res;
32       }
33     };
```

3.24 Segment Tree 2D

```
1    /*
2       2D Segment Tree (Sum over Rectangles)
3       -------------------------------------
4       Indexing: 0-based
5       Grid Size: n * m
6       Time Complexity:
7         - build: O(nm log n log m)
8         - point update: O(log n log m)
9         - range query [x1..x2][y1..y2]: O(log n log m)
10      Space Complexity: O(4n x 4m)
11   */
12
13   const int MAXN = 505;
14   int n, m;                 // grid dimensions
15   int a[MAXN][MAXN];      // input grid
16   int t[4 * MAXN][4 * MAXN]; // segment tree
17
18   // Build the tree along y-axis (internal to each x-interval)
19   void build_y(int vx, int lx, int rx, int vy, int ly, int ry) {
20     if (ly == ry) {
21       if (lx == rx)
22         t[vx][vy] = a[lx][ly];
23       else
24         t[vx][vy] = t[vx * 2][vy] + t[vx * 2 + 1][vy];
25     } else {
26       int my = (ly + ry) / 2;
27       build_y(vx, lx, rx, vy * 2, ly, my);
28       build_y(vx, lx, rx, vy * 2 + 1, my + 1, ry);
29       t[vx][vy] = t[vx][vy * 2] + t[vx][vy * 2 + 1];
30     }
31   }
32
33   // Build the tree along x-axis and call build_y for each
34   void build_x(int vx, int lx, int rx) {
35     if (lx != rx) {
36       int mx = (lx + rx) / 2;
37       build_x(vx * 2, lx, mx);
38       build_x(vx * 2 + 1, mx + 1, rx);
39     }
40     build_y(vx, lx, rx, 1, 0, m - 1);
41   }
42
43   // Query along y-axis in a fixed x-node
44   int sum_y(int vx, int vy, int tly, int try_, int ly, int ry) {
45     if (ly > ry) return 0;
46     if (ly == tly && ry == try_) return t[vx][vy];
47     int tmy = (tly + try_) / 2;
48     return sum_y(vx, vy * 2, tly, tmy, ly, min(ry, tmy))
49         + sum_y(vx, vy * 2 + 1, tmy + 1, try_, max(ly, tmy + 1), ry);
50   }
51
52   // Query sum in rectangle [lx..rx][ly..ry]
53   int sum_x(int vx, int tlx, int trx, int lx, int rx, int ly, int ry) {
54     if (lx > rx) return 0;
55     if (lx == tlx && trx == rx)
56       return sum_y(vx, 1, 0, m - 1, ly, ry);
57     int tmx = (tlx + trx) / 2;
58     return sum_x(vx * 2, tlx, tmx, lx, min(rx, tmx), ly, ry)
59         + sum_x(vx * 2 + 1, tmx + 1, trx, max(lx, tmx + 1), rx, ly, ry);
60   }
61
62   // Update along y-axis for fixed x-node
63   void update_y(int vx, int lx, int rx, int vy, int ly, int ry, int x, int
          y, int new_val) {
64     if (ly == ry) {
65       if (lx == rx)
66         t[vx][vy] = new_val;
67       else
68         t[vx][vy] = t[vx * 2][vy] + t[vx * 2 + 1][vy];
69     } else {
70       int my = (ly + ry) / 2;
71       if (y <= my)
```

```
72      update_y(vx, lx, rx, vy * 2, ly, my, x, y, new_val);
73    else
74      update_y(vx, lx, rx, vy * 2 + 1, my + 1, ry, x, y, new_val);
75    t[vx][vy] = t[vx][vy * 2] + t[vx][vy * 2 + 1];
76  }
77 }
78
79 // Update point (x, y) to new_val
80 void update_x(int vx, int lx, int rx, int x, int y, int new_val) {
81   if (lx != rx) {
82     int mx = (lx + rx) / 2;
83     if (x <= mx)
84       update_x(vx * 2, lx, mx, x, y, new_val);
85     else
86       update_x(vx * 2 + 1, mx + 1, rx, x, y, new_val);
87   }
88   update_y(vx, lx, rx, 1, 0, m - 1, x, y, new_val);
89 }
```

3.25   Segment Tree Dynamic

```
1  /*
2    Dynamic Segment Tree (Point Add, Range Sum)
3    -------------------------------------------
4    Indexing: [0, INF) or any large bounded range
5    Time Complexity:
6      - add(k, x): O(log U)
7      - get_sum(l, r): O(log U)
8        where U = range size (e.g., 1e9 if implicit bounds)
9
10   Space Complexity: O(nodes visited or created) -> worst O(log U) per
        operation
11 */
12
13 struct Vertex {
14   int left, right;        // interval [left, right)
15   int sum = 0;            // sum of elements in this interval
16   Vertex *left_child = nullptr, *right_child = nullptr;
17
18   Vertex(int lb, int rb) {
19     left = lb;
20     right = rb;
21   }
22
23   // Create children lazily only when needed
24   void extend() {
25     if (!left_child && left + 1 < right) {
26       int mid = (left + right) / 2;
27       left_child = new Vertex(left, mid);
28       right_child = new Vertex(mid, right);
29     }
30   }
31
32   // Add 'x' to position 'k'
33   void add(int k, int x) {
34     extend();
35     sum += x;
36     if (left_child) {
37       if (k < left_child->right)
38         left_child->add(k, x);
39       else
40         right_child->add(k, x);
41     }
42   }
43
44   // Get sum over interval [lq, rq)
45   int get_sum(int lq, int rq) {
46     if (lq <= left && right <= rq)
47       return sum;
48     if (rq <= left || right <= lq)
49       return 0;
50     extend();
51     return left_child->get_sum(lq, rq) + right_child->get_sum(lq, rq);
52   }
53 };
54
55 Vertex *root = new Vertex(0, 1e9); // Range [0, 1e9)
56 root->add(5, 10);         // a[5] += 10
57 root->add(1000, 20);      // a[1000] += 20
58 cout << root->get_sum(0, 10) << '\n';     // sum of [0, 10) = 10
59 cout << root->get_sum(0, 2000) << '\n';   // sum of [0, 2000) = 30
```

3.26   Segment Tree Lazy Types

```
1 struct max_t {
2   ll val;
```

```cpp
 3      static const ll null_v = -1LL << 61;
 4      max_t(): val(0) {}
 5      max_t(ll v): val(v) {}
 6      max_t op(max_t& other) {
 7        return max_t(max(val, other.val));
 8      }
 9      max_t lazy_op(max_t& v, int size) {
10        return max_t(val + v.val);
11      }
12  };
13
14  struct min_t {
15      ll val;
16      static const ll null_v = 1LL << 61;
17      min_t(): val(0) {}
18      min_t(ll v): val(v) {}
19      min_t op(min_t& other) {
20        return min_t(min(val, other.val));
21      }
22      min_t lazy_op(min_t& v, int size) {
23        return min_t(val + v.val);
24      }
25  };
26
27  struct sum_t {
28      ll val;
29      static const ll null_v = 0;
30      sum_t(): val(0) {}
31      sum_t(ll v): val(v) {}
32      sum_t op(sum_t& other) {
33        return sum_t(val + other.val);
34      }
35      sum_t lazy_op(sum_t& v, int size) {
36        return sum_t(val + v.val * size);
37      }
38  };
```

3.27   Segment Tree Lazy

```cpp
 1  /*
 2      Lazy Segment Tree (Range Update, Range Query)
 3      ----------------------------------------------
 4      Indexing: 0-based
 5      Bounds: [0, n-1]
 6      Time Complexity:
 7        - build: O(n)
 8        - update(l, r, v): O(log n)
 9        - query(l, r): O(log n)
10      Space Complexity: O(4n)
11  */
12
13  // See SegTreeLazy_types for num_t structs
14
15  const num_t num_t::null_v = num_t(0);
16
17
18  template <typename num_t>
19  struct segtree {
20      int n;
21      vector<num_t> tree, lazy;
22
23      // Initialize segment tree with array of size s
24      void init(int s, long long* arr) {
25        n = s;
26        tree.assign(4 * n, num_t());
27        lazy.assign(4 * n, num_t());
28        init(0, 0, n - 1, arr);
29      }
30
31      // Build segment tree from array
32      num_t init(int i, int l, int r, long long* arr) {
33        if (l == r) return tree[i] = num_t(arr[l]);
34
35        int mid = (l + r) / 2;
36        num_t left = init(2 * i + 1, l, mid, arr);
37        num_t right = init(2 * i + 2, mid + 1, r, arr);
38        return tree[i] = left.op(right);
39      }
40
41      // Public wrapper: update range [l, r] with value v
42      void update(int l, int r, num_t v) {
43        if (l > r) return;
44        update(0, 0, n - 1, l, r, v);
45      }
46
47      // Internal recursive update
```

```
48    num_t update(int i, int tl, int tr, int ql, int qr, num_t v) {
49      eval_lazy(i, tl, tr);
50
51      if (tr < ql || qr < tl) return tree[i]; // no overlap
52      if (ql <= tl && tr <= qr) {
53        lazy[i].val += v.val;
54        eval_lazy(i, tl, tr);
55        return tree[i];
56      }
57
58      int mid = (tl + tr) / 2;
59      num_t a = update(2 * i + 1, tl, mid, ql, qr, v);
60      num_t b = update(2 * i + 2, mid + 1, tr, ql, qr, v);
61      return tree[i] = a.op(b);
62    }
63
64    // Public wrapper: query sum in range [l, r]
65    num_t query(int l, int r) {
66      if (l > r) return num_t::null_v;
67      return query(0, 0, n - 1, l, r);
68    }
69
70    // Internal recursive query
71    num_t query(int i, int tl, int tr, int ql, int qr) {
72      eval_lazy(i, tl, tr);
73
74      if (ql <= tl && tr <= qr) return tree[i]; // total overlap
75      if (tr < ql || qr < tl) return num_t::null_v; // no overlap
76
77      int mid = (tl + tr) / 2;
78      num_t a = query(2 * i + 1, tl, mid, ql, qr);
79      num_t b = query(2 * i + 2, mid + 1, tr, ql, qr);
80      return a.op(b);
81    }
82
83    // Push down pending lazy updates to children
84    void eval_lazy(int i, int l, int r) {
85      tree[i] = tree[i].lazy_op(lazy[i], r - l + 1);
86      if (l != r) {
87        lazy[2 * i + 1].val += lazy[i].val;
88        lazy[2 * i + 2].val += lazy[i].val;
89      }
90      lazy[i] = num_t(); // reset lazy at this node
```

```
91    }
92  };
```

3.28   Segment Tree Lazy Range Set

```
1   /*
2     Lazy Segment Tree (Range Set + Range Add + Range Sum)
3     -----------------------------------------------------
4     Indexing: 0-based
5     Bounds: [0, N-1]
6
7     Features:
8       - Supports range set (assign value), range add (increment), and
           range sum queries
9       - Properly prioritizes lazy set > lazy add
10  */
11
12  const int maxN = 1e5 + 5;
13  int N, Q;
14  int a[maxN];
15
16  struct node {
17    ll val = 0;       // range sum
18    ll lzAdd = 0;     // pending addition
19    ll lzSet = 0;     // pending set (non-zero means active)
20  };
21
22  node tree[maxN << 2];
23
24  #define lc (p << 1)
25  #define rc ((p << 1) | 1)
26
27  // Update current node based on its children
28  inline void pushup(int p) {
29    tree[p].val = tree[lc].val + tree[rc].val;
30  }
31
32  // Push lazy values down to children
33  void pushdown(int p, int l, int mid, int r) {
34    // Range set overrides any pending add
35    if (tree[p].lzSet != 0) {
36      tree[lc].lzSet = tree[rc].lzSet = tree[p].lzSet;
37      tree[lc].val = (mid - l + 1) * tree[p].lzSet;
```

```
38        tree[rc].val = (r - mid) * tree[p].lzSet;
39        tree[lc].lzAdd = tree[rc].lzAdd = 0;
40        tree[p].lzSet = 0;
41      }
42      // Otherwise propagate add
43      else if (tree[p].lzAdd != 0) {
44        if (tree[lc].lzSet == 0) tree[lc].lzAdd += tree[p].lzAdd;
45        else {
46          tree[lc].lzSet += tree[p].lzAdd;
47          tree[lc].lzAdd = 0;
48        }
49        if (tree[rc].lzSet == 0) tree[rc].lzAdd += tree[p].lzAdd;
50        else {
51          tree[rc].lzSet += tree[p].lzAdd;
52          tree[rc].lzAdd = 0;
53        }
54        tree[lc].val += (mid - l + 1) * tree[p].lzAdd;
55        tree[rc].val += (r - mid) * tree[p].lzAdd;
56        tree[p].lzAdd = 0;
57      }
58    }
59
60    // Build tree from array a[0..N-1]
61    void build(int p, int l, int r) {
62      tree[p].lzAdd = tree[p].lzSet = 0;
63      if (l == r) {
64        tree[p].val = a[l];
65        return;
66      }
67      int mid = (l + r) >> 1;
68      build(lc, l, mid);
69      build(rc, mid + 1, r);
70      pushup(p);
71    }
72
73    // Add 'val' to all elements in [a, b]
74    void add(int p, int l, int r, int a, int b, ll val) {
75      if (a > r || b < l) return;
76      if (a <= l && r <= b) {
77        tree[p].val += (r - l + 1) * val;
78        if (tree[p].lzSet == 0) tree[p].lzAdd += val;
79        else tree[p].lzSet += val;
80        return;
81      }
82      int mid = (l + r) >> 1;
83      pushdown(p, l, mid, r);
84      add(lc, l, mid, a, b, val);
85      add(rc, mid + 1, r, a, b, val);
86      pushup(p);
87    }
88
89    // Set all elements in [a, b] to 'val'
90    void set(int p, int l, int r, int a, int b, ll val) {
91      if (a > r || b < l) return;
92      if (a <= l && r <= b) {
93        tree[p].val = (r - l + 1) * val;
94        tree[p].lzAdd = 0;
95        tree[p].lzSet = val;
96        return;
97      }
98      int mid = (l + r) >> 1;
99      pushdown(p, l, mid, r);
100     set(lc, l, mid, a, b, val);
101     set(rc, mid + 1, r, a, b, val);
102     pushup(p);
103   }
104
105   // Query sum over [a, b]
106   ll query(int p, int l, int r, int a, int b) {
107     if (a > r || b < l) return 0;
108     if (a <= l && r <= b) return tree[p].val;
109     int mid = (l + r) >> 1;
110     pushdown(p, l, mid, r);
111     return query(lc, l, mid, a, b) + query(rc, mid + 1, r, a, b);
112   }
113   // Example usage
114   N = 5;
115   a[0] = 2, a[1] = 4, a[2] = 3, a[3] = 1, a[4] = 5;
116   build(1, 0, N - 1);
117   set(1, 0, N - 1, 1, 3, 7);     // a[1..3] = 7
118   add(1, 0, N - 1, 2, 4, 2);     // a[2..4] += 2
119   cout << query(1, 0, N - 1, 0, 4) << '\n'; // total sum
```

3.29   Segment Tree Lazy Range Set 2

```
1   #include <bits/stdc++.h>
```

```cpp
 2  using namespace std;
 3
 4  typedef long long ll;
 5
 6  struct sum_t {
 7    ll val;
 8    static const ll null_v = 0;
 9    sum_t(): val(null_v) {}
10    sum_t(ll v): val(v) {}
11    sum_t op(sum_t& other) {
12      return sum_t(val + other.val);
13    }
14    sum_t lazy_op(sum_t& v, int size) {
15      return v.val * size;
16    }
17    sum_t lazy_op1(sum_t& v, int size) {
18      return val + v.val * size;
19    }
20  };
21
22  template <typename num_t>
23  struct segtree {
24    int n;
25    vector<num_t> tree, lazy, lazy1;
26
27    void init(int s) {
28      n = s;
29      tree.assign(4 * n, num_t(0));
30      lazy.assign(4 * n, num_t(-1));
31      lazy1.assign(4 * n, num_t(0));
32      init(0, 0, n - 1);
33    }
34
35    num_t init(int i, int l, int r) {
36      if (l == r) return tree[i] = num_t(0);
37
38      int mid = (l + r) / 2;
39      num_t left = init(2 * i + 1, l, mid);
40      num_t right = init(2 * i + 2, mid + 1, r);
41      return tree[i] = left.op(right);
42    }
43
44    void update(int l, int r, num_t v) {
45      if (l > r) return;
46      update(0, 0, n - 1, l, r, v);
47    }
48
49    num_t update(int i, int tl, int tr, int ql, int qr, num_t v) {
50      eval_lazy(i, tl, tr);
51
52      if (tr < ql || qr < tl) return tree[i];
53      if (ql <= tl && tr <= qr) {
54        lazy[i].val = v.val;
55        lazy1[i].val = 0;
56        eval_lazy(i, tl, tr);
57        return tree[i];
58      }
59
60      int mid = (tl + tr) / 2;
61      num_t a = update(2 * i + 1, tl, mid, ql, qr, v);
62      num_t b = update(2 * i + 2, mid + 1, tr, ql, qr, v);
63      return tree[i] = a.op(b);
64    }
65
66    void update1(int l, int r, num_t v) {
67      if (l > r) return;
68      update1(0, 0, n - 1, l, r, v);
69    }
70
71    num_t update1(int i, int tl, int tr, int ql, int qr, num_t v) {
72
73      if (tr < ql || qr < tl){
74        eval_lazy(i, tl, tr);
75        return tree[i];
76      }
77      if (ql <= tl && tr <= qr) {
78        lazy1[i].val += v.val;
79        if(lazy[i].val != -1){
80          lazy[i].val += lazy1[i].val;
81          lazy1[i].val = 0;
82        }
83        eval_lazy(i, tl, tr);
84        return tree[i];
85      }
86
87      eval_lazy(i, tl, tr);
```

```
88      int mid = (tl + tr) / 2;
89      num_t a = update1(2 * i + 1, tl, mid, ql, qr, v);
90      num_t b = update1(2 * i + 2, mid + 1, tr, ql, qr, v);
91      return tree[i] = a.op(b);
92    }
93
94    num_t query(int l, int r) {
95      if (l > r) return num_t::null_v;
96      return query(0, 0, n - 1, l, r);
97    }
98
99    num_t query(int i, int tl, int tr, int ql, int qr) {
100     eval_lazy(i, tl, tr);
101
102     if (ql <= tl && tr <= qr) return tree[i];
103     if (tr < ql || qr < tl) return num_t::null_v;
104
105     int mid = (tl + tr) / 2;
106     num_t a = query(2 * i + 1, tl, mid, ql, qr);
107     num_t b = query(2 * i + 2, mid + 1, tr, ql, qr);
108     return a.op(b);
109   }
110
111   void eval_lazy(int i, int l, int r) {
112     if(lazy[i].val != -1){
113       tree[i] = tree[i].lazy_op(lazy[i], r - l + 1);
114       if (l != r) {
115         lazy[2 * i + 1].val = lazy[i].val;
116         lazy[2 * i + 2].val = lazy[i].val;
117         lazy1[2 * i + 1].val = lazy1[2 * i + 2].val = 0;
118       }
119       lazy[i] = num_t(-1);
120       lazy1[i] = num_t(0);
121     }
122     else{
123       tree[i] = tree[i].lazy_op1(lazy1[i], r - l + 1);
124       if(l != r){
125         if(lazy[2 * i + 1].val == -1) lazy1[2 * i + 1].val += lazy1[i].
              val;
126         else{
127           lazy[2 * i + 1].val += lazy1[i].val + lazy1[2 * i + 1].val;
128           lazy1[2 * i + 1].val = 0;
129         }
```

```
130         if(lazy[2 * i + 2].val == -1) lazy1[2 * i + 2].val += lazy1[i].
              val;
131         else{
132           lazy[2 * i + 2].val += lazy1[i].val + lazy1[2 * i + 2].val;
133           lazy1[2 * i + 2].val = 0;
134         }
135       }
136       lazy1[i] = num_t(0);
137     }
138   }
139 };
140
141 int main(){
142   ios_base::sync_with_stdio(false); cin.tie(NULL);
143   int n, q; cin >> n >> q;
144   segtree <sum_t> st;
145   st.init(n);
146   while(q--){
147     int t; cin >> t;
148     if(t == 1){
149       int l, r, x; cin >> l >> r >> x;
150       r--;
151       st.update(l, r, x);
152     }
153     else if(t == 2){
154       int l, r, x; cin >> l >> r >> x;
155       r--;
156       st.update1(l, r, x);
157     }
158     else{
159       int l, r; cin >> l >> r;
160       r--;
161       cout << st.query(l, r).val << endl;
162     }
163   }
164 }
```

3.30   Segment Tree Max Subarray Sum

```
1 const ll inf=1e18;
2
3 struct Node {
4   ll maxi, l_max, r_max, sum;
```

```
5    Node(ll _maxi, ll _l_max, ll _r_max, ll _sum){
6      maxi=_maxi;
7      l_max=_l_max;
8      r_max=_r_max;
9      sum=_sum;
10   }
11   Node operator+(Node b) {
12     return {max(max(maxi, b.maxi), r_max + b.l_max),
13       max(l_max, sum + b.l_max), max(b.r_max, r_max + b.sum),
14       sum + b.sum};
15   }
16
17 };
18
19 struct SegmentTreeMaxSubSum{
20   int n;
21   vector<Node> t;
22   SegmentTreeMaxSubSum(int _n) : n(_n), t(2 * _n, Node(-inf, -inf, -inf,
          -inf)) {}
23   void update(int pos, ll val) {
24     t[pos += n] = Node(val, val, val, val);
25     for (pos>>=1; pos ; pos >>= 1) {
26       t[pos] = t[2*pos]+t[2*pos+1];
27     }
28   }
29   Node query(int l, int r) {
30     Node node_l = Node(-inf, -inf, -inf, -inf);
31     Node node_r = Node(-inf, -inf, -inf, -inf);
32     for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
33       if (l & 1) {
34         node_l=node_l+t[l++];
35       }
36       if (r & 1) {
37         node_r=t[--r]+node_r;
38       }
39     }
40     return node_l+node_r;
41   }
42 };
```

3.31   Segment Tree Range Update

```
1  /*
```

```
2    Segment Tree (Range Min Update, Point Query)
3    --------------------------------------------
4    Indexing: 0-based
5    Bounds: [0, n-1]
6    Time Complexity:
7      - update(l, r, val): O(log n) -> applies min(val) over [l, r)
8      - get(pos): O(log n) -> minimum affecting position pos
9    Space Complexity: O(2n)
10 */
11
12 struct SegmentTree {
13   vector<ll> a; // a[i] = min value affecting segment i
14   int n;
15
16   SegmentTree(int _n) : a(2 * _n, 1e18), n(_n) {}
17
18   // Get the effective minimum at position 'pos'
19   ll get(int pos) {
20     ll res = 1e18;
21     for (pos += n; pos > 0; pos >>= 1) {
22       res = min(res, a[pos]);
23     }
24     return res;
25   }
26
27   // Apply min(val) to all positions in [l, r)
28   void update(int l, int r, ll val) {
29     for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
30       if (l & 1) {
31         a[l] = min(a[l], val);
32         l++;
33       }
34       if (r & 1) {
35         --r;
36         a[r] = min(a[r], val);
37       }
38     }
39   }
40 };
```

3.32   Segment Tree Struct Types

```
1  // Sum segment tree
```

```cpp
struct sum_t{
  ll val;
  static const long long null_v = 0;

  sum_t(): val(null_v) {}
  sum_t(long long v): val(v) {}

  sum_t operator + (const sum_t &a) const {
    sum_t ans;
    ans.val = val + a.val;
    return ans;
  }
};
// Min segment tree
struct min_t{
  ll val;
  static const long long null_v = 1e18;

  min_t(): val(null_v) {}
  min_t(long long v): val(v) {}

  min_t operator + (const min_t &a) const {
    min_t ans;
    ans.val = min(val, a.val);
    return ans;
  }
};
// Max segment tree
struct max_t{
  ll val;
  static const long long null_v = -1e18;

  max_t(): val(null_v) {}
  max_t(long long v): val(v) {}

  max_t operator + (const max_t &a) const {
    max_t ans;
    ans.val = max(val, a.val);
    return ans;
  }
};
// GCD segment tree
struct gcd_t{
  ll val;
  static const long long null_v = 0;

  gcd_t(): val(null_v) {}
  gcd_t(long long v): val(v) {}

  gcd_t operator + (const gcd_t &a) const {
    gcd_t ans;
    ans.val = gcd(val, a.val);
    return ans;
  }
};
```

3.33   Segment Tree Struct

```cpp
// works as a 0-indexed segtree (not lazy)
template <typename num_t>
struct segtree
{
  int n, k;
  vector<num_t> tree;
  void init(int s, vector<ll> arr)
  {
    n = s;
    k = 0;
    while ((1 << k) < n)
      k++;
    tree = vector<num_t>(2 * (1 << k) + 1);
    for (int i = 0; i < n; i++)
    {
      tree[(1 << k) + i] = arr[i];
    }
    for (int i = (1 << k) - 1; i > 0; i--)
    {
      tree[i] = tree[i * 2] + tree[i * 2 + 1];
    }
  }
  void update(int a, ll b)
  {
    a += (1 << k);
    tree[a] = b;
    for (a /= 2; a >= 1; a /= 2)
    {
```

```
29      tree[a] = tree[a * 2] + tree[a * 2 + 1];
30    }
31  }
32  num_t find(int a, int b)
33  {
34    a += (1 << k);
35    b += (1 << k);
36    num_t s;
37    while (a <= b)
38    {
39      if (a % 2 == 1)
40        s = s + tree[a++];
41      if (b % 2 == 0)
42        s = s + tree[b--];
43      a /= 2;
44      b /= 2;
45    }
46    return s;
47  }
48 };
```

3.34   Segment Tree Walk

```
1  /*
2    Segment Tree Walk - Find First Position >= val
3    ----------------------------------------------
4    Indexing: 0-based
5    Bounds: [0, n-1]
6    Time Complexity:
7      - build: O(n)
8      - update(pos, val): O(log n)
9      - get(L, R): O(log n) -> min value in [L, R]
10     - query(L, R, val): O(log n) -> find first index in [L, R] where a[i
           ] >= val
11
12   Features:
13     - Stores original value array in segment tree form
14     - Maps original indices to tree positions for fast updates
15     - Allows efficient walk to find constrained elements (e.g. lower
           bound >= val)
16 */
17
18 struct SegmentTreeWalk {
```

```
19    vector<ll> a;           // segment tree values
20    vector<int> final_pos;  // maps index i to position in tree (leaf)
21    int n;
22
23    SegmentTreeWalk(int _n) : a(4 * _n, 1e18), final_pos(_n), n(_n) {}
24
25    // Build segment tree from array 'vals[0..n-1]', start with node=1, l
           =0, r=n-1
26    void build(int l, int r, int node, const vector<ll> &vals) {
27      if (l == r) {
28        final_pos[l] = node;
29        a[node] = vals[l];
30      } else {
31        int mid = (l + r) / 2;
32        build(l, mid, node * 2, vals);
33        build(mid + 1, r, node * 2 + 1, vals);
34        a[node] = min(a[node * 2], a[node * 2 + 1]);
35      }
36    }
37
38    // Update value at original index 'pos' to 'val'
39    void update(int pos, ll val) {
40      pos = final_pos[pos]; // leaf position
41      a[pos] = val;
42      for (pos /= 2; pos > 0; pos /= 2)
43        a[pos] = min(a[pos * 2], a[pos * 2 + 1]);
44    }
45
46    // Get min value in [L, R], with current node interval [l, r] and root
           'node'
47    ll get(int l, int r, int L, int R, int node) {
48      if (L > R) return 1e18;
49      if (l == L && r == R) return a[node];
50      int mid = (l + r) / 2;
51      return min(
52        get(l, mid, L, min(R, mid), node * 2),
53        get(mid + 1, r, max(L, mid + 1), R, node * 2 + 1)
54      );
55    }
56
57    // Find first position in [L, R] with a[i] >= val, starting from node
           interval [l, r]
58    pair<ll, ll> query(int l, int r, int L, int R, int node, int val) {
```

```
59      if (l > R || r < L) return {-1, 0};            // out of query
            bounds
60      if (a[node] < val) return {-1, 0};             // all values < val
61      if (l == r) return {a[node], l};               // leaf node that
            satisfies

63      int mid = (l + r) / 2;
64      auto left = query(l, mid, L, R, node * 2, val);
65      if (left.first != -1) return left;
66      return query(mid + 1, r, L, R, node * 2 + 1, val);
67    }
68  };
69  // Example usage:
70  int n = 8;
71  vector<ll> vals = {4, 2, 7, 1, 9, 5, 6, 3};
72  SegmentTreeWalk st(n);
73  st.build(0, n - 1, 1, vals);
```

3.35   Sparse Table

```
1   /*
2     Sparse Table (Range Minimum Query)
3     ----------------------------------
4     Indexing: 0-based
5     Bounds: [0, n-1]
6     Time Complexity:
7       - Build: O(n log n)
8       - Query: O(1)
9     Space Complexity: O(n log n)

11    Features:
12      - Immutable RMQ (no updates)
13      - Works for idempotent operations like min, max, gcd
14  */

16  const int MAXN = 100005;
17  const int K = 30; // floor(log2(MAXN))
18  int lg[MAXN + 1]; // log base 2 of each i
19  int st[K + 1][MAXN]; // st[k][i] = min in [i, i + 2^k - 1]

21  vector<int> a; // input array
22  int n;

```

```
24  // Returns min in range [L, R]
25  int mini(int L, int R) {
26    int len = R - L + 1;
27    int i = lg[len];
28    return min(st[i][L], st[i][R - (1 << i) + 1]);
29  }

31  int main() {
32    cin >> n;
33    a.resize(n);
34    for (int i = 0; i < n; i++) cin >> a[i];
35    // Precompute binary logs
36    lg[1] = 0;
37    for (int i = 2; i <= n; i++) {
38      lg[i] = lg[i / 2] + 1;
39    }
40    // Initialize 2^0 intervals
41    for (int i = 0; i < n; i++) {
42      st[0][i] = a[i];
43    }
44    // Build sparse table
45    for (int k = 1; k <= K; k++) {
46      for (int i = 0; i + (1 << k) <= n; i++) {
47        st[k][i] = min(st[k - 1][i], st[k - 1][i + (1 << (k - 1))]);
48      }
49    }
50    // Example usage
51    int q; cin >> q;
52    while (q--) {
53      int l, r;
54      cin >> l >> r;
55      cout << mini(l, r) << '\n';
56    }
57    return 0;
58  }
```

3.36   Sparse Table 2D

```
1   #include<bits/stdc++.h>
2   using namespace std;

4   const int N = 505, LG = 10;

```

```
6   int st[N][N][LG][LG];
7   int a[N][N], lg2[N];
8
9   int yo(int x1, int y1, int x2, int y2) {
10    x2++;
11    y2++;
12    int a = lg2[x2 - x1], b = lg2[y2 - y1];
13    return max(
14          max(st[x1][y1][a][b], st[x2 - (1 << a)][y1][a][b]),
15          max(st[x1][y2 - (1 << b)][a][b], st[x2 - (1 << a)][y2 - (1 << b
                )][a][b])
16        );
17  }
18
19  void build(int n, int m) { // 0 indexed
20    for (int i = 2; i < N; i++) lg2[i] = lg2[i >> 1] + 1;
21    for (int i = 0; i < n; i++) {
22      for (int j = 0; j < m; j++) {
23        st[i][j][0][0] = a[i][j];
24      }
25    }
26    for (int a = 0; a < LG; a++) {
27      for (int b = 0; b < LG; b++) {
28        if (a + b == 0) continue;
29        for (int i = 0; i + (1 << a) <= n; i++) {
30          for (int j = 0; j + (1 << b) <= m; j++) {
31            if (!a) {
32              st[i][j][a][b] = max(st[i][j][a][b - 1], st[i][j + (1 << (b
                    - 1))][a][b - 1]);
33            } else {
34              st[i][j][a][b] = max(st[i][j][a - 1][b], st[i + (1 << (a -
                    1))][j][a - 1][b]);
35            }
36          }
37        }
38      }
39    }
40  }
41
42  string s[N];
43  int l[N][N], u[N][N];
44
45  int32_t main() {
```

```
46    ios_base::sync_with_stdio(0);
47    cin.tie(0);
48
49    int n, m;
50    cin >> n >> m;
51    for (int i = 0; i < n; i++) {
52      cin >> s[i];
53    }
54    for (int i = 0; i < n; i++) {
55      for (int j = 0; j < m; j++) {
56        if (!j) l[i][j] = 1;
57        else l[i][j] = 1 + (s[i][j - 1] <= s[i][j] ? l[i][j - 1] : 0);
58      }
59    }
60    for (int j = 0; j < m; j++) {
61      for (int i = 0; i < n; i++) {
62        if (!i) u[i][j] = 1;
63        else u[i][j] = 1 + (s[i - 1][j] <= s[i][j] ? u[i - 1][j] : 0);
64      }
65    }
66    for (int i = 0; i < n; i++) {
67      for (int j = 0; j < m; j++) {
68        int nw = 1, mnx = u[i][j], mny = l[i][j];
69        for (int len = 1; len <= min(i, j); len++) {
70          mnx = min(mnx, u[i][j - len]);
71          mny = min(mny, l[i - len][j]);
72          if (min(mnx, mny) >= len + 1) nw++;
73          else break;
74        }
75        a[i][j] = nw;
76      }
77    }
78    build(n, m);
79    int q;
80    cin >> q;
81    while (q--) {
82      int x1, y1, x2, y2;
83      cin >> x1 >> y1 >> x2 >> y2;
84      x1--, y1--;
85      x2--;
86      y2--;
87      int l = 1, r = min(x2 - x1 + 1, y2 - y1 + 1), ans = 0;
88      while (l <= r) {
```

```
89        int mid = l + r >> 1;
90        if (yo(x1 + mid - 1, y1 + mid - 1, x2, y2) >= mid) ans = mid, l =
              mid + 1;
91        else r = mid - 1;
92      }
93      cout << ans << '\n';
94    }
95    return 0;
96  }
97  // https://www.codechef.com/problems/CENS20B
```

3.37   Square Root Decomposition

```
1   /*
2     Sqrt Decomposition (String Block Cut and Move)
3     ----------------------------------------------
4     Operation:
5       - Supports moving substrings using block cut logic
6       - Rebuilds when too many blocks (for performance)
7
8     Indexing: 0-based
9     String Bounds: [0, n)
10    Time Complexity:
11      - cut(a, b): O(sqrt(n))
12      - rebuildDecomp(): O(n)
13    When to rebuild: after too many block splits
14
15    Use case: performing multiple cut/paste operations efficiently on
            large strings
16  */
17
18  const int MAXI = 350; // = sqrt(n), for n up to 1e5
19
20  int n, numBlocks;
21  string s;
22
23  struct Block {
24    int l, r; // indices into string s
25    int sz() const { return r - l; }
26  };
27
28  Block blocks[2 * MAXI];      // current block array
29  Block newBlocks[2 * MAXI];   // used temporarily during cutting
```

```
30
31  // Rebuilds the entire decomposition into 1 block (or balanced ones)
32  void rebuildDecomp() {
33    string newS = s;
34    int k = 0;
35    // Flatten string using current block structure
36    for (int i = 0; i < numBlocks; i++) {
37      for (int j = blocks[i].l; j < blocks[i].r; j++) {
38        newS[k++] = s[j];
39      }
40    }
41    // Reset to one big block
42    numBlocks = 1;
43    blocks[0] = {0, n};
44    s = newS;
45  }
46
47  // Cut [a, b) into a separate region and reorder it to the end
48  void cut(int a, int b) {
49    int pos = 0, curBlock = 0;
50    // Pass 1: Split blocks to isolate [a, b)
51    for (int i = 0; i < numBlocks; i++) {
52      Block B = blocks[i];
53      bool containsA = (pos < a && pos + B.sz() > a);
54      bool containsB = (pos < b && pos + B.sz() > b);
55      int cutA = B.l + a - pos;
56      int cutB = B.l + b - pos;
57
58      if (containsA && containsB) {
59        newBlocks[curBlock++] = {B.l, cutA};
60        newBlocks[curBlock++] = {cutA, cutB};
61        newBlocks[curBlock++] = {cutB, B.r};
62      } else if (containsA) {
63        newBlocks[curBlock++] = {B.l, cutA};
64        newBlocks[curBlock++] = {cutA, B.r};
65      } else if (containsB) {
66        newBlocks[curBlock++] = {B.l, cutB};
67        newBlocks[curBlock++] = {cutB, B.r};
68      } else {
69        newBlocks[curBlock++] = B;
70      }
71
72      pos += B.sz();
```

```
73    }
74
75    // Pass 2: Reorder - move [a, b) to the end
76    pos = 0;
77    numBlocks = 0;
78
79    // First add all blocks not in [a, b)
80    for (int i = 0; i < curBlock; i++) {
81      if (pos < a || pos >= b)
82        blocks[numBlocks++] = newBlocks[i];
83      pos += newBlocks[i].sz();
84    }
85
86    // Then add blocks in [a, b)
87    pos = 0;
88    for (int i = 0; i < curBlock; i++) {
89      if (pos >= a && pos < b)
90        blocks[numBlocks++] = newBlocks[i];
91      pos += newBlocks[i].sz();
92    }
93  }
94  // Example usage
95  int main() {
96    cin >> s;
97    n = s.size();
98    numBlocks = 1;
99    blocks[0] = {0, n};
100
101   int q; cin >> q;
102   while (q--) {
103     int a, b;
104     cin >> a >> b;
105     cut(a, b); // move [a, b) to the end
106
107     if (numBlocks > MAXI) rebuildDecomp();
108   }
109
110   rebuildDecomp(); // flatten before output
111   cout << s << '\n';
112 }
```

3.38   Tuple

```cpp
#include <tuple>
#include <iostream>
#include <string>

int main() {
    // -----------------------------
    // DECLARING A TUPLE
    // -----------------------------
    std::tuple<int, double, std::string> t1;

    // -----------------------------
    // INITIALIZING A TUPLE
    // -----------------------------
    std::tuple<int, double, std::string> t2(42, 3.14, "hello");

    // -----------------------------
    // USING std::make_tuple
    // (automatic type deduction)
    // -----------------------------
    auto t3 = std::make_tuple(7, 2.71, "world");

    // -----------------------------
    // ACCESSING ELEMENTS USING std::get
    // -----------------------------
    int a = std::get<0>(t2);        // a = 42
    double b = std::get<1>(t2);     // b = 3.14
    std::string c = std::get<2>(t2); // c = "hello"

    // -----------------------------
    // MODIFYING ELEMENTS
    // -----------------------------
    std::get<1>(t2) = 5.67; // change second element

    // -----------------------------
    // STRUCTURED BINDINGS (C++17)
    // Unpacks the tuple into variables
    // -----------------------------
    auto [x, y, z] = t3;   // x=7, y=2.71, z="world"

    // -----------------------------
    // tuple_size and tuple_element
    // (using decltype to reference the type)
    // -----------------------------
```

```cpp
44    std::size_t n = std::tuple_size<decltype(t3)>::value; // n = 3
45    using FirstType = std::tuple_element<0, decltype(t3)>::type;
46    // FirstType is int
47
48    // ----------------------------
49    // SMALL EXAMPLE OF USE
50    // ----------------------------
51    auto person = std::make_tuple("Alice", 25, 1.70);
52
53    // Access values
54    std::cout << "Name: "  << std::get<0>(person) << "\n";
55    std::cout << "Age: "   << std::get<1>(person) << "\n";
56    std::cout << "Height: "<< std::get<2>(person) << "\n";
57
58    // Modify a value
59    std::get<1>(person) = 26;
60
61    // Print updated value
62    std::cout << "Updated age: " << std::get<1>(person) << "\n";
63
64    return 0;
65 }
```

3.39   Treap

```cpp
1  std::mt19937 rng(std::chrono::steady_clock::now().time_since_epoch().
      count());
2
3  struct Node {
4    Node *l = 0, *r = 0;
5    int val, y, c = 1;
6    Node(int val) : val(val), y(rng()) {}
7    void recalc();
8  };
9
10 int cnt(Node* n) { return n ? n->c : 0; }
11 void Node::recalc() { c = cnt(l) + cnt(r) + 1; }
12
13 template<class F> void each(Node* n, F f) {
14   if (n) { each(n->l, f); f(n->val); each(n->r, f); }
15 }
16
17 pair<Node*, Node*> split(Node* n, int k) {
18   if (!n) return {};
19   if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
20     auto pa = split(n->l, k);
21     n->l = pa.second;
22     n->recalc();
23     return {pa.first, n};
24   } else {
25     auto pa = split(n->r, k - cnt(n->l) - 1); // and just "k"
26     n->r = pa.first;
27     n->recalc();
28     return {n, pa.second};
29   }
30 }
31
32 Node* merge(Node* l, Node* r) {
33   if (!l) return r;
34   if (!r) return l;
35   if (l->y > r->y) {
36     l->r = merge(l->r, r);
37     l->recalc();
38     return l;
39   } else {
40     r->l = merge(l, r->l);
41     r->recalc();
42     return r;
43   }
44 }
45
46 Node* ins(Node* t, Node* n, int pos) {
47   auto pa = split(t, pos);
48   return merge(merge(pa.first, n), pa.second);
49 }
50
51 // Example application: move the range [l, r) to index k
52 void move(Node*& t, int l, int r, int k) {
53   Node *a, *b, *c;
54   tie(a,b) = split(t, l); tie(b,c) = split(b, r - l);
55   if (k <= l) t = merge(ins(a, b, k), c);
56   else t = merge(a, ins(c, b, k - r));
57 }
58
59 // Usage
60 // create treap
```

```
61  // Node* name=nullptr;
62  // insert element
63  // name=ins(name, new Node(val), pos);
64  // Node* x = new Node(val);
65  // name = ins(name, x, pos);
66  // merge two treaps (name before x)
67  // name=merge(name, x);
68  // split treap (this will split treap in two treaps,
69  // first with elements [0, pos) and second with elements [pos, n))
70  // pa will be pair of two treaps
71  // auto pa = split(name, pos);
72  // move range [l, r) to index k
73  // move(name, l, r, k);
74  // iterate over treap
75  // each(name, [&](int val) {
76  //      cout << val << ' ';
77  // });
```

3.40   Treap 2

```
1   typedef struct item * pitem;
2   struct item {
3       int prior, value, cnt;
4       bool rev;
5       pitem l, r;
6   };
7
8   int cnt (pitem it) {
9       return it ? it->cnt : 0;
10  }
11
12  void upd_cnt (pitem it) {
13      if (it)
14          it->cnt = cnt(it->l) + cnt(it->r) + 1;
15  }
16
17  void push (pitem it) {
18      if (it && it->rev) {
19          it->rev = false;
20          swap (it->l, it->r);
21          if (it->l)  it->l->rev ^= true;
22          if (it->r)  it->r->rev ^= true;
23      }
```

```
24  }
25
26  void merge (pitem & t, pitem l, pitem r) {
27      push (l);
28      push (r);
29      if (!l || !r)
30          t = l ? l : r;
31      else if (l->prior > r->prior)
32          merge (l->r, l->r, r),  t = l;
33      else
34          merge (r->l, l, r->l),  t = r;
35      upd_cnt (t);
36  }
37
38  void split (pitem t, pitem & l, pitem & r, int key, int add = 0) {
39      if (!t)
40          return void( l = r = 0 );
41      push (t);
42      int cur_key = add + cnt(t->l);
43      if (key <= cur_key)
44          split (t->l, l, t->l, key, add),  r = t;
45      else
46          split (t->r, t->r, r, key, add + 1 + cnt(t->l)),  l = t;
47      upd_cnt (t);
48  }
49
50  void reverse (pitem t, int l, int r) {
51      pitem t1, t2, t3;
52      split (t, t1, t2, l);
53      split (t2, t2, t3, r-l+1);
54      t2->rev ^= true;
55      merge (t, t1, t2);
56      merge (t, t, t3);
57  }
58
59  void output (pitem t) {
60      if (!t)  return;
61      push (t);
62      output (t->l);
63      printf ("%d ", t->value);
64      output (t->r);
65  }
```

3.41   Treap With Inversion

```cpp
std::mt19937 rng(std::chrono::steady_clock::now().time_since_epoch().
    count());

struct Node {
  Node *l = 0, *r = 0;
  int val, y, c = 1;
  bool rev = 0;
  Node(int val) : val(val), y(rng()) {}
  void recalc();
  void push();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }
void Node::push() {
  if (rev) {
    rev = 0;
    swap(l, r);
    if (l) l->rev ^= 1;
    if (r) r->rev ^= 1;
  }
}

template<class F> void each(Node* n, F f) {
  if (n) { n->push(); each(n->l, f); f(n->val); each(n->r, f); }
}

pair<Node*, Node*> split(Node* n, int k) {
  if (!n) return {};
  n->push();
  if (cnt(n->l) >= k) {
    auto pa = split(n->l, k);
    n->l = pa.second;
    n->recalc();
    return {pa.first, n};
  } else {
    auto pa = split(n->r, k - cnt(n->l) - 1);
    n->r = pa.first;
    n->recalc();
    return {n, pa.second};
  }
}
```

```cpp
}

Node* merge(Node* l, Node* r) {
  if (!l) return r;
  if (!r) return l;
  l->push();
  r->push();
  if (l->y > r->y) {
    l->r = merge(l->r, r);
    l->recalc();
    return l;
  } else {
    r->l = merge(l, r->l);
    r->recalc();
    return r;
  }
}

Node* ins(Node* t, Node* n, int pos) {
  auto pa = split(t, pos);
  return merge(merge(pa.first, n), pa.second);
}

// Example application: reverse the range [l, r]
void reverse(Node*& t, int l, int r) {
  Node *a, *b, *c;
  tie(a,b) = split(t, l);
  tie(b,c) = split(b, r - l + 1);
  b->rev ^= 1;
  t = merge(merge(a, b), c);
}

void move(Node*& t, int l, int r, int k) {
  Node *a, *b, *c;
  tie(a,b) = split(t, l);
  tie(b,c) = split(b, r - l);
  if (k <= l) t = merge(ins(a, b, k), c);
  else t = merge(a, ins(c, b, k - r));
}
```

3.42   Treap With Inversions and Range Updates

```cpp
#include <bits/stdc++.h>
```

```cpp
using namespace std;

typedef long long ll;

const int mod = 998244353;

template<int MOD>
struct ModInt {
  ll v;
  ModInt(ll _v = 0) {v = (-MOD < _v && _v < MOD) ? _v : _v % MOD; if (v
      < 0) v += MOD;}
  ModInt& operator += (const ModInt &other) {v += other.v; if (v >= MOD)
       v -= MOD; return *this;}
  ModInt& operator -= (const ModInt &other) {v -= other.v; if (v < 0) v
      += MOD; return *this;}
  ModInt& operator *= (const ModInt &other) {v = v * other.v % MOD;
      return *this;}
  ModInt& operator /= (const ModInt &other) {return *this *= inverse(
      other);}
  bool operator == (const ModInt &other) const {return v == other.v;}
  bool operator != (const ModInt &other) const {return v != other.v;}
  friend ModInt operator + (ModInt a, const ModInt &b) {return a += b;}
  friend ModInt operator - (ModInt a, const ModInt &b) {return a -= b;}
  friend ModInt operator * (ModInt a, const ModInt &b) {return a *= b;}
  friend ModInt operator / (ModInt a, const ModInt &b) {return a /= b;}
  friend ModInt operator - (const ModInt &a) {return 0 - a;}
  friend ModInt power(ModInt a, ll b) {ModInt ret(1); while (b > 0) {if
      (b & 1) ret *= a; a *= a; b >>= 1;} return ret;}
  friend ModInt inverse(ModInt a) {return power(a, MOD - 2);}
  friend istream& operator >> (istream &is, ModInt &m) {is >> m.v; m.v =
       (-MOD < m.v && m.v < MOD) ? m.v : m.v % MOD; if (m.v < 0) m.v +=
      MOD; return is;}
  friend ostream& operator << (ostream &os, const ModInt &m) {return os
      << m.v;}
};

struct Line{
  ModInt<mod> b = 1, c = 0;
  ModInt<mod> operator()(ModInt<mod> x){ return b * x + c;}
  Line operator()(Line other) {return Line{b * other.b, other.c * b + c
      };}
  operator bool() const{
    return b != 1 || c != 0;
```

```cpp
  };
};

std::mt19937 rng(std::chrono::steady_clock::now().time_since_epoch().
    count());

struct Node {
  Node *l = 0, *r = 0;
  int y, c = 1;
  ModInt<mod> val, sum;
  Line line;
  bool rev = 0;
  Node(int val) : val(val), sum(val), y(rng()) {}
  void recalc();
  void push();
};

int cnt(Node* n) { return n ? n->c : 0; }
ModInt<mod> sum1(Node* n) { return n ? n->sum : ModInt<mod>(0); }
void Node::recalc() { if(l) l -> push(); if(r) r -> push(); c = cnt(l) +
    cnt(r) + 1; sum = sum1(l) + sum1(r) + val;}
void Node::push(){
  if(rev){
    rev = 0;
    swap(l, r);
    if(l) l -> rev ^= 1;
    if(r) r -> rev ^= 1;
  }
  if(line){
    val = line(val);
    sum = sum * line.b + line.c * c;
    if(l){
      l -> line = line(l -> line);
    }
    if(r){
      r -> line = line(r -> line);
    }
    line = {1, 0};
  }
}

template<class F> void each(Node* n, F f) {
  if (n) { n -> push(); each(n->l, f); f(n->val); each(n->r, f); }
```

```cpp
76  }
77
78  pair<Node*, Node*> split(Node* n, int k) {
79    if (!n) return {};
80    n -> push();
81    if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
82      auto [L,R] = split(n->l, k);
83      n->l = R;
84      n->recalc();
85      return {L, n};
86    } else {
87      auto [L,R] = split(n->r,k - cnt(n->l) - 1); // and just "k"
88      n->r = L;
89      n->recalc();
90      return {n, R};
91    }
92  }
93
94  Node* merge(Node* l, Node* r) {
95    if (!l) return r;
96    if (!r) return l;
97    l -> push();
98    r -> push();
99    if (l->y > r->y) {
100     l->r = merge(l->r, r);
101     return l->recalc(), l;
102   } else {
103     r->l = merge(l, r->l);
104     return r->recalc(), r;
105   }
106 }
107
108 Node* ins(Node* t, Node* n, int pos) {
109   auto [l,r] = split(t, pos);
110   return merge(merge(l, n), r);
111 }
112
113 void move(Node*& t, int l, int r, int k){
114   Node *a, *b, *c;
115   tie(a, b) = split(t, l); tie(b, c) = split(b, r - l);
116   if(k <= l) t = merge(ins(a, b, k), c);
117   else t = merge(a, ins(c, b, k - r));
118 }
```

```cpp
119
120 void revv(Node*& t, int l, int r){
121   Node *a, *b, *c;
122   tie(a, b) = split(t, l); tie(b, c) = split(b, r - l);
123   b -> rev ^= 1;
124   t = merge(merge(a, b), c);
125 }
126
127 ModInt<mod> query(Node*& t, int l, int r){
128   Node *a, *b, *c;
129   tie(a, b) = split(t, l); tie(b, c) = split(b, r - l);
130   ModInt<mod> res = sum1(b);
131   t = merge(merge(a, b), c);
132   return res;
133 }
134
135 void remove(Node*& t, int x){
136   Node *a, *b, *c;
137   tie(a, b) = split(t, x); tie(b, c) = split(b, 1);
138   t = merge(a, c);
139 }
140
141 void update(Node*& t, int l, int r, ll X, ll Y){
142   Node *a, *b, *c;
143   tie(a, b) = split(t, l); tie(b, c) = split(b, r - 1);
144   b -> line = Line{X, Y}(b -> line);
145   t = merge(a, merge(b, c));
146 }
147
148 int main(){
149   ios_base::sync_with_stdio(false); cin.tie(NULL);
150   int n, q; cin >> n >> q;
151   vector<int> a(n);
152   for(int i = 0; i < n; i++){
153     cin >> a[i];
154   }
155   Node* treap = nullptr;
156   for(int i = 0; i < n; i++){
157     treap = ins(treap, new Node(a[i]), i);
158   }
159   while(q--){
160     int t; cin >> t;
161     if(t == 0){
```

```
162        int i, x; cin >> i >> x;
163        treap = ins(treap, new Node(x), i);
164      }
165      else if(t == 1){
166        int x; cin >> x;
167        remove(treap, x);
168      }
169      else if(t == 2){
170        int l, r; cin >> l >> r;
171        revv(treap, l, r);
172      }
173      else if(t == 3){
174        int l, r, b, c; cin >> l >> r >> b >> c;
175        update(treap, l, r, b, c);
176      }
177      else{
178        int l, r; cin >> l >> r;
179        cout << query(treap, l, r) << endl;
180      }
181    }
182 }
```

4   Dynamic Programming

4.1   CHT Deque

```
1  /*
2    Convex Hull Trick (CHT) - Min Query with Increasing Slopes
3    -----------------------------------------------------------
4    Indexing: 1-based for 'a', 'dp', 's'
5    Bounds:
6      - i from 1 to m // number of elements in the array
7      - j from 1 to p // number of transitions
8    Time Complexity: O(m * p)
9    Requires:
10     - Lines inserted in increasing slope order for min query
11     - Queries made with increasing x values
12
13   dp[i][j] = min over k < i of { dp[k][j-1] + a[i] * (i - k) + s[i] - s[
         k+1] }
14   dp[i][j] = min over k < i of { dp[k][j-1] + cost(k, i) }
15
16   We reformulate:
17     y = m * x + c
18     line: m = -k - 1, c = dp[k][j-1] - s[k+1]
19     eval: m * a[i] + c + a[i] * i + s[i]
20 */
21
22 struct Line {
23   ll a, b; // y = ax + b
24   Line(ll A, ll B) : a(A), b(B) {}
25   ll eval(ll x) const {
26     return a * x + b;
27   }
28   // Returns intersection x-coordinate with another line
29   double intersect(const Line& other) const {
30     return (double)(other.b - b) / (a - other.a);
31   }
32 };
33
34 // this finds the minimum and slope in increasing
35 // Deques for each dp stage
36 deque<Line> cht[p+1];
37 // Fill dp
38 cht[0].push_back(Line(-1, -s[1])); // base case
39 for (int i = 1; i <= m; i++) {
40   for (int j = p; j >= 1; j--) {
41     if (j > i) continue;
42     // Maintain front of deque to find minimum
43     while (cht[j - 1].size() >= 2 && cht[j - 1][1].eval(a[i]) <= cht[j -
           1][0].eval(a[i])) {
44       cht[j - 1].pop_front();
45     }
46     // Evaluate best line
47     dp[i][j] = cht[j - 1].front().eval(a[i]) + a[i] * i + s[i];
48     // Create new line for current i
49     Line curr(-i - 1, dp[i][j] - s[i + 1]);
50     // Maintain convexity: remove worse lines from back
51     while (cht[j].size() >= 2) {
52       Line& l1 = cht[j][cht[j].size() - 2];
53       Line& l2 = cht[j].back();
54       if (curr.intersect(l1) <= l2.intersect(l1)) {
55         cht[j].pop_back();
56       } else break;
57     }
58     cht[j].push_back(curr);
```

```
59    }
60  }
```

4.2   Digit DP

```
1   /*
2     Digit Dynamic Programming (Digit DP)
3     ------------------------------------
4     Goal: Count numbers in range [0, x] that do not have two adjacent
            equal digits.
5     State:
6       - pos: current digit position
7       - last: digit placed at previous position (0 to 9)
8       - f: tight flag (0 = must match prefix of x, 1 = already below x)
9       - z: leading zero flag (1 = still in leading zero zone)
10    Notes:
11      - Solve up to x using 'solve(x)'
12      - Can be modified to count palindromes, digits divisible by 3, etc.
13  */
14
15  vector<int> num;
16  ll DP[20][20][2][2]; // pos, last digit, f (tight), z (leading zero)
17
18  ll g(int pos, int last, int f, int z) {
19    if (pos == num.size()) return 1; // reached end, valid number
20    if (DP[pos][last][f][z] != -1) return DP[pos][last][f][z];
21    ll res = 0;
22    int limit = f ? 9 : num[pos]; // upper digit bound based on tight flag
23    for (int dgt = 0; dgt <= limit; dgt++) {
24      // Skip if digit equals last (unless it's a leading zero)
25      if (dgt == last && !(dgt == 0 && z == 1)) continue;
26      int nf = f;
27      if (!f && dgt < limit) nf = 1;
28      if (z && dgt == 0) res += g(pos + 1, dgt, nf, 1); // still leading
            zeros
29      else res += g(pos + 1, dgt, nf, 0); // now in significant digits
30    }
31    return DP[pos][last][f][z] = res;
32  }
33
34  ll solve(ll x) {
35    if (x == -1) return 0;
36    num.clear();
37    while (x > 0) {
38      num.push_back(x % 10);
39      x /= 10;
40    }
41    reverse(num.begin(), num.end());
42    memset(DP, -1, sizeof(DP));
43    return g(0, 0, 0, 1);
44  }
```

4.3   Divide and Conquer DP

```
1   /*
2     Divide and Conquer DP Optimization
3     ------------------------------------
4     Problem:
5       - dp[i][j] = min over k <= j of { dp[i-1][k] + C(k, j) }
6       - C(k, j) must satisfy the quadrangle inequality:
7         - C(a, c) + C(b, d) <= C(a, d) + C(b, c) for a <= b <= c <= d
8       - or monotonicity of opt[i][j] <= opt[i][j+1]
9
10    Indexing: 0-based
11    Time Complexity: O(m * n * log n)
12    Space Complexity: O(n)
13
14    dp_cur[j]: current dp[i][j] layer
15    dp_before[j]: previous dp[i-1][j] layer
16  */
17
18  int n, m;
19  vector<ll> dp_before, dp_cur;
20  ll C(int i, int j); // Cost function defined by user
21
22  // Recursively compute dp_cur[l..r] with optimal k in [optl, optr]
23  void compute(int l, int r, int optl, int optr) {
24    if (l > r) return;
25    int mid = (l + r) / 2;
26    pair<ll, int> best = {LLONG_MAX, -1};
27    for (int k = optl; k <= min(mid, optr); k++) {
28      ll val = (k > 0 ? dp_before[k - 1] : 0) + C(k, mid);
29      if (val < best.first) best = {val, k};
30    }
31    dp_cur[mid] = best.first;
32    int opt = best.second;
```

```
33    compute(l, mid - 1, optl, opt);
34    compute(mid + 1, r, opt, optr);
35  }
36
37  // Entry point: computes dp[m-1][n-1]
38  ll solve() {
39    dp_before.assign(n, 0);
40    dp_cur.assign(n, 0);
41    for (int i = 0; i < n; i++) {
42      dp_before[i] = C(0, i);
43    }
44    for (int i = 1; i < m; i++) {
45      compute(0, n - 1, 0, n - 1);
46      dp_before = dp_cur;
47    }
48    return dp_before[n - 1];
49  }
```

4.4   Edit Distance

```
1  /*
2    Given strings s and t, compute the minimum number of operations
3    (insert, delete, substitute) to convert s into t.
4    Indexing: 0-based (strings), DP is 1-based with offset
5    dp[i][j] = cost to convert s[0..i-1] into t[0..j-1]
6    Time Complexity: O(n * m)
7    Transitions:
8      - insert: dp[i][j-1] + 1
9      - delete: dp[i-1][j] + 1
10     - replace/match: dp[i-1][j-1] + (s[i-1] != t[j-1])
11 */
12 const int MAXN = 5005;
13 int dp[MAXN][MAXN];
14
15 string s, t; cin >> s >> t;
16 int n = s.length(), m = t.length();
17 // Initialize all to a large number
18 for (int i = 0; i <= n; i++) {
19   fill(dp[i], dp[i] + m + 1, 1e9);
20 }
21 dp[0][0] = 0;
22 for (int i = 0; i <= n; i++) {
23   for (int j = 0; j <= m; j++) {
```

```
24     if (j) { // insert
25       dp[i][j] = min(dp[i][j], dp[i][j - 1] + 1);
26     }
27     if (i) { // delete
28       dp[i][j] = min(dp[i][j], dp[i - 1][j] + 1);
29     }
30     if (i && j) { // replace or match
31       int cost = (s[i - 1] != t[j - 1]) ? 1 : 0;
32       dp[i][j] = min(dp[i][j], dp[i - 1][j - 1] + cost);
33     }
34   }
35 }
```

4.5   Knuth's Algorithm

```
1  #include<bits/stdc++.h>
2  using namespace std;
3
4  const int N = 1010;
5  using ll = long long;
6  /*
7  Knuths optimization works for optimization over sub arrays
8  for which optimal middle point depends monotonously on the end points.
9  Let mid[l,r] be the first middle point for (l,r) sub array which gives
       optimal result.
10 It can be proven that mid[l,r-1] <= mid[l,r] <= mid[l+1,r]
11 - this means monotonicity of mid by l and r.
12 Applying this optimization reduces time complexity from O(k^3) to O(k^2)
13 because with fixed s (sub array length) we have m_right(l) = mid[l+1][r]
       = m_left(l+1).
14 That's why nested l and m loops require not more than 2k iterations
       overall.
15 */
16
17 int n, k;
18 int a[N], mid[N][N];
19 ll res[N][N];
20 ll solve() {
21   for (int s = 0; s <= k; s++) {          // s - length of the subarray
22     for (int l = 0; l + s <= k; l++) {  // l - left point
23       int r = l + s;                      // r - right point
24       if (s < 2) {
25         res[l][r] = 0;                    // base case- nothing to break
```

```
26        mid[l][r] = l;                    // mid is equal to left border
27          continue;
28        }
29      int mleft = mid[l][r - 1];
30      int mright = mid[l + 1][r];
31      res[l][r] = 2e18;
32      for (int m = mleft; m <= mright; m++) {     // iterating for m in
               the bounds only
33          ll tmp = res[l][m] + res[m][r] + (a[r] - a[l]);
34          if (res[l][r] > tmp) {                      // relax current solution
35            res[l][r] = tmp;
36            mid[l][r] = m;
37          }
38        }
39      }
40    }
41    ll ans = res[0][k];
42    return ans;
43 }
44 int main() {
45    int i, j, m;
46    while(cin >> n >> k) {
47      for(i = 1; i <= k; i++) cin >> a[i];
48      a[0] = 0;
49      a[k + 1] = n;
50      k++;
51      cout << solve() << endl;
52    }
53    return 0;
54 }
55 // https://vjudge.net/problem/ZOJ-2860
```

4.6   LCS

```
1 string s, t; cin >> s >> t;
2 int n=s.length(), m=t.length();
3 int dp[n+1][m+1];
4 memset(dp, 0, sizeof(dp));
5 for(int i=1;i<=n;i++){
6    for(int j=1;j<=m;j++){
7        dp[i][j]=max(dp[i-1][j], dp[i][j-1]);
8        if(s[i-1]==t[j-1]){
9            dp[i][j]=dp[i-1][j-1]+1;
```

```
10        }
11    }
12 }
13 int i=n, j=m;
14 string ans="";
15 while(i && j){
16    if(s[i-1]==t[j-1]){
17        ans+=s[i-1];
18        i--; j--;
19    }
20    else if(dp[i][j-1]>=dp[i-1][j]){
21        j--;
22    }
23    else{
24        i--;
25    }
26 }
27 reverse(all(ans));
28 cout << ans << endl;
29
30 // For two permutations one can create new array that will map each
        element from the first permutation to the second.
31 // For each element a[i] in the first permutatio, you find which j is a[
        i] == b[j].
32 // After creating this new array, run LIS (Longest Increasing
        subsequence).
```

4.7   Line Container

```
1 /*
2    Line Container (Dynamic Convex Hull Trick)
3    ------------------------------------------
4    Supports:
5      - Adding lines: y = k * x + m
6      - Querying maximum y at given x
7    Indexing: arbitrary, supports any x
8    Time Complexity:
9      - add(): amortized O(log n)
10     - query(x): O(log n)
11   Space Complexity: O(n)
12   For min queries: negate slopes and intercepts on insert and result on
        query
13   Structure:
```

```
14        - Stores lines in slope-sorted order (k)
15        - Each line keeps its intersection point 'p' with the next line
16        - Binary search on 'p' to answer queries
17    */
18    struct Line {
19      mutable ll k, m, p;
20      bool operator<(const Line& o) const { return k < o.k; }  // Sort by
            slope
21      bool operator<(ll x) const { return p < x; }            // Query
            comparator
22    };
23
24    struct LineContainer : multiset<Line, less<>> {
25      // (for doubles, use inf = 1/.0, div(a,b) = a/b)
26      static const ll inf = LLONG_MAX;
27      ll div(ll a, ll b) { // Floored division
28        return a / b - ((a ^ b) < 0 && a % b);
29      }
30      // Update intersection point x->p with y
31      bool isect(iterator x, iterator y) {
32        if (y == end()) return x->p = inf, false;
33        if (x->k == y->k)
34          x->p = (x->m > y->m ? inf : -inf); // higher line wins
35        else
36          x->p = div(y->m - x->m, x->k - y->k);
37        return x->p >= y->p;
38      }
39      // Add new line: y = k * x + m
40      void add(ll k, ll m) {
41        auto z = insert({k, m, 0}), y = z++, x = y;
42        // Remove dominated lines after y
43        while (isect(y, z)) z = erase(z);
44        // Remove dominated lines before y
45        if (x != begin() && isect(--x, y))
46          isect(x, y = erase(y));
47        // Further cleanup to preserve order
48        while ((y = x) != begin() && (--x)->p >= y->p)
49          isect(x, erase(y));
50      }
51      // Query max y at given x
52      ll query(ll x) {
53        assert(!empty());
54        auto l = *lower_bound(x);
55        return l.k * x + l.m;
56      }
57    };
58    // Example usage:
59    LineContainer cht;
60    cht.add(3, 5);        // y = 3x + 5
61    cht.add(2, 7);        // y = 2x + 7
62    cout << cht.query(4) << '\n'; // max y at x = 4
```

4.8   Longest Increasing Subsequence

```
1    /*
2      Longest Increasing Subsequence + (Recover Sequence) O(n log n)
3      ------------------------------------------------
4      If no recovery is needed, use dp[] only.
5      dp.size() gives the length of LIS.
6      For non-decreasing use upper_bound instead of lower_bound.
7    */
8    vector<int> dp;     // smallest tail values of LIS length i+1
9    vector<int> dp_index;     // index in original array
10   vector<int> parent(n, -1);     // parent[i] = index of previous element
         in LIS
11   vector<int> last_pos(n + 1);  // last_pos[len] = index in v[] ending LIS
         of length len
12   for (int i = 0; i < n; i++) {
13     auto it = lower_bound(dp.begin(), dp.end(), v[i]);
14     int len = it - dp.begin();
15     if (it == dp.end()) {
16       dp.push_back(v[i]);
17       dp_index.push_back(i); // Ignore if no recovery
18     } else {
19       *it = v[i];
20       dp_index[len] = i; // Ignore if no recovery
21     }
22     if (len > 0) parent[i] = dp_index[len - 1]; // Ignore if no recovery
23   }
24   // Reconstruct LIS
25   vector<int> lis;
26   int pos = dp_index.back();
27   while (pos != -1) {
28     lis.push_back(v[pos]);
29     pos = parent[pos];
30   }
```

```
31   reverse(lis.begin(), lis.end());
```

4.9   SOS DP

```
1    vector<int> SOS_DP(vector<int> A, int k) {
2      vector<int> Ap = A;
3      for (int i = k - 1; i >= 0; i--)
4        for (int mk = 0; mk < (1 << k); mk++)
5          if (mk & (1 << i))
6            Ap[mk] += Ap[mk - (1 << i)];
7      return Ap;
8    }
9
10   vector<int> SOS_DP_inv (vector<int> Ap, int k) {
11     vector<int> A = Ap;
12     for (int i = 0; i < k; i++)
13         for (int mk = 0; mk < (1 << k); mk++)
14             if (mk & (1 << i))
15                 A[mk] -= A[mk - (1 << i)];
16     return A;
17   }
18
19   vector<int> SUPER_SOS_DP(vector<int> A, int k) {
20     vector<int> Ap = A;
21     for (int i = k - 1; i >= 0; i--)
22       for (int mk = 0; mk < (1 << k); mk++)
23         if (~mk & (1 << i))
24           Ap[mk] += Ap[mk + (1 << i)];
25     return Ap;
26   }
27
28   vector<int> SUPER_SOS_DP_inv (vector<int> Ap, int k) {
29     vector<int> A = Ap;
30     for (int i = 0; i < k; i++)
31       for (int mk = (1 << k) - 1; mk >= 0; mk--)
32             if (~mk & (1 << i))
33                 A[mk] -= A[mk + (1 << i)];
34     return A;
35   }
36
37   // or convolution
38   Ap = SOS_DP(A, k);
39   Bp = SOS_DP(B, k);
```

```
40
41   for (int i = 0; i < (1 << k); i++)
42       Cp[i] = Ap[i] * Bp[i];
43
44   C = SOS_DP_inv(Cp, k);
45
46   // and convolution
47   Ap = SUPER_SOS_DP(A, k);
48   Bp = SUPER_SOS_DP(B, k);
49
50   for (int i = 0; i < (1 << k); i++)
51       Cp[i] = Ap[i] * Bp[i];
52
53   C = SUPER_SOS_DP_inv(Cp, k);
```

5   Flow

5.1   Dinic

```
1    // Si en el grafo todos los vertices distintos
2    // de s y t cumplen que solo tienen una arista
3    // de entrada o una de salida la y dicha arista
4    // tiene capacidad 1 entonces la complejidad es
5    // O(E sqrt(v))
6
7    // si todas las aristas tinen capacidad 1
8    // el algoritmo tiene complejidad O(E sqrt(E))
9
10   // to find min cut run bfs from source and find all vertices that can be
          reached
11   // edges between vertices that can be reached and the ones that cant are
          the min cut
12   struct FlowEdge {
13     int v, u;
14     long long cap, flow = 0;
15     FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(cap) {}
16   };
17
18   struct Dinic {
19     const long long flow_inf = 1e18;
20     vector<FlowEdge> edges;
21     vector<vector<int>> adj;
22     vector<int> sz;
```

```cpp
int n, m = 0;
int s, t;
vector<int> level, ptr;
queue<int> q;

Dinic(int n, int s, int t) : n(n), s(s), t(t) {
    adj.resize(n);
    level.resize(n);
    ptr.resize(n);
    sz.resize(n);
}

void add_edge(int v, int u, long long cap) {
    edges.emplace_back(v, u, cap);
    edges.emplace_back(u, v, 0);
    adj[v].push_back(m);
    adj[u].push_back(m + 1);
    m += 2;
}

bool bfs() {
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (int id : adj[v]) {
            if (edges[id].cap - edges[id].flow < 1)
                continue;
            if (level[edges[id].u] != -1)
                continue;
            level[edges[id].u] = level[v] + 1;
            q.push(edges[id].u);
        }
    }
    return level[t] != -1;
}

long long dfs(int v, long long pushed) {
    if (pushed == 0)
        return 0;
    if (v == t)
        return pushed;
    for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {
        int id = adj[v][cid];
        int u = edges[id].u;
        if (level[v] + 1 != level[u] || edges[id].cap - edges[id].flow
                < 1)
            continue;
        long long tr = dfs(u, min(pushed, edges[id].cap - edges[id].
                flow));
        if (tr == 0)
            continue;
        edges[id].flow += tr;
        edges[id ^ 1].flow -= tr;
        return tr;
    }
    return 0;
}

long long flow() {
    long long f = 0;
    while (true) {
        fill(level.begin(), level.end(), -1);
        level[s] = 0;
        q.push(s);
        if (!bfs())
            break;
        fill(ptr.begin(), ptr.end(), 0);
        while (long long pushed = dfs(s, flow_inf)) {
            f += pushed;
        }
    }
    return f;
}

vector<pair<int, int>> minCut(){
    vector<pair<int, int>> ans;
    queue<int> q;
    q.push(s);
    vector<bool> vis(n, 0);
    while((int)q.size()){
        int cur = q.front();
        q.pop();
        vis[cur] = 1;
        for(int id: adj[cur]){
            if(edges[id].cap - edges[id].flow <= 0) continue;
```

```
107        if(vis[edges[id].u]) continue;
108        vis[edges[id].u] = 1;
109        q.push(edges[id].u);
110      }
111    }
112    for(auto edge: edges){
113      if(vis[edge.v] && !vis[edge.u] && edge.flow > 0) {
114        ans.push_back({edge.v, edge.u});
115      }
116    }
117    return ans;
118  }
119
120  bool getPath(int cur, vector<int> &path){
121    path.push_back(cur);
122    for(int &i = sz[cur]; i >= 0; i--){
123      int edge_index = adj[cur][i];
124      if(edge_index & 1) continue;
125      if(edges[edge_index].cap - edges[edge_index].flow == 0){
126        i--;
127        getPath(edges[edge_index].u, path);
128        return true;
129      }
130    }
131    return false;
132  }
133
134  vector<vector<int>> getPaths(){
135    vector<vector<int>> ans;
136    vector<int> path;
137    for(int i = 0; i < n; i++) sz[i] = (int)adj[i].size() - 1;
138    while(getPath(s, path)){
139      ans.push_back(path);
140      path.clear();
141    }
142    return ans;
143  }
144 };
```

5.2   Hopcroft-Karp

```
1  // maximum matching in bipartite graph
2  vector<int> match, dist;
3  vector<vector<int>> g;
4  int n, m, k;
5  bool bfs()
6  {
7    queue<int> q;
8    // The alternating path starts with unmatched nodes
9    for (int node = 1; node <= n; node++)
10   {
11     if (!match[node])
12     {
13       q.push(node);
14       dist[node] = 0;
15     }
16     else
17     {
18       dist[node] = INF;
19     }
20   }
21
22   dist[0] = INF;
23
24   while (!q.empty())
25   {
26     int node = q.front();
27     q.pop();
28     if (dist[node] >= dist[0])
29     {
30       continue;
31     }
32     for (int son : g[node])
33     {
34       // If the match of son is matched
35       if (dist[match[son]] == INF)
36       {
37         dist[match[son]] = dist[node] + 1;
38         q.push(match[son]);
39       }
40     }
41   }
42   // Returns true if an alternating path has been found
43   return dist[0] != INF;
44 }
45
```

```
46  // Returns true if an augmenting path has been found starting from
        vertex node
47  bool dfs(int node)
48  {
49    if (node == 0)
50    {
51      return true;
52    }
53    for (int son : g[node])
54    {
55      if (dist[match[son]] == dist[node] + 1 && dfs(match[son]))
56      {
57        match[node] = son;
58        match[son] = node;
59        return true;
60      }
61    }
62    dist[node] = INF;
63    return false;
64  }
65
66  int hopcroft_karp()
67  {
68    int cnt = 0;
69    // While there is an alternating path
70    while (bfs())
71    {
72      for (int node = 1; node <= n; node++)
73      {
74        // If node is unmatched but we can match it using an augmenting
              path
75        if (!match[node] && dfs(node))
76        {
77          cnt++;
78        }
79      }
80    }
81    return cnt;
82  }
83  // usage
84  // n numero de puntos en la izquierda
85  // m numero de puntos en la derecha
86  // las aristas se guardan en g
```

```
87  // los puntos estan 1 indexados
88  // el punto 1 de m es el punto n+1 de g
89  // hopcroft_karp() devuelve el tamano del maximo matching
90  // match contiene el match de cada punto
91  // si match de i es 0, entonces i no esta matcheado
92  //
93  // https://judge.yosupo.jp/submission/247277
```

5.3   Hungarian

```
1   #define forn(i,n) for(int i=0;i<int(n);++i)
2   #define forsn(i,s,n) for(int i=s;i<int(n);++i)
3   #define forall(i,c) for(typeof(c.begin()) i=c.begin();i!=c.end();++i)
4   #define DBG(X) cerr << #X << " = " << X << endl;
5   typedef vector<int> vint;
6   typedef vector<vint> vvint;
7
8   void showmt();
9
10  /* begin notebook */
11
12  #define MAXN 256
13  #define INFTO 0x7f7f7f7f
14  int n;
15  int mt[MAXN][MAXN]; // Matriz de costos (X * Y)
16  int xy[MAXN], yx[MAXN]; // Matching resultante (X->Y, Y->X)
17
18  int lx[MAXN], ly[MAXN], slk[MAXN], slkx[MAXN], prv[MAXN];
19  char S[MAXN], T[MAXN];
20
21  void updtree(int x) {
22    forn(y, n) if (lx[x] + ly[y] - mt[x][y] < slk[y]) {
23      slk[y] = lx[x] + ly[y] - mt[x][y];
24      slkx[y] = x;
25    }
26  }
27  int hungar() {
28    forn(i, n) {
29      ly[i] = 0;
30      lx[i] = *max_element(mt[i], mt[i]+n);
31    }
32    memset(xy, -1, sizeof(xy));
33    memset(yx, -1, sizeof(yx));
```

```
34
35     forn(m, n) {
36       memset(S, 0, sizeof(S));
37       memset(T, 0, sizeof(T));
38       memset(prv, -1, sizeof(prv));
39       memset(slk, 0x7f, sizeof(slk));
40       queue<int> q;
41       #define bpone(e, p) { q.push(e); prv[e] = p; S[e] = 1; updtree(e); }
42       forn(i, n) if (xy[i] == -1) { bpone(i, -2); break; }
43
44       int x=0, y=-1;
45       while (y==-1) {
46         while (!q.empty() && y==-1) {
47           x = q.front(); q.pop();
48           forn(j, n) if (mt[x][j] == lx[x] + ly[j] && !T[j]) {
49             if (yx[j] == -1) { y = j; break; }
50             T[j] = 1;
51             bpone(yx[j], x);
52           }
53         }
54         if (y!=-1) break;
55         int dlt = INFTO;
56         forn(j, n) if (!T[j]) dlt = min(dlt, slk[j]);
57         forn(k, n) {
58           if (S[k]) lx[k] -= dlt;
59           if (T[k]) ly[k] += dlt;
60           if (!T[k]) slk[k] -= dlt;
61         }
62 //        q = queue<int>();
63         forn(j, n) if (!T[j] && !slk[j]) {
64           if (yx[j] == -1) {
65             x = slkx[j]; y = j; break;
66           } else {
67             T[j] = 1;
68             if (!S[yx[j]]) bpone(yx[j], slkx[j]);
69           }
70         }
71       }
72       if (y!=-1) {
73         for(int p = x; p != -2; p = prv[p]) {
74           yx[y] = p;
75           int ty = xy[p]; xy[p] = y; y = ty;
76         }
```

```
77         } else break;
78       }
79       int res = 0;
80       forn(i, n) res += mt[i][xy[i]];
81       return res;
82 }
```

5.4   Max Flow Min Cost

```
1  // dado un acomodo de flujos con costos
2  // devuelve el costo minimo para un flujo especificado
3
4  struct Edge
5  {
6    int from, to, capacity, cost;
7    Edge(int _from, int _to, int _capacity, int _cost)
8    {
9      from = _from;
10     to = _to;
11     capacity = _capacity;
12     cost = _cost;
13   }
14 };
15
16 vector<vector<int>> adj, cost, capacity;
17
18 const int INF = 1e9;
19
20 void shortest_paths(int n, int v0, vector<int> &d, vector<int> &p)
21 {
22   d.assign(n, INF);
23   d[v0] = 0;
24   vector<bool> inq(n, false);
25   queue<int> q;
26   q.push(v0);
27   p.assign(n, -1);
28
29   while (!q.empty())
30   {
31     int u = q.front();
32     q.pop();
33     inq[u] = false;
34     for (int v : adj[u])
```

```
35        {
36          if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v])
37          {
38            d[v] = d[u] + cost[u][v];
39            p[v] = u;
40            if (!inq[v])
41            {
42              inq[v] = true;
43              q.push(v);
44            }
45          }
46        }
47    }
48  }
49
50  int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t)
51  {
52    adj.assign(N, vector<int>());
53    cost.assign(N, vector<int>(N, 0));
54    capacity.assign(N, vector<int>(N, 0));
55    for (Edge e : edges)
56    {
57      adj[e.from].push_back(e.to);
58      adj[e.to].push_back(e.from);
59      cost[e.from][e.to] = e.cost;
60      cost[e.to][e.from] = -e.cost;
61      capacity[e.from][e.to] = e.capacity;
62    }
63
64    int flow = 0;
65    int cost = 0;
66    vector<int> d, p;
67    while (flow < K)
68    {
69      shortest_paths(N, s, d, p);
70      if (d[t] == INF)
71        break;
72
73      // find max flow on that path
74      int f = K - flow;
75      int cur = t;
76      while (cur != s)
77      {
78        f = min(f, capacity[p[cur]][cur]);
79        cur = p[cur];
80      }
81
82      // apply flow
83      flow += f;
84      cost += f * d[t];
85      cur = t;
86      while (cur != s)
87      {
88        capacity[p[cur]][cur] -= f;
89        capacity[cur][p[cur]] += f;
90        cur = p[cur];
91      }
92    }
93
94    if (flow < K)
95      return -1;
96    else
97      return cost;
98  }
```

5.5   Max Flow

```
1  long long max_flow(vector<vector<int>> adj, vector<vector<long long>>
       capacity,
2                     int source, int sink)
3  {
4    int n = adj.size();
5    vector<int> parent(n, -1);
6    // Find a way from the source to sink on a path with non-negative
          capacities
7    auto reachable = [&]() -> bool
8    {
9      queue<int> q;
10     q.push(source);
11     while (!q.empty())
12     {
13       int node = q.front();
14       q.pop();
15       for (int son : adj[node])
16       {
17         long long w = capacity[node][son];
```

```
18        if (w <= 0 || parent[son] != -1)
19          continue;
20        parent[son] = node;
21        q.push(son);
22      }
23    }
24    return parent[sink] != -1;
25  };
26
27  long long flow = 0;
28  // While there is a way from source to sink with non-negative
          capacities
29  while (reachable())
30  {
31    int node = sink;
32    // The minimum capacity on the path from source to sink
33    long long curr_flow = LLONG_MAX;
34    while (node != source)
35    {
36      curr_flow = min(curr_flow, capacity[parent[node]][node]);
37      node = parent[node];
38    }
39    node = sink;
40    while (node != source)
41    {
42      // Subtract the capacity from capacity edges
43      capacity[parent[node]][node] -= curr_flow;
44      // Add the current flow to flow backedges
45      capacity[node][parent[node]] += curr_flow;
46      node = parent[node];
47    }
48    flow += curr_flow;
49    fill(parent.begin(), parent.end(), -1);
50  }
51
52  return flow;
53 }
54
55
56
57 //vector<vector<long long>> capacity(n, vector<long long>(n));
58 //vector<vector<int>> adj(n);
59 //adj[a].push_back(b);
```

```
60 //adj[b].push_back(a);
61 //capacity[a][b] += c;
```

5.6   Min Cost Max Flow

```
1  /**
2   *  If costs can be negative, call setpi before maxflow, but note that
        negative cost cycles are not supported.
3   *  To obtain the actual flow, look at positive values only
4   * Time: $O(F E \log(V))$ where F is max flow. $O(VE)$ for setpi.
5   */
6  #include <bits/stdc++.h>
7  using namespace std;
8
9  #include <ext/pb_ds/priority_queue.hpp>
10 using namespace __gnu_pbds;
11
12 #define rep(i, a, b) for(int i = a; i < (b); ++i)
13 #define all(x) begin(x), end(x)
14 #define sz(x) (int)(x).size()
15 typedef long long ll;
16 typedef pair<int, int> pii;
17 typedef vector<int> vi;
18
19 #pragma once
20
21 // #include <bits/extc++.h> /// include-line, keep-include
22
23 const ll INF = numeric_limits<ll>::max() / 4;
24
25 struct MCMF {
26   struct edge {
27     int from, to, rev;
28     ll cap, cost, flow;
29   };
30   int N;
31   vector<vector<edge>> ed;
32   vi seen;
33   vector<ll> dist, pi;
34   vector<edge*> par;
35
36   MCMF(int N) : N(N), ed(N), seen(N), dist(N), pi(N), par(N) {}
37
```

```cpp
void addEdge(int from, int to, ll cap, ll cost) {
  if (from == to) return;
  ed[from].push_back(edge{ from,to,sz(ed[to]),cap,cost,0 });
  ed[to].push_back(edge{ to,from,sz(ed[from])-1,0,-cost,0 });
}

void path(int s) {
  fill(all(seen), 0);
  fill(all(dist), INF);
  dist[s] = 0; ll di;

  __gnu_pbds::priority_queue<pair<ll, int>> q;
  vector<decltype(q)::point_iterator> its(N);
  q.push({ 0, s });

  while (!q.empty()) {
    s = q.top().second; q.pop();
    seen[s] = 1; di = dist[s] + pi[s];
    for (edge& e : ed[s]) if (!seen[e.to]) {
      ll val = di - pi[e.to] + e.cost;
      if (e.cap - e.flow > 0 && val < dist[e.to]) {
        dist[e.to] = val;
        par[e.to] = &e;
        if (its[e.to] == q.end())
          its[e.to] = q.push({ -dist[e.to], e.to });
        else
          q.modify(its[e.to], { -dist[e.to], e.to });
      }
    }
  }
  rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
}

pair<ll, ll> maxflow(int s, int t) {
  ll totflow = 0, totcost = 0;
  while (path(s), seen[t]) {
    ll fl = INF;
    for (edge* x = par[t]; x; x = par[x->from])
      fl = min(fl, x->cap - x->flow);

    totflow += fl;
    for (edge* x = par[t]; x; x = par[x->from]) {
      x->flow += fl;
      ed[x->to][x->rev].flow -= fl;
    }
  }
  rep(i,0,N) for(edge& e : ed[i]) totcost += e.cost * e.flow;
  return {totflow, totcost/2};
}

// If some costs can be negative, call this before maxflow:
void setpi(int s) { // (otherwise, leave this out)
  fill(all(pi), INF); pi[s] = 0;
  int it = N, ch = 1; ll v;
  while (ch-- && it--)
    rep(i,0,N) if (pi[i] != INF)
      for (edge& e : ed[i]) if (e.cap)
        if ((v = pi[i] + e.cost) < pi[e.to])
          pi[e.to] = v, ch = 1;
  assert(it >= 0); // negative cost cycle
}
};
```

5.7   Push Relabel

```cpp
const int inf = 1000000000;

int n;
vector<vector<int>> capacity, flow;
vector<int> height, excess, seen;
queue<int> excess_vertices;

void push(int u, int v) {
  int d = min(excess[u], capacity[u][v] - flow[u][v]);
  flow[u][v] += d;
  flow[v][u] -= d;
  excess[u] -= d;
  excess[v] += d;
  if (d && excess[v] == d)
    excess_vertices.push(v);
}

void relabel(int u) {
  int d = inf;
  for (int i = 0; i < n; i++) {
    if (capacity[u][i] - flow[u][i] > 0)
```

```
22        d = min(d, height[i]);
23    }
24    if (d < inf)
25      height[u] = d + 1;
26 }
27
28 void discharge(int u) {
29    while (excess[u] > 0) {
30      if (seen[u] < n) {
31        int v = seen[u];
32        if (capacity[u][v] - flow[u][v] > 0 && height[u] > height[v])
33          push(u, v);
34        else
35          seen[u]++;
36      } else {
37        relabel(u);
38        seen[u] = 0;
39      }
40    }
41 }
42
43 int max_flow(int s, int t) {
44    height.assign(n, 0);
45    height[s] = n;
46    flow.assign(n, vector<int>(n, 0));
47    excess.assign(n, 0);
48    excess[s] = inf;
49    for (int i = 0; i < n; i++) {
50      if (i != s)
51        push(s, i);
52    }
53    seen.assign(n, 0);
54
55    while (!excess_vertices.empty()) {
56      int u = excess_vertices.front();
57      excess_vertices.pop();
58      if (u != s && u != t)
59        discharge(u);
60    }
61
62    int max_flow = 0;
63    for (int i = 0; i < n; i++)
64      max_flow += flow[i][t];
```

```
65    return max_flow;
66 }
```

6   Geometry

6.1   Point Struct

```
1 typedef long long T;
2 struct pt {
3    T x,y;
4    pt operator+(pt p) {return {x+p.x, y+p.y};}
5    pt operator-(pt p) {return {x-p.x, y-p.y};}
6    pt operator*(T d) {return {x*d, y*d};}
7    pt operator/(T d) {return {x/d, y/d};}
8 };
9
10 // cross product
11 // positivo si el segundo esta en sentido antihorario
12 // 0 si el angulo es 180
13 // negativo si el segundo esta en sentido horario
14 T cross(pt v, pt w) {return v.x*w.y - v.y*w.x;}
15
16 // dot product
17 // positivo si el angulo entre los vectores es agudo
18 // 0 si son perpendiculares
19 // negativo si el angulo es obtuso
20 T dot(pt v, pt w) {return v.x*w.x + v.y*w.y;}
21
22 T orient(pt a, pt b, pt c) {return cross(b-a,c-a);}
23
24 T dist(pt a,pt b){
25    pt aux=b-a;
26    return sqrtl(aux.x*aux.x+aux.y*aux.y);
27 }
```

6.2   Sort Points

```
1 // This comparator sorts the points clockwise
2 // starting from the first quarter
3
4 bool getQ(Point a){
5    if(a.y!=0){
6      if(a.y>0)return 0;
```

```
7        return 1;
8      }
9      if(a.x>0)return 0;
10     return 1;
11   }
12   bool comp(Point a, Point b){
13     if(getQ(a)!=getQ(b))return getQ(a)<getQ(b);
14     return a*b>0;
15   }
```

6.3   Point Struct2

```
1    #include <bits/stdc++.h>
2    using namespace std;
3    using ld = long double;
4    const ld eps = 1e-9, inf = numeric_limits<ld>::max(), pi = acos(-1);
5    // For use with integers, just set eps=0 and everything remains the same
6    bool geq(ld a, ld b){return a-b >= -eps;}      //a >= b
7    bool leq(ld a, ld b){return b-a >= -eps;}      //a <= b
8    bool ge(ld a, ld b){return a-b > eps;}         //a > b
9    bool le(ld a, ld b){return b-a > eps;}         //a < b
10   bool eq(ld a, ld b){return abs(a-b) <= eps;}   //a == b
11   bool neq(ld a, ld b){return abs(a-b) > eps;}   //a != b
12
13   struct point{
14     ld x, y;
15     point(): x(0), y(0){}
16     point(ld x, ld y): x(x), y(y){}
17
18     point operator+(const point & p) const{return point(x + p.x, y + p.y)
            ;}
19     point operator-(const point & p) const{return point(x - p.x, y - p.y)
            ;}
20     point operator*(const ld & k) const{return point(x * k, y * k);}
21     point operator/(const ld & k) const{return point(x / k, y / k);}
22
23     point operator+=(const point & p){*this = *this + p; return *this;}
24     point operator-=(const point & p){*this = *this - p; return *this;}
25     point operator*=(const ld & p){*this = *this * p; return *this;}
26     point operator/=(const ld & p){*this = *this / p; return *this;}
27
28     point rotate(const ld & a) const{return point(x*cos(a) - y*sin(a), x*
            sin(a) + y*cos(a));}
29     point perp() const{return point(-y, x);}
30     ld ang() const{
31       ld a = atan2l(y, x); a += le(a, 0) ? 2*pi : 0; return a;
32     }
33     ld dot(const point & p) const{return x * p.x + y * p.y;}
34     ld cross(const point & p) const{return x * p.y - y * p.x;}
35     ld norm() const{return x * x + y * y;}
36     ld length() const{return sqrtl(x * x + y * y);}
37     point unit() const{return (*this) / length();}
38
39     bool operator==(const point & p) const{return eq(x, p.x) && eq(y, p.y)
            ;}
40     bool operator!=(const point & p) const{return !(*this == p);}
41     bool operator<(const point & p) const{return le(x, p.x) || (eq(x, p.x)
            && le(y, p.y));}
42     bool operator>(const point & p) const{return ge(x, p.x) || (eq(x, p.x)
            && ge(y, p.y));}
43     bool half(const point & p) const{return le(p.cross(*this), 0) || (eq(p
            .cross(*this), 0) && le(p.dot(*this), 0));}
44   };
45
46   istream &operator>>(istream &is, point & p){return is >> p.x >> p.y;}
47   ostream &operator<<(ostream &os, const point & p){return os << "(" << p.
        x << "," << p.y << ")";}
48
49   int sgn(ld x){
50     if(ge(x, 0)) return 1;
51     if(le(x, 0)) return -1;
52     return 0;
53   }
54
55   void polarSort(vector<point> & P, const point & o, const point & v){
56     //sort points in P around o, taking the direction of v as first angle
57     sort(P.begin(), P.end(), [&](const point & a, const point & b){
58       return point((a - o).half(v), 0) < point((b - o).half(v), (a - o).
            cross(b - o));
59     });
60   }
```

6.4   Find Triangle With Given Area

```
1    #include <algorithm>
2    #include <cassert>
```

```cpp
#include <iostream>
#include <vector>
using namespace std;

struct point {
    int x, y;
    int index;

    point(int _x = 0, int _y = 0) : x(_x), y(_y) {
        index = -1;
    }

    point& operator+=(const point &other) {
        x += other.x;
        y += other.y;
        return *this;
    }

    point& operator-=(const point &other) {
        x -= other.x;
        y -= other.y;
        return *this;
    }

    point operator+(const point &other) const {
        return point(*this) += other;
    }

    point operator-(const point &other) const {
        return point(*this) -= other;
    }

    bool operator<(const point &other) const {
        return make_pair(y, x) > make_pair(other.y, other.x);
    }
};

long long cross(const point &a, const point &b) {
    return (long long) a.x * b.y - (long long) b.x * a.y;
}

long long triangle_area(const point &a, const point &b, const point &c)
    {
    return abs(cross(a, b) + cross(b, c) + cross(c, a));
}

struct segment {
    point p;
    int index1, index2;

    segment(const point &_p, int _index1, int _index2) : p(_p), index1(
        _index1), index2(_index2) {}

    bool operator<(const segment &other) const {
        return cross(p, other.p) > 0;
    }
};

int N;
long long S;
vector<point> points;
vector<segment> segments;
vector<int> location;

void yes(point a, point b, point c) {
    cout << "Yes" << '\n';
    cout << a.x << ' ' << a.y << '\n';
    cout << b.x << ' ' << b.y << '\n';
    cout << c.x << ' ' << c.y << '\n';
    assert(triangle_area(a, b, c) == 2 * S);
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);

    cin >> N >> S;
    points.resize(N);

    for (int i = 0; i < N; i++)
        cin >> points[i].x >> points[i].y;

    sort(points.begin(), points.end());
    location.resize(N);

    for (int i = 0; i < N; i++) {
```

```
87          points[i].index = i;
88          location[i] = i;
89      }
90
91      for (int i = 0; i < N; i++)
92          for (int j = 0; j < i; j++)
93              segments.emplace_back(points[j] - points[i], i, j);
94
95      sort(segments.begin(), segments.end());
96
97      for (segment &s : segments) {
98          int &a = location[s.index1];
99          int &b = location[s.index2];
100         assert(b == a - 1);
101
102         int low = 0, high = b - 1;
103
104         while (low <= high) {
105             int mid = (low + high) / 2;
106             long long area = triangle_area(points[mid], points[a],
                    points[b]);
107
108             if (area == 2 * S) {
109                 yes(points[mid], points[a], points[b]);
110                 return 0;
111             }
112
113             if (area > 2 * S)
114                 low = mid + 1;
115             else
116                 high = mid - 1;
117         }
118
119         low = a + 1;
120         high = N - 1;
121
122         while (low <= high) {
123             int mid = (low + high) / 2;
124             long long area = triangle_area(points[mid], points[a],
                    points[b]);
125
126             if (area == 2 * S) {
127                 yes(points[mid], points[a], points[b]);
```

```
128                 return 0;
129             }
130
131             if (area > 2 * S)
132                 high = mid - 1;
133             else
134                 low = mid + 1;
135         }
136
137         swap(points[a], points[b]);
138         swap(a, b);
139     }
140
141     cout << "No" << '\n';
142     return 0;
143 }
```

6.5   Antipodal Pairs

```
1  vector<pair<int, int>> antipodalPairs(vector<point> & P){
2    vector<pair<int, int>> ans;
3    int n = P.size(), k = 1;
4    auto f = [&](int u, int v, int w){return abs((P[v%n]-P[u%n]).cross(P[w
         %n]-P[u%n]));};
5    while(ge(f(n-1, 0, k+1), f(n-1, 0, k))) ++k;
6    for(int i = 0, j = k; i <= k && j < n; ++i){
7      ans.emplace_back(i, j);
8      while(j < n-1 && ge(f(i, i+1, j+1), f(i, i+1, j)))
9        ans.emplace_back(i, ++j);
10   }
11   return ans;
12 }
```

6.6   Area and Perimeter

```
1  ld perimeter(vector<point> & P){
2    int n = P.size();
3    ld ans = 0;
4    for(int i = 0; i < n; i++){
5      ans += (P[i] - P[(i + 1) % n]).length();
6    }
7    return ans;
8  }
9
```

```
10  ld area(vector<point> & P){
11    int n = P.size();
12    ld ans = 0;
13    for(int i = 0; i < n; i++){
14      ans += P[i].cross(P[(i + 1) % n]);
15    }
16    return abs(ans / 2);
17  }
```

6.7   Area Union Circles

```
1   struct circ{
2     point c;
3     ld r;
4     circ() {}
5     circ(const point & c, ld r): c(c), r(r) {}
6     set<pair<ld, ld>> ranges;
7
8     void disable(ld l, ld r){
9       ranges.emplace(l, r);
10    }
11
12    auto getActive() const{
13      vector<pair<ld, ld>> ans;
14      ld maxi = 0;
15      for(const auto & dis : ranges){
16        ld l, r;
17        tie(l, r) = dis;
18        if(l > maxi){
19          ans.emplace_back(maxi, l);
20        }
21        maxi = max(maxi, r);
22      }
23      if(!eq(maxi, 2*pi)){
24        ans.emplace_back(maxi, 2*pi);
25      }
26      return ans;
27    }
28  };
29
30  ld areaUnionCircles(const vector<circ> & circs){
31    vector<circ> valid;
32    for(const circ & curr : circs){
```

```
33      if(eq(curr.r, 0)) continue;
34      circ nuevo = curr;
35      for(circ & prev : valid){
36        if(circleInsideCircle(prev.c, prev.r, nuevo.c, nuevo.r)){
37          nuevo.disable(0, 2*pi);
38        }else if(circleInsideCircle(nuevo.c, nuevo.r, prev.c, prev.r)){
39          prev.disable(0, 2*pi);
40        }else{
41          auto cruce = intersectionCircles(prev.c, prev.r, nuevo.c, nuevo.r);
42          if(cruce.size() == 2){
43            ld a1 = (cruce[0] - prev.c).ang();
44            ld a2 = (cruce[1] - prev.c).ang();
45            ld b1 = (cruce[1] - nuevo.c).ang();
46            ld b2 = (cruce[0] - nuevo.c).ang();
47            if(a1 < a2){
48              prev.disable(a1, a2);
49            }else{
50              prev.disable(a1, 2*pi);
51              prev.disable(0, a2);
52            }
53            if(b1 < b2){
54              nuevo.disable(b1, b2);
55            }else{
56              nuevo.disable(b1, 2*pi);
57              nuevo.disable(0, b2);
58            }
59          }
60        }
61      }
62      valid.push_back(nuevo);
63    }
64    ld ans = 0;
65    for(const circ & curr : valid){
66      for(const auto & range : curr.getActive()){
67        ld l, r;
68        tie(l, r) = range;
69        ans += curr.r*(curr.c.x * (sin(r) - sin(l)) - curr.c.y * (cos(r) -
              cos(l))) + curr.r*curr.r*(r-l);
70      }
71    }
72    return ans/2;
73  };
```

6.8   Centroid

```
point centroid(vector<point> & P){
  point num;
  ld den = 0;
  int n = P.size();
  for(int i = 0; i < n; ++i){
    ld cross = P[i].cross(P[(i + 1) % n]);
    num += (P[i] + P[(i + 1) % n]) * cross;
    den += cross;
  }
  return num / (3 * den);
}
```

6.9   Circle Inside Circle

```
int circleInsideCircle(const point & c1, ld r1, const point & c2, ld r2)
    {
  //test if circle 2 is inside circle 1
  //returns "-1" if 2 touches internally 1, "1" if 2 is inside 1, "0" if
      they overlap
  ld l = r1 - r2 - (c1 - c2).length();
  return (ge(l, 0) ? 1 : (eq(l, 0) ? -1 : 0));
}
```

6.10   Circle Outside Circle

```
int circleOutsideCircle(const point & c1, ld r1, const point & c2, ld r2
    ){
  //test if circle 2 is outside circle 1
  //returns "-1" if they touch externally, "1" if 2 is outside 1, "0" if
      they overlap
  ld l = (c1 - c2).length() - (r1 + r2);
  return (ge(l, 0) ? 1 : (eq(l, 0) ? -1 : 0));
}
```

6.11   Closest Pair of Points

```
bool comp1(const point & a, const point & b){
  return le(a.y, b.y);
}
pair<point, point> closestPairOfPoints(vector<point> P){
  sort(P.begin(), P.end(), comp1);
  set<point> S;
```

```
  ld ans = inf;
  point p, q;
  int pos = 0;
  for(int i = 0; i < P.size(); ++i){
    while(pos < i && geq(P[i].y - P[pos].y, ans)){
      S.erase(P[pos++]);
    }
    auto lower = S.lower_bound({P[i].x - ans - eps, -inf});
    auto upper = S.upper_bound({P[i].x + ans + eps, -inf});
    for(auto it = lower; it != upper; ++it){
      ld d = (P[i] - *it).length();
      if(le(d, ans)){
        ans = d;
        p = P[i];
        q = *it;
      }
    }
    S.insert(P[i]);
  }
  return {p, q};
}
```

6.12   Common Tangents

```
vector<vector<point>> tangents(const point & c1, ld r1, const point & c2
    , ld r2, bool inner){
  //returns a vector of segments or a single point
  if(inner) r2 = -r2;
  point d = c2 - c1;
  ld dr = r1 - r2, d2 = d.norm(), h2 = d2 - dr*dr;
  if(eq(d2, 0) || le(h2, 0)) return {};
  point v = d*dr/d2;
  if(eq(h2, 0)) return {{c1 + v*r1}};
  else{
    point u = d.perp()*sqrt(h2)/d2;
    return {{c1 + (v - u)*r1, c2 + (v - u)*r2}, {c1 + (v + u)*r1, c2 + (
        v + u)*r2}};
  }
}
```

6.13   Convex Hull

```
vector<point> convexHull(vector<point> P){
  sort(P.begin(), P.end());
```

```
3    vector<point> L, U;
4    for(int i = 0; i < P.size(); i++){
5      while(L.size() >= 2 && leq((L[L.size() - 2] - P[i]).cross(L[L.size()
           - 1] - P[i]), 0)){
6        L.pop_back();
7      }
8      L.push_back(P[i]);
9    }
10   for(int i = P.size() - 1; i >= 0; i--){
11     while(U.size() >= 2 && leq((U[U.size() - 2] - P[i]).cross(U[U.size()
           - 1] - P[i]), 0)){
12       U.pop_back();
13     }
14     U.push_back(P[i]);
15   }
16   L.pop_back();
17   U.pop_back();
18   L.insert(L.end(), U.begin(), U.end());
19   return L;
20 }
```

6.14   Segment Intersection with Ray

```
1 bool crossesRay(const point & a, const point & b, const point & p){
2   return (geq(b.y, p.y) - geq(a.y, p.y)) * sgn((a - p).cross(b - p)) >
        0;
3 }
```

6.15   Cut Polygon

```
1 vector<point> cutPolygon(const vector<point> & P, const point & a, const
      point & v){
2   //returns the part of the convex polygon P on the left side of line a+
       tv
3   int n = P.size();
4   vector<point> lhs;
5   for(int i = 0; i < n; ++i){
6     if(geq(v.cross(P[i] - a), 0)){
7       lhs.push_back(P[i]);
8     }
9     if(intersectLineSegmentInfo(a, v, P[i], P[(i+1)%n]) == 1){
10      point p = intersectLines(a, v, P[i], P[(i+1)%n] - P[i]);
11      if(p != P[i] && p != P[(i+1)%n]){
12        lhs.push_back(p);
```

```
13      }
14    }
15  }
16  return lhs;
17 }
```

6.16   Delaunay Triangulation

```
1 //Delaunay triangulation in O(n log n)
2 const point inf_pt(inf, inf);
3
4 struct QuadEdge{
5   point origin;
6   QuadEdge* rot = nullptr;
7   QuadEdge* onext = nullptr;
8   bool used = false;
9   QuadEdge* rev() const{return rot->rot;}
10  QuadEdge* lnext() const{return rot->rev()->onext->rot;}
11  QuadEdge* oprev() const{return rot->onext->rot;}
12  point dest() const{return rev()->origin;}
13 };
14
15 QuadEdge* make_edge(const point & from, const point & to){
16   QuadEdge* e1 = new QuadEdge;
17   QuadEdge* e2 = new QuadEdge;
18   QuadEdge* e3 = new QuadEdge;
19   QuadEdge* e4 = new QuadEdge;
20   e1->origin = from;
21   e2->origin = to;
22   e3->origin = e4->origin = inf_pt;
23   e1->rot = e3;
24   e2->rot = e4;
25   e3->rot = e2;
26   e4->rot = e1;
27   e1->onext = e1;
28   e2->onext = e2;
29   e3->onext = e4;
30   e4->onext = e3;
31   return e1;
32 }
33
34 void splice(QuadEdge* a, QuadEdge* b){
35   swap(a->onext->rot->onext, b->onext->rot->onext);
```

```
36      swap(a->onext, b->onext);
37  }
38
39  void delete_edge(QuadEdge* e){
40      splice(e, e->oprev());
41      splice(e->rev(), e->rev()->oprev());
42      delete e->rot;
43      delete e->rev()->rot;
44      delete e;
45      delete e->rev();
46  }
47
48  QuadEdge* connect(QuadEdge* a, QuadEdge* b){
49      QuadEdge* e = make_edge(a->dest(), b->origin);
50      splice(e, a->lnext());
51      splice(e->rev(), b);
52      return e;
53  }
54
55  bool left_of(const point & p, QuadEdge* e){
56      return ge((e->origin - p).cross(e->dest() - p), 0);
57  }
58
59  bool right_of(const point & p, QuadEdge* e){
60      return le((e->origin - p).cross(e->dest() - p), 0);
61  }
62
63  ld det3(ld a1, ld a2, ld a3, ld b1, ld b2, ld b3, ld c1, ld c2, ld c3) {
64      return a1 * (b2 * c3 - c2 * b3) - a2 * (b1 * c3 - c1 * b3) + a3 * (b1
            * c2 - c1 * b2);
65  }
66
67  bool in_circle(const point & a, const point & b, const point & c, const
        point & d) {
68      ld det = -det3(b.x, b.y, b.norm(), c.x, c.y, c.norm(), d.x, d.y, d.
            norm());
69      det += det3(a.x, a.y, a.norm(), c.x, c.y, c.norm(), d.x, d.y, d.norm()
            );
70      det -= det3(a.x, a.y, a.norm(), b.x, b.y, b.norm(), d.x, d.y, d.norm()
            );
71      det += det3(a.x, a.y, a.norm(), b.x, b.y, b.norm(), c.x, c.y, c.norm()
            );
72      return ge(det, 0);
```

```
73  }
74
75  pair<QuadEdge*, QuadEdge*> build_tr(int l, int r, vector<point> & P){
76      if(r - l + 1 == 2){
77          QuadEdge* res = make_edge(P[l], P[r]);
78          return {res, res->rev()};
79      }
80      if(r - l + 1 == 3){
81          QuadEdge *a = make_edge(P[l], P[l + 1]), *b = make_edge(P[l + 1], P[
                r]);
82          splice(a->rev(), b);
83          int sg = sgn((P[l + 1] - P[l]).cross(P[r] - P[l]));
84          if(sg == 0)
85              return {a, b->rev()};
86          QuadEdge* c = connect(b, a);
87          if(sg == 1)
88              return {a, b->rev()};
89          else
90              return {c->rev(), c};
91      }
92      int mid = (l + r) / 2;
93      QuadEdge *ldo, *ldi, *rdo, *rdi;
94      tie(ldo, ldi) = build_tr(l, mid, P);
95      tie(rdi, rdo) = build_tr(mid + 1, r, P);
96      while(true){
97          if(left_of(rdi->origin, ldi)){
98              ldi = ldi->lnext();
99              continue;
100         }
101         if(right_of(ldi->origin, rdi)){
102             rdi = rdi->rev()->onext;
103             continue;
104         }
105         break;
106     }
107     QuadEdge* basel = connect(rdi->rev(), ldi);
108     auto valid = [&basel](QuadEdge* e){return right_of(e->dest(), basel)
            ;};
109     if(ldi->origin == ldo->origin)
110         ldo = basel->rev();
111     if(rdi->origin == rdo->origin)
112         rdo = basel;
113     while(true){
```

```
114    QuadEdge* lcand = basel->rev()->onext;
115    if(valid(lcand)){
116      while(in_circle(basel->dest(), basel->origin, lcand->dest(), lcand
             ->onext->dest())){
117        QuadEdge* t = lcand->onext;
118        delete_edge(lcand);
119        lcand = t;
120      }
121    }
122    QuadEdge* rcand = basel->oprev();
123    if(valid(rcand)){
124      while(in_circle(basel->dest(), basel->origin, rcand->dest(), rcand
             ->oprev()->dest())){
125        QuadEdge* t = rcand->oprev();
126        delete_edge(rcand);
127        rcand = t;
128      }
129    }
130    if(!valid(lcand) && !valid(rcand))
131      break;
132    if(!valid(lcand) || (valid(rcand) && in_circle(lcand->dest(), lcand
           ->origin, rcand->origin, rcand->dest())))
133      basel = connect(rcand, basel->rev());
134    else
135      basel = connect(basel->rev(), lcand->rev());
136  }
137  return {ldo, rdo};
138 }
139
140 vector<tuple<point, point, point>> delaunay(vector<point> & P){
141   sort(P.begin(), P.end());
142   auto res = build_tr(0, (int)P.size() - 1, P);
143   QuadEdge* e = res.first;
144   vector<QuadEdge*> edges = {e};
145   while(le((e->dest() - e->onext->dest()).cross(e->origin - e->onext->
          dest()), 0))
146     e = e->onext;
147   auto add = [&P, &e, &edges](){
148     QuadEdge* curr = e;
149     do{
150       curr->used = true;
151       P.push_back(curr->origin);
152       edges.push_back(curr->rev());
153       curr = curr->lnext();
154     }while(curr != e);
155   };
156   add();
157   P.clear();
158   int kek = 0;
159   while(kek < (int)edges.size())
160     if(!(e = edges[kek++])->used)
161       add();
162   vector<tuple<point, point, point>> ans;
163   for(int i = 0; i < (int)P.size(); i += 3){
164     ans.emplace_back(P[i], P[i + 1], P[i + 2]);
165   }
166   return ans;
167 }
```

6.17   Diameter and Width

```
1 pair<ld, ld> diameterAndWidth(vector<point> & P){
2   int n = P.size(), k = 0;
3   auto dot = [&](int a, int b){return (P[(a+1)%n]-P[a]).dot(P[(b+1)%n]-P
        [b]);};
4   auto cross = [&](int a, int b){return (P[(a+1)%n]-P[a]).cross(P[(b+1)%
        n]-P[b]);};
5   ld diameter = 0;
6   ld width = inf;
7   while(ge(dot(0, k), 0)) k = (k+1) % n;
8   for(int i = 0; i < n; ++i){
9     while(ge(cross(i, k), 0)) k = (k+1) % n;
10    //pair: (i, k)
11    diameter = max(diameter, (P[k] - P[i]).length());
12    width = min(width, distancePointLine(P[i], P[(i+1)%n] - P[i], P[k]))
          ;
13  }
14  return {diameter, width};
15 }
```

6.18   Distance Between Point and Circle

```
1 ld distancePointCircle(const point & c, ld r, const point & p){
2   //point p, circle with center c and radius r
3   return max((ld)0, (p - c).length() - r);
4 }
```

6.19   Distance Between Point and Line

```
1  ld distancePointLine(const point & a, const point & v, const point & p){
2    //line: a + tv, point p
3    return abs(v.cross(p - a)) / v.length();
4  }
```

6.20   Example of Geometry

```
1  int main(){
2    /*vector<pair<point, point>> centers = {{point(-2, 5), point(-8, -7)},
          {point(14, 4), point(18, 6)}, {point(9, 20), point(9, 28)},
3                      {point(21, 20), point(21, 29)}, {point(8, -10),
                          point(14, -10)}, {point(24, -6), point(34, -6)
                          },
4                      {point(34, 8), point(36, 9)}, {point(50, 20),
                          point(56, 24.5)}};
5    vector<pair<ld, ld>> radii = {{7, 4}, {3, 5}, {4, 4}, {4, 5}, {3, 3},
          {4, 6}, {5, 1}, {10, 2.5}};
6    int n = centers.size();
7    for(int i = 0; i < n; ++i){
8      cout << "\n" << centers[i].first << " " << radii[i].first << " " <<
            centers[i].second << " " << radii[i].second << "\n";
9      auto extLines = tangents(centers[i].first, radii[i].first, centers[i
            ].second, radii[i].second, false);
10     cout << "Exterior tangents:\n";
11     for(auto par : extLines){
12       for(auto p : par){
13         cout << p << " ";
14       }
15       cout << "\n";
16     }
17     auto intLines = tangents(centers[i].first, radii[i].first, centers[i
            ].second, radii[i].second, true);
18     cout << "Interior tangents:\n";
19     for(auto par : intLines){
20       for(auto p : par){
21         cout << p << " ";
22       }
23       cout << "\n";
24     }
25   }*/
26
27   /*int n;
28   cin >> n;
29   vector<point> P(n);
30   for(auto & p : P) cin >> p;
31   auto triangulation = delaunay(P);
32   for(auto triangle : triangulation){
33     cout << get<0>(triangle) << " " << get<1>(triangle) << " " << get
            <2>(triangle) << "\n";
34   }*/
35
36   /*int n;
37   cin >> n;
38   vector<point> P(n);
39   for(auto & p : P) cin >> p;
40   auto ans = smallestEnclosingCircle(P);
41   cout << ans.first << " " << ans.second << "\n";*/
42
43   /*vector<point> P;
44   srand(time(0));
45   for(int i = 0; i < 1000; ++i){
46     P.emplace_back(rand() % 1000000000, rand() % 1000000000);
47   }
48   point o(rand() % 1000000000, rand() % 1000000000), v(rand() %
            1000000000, rand() % 1000000000);
49   polarSort(P, o, v);
50   auto ang = [&](point p){
51     ld th = atan2(p.y, p.x);
52     if(th < 0) th += acosl(-1)*2;
53     ld t = atan2(v.y, v.x);
54     if(t < 0) t += acosl(-1)*2;
55     if(th < t) th += acosl(-1)*2;
56     return th;
57   };
58   for(int i = 0; i < P.size()-1; ++i){
59     assert(leq(ang(P[i] - o), ang(P[i+1] - o)));
60   }*/
61   return 0;
62 }
```

6.21   Half Plane Intersection

```
1  struct plane{
2    point a, v;
```

```cpp
 3    plane(): a(), v(){}
 4    plane(const point& a, const point& v): a(a), v(v){}
 5
 6    point intersect(const plane& p) const{
 7      ld t = (p.a - a).cross(p.v) / v.cross(p.v);
 8      return a + v*t;
 9    }
10
11    bool outside(const point& p) const{ // test if point p is strictly
          outside
12      return le(v.cross(p - a), 0);
13    }
14
15    bool inside(const point& p) const{ // test if point p is inside or in
          the boundary
16      return geq(v.cross(p - a), 0);
17    }
18
19    bool operator<(const plane& p) const{ // sort by angle
20      auto lhs = make_tuple(v.half({1, 0}), ld(0), v.cross(p.a - a));
21      auto rhs = make_tuple(p.v.half({1, 0}), v.cross(p.v), ld(0));
22      return lhs < rhs;
23    }
24
25    bool operator==(const plane& p) const{ // paralell and same directions
          , not really equal
26      return eq(v.cross(p.v), 0) && ge(v.dot(p.v), 0);
27    }
28  };
29
30  vector<point> halfPlaneIntersection(vector<plane> planes){
31    planes.push_back({{0, -inf}, {1, 0}});
32    planes.push_back({{inf, 0}, {0, 1}});
33    planes.push_back({{0, inf}, {-1, 0}});
34    planes.push_back({{-inf, 0}, {0, -1}});
35    sort(planes.begin(), planes.end());
36    planes.erase(unique(planes.begin(), planes.end()), planes.end());
37    deque<plane> ch;
38    deque<point> poly;
39    for(const plane& p : planes){
40      while(ch.size() >= 2 && p.outside(poly.back())) ch.pop_back(), poly.
            pop_back();
41      while(ch.size() >= 2 && p.outside(poly.front())) ch.pop_front(),
            poly.pop_front();
42      if(p.v.half({1, 0}) && poly.empty()) return {};
43      ch.push_back(p);
44      if(ch.size() >= 2) poly.push_back(ch[ch.size()-2].intersect(ch[ch.
            size()-1]));
45    }
46    while(ch.size() >= 3 && ch.front().outside(poly.back())) ch.pop_back()
          , poly.pop_back();
47    while(ch.size() >= 3 && ch.back().outside(poly.front())) ch.pop_front
          (), poly.pop_front();
48    poly.push_back(ch.back().intersect(ch.front()));
49    return vector<point>(poly.begin(), poly.end());
50  }
51
52  vector<point> halfPlaneIntersectionRandomized(vector<plane> planes){
53    point p = planes[0].a;
54    int n = planes.size();
55    random_shuffle(planes.begin(), planes.end());
56    for(int i = 0; i < n; ++i){
57      if(planes[i].inside(p)) continue;
58      ld lo = -inf, hi = inf;
59      for(int j = 0; j < i; ++j){
60        ld A = planes[j].v.cross(planes[i].v);
61        ld B = planes[j].v.cross(planes[j].a - planes[i].a);
62        if(ge(A, 0)){
63          lo = max(lo, B/A);
64        }else if(le(A, 0)){
65          hi = min(hi, B/A);
66        }else{
67          if(ge(B, 0)) return {};
68        }
69        if(ge(lo, hi)) return {};
70      }
71      p = planes[i].a + planes[i].v*lo;
72    }
73    return {p};
74  }
```

6.22   Incircle

```cpp
 1  pair<point, ld> getCircle(const point & m, const point & n, const point
        & p){
 2    //find circle that passes through points p, q, r
```

```
3    point c = intersectLines((n + m) / 2, (n - m).perp(), (p + n) / 2, (p
         - n).perp());
4    ld r = (c - m).length();
5    return {c, r};
6  }
```

6.23   Intersection of Two Circles

```
1  vector<point> intersectionCircles(const point & c1, ld r1, const point &
        c2, ld r2){
2    //circle 1 with center c1 and radius r1
3    //circle 2 with center c2 and radius r2
4    point d = c2 - c1;
5    ld d2 = d.norm();
6    if(eq(d2, 0)) return {}; //concentric circles
7    ld pd = (d2 + r1*r1 - r2*r2) / 2;
8    ld h2 = r1*r1 - pd*pd/d2;
9    point p = c1 + d*pd/d2;
10   if(eq(h2, 0)) return {p}; //circles touch at one point
11   else if(le(h2, 0)) return {}; //circles don't intersect
12   else{
13     point u = d.perp() * sqrt(h2/d2);
14     return {p - u, p + u};
15   }
16 }
```

6.24   Intersection Line and Circle

```
1  vector<point> intersectLineCircle(const point & a, const point & v,
        const point & c, ld r){
2    //line a+tv, circle with center c and radius r
3    ld h2 = r*r - v.cross(c - a) * v.cross(c - a) / v.norm();
4    point p = a + v * v.dot(c - a) / v.norm();
5    if(eq(h2, 0)) return {p}; //line tangent to circle
6    else if(le(h2, 0)) return {}; //no intersection
7    else{
8      point u = v.unit() * sqrt(h2);
9      return {p - u, p + u}; //two points of intersection (chord)
10   }
11 }
```

6.25   Intersection Polygon and Circle

```
1  ld signed_angle(const point & a, const point & b){
```

```
2    return sgn(a.cross(b)) * acosl(a.dot(b) / (a.length() * b.length()));
3  }

5  ld intersectPolygonCircle(const vector<point> & P, const point & c, ld r
        ){
6    //Gets the area of the intersection of the polygon with the circle
7    int n = P.size();
8    ld ans = 0;
9    for(int i = 0; i < n; ++i){
10     point p = P[i], q = P[(i+1)%n];
11     bool p_inside = (pointInCircle(c, r, p) != 0);
12     bool q_inside = (pointInCircle(c, r, q) != 0);
13     if(p_inside && q_inside){
14       ans += (p - c).cross(q - c);
15     }else if(p_inside && !q_inside){
16       point s1 = intersectSegmentCircle(p, q, c, r)[0];
17       point s2 = intersectSegmentCircle(c, q, c, r)[0];
18       ans += (p - c).cross(s1 - c) + r*r * signed_angle(s1 - c, s2 - c);
19     }else if(!p_inside && q_inside){
20       point s1 = intersectSegmentCircle(c, p, c, r)[0];
21       point s2 = intersectSegmentCircle(p, q, c, r)[0];
22       ans += (s2 - c).cross(q - c) + r*r * signed_angle(s1 - c, s2 - c);
23     }else{
24       auto info = intersectSegmentCircle(p, q, c, r);
25       if(info.size() <= 1){
26         ans += r*r * signed_angle(p - c, q - c);
27       }else{
28         point s2 = info[0], s3 = info[1];
29         point s1 = intersectSegmentCircle(c, p, c, r)[0];
30         point s4 = intersectSegmentCircle(c, q, c, r)[0];
31         ans += (s2 - c).cross(s3 - c) + r*r * (signed_angle(s1 - c, s2 -
                 c) + signed_angle(s3 - c, s4 - c));
32       }
33     }
34   }
35   return abs(ans)/2;
36 }
```

6.26   Intersection Segment and Circle

```
1  vector<point> intersectSegmentCircle(const point & a, const point & b,
        const point & c, ld r){
2    //segment ab, circle with center c and radius r
```

```
3    vector<point> P = intersectLineCircle(a, b - a, c, r), ans;
4    for(const point & p : P){
5      if(pointInSegment(a, b, p)) ans.push_back(p);
6    }
7    return ans;
8  }
```

6.27 Line Intersection

```
1  int intersectLinesInfo(const point & a1, const point & v1, const point &
       a2, const point & v2){
2    //lines a1+tv1 and a2+tv2
3    ld det = v1.cross(v2);
4    if(eq(det, 0)){
5      if(eq((a2 - a1).cross(v1), 0)){
6        return -1; //infinity points
7      }else{
8        return 0; //no points
9      }
10   }else{
11     return 1; //single point
12   }
13 }
14
15 point intersectLines(const point & a1, const point & v1, const point &
       a2, const point & v2){
16   //lines a1+tv1, a2+tv2
17   //assuming that they intersect
18   ld det = v1.cross(v2);
19   return a1 + v1 * ((a2 - a1).cross(v2) / det);
20 }
```

6.28 Minkowski Sum

```
1  vector<point> minkowskiSum(vector<point> A, vector<point> B){
2    int na = (int)A.size(), nb = (int)B.size();
3    if(A.empty() || B.empty()) return {};
4
5    rotate(A.begin(), min_element(A.begin(), A.end()), A.end());
6    rotate(B.begin(), min_element(B.begin(), B.end()), B.end());
7
8    int pa = 0, pb = 0;
9    vector<point> M;
10
```

```
11   while(pa < na && pb < nb){
12     M.push_back(A[pa] + B[pb]);
13     ld x = (A[(pa + 1) % na] - A[pa]).cross(B[(pb + 1) % nb] - B[pb]);
14     if(leq(x, 0)) pb++;
15     if(geq(x, 0)) pa++;
16   }
17
18   while(pa < na) M.push_back(A[pa++] + B[0]);
19   while(pb < nb) M.push_back(B[pb++] + A[0]);
20
21   return M;
22 }
```

6.29 Point in Circle

```
1  int pointInCircle(const point & c, ld r, const point & p){
2    //test if point p is inside the circle with center c and radius r
3    //returns "0" if it's outside, "-1" if it's in the perimeter, "1" if
        it's inside
4    ld l = (p - c).length() - r;
5    return (le(l, 0) ? 1 : (eq(l, 0) ? -1 : 0));
6  }
```

6.30 Point in Convex Hull

```
1  //point in convex polygon in O(log n)
2  //make sure that P is convex and in ccw
3  //before the queries, do the preprocess on P:
4  // rotate(P.begin(), min_element(P.begin(), P.end()), P.end());
5  // int right = max_element(P.begin(), P.end()) - P.begin();
6  //returns 0 if p is outside, 1 if p is inside, -1 if p is in the
       perimeter
7  int pointInConvexPolygon(const vector<point> & P, const point & p, int
       right){
8    if(p < P[0] || P[right] < p) return 0;
9    int orientation = sgn((P[right] - P[0]).cross(p - P[0]));
10   if(orientation == 0){
11     if(p == P[0] || p == P[right]) return -1;
12     return (right == 1 || right + 1 == P.size()) ? -1 : 1;
13   }else if(orientation < 0){
14     auto r = lower_bound(P.begin() + 1, P.begin() + right, p);
15     int det = sgn((p - r[-1]).cross(r[0] - r[-1])) - 1;
16     if(det == -2) det = 1;
17     return det;
```

```
18    }else{
19      auto l = upper_bound(P.rbegin(), P.rend() - right - 1, p);
20      int det = sgn((p - l[0]).cross((l == P.rbegin() ? P[0] : l[-1]) - l
            [0])) - 1;
21      if(det == -2) det = 1;
22      return det;
23    }
24  }
```

6.31   Point in Line

```
1  bool pointInLine(const point & a, const point & v, const point & p){
2    //line a+tv, point p
3    return eq((p - a).cross(v), 0);
4  }
```

6.32   Point in Perimeter

```
1  bool pointInPerimeter(const vector<point> & P, const point & p){
2    int n = P.size();
3    for(int i = 0; i < n; i++){
4      if(pointInSegment(P[i], P[(i + 1) % n], p)){
5        return true;
6      }
7    }
8    return false;
9  }
```

6.33   Point in Polygon

```
1  int pointInPolygon(const vector<point> & P, const point & p){
2    if(pointInPerimeter(P, p)){
3      return -1; //point in the perimeter
4    }
5    int n = P.size();
6    int rays = 0;
7    for(int i = 0; i < n; i++){
8      rays += crossesRay(P[i], P[(i + 1) % n], p);
9    }
10   return rays & 1; //0: point outside, 1: point inside
11 }
```

6.34   Point in Segment

```
1  bool pointInSegment(const point & a, const point & b, const point & p){
2    //segment ab, point p
3    return pointInLine(a, b - a, p) && leq((a - p).dot(b - p), 0);
4  }
```

6.35   Points Tangency

```
1  pair<point, point> pointsOfTangency(const point & c, ld r, const point &
        p){
2    //point p (outside the circle), circle with center c and radius r
3    point v = (p - c).unit() * r;
4    ld d2 = (p - c).norm(), d = sqrt(d2);
5    point v1 = v * (r / d), v2 = v.perp() * (sqrt(d2 - r*r) / d);
6    return {c + v1 - v2, c + v1 + v2};
7  }
```

6.36   Projection Point Circle

```
1  point projectionPointCircle(const point & c, ld r, const point & p){
2    //point p (outside the circle), circle with center c and radius r
3    return c + (p - c).unit() * r;
4  }
```

6.37   Segment Intersection

```
1  int intersectLineSegmentInfo(const point & a, const point & v, const
        point & c, const point & d){
2    //line a+tv, segment cd
3    point v2 = d - c;
4    ld det = v.cross(v2);
5    if(eq(det, 0)){
6      if(eq((c - a).cross(v), 0)){
7        return -1; //infinity points
8      }else{
9        return 0; //no point
10     }
11   }else{
12     return sgn(v.cross(c - a)) != sgn(v.cross(d - a)); //1: single point
          , 0: no point
13   }
14 }
15
16 int intersectSegmentsInfo(const point & a, const point & b, const point
        & c, const point & d){
```

```
17    //segment ab, segment cd
18    point v1 = b - a, v2 = d - c;
19    int t = sgn(v1.cross(c - a)), u = sgn(v1.cross(d - a));
20    if(t == u){
21      if(t == 0){
22        if(pointInSegment(a, b, c) || pointInSegment(a, b, d) ||
              pointInSegment(c, d, a) || pointInSegment(c, d, b)){
23          return -1; //infinity points
24        }else{
25          return 0; //no point
26        }
27      }else{
28        return 0; //no point
29      }
30    }else{
31      return sgn(v2.cross(a - c)) != sgn(v2.cross(b - c)); //1: single
              point, 0: no point
32    }
33  }
```

6.38   Smallest Enclosing Circle

```
1  pair<point, ld> mec2(vector<point> & S, const point & a, const point & b
       , int n){
2    ld hi = inf, lo = -hi;
3    for(int i = 0; i < n; ++i){
4      ld si = (b - a).cross(S[i] - a);
5      if(eq(si, 0)) continue;
6      point m = getCircle(a, b, S[i]).first;
7      ld cr = (b - a).cross(m - a);
8      if(le(si, 0)) hi = min(hi, cr);
9      else lo = max(lo, cr);
10   }
11   ld v = (ge(lo, 0) ? lo : le(hi, 0) ? hi : 0);
12   point c = (a + b) / 2 + (b - a).perp() * v / (b - a).norm();
13   return {c, (a - c).norm()};
14 }
15
16 pair<point, ld> mec(vector<point> & S, const point & a, int n){
17   random_shuffle(S.begin(), S.begin() + n);
18   point b = S[0], c = (a + b) / 2;
19   ld r = (a - c).norm();
20   for(int i = 1; i < n; ++i){
```

```
21     if(ge((S[i] - c).norm(), r)){
22       tie(c, r) = (n == S.size() ? mec(S, S[i], i) : mec2(S, a, S[i], i)
              );
23     }
24   }
25   return {c, r};
26 }
27
28 pair<point, ld> smallestEnclosingCircle(vector<point> S){
29   assert(!S.empty());
30   auto r = mec(S, S[0], S.size());
31   return {r.first, sqrt(r.second)};
32 }
```

6.39   Smallest Enclosing Rectangle

```
1  pair<ld, ld> smallestEnclosingRectangle(vector<point> & P){
2    int n = P.size();
3    auto dot = [&](int a, int b){return (P[(a+1)%n]-P[a]).dot(P[(b+1)%n]-P
       [b]);};
4    auto cross = [&](int a, int b){return (P[(a+1)%n]-P[a]).cross(P[(b+1)%
       n]-P[b]);};
5    ld perimeter = inf, area = inf;
6    for(int i = 0, j = 0, k = 0, m = 0; i < n; ++i){
7      while(ge(dot(i, j), 0)) j = (j+1) % n;
8      if(!i) k = j;
9      while(ge(cross(i, k), 0)) k = (k+1) % n;
10     if(!i) m = k;
11     while(le(dot(i, m), 0)) m = (m+1) % n;
12     //pairs: (i, k) , (j, m)
13     point v = P[(i+1)%n] - P[i];
14     ld h = distancePointLine(P[i], v, P[k]);
15     ld w = distancePointLine(P[j], v.perp(), P[m]);
16     perimeter = min(perimeter, 2 * (h + w));
17     area = min(area, h * w);
18   }
19   return {area, perimeter};
20 }
```

6.40   Vantage Point Tree

```
1  struct vantage_point_tree{
2    struct node
3    {
```

```
4        point p;
5        ld th;
6        node *l, *r;
7      }*root;
8
9      vector<pair<ld, point>> aux;
10
11     vantage_point_tree(vector<point> &ps){
12       for(int i = 0; i < ps.size(); ++i)
13         aux.push_back({ 0, ps[i] });
14       root = build(0, ps.size());
15     }
16
17     node *build(int l, int r){
18       if(l == r)
19         return 0;
20       swap(aux[l], aux[l + rand() % (r - l)]);
21       point p = aux[l++].second;
22       if(l == r)
23         return new node({ p });
24       for(int i = l; i < r; ++i)
25         aux[i].first = (p - aux[i].second).dot(p - aux[i].second);
26       int m = (l + r) / 2;
27       nth_element(aux.begin() + l, aux.begin() + m, aux.begin() + r);
28       return new node({ p, sqrt(aux[m].first), build(l, m), build(m, r) })
            ;
29     }
30
31     priority_queue<pair<ld, node*>> que;
32
33     void k_nn(node *t, point p, int k){
34       if(!t)
35         return;
36       ld d = (p - t->p).length();
37       if(que.size() < k)
38         que.push({ d, t });
39       else if(ge(que.top().first, d)){
40         que.pop();
41         que.push({ d, t });
42       }
43       if(!t->l && !t->r)
44         return;
45       if(le(d, t->th)){
```

```
46         k_nn(t->l, p, k);
47         if(leq(t->th - d, que.top().first))
48           k_nn(t->r, p, k);
49       }else{
50         k_nn(t->r, p, k);
51         if(leq(d - t->th, que.top().first))
52           k_nn(t->l, p, k);
53       }
54     }
55
56     vector<point> k_nn(point p, int k){
57       k_nn(root, p, k);
58       vector<point> ans;
59       for(; !que.empty(); que.pop())
60         ans.push_back(que.top().second->p);
61       reverse(ans.begin(), ans.end());
62       return ans;
63     }
64   };
```

7   Graphs

7.1   2Sat

```
1    struct TwoSatSolver {
2      int n_vars;                    // Number of boolean variables
3      int n_vertices;                // Total vertices in the implication
           graph (2 per variable)
4      vector<vector<int>> adj;       // Implication graph: adj[i] contains
           edges from node i
5      vector<vector<int>> adj_t;     // Transposed graph for Kosaraju's
           algorithm
6      vector<bool> used;             // Visited marker for DFS
7      vector<int> order;             // Finishing order of vertices (DFS1)
8      vector<int> comp;              // Component ID for each node (DFS2)
9      vector<bool> assignment;       // Final truth assignment for each
           variable
10
11     // Constructor initializes all data structures
12     TwoSatSolver(int _n_vars)
13       : n_vars(_n_vars),
14         n_vertices(2 * _n_vars),
15         adj(n_vertices),
```

```
16        adj_t(n_vertices),
17        used(n_vertices),
18        comp(n_vertices, -1),
19        assignment(n_vars) {
20      order.reserve(n_vertices); // Pre-allocate memory for efficiency
21    }
22
23    // First DFS pass for Kosaraju's algorithm (on original graph)
24    void dfs1(int v) {
25      used[v] = true;
26      for (int u : adj[v]) {
27        if (!used[u])
28          dfs1(u);
29      }
30      order.push_back(v); // Save the vertex post-DFS for reverse ordering
31    }
32
33    // Second DFS pass on the transposed graph to label components
34    void dfs2(int v, int cl) {
35      comp[v] = cl;
36      for (int u : adj_t[v]) {
37        if (comp[u] == -1)
38          dfs2(u, cl);
39      }
40    }
41
42    // Solves the 2-SAT problem using Kosaraju's algorithm
43    bool solve_2SAT() {
44      // 1st pass: fill the order vector
45      order.clear();
46      used.assign(n_vertices, false);
47      for (int i = 0; i < n_vertices; ++i) {
48        if (!used[i])
49          dfs1(i);
50      }
51
52      // 2nd pass: find SCCs in reverse postorder
53      comp.assign(n_vertices, -1);
54      for (int i = 0, j = 0; i < n_vertices; ++i) {
55        int v = order[n_vertices - i - 1]; // Reverse postorder
56        if (comp[v] == -1)
57          dfs2(v, j++);
58      }
```

```
59
60      // Assign values to variables based on component comparison
61      assignment.assign(n_vars, false);
62      for (int i = 0; i < n_vertices; i += 2) {
63        if (comp[i] == comp[i + 1])
64          return false; // Contradiction: variable and its negation are in
                  the same SCC
65        assignment[i / 2] = comp[i] > comp[i + 1]; // True if var's
                  component comes after its negation
66      }
67      return true;
68    }
69
70    // Adds a disjunction (a v b) to the implication graph
71    // 'na' and 'nb' indicate negation: if true means !a or !b
72    // Variables are 0-indexed. Bounds are inclusive for each literal (i.e
          ., 0 to n_vars - 1)
73    void add_disjunction(int a, bool na, int b, bool nb) {
74      // Each variable 'x' has two nodes:
75      // x => 2*x, !x => 2*x + 1
76      // We encode (a v b) as (!a -> b) and (!b -> a)
77      a = 2 * a ^ na;
78      b = 2 * b ^ nb;
79      int neg_a = a ^ 1;
80      int neg_b = b ^ 1;
81
82      adj[neg_a].push_back(b);
83      adj[neg_b].push_back(a);
84      adj_t[b].push_back(neg_a);
85      adj_t[a].push_back(neg_b);
86    }
87  };
```

7.2   Articulation Points

```
1  /*
2    Articulation Points (Cut Vertices) in an Undirected Graph
3    ----------------------------------------------------------
4    Indexing: 0-based
5    Node Bounds: [0, n-1] inclusive
6    Time Complexity: O(V + E)
7    Space Complexity: O(V)
8
```

```
 9    Use Case:
10      - Identifies vertices whose removal increases the number of
           connected components.
11      - Works on undirected graphs (connected or disconnected).
12  */
13
14  int n; // Number of nodes in the graph
15  vector<vector<int>> adj; // Adjacency list of the undirected graph
16
17  vector<bool> visited; // Marks if a node was visited during DFS
18  vector<int> tin, low; // tin[v]: discovery time; low[v]: lowest
        discovery time reachable from subtree
19  int timer; // Global time counter for DFS
20
21  // DFS traversal to identify articulation points
22  void dfs(int v, int p = -1) {
23    visited[v] = true;
24    tin[v] = low[v] = timer++;
25    int children = 0;
26    for (int to : adj[v]) {
27      if (to == p) continue; // Skip the parent edge
28      if (visited[to]) {
29        // Back edge
30        low[v] = min(low[v], tin[to]);
31      } else {
32        dfs(to, v);
33        low[v] = min(low[v], low[to]);
34        // Articulation point condition for non-root
35        if (low[to] >= tin[v] && p != -1) {
36          // v is an articulation point
37          // handle_cutpoint(v);
38        }
39        ++children;
40      }
41    }
42    // Articulation point condition for root
43    if (p == -1 && children > 1) {
44      // v is an articulation point
45      // handle_cutpoint(v);
46    }
47  }
48
49  // Initializes structures and launches DFS
```

```
50  void find_cutpoints() {
51    timer = 0;
52    visited.assign(n, false);
53    tin.assign(n, -1);
54    low.assign(n, -1);
55
56    for (int i = 0; i < n; ++i) {
57      if (!visited[i])
58        dfs(i);
59    }
60  }
```

7.3   Bellman-Ford

```
 1  /*
 2    Bellman-Ford (SPFA variant) for Shortest Paths
 3    ------------------------------------------------
 4    Indexing: 0-based
 5    Node Bounds: [0, n-1] inclusive
 6    Time Complexity: O(V * E) worst-case (amortized better)
 7    Space Complexity: O(V + E)
 8
 9    Features:
10      - Handles negative edge weights
11      - Detects negative weight cycles (returns false if one exists)
12      - Works on directed or undirected graphs
13
14    Path Reconstruction:
15      - To recover the path from source 's' to any node 'u':
16          vector<int> path;
17          for (int v = u; v != -1; v = parent[v])
18            path.push_back(v);
19          reverse(path.begin(), path.end());
20  */
21
22  const int INF = 1<<30; // Large value to represent "infinity"
23  vector<vector<pair<int, int>>> adj; // adj[v] = list of (neighbor,
        weight) pairs
24  vector<int> parent; // parent(n, -1) for path reconstruction
25
26  // SPFA implementation to find shortest paths from source s
27  // d[i] will contain shortest distance from s to i
28  // Returns false if a negative cycle is detected
```

```cpp
// For path reconstruction add vector<int>& parent as parameter
bool spfa(int s, vector<int>& d, vector<int>& parent) {
  int n = adj.size();
  d.assign(n, INF);
  vector<int> cnt(n, 0);        // Count how many times each node has
      been relaxed
  vector<bool> inqueue(n, false); // Tracks if a node is currently in
      queue
  queue<int> q;

  d[s] = 0;
  q.push(s);
  inqueue[s] = true;

  while (!q.empty()) {
    int v = q.front();
    q.pop();
    inqueue[v] = false;

    for (auto edge : adj[v]) {
      int to = edge.first;
      int len = edge.second;

      if (d[v] + len < d[to]) {
        parent[to] = v; // For path reconstruction
        d[to] = d[v] + len;
        if (!inqueue[to]) {
          q.push(to);
          inqueue[to] = true;
          cnt[to]++;
          if (cnt[to] > n)
            return false; // Negative weight cycle detected
        }
      }
    }
  }

  return true; // No negative cycles; shortest paths computed
}
```

7.4   Bipartite Checker

```cpp
/*
```

```cpp
  Bipartite Graph Checker (BFS-based)
  -----------------------------------
  Indexing: 0-based
  Time Complexity: O(V + E)
  Space Complexity: O(V)

  Handles disconnected graphs
*/

int n; // Number of nodes
vector<vector<int>> adj; // Adjacency list of the undirected graph

vector<int> side(n, -1); // -1 = unvisited, 0/1 = sides of bipartition
bool is_bipartite = true;
queue<int> q;

for (int st = 0; st < n; ++st) {
  if (side[st] == -1) {
    q.push(st);
    side[st] = 0; // Start with side 0
    while (!q.empty()) {
      int v = q.front();
      q.pop();
      for (int u : adj[v]) {
        if (side[u] == -1) {
          // Assign opposite side to neighbor
          side[u] = side[v] ^ 1;
          q.push(u);
        } else {
          // Conflict: adjacent nodes on same side
          is_bipartite &= side[u] != side[v];
        }
      }
    }
  }
}

cout << (is_bipartite ? "YES" : "NO") << endl;
```

7.5   Bipartite Maximum Matching

```cpp
/*
  Maximum Bipartite Matching (Kuhn's Algorithm)
```

```
3      ------------------------------------------------
4      Indexing: 0-based
5      Time Complexity: O(N * (E + N)) worst case
6      Space Complexity: O(N + K + E)
7
8      Input:
9        - n: number of nodes on the left side
10       - k: number of nodes on the right side
11       - g: adjacency list where g[v] contains all right nodes adjacent to
             left node v
12
13     Output:
14       - Prints the pairs (left, right) in the matching
15       - mt[r] = l means right node r is matched to left node l
16   */
17
18   int n, k; // n: number of left nodes, k: number of right nodes
19   vector<vector<int>> g; // g[l]: list of right-side neighbors of left
          node l
20   vector<int> mt;        // mt[r]: matched left node for right node r (or
          -1 if unmatched)
21   vector<bool> used;     // used[l]: visited status for left node l during
          DFS
22
23   // Try to find an augmenting path from left node v
24   bool try_kuhn(int v) {
25     if (used[v])
26       return false;
27     used[v] = true;
28     for (int to : g[v]) {
29       if (mt[to] == -1 || try_kuhn(mt[to])) {
30         mt[to] = v;
31         return true;
32       }
33     }
34     return false;
35   }
36
37   int main() {
38     //... reading the graph ...
39
40     mt.assign(k, -1); // Right-side nodes initially unmatched
41     for (int v = 0; v < n; ++v) {
42       used.assign(n, false); // Reset visited for each left node
43       try_kuhn(v);
44     }
45     // Output matched pairs (left+1, right+1 for 1-based output)
46     for (int i = 0; i < k; ++i) {
47       if (mt[i] != -1)
48         printf("%d %d\n", mt[i] + 1, i + 1);
49     }
50     return 0;
51   }
```

7.6   Bipartite Minimum Maximum Matching2

```
1    /*
2      THIS CODE HAS NOT BEEN TESTED BEFOREHAND
3      Maximum Bipartite Matching - Hopcroft-Karp Algorithm
4      ------------------------------------------------------
5      Indexing: 0-based
6      Time Complexity: O(sqrt(V) * E)
7      Space Complexity: O(V + E)
8
9      Input:
10       - n: number of nodes on the left
11       - m: number of nodes on the right
12       - g[l]: adjacency list for left node l (list of right-side neighbors
             )
13
14     Output:
15       - match_right[r] = l means right node r is matched to left node l
16       - match_left[l] = r means left node l is matched to right node r
17       - Function returns total number of matching pairs
18   */
19
20   const int INF = 1e9;
21
22   int n, m; // Number of nodes on left and right
23   vector<vector<int>> g;          // Adjacency list: g[l] contains neighbors
          of left node l
24   vector<int> match_left, match_right; // match_left[l], match_right[r]
25   vector<int> dist;               // Distance levels for BFS
26
27   // BFS to build layer graph, returns true if there's at least one
          unmatched node on the right reachable from left
```

```cpp
bool bfs() {
  queue<int> q;
  dist.assign(n, INF);

  for (int l = 0; l < n; ++l) {
    if (match_left[l] == -1) {
      dist[l] = 0;
      q.push(l);
    }
  }

  bool found = false;

  while (!q.empty()) {
    int l = q.front(); q.pop();
    for (int r : g[l]) {
      int matched_l = match_right[r];
      if (matched_l == -1) {
        found = true; // Free right node reachable -> potential
              augmenting path
      } else if (dist[matched_l] == INF) {
        dist[matched_l] = dist[l] + 1;
        q.push(matched_l);
      }
    }
  }

  return found;
}

// DFS to find augmenting paths in the layered graph
bool dfs(int l) {
  for (int r : g[l]) {
    int matched_l = match_right[r];
    if (matched_l == -1 || (dist[matched_l] == dist[l] + 1 && dfs(
        matched_l))) {
      match_left[l] = r;
      match_right[r] = l;
      return true;
    }
  }
  dist[l] = INF;
  return false;
}
```

```cpp
}

// Main function to compute maximum matching
int hopcroft_karp() {
  match_left.assign(n, -1);
  match_right.assign(m, -1);

  int matching = 0;
  while (bfs()) {
    for (int l = 0; l < n; ++l) {
      if (match_left[l] == -1 && dfs(l)) {
        ++matching;
      }
    }
  }
  return matching;
}

int main() {
  cin >> n >> m;
  g.assign(n, {});
  int e; cin >> e;
  while (e--) {
    int u, v;
    cin >> u >> v;
    g[u].push_back(v); // u in [0, n-1], v in [0, m-1]
  }

  int res = hopcroft_karp();
  cout << "Maximum matching size: " << res << '\n';
  for (int r = 0; r < m; ++r) {
    if (match_right[r] != -1) {
      cout << match_right[r] + 1 << " " << r + 1 << '\n'; // 1-based
            output
    }
  }
}
```

7.7   Block Cut Tree

```cpp
/*
  Block-Cut Tree from Biconnected Components
  -----------------------------------------
```

```
4      Indexing: 0-based
5      Node Bounds: [0, n-1] inclusive
6      Time Complexity: O(V + E)
7      Space Complexity: O(V + E)
8
9      Features:
10       - Identifies articulation points (cut vertices)
11       - Extracts all biconnected components (BCCs)
12       - Constructs the Block-Cut Tree:
13           - Each BCC becomes a node in the tree
14           - Each articulation point becomes its own node
15           - An edge connects a BCC-node to each cutpoint in it
16
17     Output:
18       - 'is_cutpoint': true if node is an articulation point
19       - 'id[v]': node ID of 'v' in the block-cut tree
20       - Returns the block-cut tree as an adjacency list
21   */
22
23   vector<vector<int>> biconnected_components(vector<vector<int>> &g, //
         Adjacency list of the undirected graph
24                                             vector<bool> &is_cutpoint, //
                                                 Output vector (resized
                                                 internally)
25                                             vector<int> &id) { // Output
                                                 vector (resized
                                                 internally)
26     int n = g.size();
27     vector<vector<int>> comps; // Stores all biconnected components
28     vector<int> stk;           // Stack of visited nodes for current
         component
29     vector<int> num(n), low(n); // DFS discovery time and low-link values
30     is_cutpoint.assign(n, false);
31
32     // DFS to find BCCs and articulation points
33     function<void(int, int, int&)> dfs = [&](int node, int parent, int &
         timer) {
34       num[node] = low[node] = ++timer;
35       stk.push_back(node);
36       for (int son : g[node]) {
37         if (son == parent) continue;
38         if (num[son]) {
39           // Back edge
40           low[node] = min(low[node], num[son]);
41         } else {
42           dfs(son, node, timer);
43           low[node] = min(low[node], low[son]);
44           // Check articulation point condition
45           if (low[son] >= num[node]) {
46             is_cutpoint[node] = (num[node] > 1 || num[son] > 2); // For
                   root and non-root
47             comps.push_back({node});
48             while (comps.back().back() != son) {
49               comps.back().push_back(stk.back());
50               stk.pop_back();
51             }
52           }
53         }
54       }
55     };
56
57   int timer = 0;
58   dfs(0, -1, timer);
59
60   id.resize(n); // Maps each original node to its block-cut tree node ID
61
62   // Build block-cut tree using articulation points and BCCs
63   function<vector<vector<int>>()> build_tree = [&]() {
64     vector<vector<int>> t(1); // Dummy index 0 (not used)
65     int node_id = 1; // Start assigning block-cut tree IDs from 1
66     // Assign unique tree node IDs to cutpoints
67     for (int node = 0; node < n; ++node) {
68       if (is_cutpoint[node]) {
69         id[node] = node_id++;
70         t.push_back({});
71       }
72     }
73     // Assign each component a new node and connect it to its cutpoints
74     for (auto &comp : comps) {
75       int bcc_node = node_id++;
76       t.push_back({});
77       for (int u : comp) {
78         if (!is_cutpoint[u]) {
79           id[u] = bcc_node;
80         } else {
```

```
82          t[bcc_node].push_back(id[u]);
83          t[id[u]].push_back(bcc_node);
84        }
85      }
86    }
87    return t;
88  };
89
90  return build_tree(); // Return the block-cut tree
91 }
```

7.8   Blossom

```
1  /*
2    Edmonds' Blossom Algorithm (Maximum Matching in General Graphs)
3    ----------------------------------------------------------------
4    Indexing: 1-based
5    Node Bounds: [1, n]
6    Time Complexity: O(n^3) in worst case
7    Space Complexity: O(n^2)
8
9    Features:
10      - Handles odd-length cycles (blossoms)
11      - Works on any undirected graph (not just bipartite)
12      - Uses BFS with blossom contraction and path augmentation
13
14    Input:
15      - n: number of vertices
16      - add_edge(u, v): undirected edges between nodes (1 <= u,v <= n)
17
18    Output:
19      - maximum_matching(): returns size of max matching
20      - match[u]: matched vertex for node u (or 0 if unmatched)
21  */
22
23  const int N = 2009;
24  mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
25
26  struct Blossom {
27    int vis[N];        // vis[u]: -1 = unvisited, 0 = in queue, 1 = outer
         layer
28    int par[N];        // par[u]: parent in alternating tree
29    int orig[N];       // orig[u]: base of blossom u belongs to
```

```
30    int match[N];      // match[u]: matched partner of u (0 if unmatched)
31    int aux[N];        // aux[u]: visit marker for LCA
32    int t;             // global timestamp for LCA markers
33    int n;             // number of nodes
34    bool ad[N];        // ad[u]: whether u is reachable in an alternating
         path
35    vector<int> g[N];  // g[u]: adjacency list
36    queue<int> Q;      // BFS queue
37
38    // Constructor: initializes data for n nodes
39    Blossom() {}
40    Blossom(int _n) {
41      n = _n;
42      t = 0;
43      for (int i = 0; i <= n; ++i) {
44        g[i].clear();
45        match[i] = par[i] = vis[i] = aux[i] = ad[i] = orig[i] = 0;
46      }
47    }
48
49    void add_edge(int u, int v) {
50      g[u].push_back(v);
51      g[v].push_back(u);
52    }
53
54    // Augment the matching along the alternating path from u to v
55    void augment(int u, int v) {
56      int pv = v, nv;
57      do {
58        pv = par[v];
59        nv = match[pv];
60        match[v] = pv;
61        match[pv] = v;
62        v = nv;
63      } while (u != pv);
64    }
65
66    int lca(int v, int w) {
67      ++t; // Increment timestamp for LCA markers
68      while (true) {
69        if (v) {
70          if (aux[v] == t) return v;
71          aux[v] = t;
```

```
 72       v = orig[par[match[v]]]; // Move to the parent in the
              alternating tree
 73     }
 74     swap(v, w);
 75   }
 76 }
 77
 78 // Contract a blossom from v and w with common ancestor a
 79 void blossom(int v, int w, int a) {
 80   while (orig[v] != a) {
 81     par[v] = w;
 82     w = match[v];
 83     ad[v] = true;
 84     if (vis[w] == 1) Q.push(w), vis[w] = 0;
 85     orig[v] = orig[w] = a;
 86     v = par[w];
 87   }
 88 }
 89
 90 // Find augmenting path starting from unmatched node u
 91 bool bfs(int u) {
 92   fill(vis + 1, vis + n + 1, -1);
 93   iota(orig + 1, orig + n + 1, 1);
 94   Q = queue<int>();
 95   Q.push(u);
 96   vis[u] = 0;
 97
 98   while (!Q.empty()) {
 99     int v = Q.front(); Q.pop();
100     ad[v] = true;
101     for (int x : g[v]) {
102       if (vis[x] == -1) {
103         par[x] = v;
104         vis[x] = 1;
105         if (!match[x]) {
106           augment(u, x);
107           return true;
108         }
109         Q.push(match[x]);
110         vis[match[x]] = 0;
111       } else if (vis[x] == 0 && orig[v] != orig[x]) {
112         int a = lca(orig[v], orig[x]);
113         blossom(x, v, a);
```

```
114         blossom(v, x, a);
115       }
116     }
117   }
118   return false;
119 }
120
121 // Computes maximum matching and returns the size
122 int maximum_matching() {
123   int ans = 0;
124   vector<int> p(n - 1);
125   iota(p.begin(), p.end(), 1);
126   shuffle(p.begin(), p.end(), rnd);
127   for (int i = 1; i <= n; ++i) {
128     shuffle(g[i].begin(), g[i].end(), rnd);
129   }
130
131   // Greedy matching: try to match unmatched nodes directly
132   for (int u : p) {
133     if (!match[u]) {
134       for (int v : g[u]) {
135         if (!match[v]) {
136           match[u] = v;
137           match[v] = u;
138           ++ans;
139           break;
140         }
141       }
142     }
143   }
144
145   // Augmenting path phase
146   for (int i = 1; i <= n; ++i) {
147     if (!match[i] && bfs(i)) ++ans;
148   }
149
150   return ans;
151 }
152 } M;
153
154 int main() {
155   ios_base::sync_with_stdio(0);
156   cin.tie(0);
```

```
157
158    int t;
159    cin >> t;
160    while (t--) {
161      int n, m;
162      cin >> n >> m;
163      M = Blossom(n);
164      // Read all edges
165      for (int i = 0; i < m; i++) {
166        int u, v;
167        cin >> u >> v;
168        M.add_edge(u, v);
169      }
170      // Compute max matching
171      int matched = M.maximum_matching();
172      if (matched * 2 == n) {
173        // Perfect matching
174        cout << 0 << '\n';
175      } else {
176        // Find reachable unmatched nodes in alternating trees
177        memset(M.ad, 0, sizeof M.ad);
178        for (int i = 1; i <= n; i++) {
179          if (M.match[i] == 0) M.bfs(i);
180        }
181        int unmatched_reachable = 0;
182        for (int i = 1; i <= n; i++) {
183          unmatched_reachable += M.ad[i];
184        }
185        cout << unmatched_reachable << '\n';
186      }
187    }
188    return 0;
189 }
```

7.9   Bridges

```
1  /*
2    Bridge-Finding in an Undirected Graph
3    -------------------------------------
4    Indexing: 0-based
5    Node Bounds: [0, n-1] inclusive
6    Time Complexity: O(V + E)
7    Space Complexity: O(V)
8
9    Input:
10     n    - Number of nodes in the graph
11     adj  - Adjacency list of the undirected graph
12
13   Output:
14     - Call 'find_bridges()' to populate bridge information.
15     - Modify the DFS 'Bridge' section to store or print the bridges.
16       A bridge is an edge (v, to) such that removing it increases the
17               number of connected components.
17  */
18
19  int n; // Number of nodes
20  vector<vector<int>> adj; // Adjacency list
21
22  vector<bool> visited; // Marks visited nodes
23  vector<int> tin, low; // tin[v]: discovery time; low[v]: lowest ancestor
          reachable
24  int timer; // Global DFS timer
25
26  // DFS to detect bridges
27  void dfs(int v, int p = -1) {
28    visited[v] = true;
29    tin[v] = low[v] = timer++;
30    for (int to : adj[v]) {
31      if (to == p) continue; // Skip edge to parent
32      if (visited[to]) {
33        // Back edge
34        low[v] = min(low[v], tin[to]);
35      } else {
36        dfs(to, v);
37        low[v] = min(low[v], low[to]);
38        // Bridge condition: if no back edge connects subtree rooted at '
              to' to ancestors of 'v'
39        if (low[to] > tin[v]) {
40          // (v, to) is a bridge
41          // Example: bridges.push_back({v, to});
42        }
43      }
44    }
45  }
46
47  // Initialize tracking structures and run DFS
```

```
48  void find_bridges() {
49    timer = 0;
50    visited.assign(n, false);
51    tin.assign(n, -1);
52    low.assign(n, -1);
53    for (int i = 0; i < n; ++i) {
54      if (!visited[i])
55        dfs(i);
56    }
57  }
```

7.10   Bridges Online

```
1   /*
2     Online Bridge-Finding (Dynamic Edge Insertion)
3     ----------------------------------------------
4     Indexing: 0-based
5     Node Bounds: [0, n-1] inclusive
6     Time Complexity:
7       - Amortized O(log^2N) per edge addition
8     Space Complexity: O(V)
9
10    Features:
11      - Maintains the number of bridges dynamically as edges are added one
             by one.
12      - Detects if adding an edge merges different 2-edge-connected
             components.
13      - No deletions supported.
14
15    Input:
16      init(n)     - Initializes the data structure for a graph with n
             nodes.
17      add_edge(a, b) - Adds an undirected edge between nodes a and b.
18
19    Output:
20      'bridges' - Global variable representing the current number of
             bridges.
21  */
22
23  vector<int> par, dsu_2ecc, dsu_cc, dsu_cc_size;
24  int bridges; // Number of bridges in the graph
25  int lca_iteration;
26  vector<int> last_visit;
```

```
27
28  // Initializes the data structures
29  void init(int n) {
30    par.resize(n);
31    dsu_2ecc.resize(n);
32    dsu_cc.resize(n);
33    dsu_cc_size.resize(n);
34    last_visit.assign(n, 0);
35    lca_iteration = 0;
36    bridges = 0;
37
38    for (int i = 0; i < n; ++i) {
39      par[i] = -1;
40      dsu_2ecc[i] = i;
41      dsu_cc[i] = i;
42      dsu_cc_size[i] = 1;
43    }
44  }
45
46  // Finds the representative of the 2-edge-connected component of node v
47  int find_2ecc(int v) {
48    if (v == -1) return -1;
49    return dsu_2ecc[v] == v ? v : dsu_2ecc[v] = find_2ecc(dsu_2ecc[v]);
50  }
51
52  // Finds the connected component representative of the component
          containing v
53  int find_cc(int v) {
54    v = find_2ecc(v);
55    return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(dsu_cc[v]);
56  }
57
58  // Makes node v the root of its tree, rerouting parent pointers upward
59  void make_root(int v) {
60    int root = v;
61    int child = -1;
62    while (v != -1) {
63      int p = find_2ecc(par[v]);
64      par[v] = child;
65      dsu_cc[v] = root;
66      child = v;
67      v = p;
68    }
```

```cpp
69    dsu_cc_size[root] = dsu_cc_size[child];
70  }
71
72  // Merges paths from a and b to their lowest common ancestor in the 2ECC
        forest
73  void merge_path(int a, int b) {
74    ++lca_iteration;
75    vector<int> path_a, path_b;
76    int lca = -1;
77
78    while (lca == -1) {
79      if (a != -1) {
80        a = find_2ecc(a);
81        path_a.push_back(a);
82        if (last_visit[a] == lca_iteration) {
83          lca = a;
84          break;
85        }
86        last_visit[a] = lca_iteration;
87        a = par[a];
88      }
89      if (b != -1) {
90        b = find_2ecc(b);
91        path_b.push_back(b);
92        if (last_visit[b] == lca_iteration) {
93          lca = b;
94          break;
95        }
96        last_visit[b] = lca_iteration;
97        b = par[b];
98      }
99    }
100
101   // Merge all nodes on path_a and path_b into the same 2ECC
102   for (int v : path_a) {
103     dsu_2ecc[v] = lca;
104     if (v == lca) break;
105     --bridges;
106   }
107   for (int v : path_b) {
108     dsu_2ecc[v] = lca;
109     if (v == lca) break;
110     --bridges;
```

```cpp
111   }
112 }
113
114 // Adds an undirected edge between a and b and updates bridge count
115 void add_edge(int a, int b) {
116   a = find_2ecc(a);
117   b = find_2ecc(b);
118   if (a == b) return; // Already in the same 2ECC
119
120   int ca = find_cc(a);
121   int cb = find_cc(b);
122
123   if (ca != cb) {
124     // Bridge found - connects two different components
125     ++bridges;
126     // Union by size
127     if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
128       swap(a, b);
129       swap(ca, cb);
130     }
131     make_root(a);
132     par[a] = b;
133     dsu_cc[a] = b;
134     dsu_cc_size[cb] += dsu_cc_size[a];
135   } else {
136     // No new bridge, but must merge paths to unify 2ECCs
137     merge_path(a, b);
138   }
139 }
140
141 // Example usage
142 int main() {
143   init(n);
144   for (auto [u, v] : edges) {
145     add_edge(u, v);
146     cout << "Current bridge count: " << bridges << '\n';
147   }
148 }
```

7.11   Dijkstra

```cpp
1  vector<vector<pair<int, int>>> adj(n); // Adjacency list (node, weight)
2  vector<ll> dist(n, 1LL << 61); // Distance array initialized to infinity
```

```
3   priority_queue<pair<ll, int>> q; // Max-heap, so we push negative
4       weights to simulate min-heap
5   dist[0] = 0; // Starting node distance
6   q.push({0, 0}); // (distance, vertex)
7
8   while (!q.empty()) {
9     auto [w, v] = q.top(); q.pop();
10    w = -w; // Convert back to positive
11    if (w > dist[v]) continue; // Skip outdated entry
12    for (auto [u, cost] : adj[v]) {
13      if (dist[v] + cost < dist[u]) {
14        dist[u] = dist[v] + cost;
15        q.push({-dist[u], u}); // Push updated distance (negated)
16      }
17    }
18  }
```

7.12   Eulerian Path

An Eulerian Path is a path that passes through every edge once. For an undirected graph an eulerian path exists if the degree of every node is even or the degree of exactly two nodes is odd. In the first case, the eulerian path is also an eulerian circuit or cycle. In a directed graph, an eulerian path exists if at most one noda has $out_i - in_i = 1$ and at most one node has $in_i - out_i = 1$. A cycle exists if $in_i - out_i = 0$ for all i.

```
1   /*
2     Eulerian Path (Hierholzer's Algorithm)
3     This will find an Eurelian Circuit.
4     For directed graph eulerian path must start on vertex with outdegree[i
5       ] == indegree[i] + 1 and ends on indegree[i] = outdegree[i] + 1.
6     For directed graph eulerian Cycle only exists if indegree[i] =
7       outdegree[i] for all i.
8     For undirected graph eulerian path must start on vertex with odd
9       degree and will end in the other vertex with odd degree.
10    For undirected graph eulerian Cycle only exists degree is even for all
11      vertices.
12
13    --------------------------------------
14    Time Complexity: O(E)
15    Space Complexity: O(V + E)
16
17    Input:
```

```
14    - g: adjacency list of the graph
15      * Directed: vector<vector<pair<int, int>>> g
16        where g[v] = list of {to, edge_index}
17      * Undirected: vector<vector<int>> g
18        where g[v] = list of neighbors
19    - seen: vector<bool> seen(E) - only needed for directed version
20    - path: vector<int> path - will be filled in reverse order of
21      traversal
      reverse(path.begin(), path.end());
22  */
23
24  // Directed Version //
25  void dfs_directed(int node) {
26    while (!g[node].empty()) {
27      auto [son, idx] = g[node].back();
28      g[node].pop_back();
29      if (seen[idx]) continue;  // Skip if edge already visited
30      seen[idx] = true;
31      dfs_directed(son);
32    }
33    path.push_back(node); // Post-order insertion (reverse of actual path)
34  }
35
36  // Undirected Version //
37  // Be careful with graph not being connected. Add set to not repeat
      edges(going from u to v and then from v to u)
38  void dfs_undirected(int node) {
39    while (!g[node].empty()) {
40      int son = g[node].back();
41      g[node].pop_back();
42      dfs_undirected(son);
43    }
44    path.push_back(node); // Post-order insertion
45  }
```

7.13   Floyd-Warshall

```
1   /*
2     Floyd-Warshall Algorithm (All-Pairs Shortest Paths)
3     ---------------------------------------------------
4     Indexing: 0-based
5     Time Complexity: O(V^3)
```

```
 6    Space Complexity: O(V^2)
 7
 8    Input:
 9      - d: distance matrix of size n x n
10          * d[i][j] should be initialized as:
11              - 0 if i == j
12              - weight of edge (i, j) if exists
13              - INF (e.g. 1e18) otherwise
14  */
15
16  vector<vector<ll>> d(n, vector<ll>(n, 1e18)); // distance matrix
17
18  // This version is by default adapted for UNDIRECTED graphs.
19  for (int k = 0; k < n; k++) {
20    for (int i = 0; i < n; i++) {
21      for (int j = i + 1; j < n; j++) { // For directed graphs, use j = 0;
              j < n; j++
22        long long new_dist = d[i][k] + d[k][j];
23        if (new_dist < d[i][j]) {
24          d[i][j] = d[j][i] = new_dist; // update both directions for
                undirected graph
25        }
26      }
27    }
28  }
```

7.14   Kruskal

```
 1  /*
 2    Kruskal's Algorithm (Minimum Spanning Tree - MST)
 3    --------------------------------------------------
 4    Indexing: 0-based for nodes in edges
 5    Time Complexity: O(E log E)
 6    Space Complexity: O(N)
 7
 8    Input:
 9      - N: number of nodes
10      - edges: list of weighted edges in form {weight, {u, v}}
11
12    Output:
13      - Returns total weight of the MST if the graph is connected
14      - Returns -1 if MST cannot be formed (i.e., graph is disconnected)
15
```

```
16    Note:
17      - Requires a Disjoint Set Union (DSU) / Union-Find data structure
            with:
18        - unite(a, b): merges components, returns true if successful
19        - size(v): returns size of component containing v
20  */
21
22  template <class T>
23  T kruskal(int N, vector<pair<T, pair<int, int>>> edges) {
24    sort(edges.begin(), edges.end()); // Sort by weight (non-decreasing)
25    T ans = 0;
26    DSU D(N); // Disjoint Set Union for N nodes
27    for (auto &[w, uv] : edges) {
28      int u = uv.first, v = uv.second;
29      if (D.unite(u, v)) {
30        ans += w; // Add edge to MST if u and v are in different
              components
31      }
32    }
33    // Check if MST spans all nodes (i.e., one component of size N)
34    return (D.size(0) == N ? ans : -1);
35  }
```

7.15   Marriage

```
 1  /*
 2    Male-Optimal Stable Marriage Problem (Gale-Shapley Algorithm)
 3    --------------------------------------------------------------
 4    Indexing: 0-based
 5    Bounds: 0 <= i, j < n
 6    Time Complexity: O(n^2)
 7    Space Complexity: O(n^2)
 8
 9    Input:
10      - n: Number of men/women (equal)
11      - gv[i][j]: j-th most preferred woman for man i
12      - om[i][j]: j-th most preferred man for woman i
13        * Both are permutations of {0, ..., n-1}
14        * om must be inverted to get om[w][m] = woman w's ranking of man
              m
15
16    Output:
17      - pm[i]: Woman matched to man i (i.e. pairings)
```

```
18      - pv[i]: Man matched to woman i
19   */
20
21   #define MAXN 1000
22   int gv[MAXN][MAXN], om[MAXN][MAXN]; // Male and female preference lists
23   int pv[MAXN], pm[MAXN];              // pv[woman] = man, pm[man] = woman
24   int pun[MAXN];                       // pun[man] = next woman to propose
         to
25
26   void stableMarriage(int n) {
27     fill_n(pv, n, -1);    // All women initially unmatched
28     fill_n(pm, n, -1);    // All men initially unmatched
29     fill_n(pun, n, 0);    // Each man starts at his top preference
30
31     int unmatched = n;    // Number of free men
32     int i = n - 1;        // Current man index (rotates over all men)
33
34     #define engage pm[j] = i; pv[i] = j;
35
36     while (unmatched) {
37       while (pm[i] == -1) {
38         int j = gv[i][pun[i]++]; // Next woman on man i's list
39
40         if (pv[j] == -1) {
41           // Woman j is free -> engage with man i
42           unmatched--;
43           engage;
44         } else if (om[j][i] < om[j][pv[j]]) {
45           // Woman j prefers i over her current partner
46           int loser = pv[j];
47           pm[loser] = -1;
48           engage;
49           i = loser; // Reconsider the rejected man
50         }
51       }
52
53       // Move to next unmatched man
54       i--;
55       if (i < 0) i = n - 1;
56     }
57
58     #undef engage
59   }
```

7.16   SCC

```
1   /*
2     Strongly Connected Components (Kosaraju's Algorithm)
3     ----------------------------------------------------
4     Indexing: 0-based
5     Time Complexity: O(V + E)
6     Space Complexity: O(V + E)
7
8     Input:
9       - n: number of nodes
10      - m: number of directed edges
11      - adj: original graph
12      - adjr: reversed graph
13
14    Output:
15      - comp[i]: component ID of node i
16      - order[]: nodes in reverse post-order (1st DFS)
17      - nc: is the number of unique comp values
18   */
19
20   vector<vector<int>> adj, adjr;
21   vector<bool> vis;
22   vector<int> order, comp;
23
24   // First DFS: post-order on original graph
25   void dfs(int v) {
26     vis[v] = true;
27     for (int u : adj[v]) {
28       if (!vis[u])
29         dfs(u);
30     }
31     order.push_back(v); // Record post-order
32   }
33
34   // Second DFS: assign component IDs on reversed graph
35   void dfsr(int v, int k) {
36     vis[v] = true;
37     comp[v] = k;
38     for (int u : adjr[v]) {
39       if (!vis[u])
40         dfsr(u, k);
41   }
```

```
42  }
43
44  void solve() {
45    int n, m;
46    cin >> n >> m;
47    adj.assign(n, vector<int>());
48    adjr.assign(n, vector<int>());
49    comp.resize(n);
50    // Read edges and build both original and reversed graphs
51    for (int i = 0; i < m; i++) {
52      int a, b;
53      cin >> a >> b;
54      a--; b--;
55      adj[a].push_back(b);
56      adjr[b].push_back(a);
57    }
58    // First pass: DFS on original graph to get order
59    vis.assign(n, false);
60    order.clear();
61    for (int i = 0; i < n; i++) {
62      if (!vis[i]) dfs(i);
63    }
64    // Second pass: DFS on reversed graph using reverse post-order
65    vis.assign(n, false);
66    int nc = 0;
67    for (int i = n - 1; i >= 0; i--) {
68      int v = order[i];
69      if (!vis[v]) {
70        dfsr(v, nc++);
71      }
72    }
73    // comp[i] now holds the component ID for node i (0-based)
74    // nc = number of strongly connected components
75  }
```

7.17   Stoer-Wagner

```
1  /*
2                                Stoer - Wagner
3  ------------------------------------------------------------------------------------
4    Solves the minimum cut problem in undirected weighted graphs with non-
         negative weights.
5
6    Time Complexity: O(V ^ 3)
7    Space Complexity: O(V ^ 2)
8  */
9
10 mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
11 struct StoerWagner {
12   int n;
13   long long G[N][N], dis[N];
14   int idx[N];
15   bool vis[N];
16   const long long inf = 1e18;
17   StoerWagner() {}
18   StoerWagner(int _n) {
19     n = _n;
20     memset(G, 0, sizeof G);
21   }
22   void add_edge(int u, int v, long long w) { //undirected edge, multiple
         edges are merged into one edge
23     if (u != v) {
24       G[u][v] += w;
25       G[v][u] += w;
26     }
27   }
28   long long solve() {
29     long long ans = inf;
30     for (int i = 0; i < n; ++ i) idx[i] = i + 1;
31     shuffle(idx, idx + n, rnd);
32     while (n > 1) {
33       int t = 1, s = 0;
34       for (int i = 1; i < n; ++ i) {
35         dis[idx[i]] = G[idx[0]][idx[i]];
36         if (dis[idx[i]] > dis[idx[t]]) t = i;
37       }
38       memset(vis, 0, sizeof vis);
39       vis[idx[0]] = true;
40       for (int i = 1; i < n; ++ i) {
41         if (i == n - 1) {
42           if (ans > dis[idx[t]]) ans = dis[idx[t]]; //idx[s] - idx[t] is
                 in two halves of the  mincut
43           if (ans == 0) return 0;
44           for (int j = 0; j < n; ++ j) {
45             G[idx[s]][idx[j]] += G[idx[j]][idx[t]];
```

```
46            G[idx[j]][idx[s]] += G[idx[j]][idx[t]];
47          }
48          idx[t] = idx[-- n];
49        }
50        vis[idx[t]] = true;
51        s = t;
52        t = -1;
53        for (int j = 1; j < n; ++ j) {
54          if (!vis[idx[j]]) {
55            dis[idx[j]] += G[idx[s]][idx[j]];
56            if (t == -1 || dis[idx[t]] < dis[idx[j]]) t = j;
57          }
58        }
59      }
60    }
61    return ans;
62  }
63 };
64
65 int32_t main() {
66   ios_base::sync_with_stdio(0);
67   cin.tie(0);
68   int t, cs = 0;
69   cin >> t;
70   while (t--) {
71     int n, m;
72     cin >> n >> m;
73     StoerWagner st(n);
74     while (m--) {
75       int u, v, w;
76       cin >> u >> v >> w;
77       st.add_edge(u, v, w);
78     }
79     cout << "Case #" << ++cs << ": " << st.solve() << '\n';
80   }
81   return 0;
82 }
```

8   Linear Algebra

8.1   Gaussian Elimination

```
1 /*
2    Gaussian elimination
3    -----------------------------------
4    Utilized for solving linear systems of equations.
5
6    If the system is modulo 2, use GaussianEliminationModulo instead.
7
8    Time Complexity: O(n ^ 3)
9 */
10
11
12 const double eps = 1e-9;
13 int Gauss(vector<vector<double>> a, vector<double> &ans) {
14   int n = (int)a.size(), m = (int)a[0].size() - 1;
15   vector<int> pos(m, -1);
16   double det = 1; int rank = 0;
17   for(int col = 0, row = 0; col < m && row < n; ++col) {
18     int mx = row;
19     for(int i = row; i < n; i++) if(fabs(a[i][col]) > fabs(a[mx][col]))
20             mx = i;
21     if(fabs(a[mx][col]) < eps) {det = 0; continue;}
22     for(int i = col; i <= m; i++) swap(a[row][i], a[mx][i]);
23     if (row != mx) det = -det;
24     det *= a[row][col];
25     pos[col] = row;
26     for(int i = 0; i < n; i++) {
27       if(i != row && fabs(a[i][col]) > eps) {
28         double c = a[i][col] / a[row][col];
29         for(int j = col; j <= m; j++) a[i][j] -= a[row][j] * c;
30       }
31     }
32     ++row; ++rank;
33   }
34   ans.assign(m, 0);
35   for(int i = 0; i < m; i++) {
36     if(pos[i] != -1) ans[i] = a[pos[i]][m] / a[pos[i]][i];
37   }
38   for(int i = 0; i < n; i++) {
39     double sum = 0;
40     for(int j = 0; j < m; j++) sum += ans[j] * a[i][j];
41     if(fabs(sum - a[i][m]) > eps) return -1; //no solution
42   }
43   for(int i = 0; i < m; i++) if(pos[i] == -1) return 2; //infinte
44         solutions
```

```
43     return 1; //unique solution
44  }
45  int main() {
46     int n, m; cin >> n >> m;
47     vector< vector<double> > v(n);
48     for(int i = 0; i < n; i++) {
49       for(int j = 0; j <= m; j++) {
50         double x; cin >> x; v[i].push_back(x);
51       }
52     }
53     vector<double> ans;
54     int k = Gauss(v, ans);
55     if(k) for(int i = 0; i < n; i++) cout << fixed << setprecision(5) <<
           ans[i] << ' ';
56     else cout << "no solution\n";
57     return 0;
58  }
```

8.2   Gaussian Elimination Modulo

```
1  /*
2    Gaussian elimination Modulo 2
3    ----------------------------------
4    Utilized for solving linear systems of equations modulo2.
5
6    Time Complexity: O(n ^ 3)
7  */
8
9  //n = number of equations, m = number of variables
10 int Gauss(int n, int m, vector<bitset<N>> a, bitset<N> &ans) {
11   //reversing for lexocgraphically largest solution
12   for (int i = 0; i < n; i++) {
13       bitset<N> tmp;
14       for (int j = 0; j < m; j++) tmp[j] = a[i][m - j - 1];
15       tmp[m] = a[i][m];
16       a[i] = tmp;
17   }
18   int rank = 0, det = 1;
19   vector<int> pos(N, -1);
20   for(int col = 0, row = 0; col < m && row < n; ++col) {
21     int mx = row;
22     for(int i = row; i < n; ++i) if(a[i][col]) { mx = i; break; }
23     if(!a[mx][col]) { det = 0; continue; }
24       swap(a[mx], a[row]);
25       if(row != mx) det = det == 0 ? 0 : 1;
26       det &= a[row][col];
27       pos[col] = row;
28       //forward elimination
29       for(int i = row + 1; i < n; ++i) if (i != row && a[i][col]) a[i] ^=
             a[row];
30       ++row, ++rank;
31   }
32   ans.reset();
33   //backward substituition
34   for (int i = m - 1; i >= 0; i--) {
35       if (pos[i] == -1) ans[i] = true;
36       else {
37           int k = pos[i];
38           for (int j = i + 1; j < m; j++) if (a[k][j]) ans[i] = ans[i] ^
                 ans[j];
39           ans[i] = ans[i] ^ a[k][m];
40       }
41   }
42   for(int i = rank; i < n; ++i) if(a[i][m]) return -1; //no solution
43   //reversing again beacuse we reversed earlier
44   bitset<N> tmp;
45   for (int j = 0; j < m; j++) tmp[j] = ans[m - j - 1];
46   ans = tmp;
47   int free_var = 0;
48   for(int i = 0; i < m; ++i) if(pos[i] == -1) free_var++;
49   return free_var; //has solution
50 }
51 string read() {
52    string t;
53    if(!(cin >> t)) return "";
54    if(t.empty() || t == "and") return "";
55    while(t[0] == '(') t.erase(t.begin());
56    while(t.back() == ')') t.pop_back();
57    return t;
58 }
59 bool is_var(string t) { return t.size() > 0 && t[0] == 'x'; }
60 int get_var(string t) { return atoi(t.substr(1).c_str()) - 1; }
61 int32_t main() {
62   ios_base::sync_with_stdio(0);
63   cin.tie(0);
64   int n, m; cin >> n >> m;
```

```
65    vector<bitset<N>> bs(n, bitset<N>(0));
66    for(int i = 0; i < n; i++) {
67        string s;
68        bool eq = 1;
69        while((s = read()).size() > 0) {
70            if(is_var(s)) {
71                int x = get_var(s);
72                bs[i][x] = bs[i][x] ^ 1;
73            }
74            else if(s == "not") eq ^= 1;
75        }
76        bs[i][m] = eq;
77    }
78    bitset<N> ans;
79    int ok = Gauss(n, m, bs, ans);
80    if (ok == -1) cout << "impossible\n";
81    else {
82        for (int i = 0; i < m; i++) cout << "FT"[ans[i]]; cout << '\n';
83    }
84     return 0;
85 }
86 //https://codeforces.com/gym/101908/problem/M
```

8.3   Simplex

```
1  /*
2  Parametric Self-Dual Simplex method
3  Solve a canonical LP:
4    min or max. c x
5    s.t. A x <= b
6      x >= 0
7  */
8  #include <bits/stdc++.h>
9  using namespace std;
10 const double eps = 1e-9, oo = numeric_limits<double>::infinity();
11
12 typedef vector<double> vec;
13 typedef vector<vec> mat;
14
15 pair<vec, double> simplexMethodPD(const mat &A, const vec &b, const vec
       &c, bool mini = true){
16   int n = c.size(), m = b.size();
17   mat T(m + 1, vec(n + m + 1));
18   vector<int> base(n + m), row(m);
19
20   for(int j = 0; j < m; ++j){
21     for(int i = 0; i < n; ++i)
22       T[j][i] = A[j][i];
23     row[j] = n + j;
24     T[j][n + j] = 1;
25     base[n + j] = 1;
26     T[j][n + m] = b[j];
27   }
28
29   for(int i = 0; i < n; ++i)
30     T[m][i] = c[i] * (mini ? 1 : -1);
31
32   while(true){
33     int p = 0, q = 0;
34     for(int i = 0; i < n + m; ++i)
35       if(T[m][i] <= T[m][p])
36         p = i;
37
38     for(int j = 0; j < m; ++j)
39       if(T[j][n + m] <= T[q][n + m])
40         q = j;
41
42     double t = min(T[m][p], T[q][n + m]);
43
44     if(t >= -eps){
45       vec x(n);
46       for(int i = 0; i < m; ++i)
47         if(row[i] < n) x[row[i]] = T[i][n + m];
48       return {x, T[m][n + m] * (mini ? -1 : 1)}; // optimal
49     }
50
51     if(t < T[q][n + m]){
52       // tight on c -> primal update
53       for(int j = 0; j < m; ++j)
54         if(T[j][p] >= eps)
55           if(T[j][p] * (T[q][n + m] - t) >= T[q][p] * (T[j][n + m] - t))
56             q = j;
57
58       if(T[q][p] <= eps)
59         return {vec(n), oo * (mini ? 1 : -1)}; // primal infeasible
60     }else{
```

```cpp
61        // tight on b -> dual update
62        for(int i = 0; i < n + m + 1; ++i)
63          T[q][i] = -T[q][i];
64
65        for(int i = 0; i < n + m; ++i)
66          if(T[q][i] >= eps)
67            if(T[q][i] * (T[m][p] - t) >= T[q][p] * (T[m][i] - t))
68              p = i;
69
70        if(T[q][p] <= eps)
71          return {vec(n), oo * (mini ? -1 : 1)}; // dual infeasible
72      }
73
74      for(int i = 0; i < m + n + 1; ++i)
75        if(i != p) T[q][i] /= T[q][p];
76
77      T[q][p] = 1; // pivot(q, p)
78      base[p] = 1;
79      base[row[q]] = 0;
80      row[q] = p;
81
82      for(int j = 0; j < m + 1; ++j){
83        if(j != q){
84          double alpha = T[j][p];
85          for(int i = 0; i < n + m + 1; ++i)
86            T[j][i] -= T[q][i] * alpha;
87        }
88      }
89    }
90
91    return {vec(n), oo};
92  }
93
94  int main(){
95    int m, n;
96    bool mini = true;
97    cout << "Numero de restricciones: ";
98    cin >> m;
99    cout << "Numero de incognitas: ";
100   cin >> n;
101   mat A(m, vec(n));
102   vec b(m), c(n);
103   for(int i = 0; i < m; ++i){
104     cout << "Restriccion #" << (i + 1) << ": ";
105     for(int j = 0; j < n; ++j){
106       cin >> A[i][j];
107     }
108     cin >> b[i];
109   }
110   cout << "[0]Max o [1]Min?: ";
111   cin >> mini;
112   cout << "Coeficientes de " << (mini ? "min" : "max") << " z: ";
113   for(int i = 0; i < n; ++i){
114     cin >> c[i];
115   }
116   cout.precision(6);
117   auto ans = simplexMethodPD(A, b, c, mini);
118   cout << (mini ? "Min" : "Max") << " z = " << ans.second << ", cuando:
          \n";
119   for(int i = 0; i < ans.first.size(); ++i){
120     cout << "x_" << (i + 1) << " = " << ans.first[i] << "\n";
121   }
122   return 0;
123 }
```

9 Math

9.1 BinPow

```cpp
1  ll binpow(ll a, ll b){
2      ll r=1;
3      while(b){
4          if(b%2)
5              r=(r*a)%MOD;
6          a=(a*a)%MOD;
7          b/=2;
8      }
9      return r;
10 }
11
12 ll divide(ll a, ll b){
13     return ((a%MOD)*binpow(b, MOD-2))%MOD;
14 }
15 void inverses(long long p) {
16   inv[MAXN] = exp(fac[MAXN], p - 2, p);
17   for (int i = MAXN; i >= 1; i--) { inv[i - 1] = inv[i] * i % p; }
```

```
18  }
```

9.2 Combination Rank

```
1   // Compute nCr (binomial coefficient)
2   long long nCr(int n, int r) {
3       if (r > n) return 0;
4       if (r == 0 || r == n) return 1;
5       long long res = 1;
6       for (int i = 1; i <= r; ++i) {
7           res = res * (n - i + 1) / i;
8       }
9       return res;
10  }
11
12  // Compute 0-indexed lexicographic rank of a combination
13  long long combinationRank0(const vector<int>& comb) {
14      long long rank = 0;
15      for (int i = 0; i < (int)comb.size(); ++i) {
16          rank += nCr(comb[i] - 1, i + 1);
17      }
18      return rank; // 0-indexed
19  }
20
21  // Given n, r, and rank (0-indexed), reconstruct the combination
22  vector<int> combinationUnrank0(int n, int r, long long rank) {
23      vector<int> comb;
24      int x = 1;
25
26      for (int i = r; i >= 1; --i) {
27          while (nCr(n - x, i) > rank) {
28              x++;
29          }
30          comb.push_back(x);
31          rank -= nCr(n - x, i);
32          x++;
33      }
34
35      return comb;
36  }
```

9.3 Chinese Remainder Theorem

```
1   ll euclid(ll a, ll b, ll &x, ll &y) {
```

```
2       if (!b) return x = 1, y = 0, a;
3       ll d = euclid(b, a % b, y, x);
4       return y -= a/b * x, d;
5   }
6
7   ll crt(ll a, ll m, ll b, ll n) {
8       if (n > m) swap(a, b), swap(m, n);
9       ll x, y, g = euclid(m, n, x, y);
10      if((a - b) % g != 0)return -1;
11      x = (b - a) % n * x % n / g * m + a;
12      return x < 0 ? x + m*n/g : x;
13  }
```

9.4 Diophantine

If one solution is (x_0, y_0) all solutions can be obtained by $x = x_0 + k * \frac{b}{gcd(a,b)}$ and
$$y = y_0 - k * \frac{a}{gcd(a,b)}.$$

```
1   int gcd(int a, int b, int& x, int& y) {
2       if (b == 0) {
3           x = 1;
4           y = 0;
5           return a;
6       }
7       int x1, y1;
8       int d = gcd(b, a % b, x1, y1);
9       x = y1;
10      y = x1 - y1 * (a / b);
11      return d;
12  }
13
14  bool find_any_solution(int a, int b, int c, int &x0, int &y0, int &g) {
15      g = gcd(abs(a), abs(b), x0, y0);
16      if (c % g) {
17          return false;
18      }
19
20      x0 *= c / g;
21      y0 *= c / g;
22      if (a < 0) x0 = -x0;
23      if (b < 0) y0 = -y0;
24      return true;
25  }
26
```

```
27
28
29  //n variables
30  vector<ll> find_any_solution(vector<ll> a, ll c) {
31    int n = a.size();
32    vector<ll> x;
33    bool all_zero = true;
34    for (int i = 0; i < n; i++) {
35      all_zero &= a[i] == 0;
36    }
37    if (all_zero) {
38      if (c) return {};
39      x.assign(n, 0);
40      return x;
41    }
42    ll g = 0;
43    for (int i = 0; i < n; i++) {
44      g = __gcd(g, a[i]);
45    }
46    if (c % g != 0) return {};
47    if (n == 1) {
48      return {c / a[0]};
49    }
50    vector<ll> suf_gcd(n);
51    suf_gcd[n - 1] = a[n - 1];
52    for (int i = n - 2; i >= 0; i--) {
53      suf_gcd[i] = __gcd(suf_gcd[i + 1], a[i]);
54    }
55    ll cur = c;
56    for (int i = 0; i + 1 < n; i++) {
57      ll x0, y0, g;
58      // solve for a[i] * x + suf_gcd[i + 1] * (y / suf_gcd[i + 1]) = cur
59      bool ok = find_any_solution(a[i], suf_gcd[i + 1], cur, x0, y0, g);
60      assert(ok);
61      {
62        // trying to minimize x0 in case x0 becomes big
63        // it is needed for this problem, not needed in general
64        ll shift = abs(suf_gcd[i + 1] / g);
65        x0 = (x0 % shift + shift) % shift;
66      }
67      x.push_back(x0);
68
69      // now solve for the next suffix
70      cur -= a[i] * x0;
71    }
72    x.push_back(a[n - 1] == 0 ? 0 : cur / a[n - 1]);
73    return x;
74  }
```

9.5   Discrete Logarithm

Finds discrete logarithm in $O(\sqrt{(m)})$.

```
1   // Returns minimum x for which a ^ x % m = b % m, a and m are coprime.
2   int solve(int a, int b, int m) {
3       a %= m, b %= m;
4       int n = sqrt(m) + 1;
5
6       int an = 1;
7       for (int i = 0; i < n; ++i)
8           an = (an * 1ll * a) % m;
9
10      unordered_map<int, int> vals;
11      for (int q = 0, cur = b; q <= n; ++q) {
12          vals[cur] = q;
13          cur = (cur * 1ll * a) % m;
14      }
15
16      for (int p = 1, cur = 1; p <= n; ++p) {
17          cur = (cur * 1ll * an) % m;
18          if (vals.count(cur)) {
19              int ans = n * p - vals[cur];
20              return ans;
21          }
22      }
23      return -1;
24  }
25
26  // Returns minimum x for which a ^ x % m = b % m.
27  int solve(int a, int b, int m) {
28      a %= m, b %= m;
29      int k = 1, add = 0, g;
30      while ((g = gcd(a, m)) > 1) {
31          if (b == k)
32              return add;
33          if (b % g)
```

```
34            return -1;
35        b /= g, m /= g, ++add;
36        k = (k * 1ll * a / g) % m;
37    }
38
39    int n = sqrt(m) + 1;
40    int an = 1;
41    for (int i = 0; i < n; ++i)
42        an = (an * 1ll * a) % m;
43
44    unordered_map<int, int> vals;
45    for (int q = 0, cur = b; q <= n; ++q) {
46        vals[cur] = q;
47        cur = (cur * 1ll * a) % m;
48    }
49
50    for (int p = 1, cur = k; p <= n; ++p) {
51        cur = (cur * 1ll * an) % m;
52        if (vals.count(cur)) {
53            int ans = n * p - vals[cur] + add;
54            return ans;
55        }
56    }
57    return -1;
58 }
```

9.6   Divisors

```
1  ll numberOfDivisors(ll num) {
2    ll total = 1;
3    for (int i = 2; (ll)i * i <= num; i++) {
4      if (num % i == 0){
5        int e = 0;
6        do{
7          e++;
8          num /= i;
9        } while (num % i == 0);
10        total *= e + 1;
11      }
12    }
13    if (num > 1) total *= 2;
14    return total;
```

```
15 }
16
17 ll SumOfDivisors(ll num) {
18    ll total = 1;
19    for (int i = 2; (ll)i * i <= num; i++) {
20      if (num % i == 0) {
21        int e = 0;
22        do {
23          e++;
24          num /= i;
25        } while (num % i == 0);
26        ll sum = 0, pow = 1;
27        do {
28          sum += pow;
29          pow *= i;
30        } while (e-- > 0);
31        total *= sum;
32      }
33    }
34    if (num > 1) total *= (1 + num);
35    return total;
36 }
```

9.7   Euler Totient (Phi)

```
1  //counts coprimes to each number from 1 to n
2  vector<int> phi1(int n) {
3    vector<int> phi(n + 1);
4    for (int i = 0; i <= n; i++) phi[i] = i;
5    for (int i = 2; i <= n; i++) {
6      if (phi[i] == i) {
7        for (int j = i; j <= n; j += i)
8          phi[j] -= phi[j] / i;
9      }
10    }
11    return phi;
12 }
```

9.8   Fibonacci

```
1  // Fibonacci in O(log n)
2  void fib(ll n, ll&x, ll&y){
3    if(n==0){
4      x = 0;
```

```
5       y = 1;
6       return ;
7     }
8     if(n&1){
9       fib(n-1, y, x);
10      y=(y+x)%MOD;
11    } else {
12      ll a, b;
13      fib(n>>1, a, b);
14      y = (a*a+b*b)%MOD;
15      x = (a*b + a*(b-a+MOD))%MOD;
16    }
17  }
18  // Usage
19  ll x, y;
20  fib(10, x, y);
21  cout << x << "␣" << y << endl;
22  // This will output 55 89
```

9.9   Matrix Exponentiation

```
1   struct Mat {
2     int n, m;
3     vector<vector<int>> a;
4     Mat() { }
5     Mat(int _n, int _m) {n = _n; m = _m; a.assign(n, vector<int>(m, 0)); }
6     Mat(vector< vector<int> > v) { n = v.size(); m = n ? v[0].size() : 0;
          a = v; }
7     inline void make_unit() {
8       assert(n == m);
9       for (int i = 0; i < n; i++)  {
10        for (int j = 0; j < n; j++) a[i][j] = i == j;
11      }
12    }
13    inline Mat operator + (const Mat &b) {
14      assert(n == b.n && m == b.m);
15      Mat ans = Mat(n, m);
16      for(int i = 0; i < n; i++) {
17        for(int j = 0; j < m; j++) {
18          ans.a[i][j] = (a[i][j] + b.a[i][j]) % mod;
19        }
20      }
21      return ans;
22    }
23    inline Mat operator - (const Mat &b) {
24      assert(n == b.n && m == b.m);
25      Mat ans = Mat(n, m);
26      for(int i = 0; i < n; i++) {
27        for(int j = 0; j < m; j++) {
28          ans.a[i][j] = (a[i][j] - b.a[i][j] + mod) % mod;
29        }
30      }
31      return ans;
32    }
33    inline Mat operator * (const Mat &b) {
34      assert(m == b.n);
35      Mat ans = Mat(n, b.m);
36      for(int i = 0; i < n; i++) {
37        for(int j = 0; j < b.m; j++) {
38          for(int k = 0; k < m; k++) {
39            ans.a[i][j] = (ans.a[i][j] + 1LL * a[i][k] * b.a[k][j] % mod)
                % mod;
40          }
41        }
42      }
43      return ans;
44    }
45    inline Mat pow(long long k) {
46      assert(n == m);
47      Mat ans(n, n), t = a; ans.make_unit();
48      while (k) {
49        if (k & 1) ans = ans * t;
50        t = t * t;
51        k >>= 1;
52      }
53      return ans;
54    }
55    inline Mat& operator += (const Mat& b) { return *this = (*this) + b; }
56    inline Mat& operator -= (const Mat& b) { return *this = (*this) - b; }
57    inline Mat& operator *= (const Mat& b) { return *this = (*this) * b; }
58    inline bool operator == (const Mat& b) { return a == b.a; }
59    inline bool operator != (const Mat& b) { return a != b.a; }
60  };
61
62  // Usage
63  // Mat a(n, n);
```

```
64  // Mat b(n, n);
65  // Mat c = a * b;
66  // Mat d = a + b;
67  // Mat e = a - b;
68  // Mat f = a.pow(k);
69  // a.a[i][j] = x;
```

9.10   Miller Rabin Deterministic

```
1  using u64 = uint64_t;
2  using u128 = __uint128_t;
3
4  u64 binpower(u64 base, u64 e, u64 mod) {
5      u64 result = 1;
6      base %= mod;
7      while (e) {
8          if (e & 1)
9              result = (u128)result * base % mod;
10         base = (u128)base * base % mod;
11         e >>= 1;
12     }
13     return result;
14 }
15
16 bool check_composite(u64 n, u64 a, u64 d, int s) {
17     u64 x = binpower(a, d, n);
18     if (x == 1 || x == n - 1)
19         return false;
20     for (int r = 1; r < s; r++) {
21         x = (u128)x * x % n;
22         if (x == n - 1)
23             return false;
24     }
25     return true;
26 };
27
28
29 bool MillerRabin(ll n) {
30     if (n < 2)
31         return false;
32
33     int r = 0;
34     ll d = n - 1;
```

```
35         while ((d & 1) == 0) {
36             d >>= 1;
37             r++;
38         }
39
40         for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
41             if (n == a)
42                 return true;
43             if (check_composite(n, a, d, r))
44                 return false;
45         }
46         return true;
47 }
```

9.11   Mobius

```
1  int mob[N];
2  void mobius() {
3    mob[1] = 1;
4    for (int i = 2; i < N; i++){
5      mob[i]--;
6      for (int j = i + i; j < N; j += i) {
7        mob[j] -= mob[i];
8      }
9    }
10 }
```

9.12   Permutation Rank

```
1  // Compute factorial (up to reasonable limits)
2  long long fact(int n) {
3      static vector<long long> f(21, -1); // cache up to 20!
4      if (n <= 1) return 1;
5      if (f[n] != -1) return f[n];
6      return f[n] = n * fact(n - 1);
7  }
8
9  // Compute 0-indexed lexicographic rank of a permutation
10 long long permutationRank0(const vector<int>& perm) {
11     int n = perm.size();
12     long long rank = 0;
13     vector<int> used(n + 1, 0);
14
15     for (int i = 0; i < n; ++i) {
```

```cpp
16          int smaller_unused = 0;
17          for (int j = 1; j < perm[i]; ++j) {
18              if (!used[j]) smaller_unused++;
19          }
20          rank += smaller_unused * fact(n - i - 1);
21          used[perm[i]] = 1;
22      }
23      return rank;
24  }
25
26  // Given n and rank (0-indexed), reconstruct the permutation
27  vector<int> permutationUnrank0(int n, long long rank) {
28      vector<int> elems(n);
29      iota(elems.begin(), elems.end(), 1); // {1, 2, 3, ..., n}
30      vector<int> perm;
31
32      for (int i = n; i >= 1; --i) {
33          long long f = fact(i - 1);
34          int idx = rank / f;
35          rank %= f;
36          perm.push_back(elems[idx]);
37          elems.erase(elems.begin() + idx);
38      }
39
40      return perm;
41  }
```

9.13   Prefix Sum Phi

```cpp
1  vector<ll> sieve(kMaxV + 1,0);
2  vector<ll> phi(kMaxV + 1,0);
3
4  void primes()
5  {
6    phi[1]=1;
7    vector<ll> pr;
8    for(int i=2;i<kMaxV;i++){
9      if(sieve[i]==0){
10       sieve[i]=i;
11       pr.pb(i);
12       phi[i]=i-1;
13     }
14     for(auto p:pr){
```

```cpp
15       if(p>sieve[i]||i*p>=kMaxV)break;
16       sieve[i*p]=p;
17       phi[i*p]=(p==sieve[i]?p:p-1)*phi[i];
18     }
19   }
20   for(int i=1;i<kMaxV;i++){
21     phi[i]+=phi[i-1];
22     phi[i]%=MOD;
23   }
24  }
25
26  map<ll,ll> m;
27  ll PHI(ll a){
28    if(a<kMaxV)return phi[a];
29    if(m.count(a))return m[a];
30    // if(a<3)return 1;
31    m[a]=((((a%MOD)*((a+1)%MOD))%MOD)*inverse(2));
32    m[a]%=MOD;
33    long long i=2;
34    while(i<=a){
35        long long j=a/i;
36        j=a/j;
37        m[a]+=MOD;
38        m[a]-=((j-i+1)*PHI(a/i))%MOD;
39        m[a]%=MOD;
40        i=j+1;
41    }
42    m[a]%=MOD;
43    return m[a];
44  }
```

9.14   Sieve

```cpp
1  const int kMaxV = 1e6;
2
3  int sieve[kMaxV + 1];
4
5  //stores some prime (not necessarly the minimum one)
6  void primes()
7  {
8    for (int i = 4; i <= kMaxV; i += 2)
9      sieve[i] = 2;
10   for (int i = 3; i <= kMaxV / i; i += 2)
```

```
11      {
12        if (sieve[i])
13          continue;
14        for (int j = i * i; j <= kMaxV; j += i)
15          sieve[j] = i;
16      }
17    }
18
19    vector<int> PrimeFactors(int x)
20    {
21      if (x == 1)
22        return {};
23
24      unordered_set<int> primes;
25      while (sieve[x])
26      {
27        primes.insert(sieve[x]);
28        x /= sieve[x];
29      }
30      primes.insert(x);
31      return {primes.begin(), primes.end()};
32    }
```

9.15   Identities

$$C_n = \frac{2(2n-1)}{n+1} C_{n-1}$$
$$C_n = \frac{1}{n+1}\binom{2n}{n}$$
$$C_n \sim \frac{4^n}{n^{3/2}\sqrt{\pi}}$$
$$\sigma(n) = O(\log(\log(n)))\ \text{(number of divisors of } n)$$
$$F_{2n+1} = F_n^2 + F_{n+1}^2$$
$$F_{2n} = F_{n+1}^2 - F_{n-1}^2$$
$$\sum_{i=1}^n F_i = F_{n+2} - 1$$
$$F_{n+i}F_{n+j} - F_n F_{n+i+j} = (-1)^n F_i F_j$$

(Möbius Inv. Formula) $\mu(p^k) = [k = 0] - [k = 1]$ Let $g(n) = \sum_{d|n} f(d)$, then
$$f(n) = \sum_{d|n} g(d)\mu\left(\frac{n}{d}\right).$$
(Dirichlet Convolution) Let f, g be arithmetic functions, then
$$(f * g)(n) = \sum_{d|n} f(d)g\left(\frac{n}{d}\right). \text{ If } f, g \text{ are multiplicative, then so is } f * g.$$
$$n = \sum_{d|n} \phi(d)$$
Lucas' Theorem: $\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$ where $m = \sum_{i=0}^k m_i p^i$ and
$$n = \sum_{i=0}^k n_i p^i.$$

9.16   Burnside's Lemma

Dado un grupo G de permutaciones y un conjunto X de n elementos, el número de órbitas de X bajo la acción de G es igual al promedio del número de puntos fijos de las permutaciones en G.

Formalmente, el número de órbitas es $\frac{1}{|G|}\sum_{g\in G} f(g)$ donde $f(g)$ es el número de puntos fijos de g.

Ejemplo: Dado un collar con n cuentas y 2 colores, el número de collares diferentes que se pueden formar es $\frac{1}{n}\sum_{i=0}^n f(i)$ donde $f(i)$ es el número de collares que quedan fijos bajo una rotación de i posiciones.

Para contar el número de collares que quedan fijos bajo una rotación de i posiciones, se puede usar la fórmula $f(i) = 2^{\gcd(i,n)}$.

Para un collar de n cuentas y k colores, el número de collares diferentes que se pueden formar es $\frac{1}{n}\sum_{i=0}^n k^{\gcd(i,n)}$

Ejemplo: Dado un cubo con 6 caras y k colores, el número de cubos diferentes que se pueden formar es $\frac{1}{24}\sum_{i=0}^{24} f(i)$ donde $f(i)$ es el número de cubos que quedan fijos bajo una rotación de i posiciones. Esta formula es igual a $\frac{1}{24}(n^6 + 3n^4 + 12n^3 + 8n^2)$

9.17   Recursion

Sea $f(n) = \sum_{i=1}^k a_i f(n-i)$ entonces podemos considerar la matriz:

$$\begin{bmatrix} f(n) \\ f(n-1) \\ \vdots \\ f(n-k+1) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \cdots & a_{k-1} & a_k \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} f(n-1) \\ f(n-2) \\ \vdots \\ f(n-k) \end{bmatrix}$$

De aqui podemos calcular $f(n)$ con exponenciación de matrices.

$$\begin{bmatrix} f(n) \\ f(n-1) \\ \vdots \\ f(n-k+1) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \cdots & a_{k-1} & a_k \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}^{n-k} \begin{bmatrix} f(k) \\ f(k-1) \\ \vdots \\ f(1) \end{bmatrix}$$

9.18   Theorems

Koeing's Theorem: La cardinalidad del emparejamiento maximo de una grafica bipartita es igual al minimum vertex cover.

Hall's Theorem: Una grafica bipartita G tiene un emparejamiento que cubre todos los nodos de G si y solo si para todo subconjunto S de nodos de G, el número de vecinos de S es mayor o igual a $|S|$.

Kuratowski's Theorem: Una grafica es plana si y solo si no contiene un subgrafo homeomorfo a $K_{3,3}$ o K_5.

9.19   Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

9.20   Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \ C_{n+1} = \frac{2(2n+1)}{n+2}C_n, \ C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$$

- sub-diagonal monotone paths in an $n \times n$ grid.

- strings with n pairs of parenthesis, correctly nested. If prefix is given, number of ways is $\binom{n}{remaining_closed}$-$\binom{n}{remaining_closed+1}$.

- binary trees with with $n+1$ leaves (0 or 2 children).

- ordered trees with $n+1$ vertices.

- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.

- permutations of $[n]$ with no 3-term increasing subseq.

9.21   Cayley's formula

Number of labeled trees of n vertices is n^{n-2}. Number of rooted forest of n vertices is $(n+1)^{n-1}$.

9.22   Geometric series

Infinite
$$a + ar + ar^2 + ar^3 + \ldots\ldots + \sum_{k=0}^{\infty} ar^k$$
$$\text{Sum} = \frac{a}{1-r}$$
Finite
$$a + ar + ar^2 + ar^3 + \ldots\ldots + \sum_{k=0}^{n} ar^k$$
$$\text{Sum} = \frac{a(1-r^{n+1})}{1-r}$$

9.23   Estimates For Divisors

$$\sum_{d|n} d = O(n \log \log n).$$

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, $200\,000$ for $n < 1e19$.

9.24   Sum of divisors

$$\sum d|n = \frac{p_1^{\alpha_1+1}-1}{p_1-1} + \frac{p_2^{\alpha_2+1}-1}{p_2-1} + \ldots + \frac{p_n^{\alpha_n+1}-1}{p_n-1}$$

9.25   Pythagorean Triplets

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \ \ b = k \cdot (2mn), \ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

9.26   Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

10   Game Theory

10.1   Sprague-Grundy theorem

https://codeforces.com/blog/entry/66040 Dado un juego con pilas p_1, p_2, ..., p_n sea $g(p)$ el nimber de la pila p, entonces el nimber del juego es $g(p_1) \oplus g(p_2) \oplus \ldots \oplus g(p_n)$. Para calcular el nimber de una pila, se puede usar la fórmula $g(r) = mex(\{g(r_1), g(r_2), ..., g(r_k)\})$ donde $r_1, r_2, ..., r_k$ son los posibles estados a los que se puede llegar desde r y $g(r) = 0$ si r es un estado perdedor.

11   Fórmulas y notas

11.1   Números de Stirling del primer tipo

$\left[{n \atop k}\right]$ representa el número de permutaciones de n elementos en exactamente k ciclos disjuntos.

$$\left[{0 \atop 0}\right] = 1$$

$$\left[{0 \atop n}\right] = \left[{n \atop 0}\right] = 0 \qquad\qquad , \quad n > 0$$

$$\left[{n \atop k}\right] = (n-1)\left[{n-1 \atop k}\right] + \left[{n-1 \atop k-1}\right] \qquad , \quad k > 0$$

$$\sum_{k=0}^{n} \left[{n \atop k}\right] = n!$$

$$\sum_{k=0}^{\infty} \left[{n \atop k}\right] x^k = \prod_{k=0}^{n-1} (x+k)$$

11.2   Números de Stirling del segundo tipo

$\left\{{n \atop k}\right\}$ representa el número de formas de particionar un conjunto de n objetos distinguibles en k subconjuntos no vacíos.

$$\left\{{0 \atop 0}\right\} = 1$$

$$\left\{{0 \atop n}\right\} = \left\{{n \atop 0}\right\} = 0 \qquad\qquad , \quad n > 0$$

$$\left\{{n \atop k}\right\} = k\left\{{n-1 \atop k}\right\} + \left\{{n-1 \atop k-1}\right\} \qquad , \quad k > 0$$

$$= \sum_{j=0}^{k} \frac{j^n}{j!} \cdot \frac{(-1)^{k-j}}{(k-j)!}$$

11.3   Números de Euler

$\left\langle\!\left\langle {n \atop k} \right\rangle\!\right\rangle$ representa el número de permutaciones de 1 a n en donde exactamente k números son mayores que el número anterior, es decir, cuántas permutaciones tienen k "ascensos".

$$\left\langle\!\left\langle {1 \atop 0} \right\rangle\!\right\rangle = 1$$

$$\left\langle\!\left\langle {n \atop k} \right\rangle\!\right\rangle = (n-k)\left\langle\!\left\langle {n-1 \atop k-1} \right\rangle\!\right\rangle + (k+1)\left\langle\!\left\langle {n-1 \atop k} \right\rangle\!\right\rangle \qquad , \quad n \geq 2$$

$$= \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

$$\sum_{k=0}^{n-1} \left\langle\!\left\langle {n \atop k} \right\rangle\!\right\rangle = n!$$

11.4   Números de Catalan

$$C_0 = 1$$

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \sum_{j=0}^{n-1} C_j C_{n-1-j}$$

$$\sum_{n=0}^{\infty} C_n x^n = \frac{1 - \sqrt{1-4x}}{2x}$$

11.5   Números de Bell

B_n representa el número de formas de particionar un conjunto de n elementos.

$$B_n = \sum_{k=0}^{n} \left\{{n \atop k}\right\} = \sum_{k=0}^{n-1} \binom{n-1}{k} B_k$$

$$\sum_{n=0}^{\infty} \frac{B_n}{n!} x^n = e^{e^x - 1}$$

11.6   Números de Bernoulli

$$B_0{}^+ = 1$$

$$B_n{}^+ = 1 - \sum_{k=0}^{n-1} \binom{n}{k} \frac{B_k{}^+}{n-k+1}$$

$$\sum_{n=0}^{\infty} \frac{B_n{}^+ x^n}{n!} = \frac{x}{1 - e^{-x}} = \frac{1}{\frac{1}{1!} - \frac{x}{2!} + \frac{x^2}{3!} - \frac{x^3}{4!} + \cdots}$$

11.7   Fórmula de Faulhaber

$$S_p(n) = \sum_{k=1}^{n} k^p = \frac{1}{p+1} \sum_{k=0}^{p} \binom{p+1}{k} B_k{}^+ n^{p+1-k}$$

11.8   Función Beta

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} = 2\int_0^{\pi/2} \sin^{2x-1}(\theta)\cos^{2x-1}(\theta)d\theta$$

$$= \int_0^1 t^{x-1}(1-t)^{y-1}dt = \int_0^\infty \frac{t^{x-1}}{(1+t)^{x+y}}dt$$

11.9   Función zeta de Riemann

La siguiente fórmula converge rápido para valores pequeños de n ($n \approx 20$):

$$\zeta(s) \approx \frac{1}{d_0(1-2^{1-s})} \sum_{k=1}^{n} \frac{(-1)^{k-1}d_k}{k^s}$$

$$d_k = \sum_{j=k}^{n} \frac{4^j}{n+j} \binom{n+j}{2j}$$

11.10   Funciones generadoras

$$\sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} a_k \right) x^n = \frac{1}{1-x} \sum_{n=0}^{\infty} a_n x^n$$

$$\sum_{n=0}^{\infty} \binom{n+k-1}{k-1} x^n = \frac{1}{(1-x)^k}$$

$$\sum_{n=0}^{\infty} p_n x^n = \frac{1}{\displaystyle\prod_{k=1}^{\infty}(1-x^k)} = \frac{1}{\displaystyle\sum_{n=-\infty}^{\infty}(-1)^n x^{\frac{1}{2}n(3n+1)}}$$

$$\sum_{p=0}^{\infty} \frac{S_p(n)}{p!} x^p = \frac{e^{x(n+1)} - e^x}{e^x - 1}$$

$$\sum_{n=0}^{\infty} n^k x^n = \frac{\displaystyle\sum_{i=0}^{k-1} \left\langle\!\!\left\langle \begin{matrix} k \\ i \end{matrix} \right\rangle\!\!\right\rangle x^{i+1}}{(1-x)^{k+1}} \quad , \quad k \geq 1$$

Sean $a_1, a_2, \ldots, a_n$ números complejos. Sean $p_m = \sum_{i=1}^{n} a_i^m$ y s_m el m-ésimo polinomio elemental simétrico de $a_1, a_2, \ldots, a_n$. Entonces se cumple que $xS'(x) + P(x)S(x) = 0$, donde $P(x) = \sum_{m=1}^{\infty} p_m x^m$ y $S(x) = \prod_{i=1}^{n}(1-a_i x) = \sum_{m=0}^{n}(-1)^m s_m x^m$.

11.11   Números armónicos

$$H_n = \sum_{k=1}^{n} \frac{1}{k} \approx \ln(n) + \gamma + \frac{1}{2n} - \frac{1}{12n^2}$$

$$\gamma \approx 0.5772156649015328606066512$$

11.12   Aproximación de Stirling

$$\ln(n!) \approx n\ln(n) - n + \frac{1}{2}\ln(2\pi n)$$

$$\# \text{ de dígitos de } n! = 1 + \left\lfloor n\log\left(\frac{n}{e}\right) + \frac{1}{2}\log(2\pi n) \right\rfloor \quad (n \geq 30)$$

11.13   Ternas pitagóricas

- Una terna de enteros positivos (a, b, c) es pitagórica si $a^2 + b^2 = c^2$. Además es primitiva si $\gcd(a, b, c) = 1$.

- Generador de ternas primitivas:

$$a = m^2 - n^2$$
$$b = 2mn$$
$$c = m^2 + n^2$$

donde $n \geq 1$, $m > n$, $\gcd(m, n) = 1$ y m, n tienen distinta paridad.

- Árbol de ternas pitagóricas primitivas: al multiplicar cualquiera de estas matrices:

$$\begin{pmatrix} 1 & -2 & 2 \\ 2 & -1 & 2 \\ 2 & -2 & 3 \end{pmatrix} \quad , \quad \begin{pmatrix} -1 & 2 & 2 \\ -2 & 1 & 2 \\ -2 & 2 & 3 \end{pmatrix} \quad , \quad \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{pmatrix}$$

por una terna primitiva $\mathbf{v^T}$, obtenemos otra terna primitiva diferente. En particular, si empezamos con $\mathbf{v} = (3, 4, 5)$, podremos generar todas las ternas primitivas.

11.14   Árbol de Stern–Brocot

Todos los racionales positivos se pueden representar como un árbol binario de búsqueda completo infinito con raíz $\frac{1}{1}$.

- Dado un racional $q = [a_0; a_1, a_2, \ldots, a_k]$ donde $a_k \neq 1$, sus hijos serán $[a_0; a_1, a_2, \ldots, a_k+1]$ y $[a_0; a_1, a_2, \ldots, a_k-1, 2]$, y su padre será $[a_0; a_1, a_2, \ldots, a_k-1]$.

- Para hallar el camino de la raíz $\frac{1}{1}$ a un racional q, se usa búsqueda binaria iniciando con $L = \frac{0}{1}$ y $R = \frac{1}{0}$. Para hallar M se supone que $L = \frac{a}{b}$ y $R = \frac{c}{d}$, entonces $M = \frac{a+c}{b+d}$.

11.15   Combinatoria

- Principio de las casillas: al colocar n objetos en k lugares hay al menos $\lceil \frac{n}{k} \rceil$ objetos en un mismo lugar.

- Número de funciones: sean A y B conjuntos con $m = |A|$ y $n = |B|$. Sea $f : A \to B$:

    - Si $m \leq n$, entonces hay $m!\binom{n}{m}$ funciones inyectivas f.

    - Si $m = n$, entonces hay $n!$ funciones biyectivas f.

    - Si $m \geq n$, entonces hay $n!\left\{{m \atop n}\right\}$ funciones suprayectivas f.

- Barras y estrellas: ¿cuántas soluciones en los enteros no negativos tiene la ecuación $\sum_{i=1}^{k} x_i = n$? Tiene $\binom{n+k-1}{k-1}$ soluciones.

- ¿Cuántas soluciones en los enteros positivos tiene la ecuación $\sum_{i=1}^{k} x_i = n$? Tiene $\binom{n-1}{k-1}$ soluciones.

- Desordenamientos: $a_0 = 1$, $a_1 = 0$, $a_n = (n-1)(a_{n-1} + a_{n-2}) = na_{n-1} + (-1)^n$.

- Sea $f(x)$ una función. Sea $g_n(x) = xg'_{n-1}(x)$ con $g_0(x) = f(x)$. Entonces $g_n(x) = \sum_{k=0}^{n} \left\{{n \atop k}\right\} x^k f^{(k)}(x)$.

- Supongamos que tenemos $m+1$ puntos: $(0, y_0)$, $(1, y_1)$, ..., (m, y_m). Entonces el polinomio $P(x)$ de grado m que pasa por todos ellos es:

$$P(x) = \left[\prod_{i=0}^{m}(x-i)\right](-1)^m \sum_{i=0}^{m} \frac{y_i(-1)^i}{(x-i)i!(m-i)!}$$

- Sea $a_0, a_1, \ldots$ una recurrencia lineal homogénea de grado d dada por $a_n = \sum_{i=1}^{d} b_i a_{n-i}$ para $n \geq d$ con términos iniciales $a_0, a_1, \ldots, a_{d-1}$. Sean $A(x)$ y $B(x)$ las funciones generadoras de las sucesiones a_n y b_n respectivamente, entonces se cumple que $A(x) = \frac{A_0(x)}{1 - B(x)}$, donde $A_0(x) = \sum_{i=0}^{d-1}\left[a_i - \sum_{j=0}^{i-1} a_j b_{i-j}\right]x^i$.

- Si queremos obtener otra recurrencia c_n tal que $c_n = a_{kn}$, las raíces del polinomio característico de c_n se obtienen al elevar todas las raíces del polinomio característico de a_n a la k-ésima potencia; y sus términos iniciales serán $a_0, a_k, \ldots, a_{k(d-1)}$.

11.16   Grafos

- Sea d_n el número de grafos con n vértices etiquetados: $d_n = 2^{\binom{n}{2}}$.

- Sea c_n el número de grafos conexos con n vértices etiquetados. Tenemos la recurrencia: $c_1 = 1$ y $d_n = \sum_{k=1}^{n}\binom{n-1}{k-1}c_k d_{n-k}$. También se cumple, usando funciones generadoras exponenciales, que $C(x) = 1 + \ln(D(x))$.

- Sea t_n el número de torneos fuertemente conexos en n nodos etiquetados. Tenemos la recurrencia $t_1 = 1$ y $d_n = \sum_{k=1}^{n}\binom{n}{k}t_k d_{n-k}$. Usando funciones generadoras exponenciales, tenemos que $T(x) = 1 - \frac{1}{D(x)}$.

- Número de spanning trees en un grafo completo con n vértices etiquetados: n^{n-2}.

- Número de bosques etiquetados con n vértices y k componentes conexas: kn^{n-k-1}.

- Para un grafo no dirigido simple G con n vértices etiquetados de 1 a n, sea $Q = D - A$, donde D es la matriz diagonal de los grados de cada nodo de G y A es la matriz de adyacencia de G. Entonces el número de spanning trees de G es igual a cualquier cofactor de Q.

- Sea G un grafo. Se define al polinomio $P_G(x)$ como el polinomio cromático de G, en donde $P_G(k)$ nos dice cuántas k-coloraciones de los vértices admite G. Ejemplos comunes:

    - Grafo completo de n nodos: $P(x) = x(x-1)(x-2)\ldots(x-(n-1))$
    - Grafo vacío de n nodos: $P(x) = x^n$
    - Árbol de n nodos: $P(x) = x(x-1)^{n-1}$
    - Ciclo de n nodos: $P(x) = (x-1)^n + (-1)^n(x-1)$

11.17 Teoría de números

$$(f * e)(n) = f(n)$$
$$(\varphi * \mathbf{1})(n) = n$$
$$(\mu * \mathbf{1})(n) = e(n)$$
$$\varphi(n^k) = n^{k-1}\varphi(n)$$

$$\sum_{\substack{k=1 \\ \gcd(k,n)=1}}^{n} k = \frac{n\varphi(n)}{2} \quad , \quad n \geq 2$$

$$\sum_{k=1}^{n} \mathrm{lcm}(k,n) = \frac{n}{2} + \frac{n}{2}\sum_{d|n} d\varphi(d) = \frac{n}{2} + \frac{n}{2}\prod_{p^a|n}\frac{p^{2a+1}+1}{p+1}$$

$$\sum_{k=1}^{n} \gcd(k,n) = \sum_{d|n} d\varphi\left(\frac{n}{d}\right) = \prod_{p^a|n} p^{a-1}(1+(a+1)(p-1))$$

- Lifting the exponent: sea p un primo, x,y enteros y n un entero positivo tal que $p \mid x - y$ pero $p \nmid x$ ni $p \nmid y$. Entonces:
  - Si p es impar: $v_p(x^n - y^n) = v_p(x - y) + v_p(n)$
  - Si $p = 2$ y n es par: $v_p(x^n - y^n) = v_p(x - y) + v_p(n) + v_p(x + y) - 1$

  donde $v_p(n)$ es el exponente de p en la factorización en primos de n.

- Suma de dos cuadrados: sea $\chi_4(n)$ una función multiplicativa igual a 1 si $n \equiv 1 \mod 4$, -1 si $n \equiv 3 \mod 4$ y cero en otro caso. Entonces, el número de soluciones enteras (a,b) de la ecuación $a^2 + b^2 = n$ es $4(\chi_4 * 1)(n) = 4\sum_{d|n}\chi_4(d)$.

- Teorema de Lucas:
$$\binom{m}{n} \equiv \prod_{i=0}^{k}\binom{m_i}{k_i} \pmod{p}$$
$$m = \sum_{i=0}^{k} m_i p^i \quad , \quad n = \sum_{i=0}^{k} n_i p^i$$
$$0 \leq m_i, n_i < p$$

- Sean $a,b,c \in \mathbb{Z}$ con $a \neq 0$ y $b \neq 0$. La ecuación $ax + by = c$ tiene como soluciones:
$$x = \frac{x_0 c - bk}{d}$$
$$y = \frac{y_0 c + ak}{d}$$

para toda $k \in \mathbb{Z}$ si y solo si $d|c$, donde $ax_0 + by_0 = \gcd(a,b) = d$ (Euclides extendido). Si a y b tienen el mismo signo, hay exactamente $\max\left(\left\lfloor\frac{x_0 c}{|b|}\right\rfloor + \left\lfloor\frac{y_0 c}{|a|}\right\rfloor + 1, 0\right)$ soluciones no negativas. Si tienen el signo distinto, hay infinitas soluciones no negativas.

- Dada una función aritmética f con $f(1) \neq 0$, existe otra función aritmética g tal que $(f * g)(n) = e(n)$, dada por:

$$g(1) = \frac{1}{f(1)}$$
$$g(n) = -\frac{1}{f(1)}\sum_{d|n,d<n} f\left(\frac{n}{d}\right)g(d) \quad , \quad n > 1$$

- Sean $h(n) = \sum_{k=1}^{n} f\left(\left\lfloor\frac{n}{k}\right\rfloor\right)g(k)$, $G(n) = \sum_{k=1}^{n} g(k)$ y $m = \lfloor\sqrt{n}\rfloor$, entonces:

$$h(n) = \sum_{k=1}^{\lfloor n/m\rfloor} f\left(\left\lfloor\frac{n}{k}\right\rfloor\right)g(k) + \sum_{k=1}^{m-1}\left(G\left(\left\lfloor\frac{n}{k}\right\rfloor\right) - G\left(\left\lfloor\frac{n}{k+1}\right\rfloor\right)\right)f(k)$$

- Sean $F(n) = \sum_{k=1}^{n} f(k)$, $G(n) = \sum_{k=1}^{n} g(k)$, $h(n) = (f * g)(n) = \sum_{d|n} f(d)g\left(\frac{n}{d}\right)$ y $H(n) = \sum_{k=1}^{n} h(k)$, entonces:

$$H(n) = \sum_{k=1}^{n} f(k)G\left(\left\lfloor\frac{n}{k}\right\rfloor\right)$$

- Sean $\Phi_p(n) = \sum_{k=1}^{n} k^p \varphi(k)$ y $M_p(n) = \sum_{k=1}^{n} k^p \mu(k)$. Aplicando lo anterior, podemos calcular $\Phi_p(n)$ y $M_p(n)$ con complejidad $O(n^{2/3})$ si precalculamos con fuerza bruta los primeros $\lfloor n^{2/3}\rfloor$ valores, y para los demás, usamos las siguientes recurrencias (DP con `map`):

$$\Phi_p(n) = S_{p+1}(n) - \sum_{k=2}^{\lfloor n/m\rfloor} k^p \Phi_p\left(\left\lfloor\frac{n}{k}\right\rfloor\right) - \sum_{k=1}^{m-1}\left(S_p\left(\left\lfloor\frac{n}{k}\right\rfloor\right) - S_p\left(\left\lfloor\frac{n}{k+1}\right\rfloor\right)\right)\Phi_p(k)$$

$$M_p(n) = 1 - \sum_{k=2}^{\lfloor n/m\rfloor} k^p M_p\left(\left\lfloor\frac{n}{k}\right\rfloor\right) - \sum_{k=1}^{m-1}\left(S_p\left(\left\lfloor\frac{n}{k}\right\rfloor\right) - S_p\left(\left\lfloor\frac{n}{k+1}\right\rfloor\right)\right)M_p(k)$$

- En general, si queremos hallar $F(n)$ y existe una función mágica $g(n)$ tal que $G(n)$ y $H(n)$ se puedan calcular en $O(1)$, entonces:

$$F(n) = \frac{1}{g(1)} \left[H(n) - \sum_{k=2}^{\lfloor n/m \rfloor} g(k) F\left(\left\lfloor \frac{n}{k} \right\rfloor \right) - \sum_{k=1}^{m-1} \left(G\left(\left\lfloor \frac{n}{k} \right\rfloor \right) - G\left(\left\lfloor \frac{n}{k+1} \right\rfloor \right) \right) F(k) \right]$$

11.18 Primos

$10^2 + 1$, $10^3 + 9$, $10^4 + 7$, $10^5 + 3$, $10^6 + 3$, $10^7 + 19$, $10^8 + 7$, $10^9 + 7$, $10^{10} + 19$, $10^{11} + 3$, $10^{12} + 39$, $10^{13} + 37$, $10^{14} + 31$, $10^{15} + 37$, $10^{16} + 61$, $10^{17} + 3$, $10^{18} + 3$. $10^2 - 3$, $10^3 - 3$, $10^4 - 27$, $10^5 - 9$, $10^6 - 17$, $10^7 - 9$, $10^8 - 11$, $10^9 - 63$, $10^{10} - 33$, $10^{11} - 23$, $10^{12} - 11$, $10^{13} - 29$, $10^{14} - 27$, $10^{15} - 11$, $10^{16} - 63$, $10^{17} - 3$, $10^{18} - 11$.

11.19 Números primos de Mersenne

Números primos de la forma $M_p = 2^p - 1$ con p primo. Todos los números perfectos pares son de la forma $2^{p-1} M_p$ y viceversa.
Los primeros 47 valores de p son: 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941, 11213, 19937, 21701, 23209, 44497, 86243, 110503, 132049, 216091, 756839, 859433, 1257787, 1398269, 2976221, 3021377, 6972593, 13466917, 20996011, 24036583, 25964951, 30402457, 32582657, 37156667, 42643801, 43112609.

11.20 Números primos de Fermat

Números primos de la forma $F_p = 2^{2^p} + 1$, solo se conocen cinco: 3, 5, 17, 257, 65537.
Un polígono de n lados es construible si y solo si n es el producto de algunas potencias de dos y distintos primos de Fermat.

12 More Topics

12.1 2D Prefix Sum

```
int b[MAXN][MAXN];
int a[MAXN][MAXN];
for (int i = 1; i <= N; i++) {
  for (int j = 1; j <= N; j++) {
    b[i][j] = a[i][j] + b[i - 1][j] +
              b[i][j - 1] - b[i - 1][j - 1];
  }
}
for (int q = 0; q < Q; q++) {
  int from_row, to_row, from_col, to_col;
  cin >> from_row >> from_col >> to_row >> to_col;
  cout << b[to_row][to_col] - b[from_row - 1][to_col] -
          b[to_row][from_col - 1] +
          b[from_row - 1][from_col - 1]
       << '\n';
}
```

12.2 CDQ D&C FFT

12.3 CDQ Divide and Conquer

```
// given x[i], y[i], z[i]
// count for each i number of j such that x[j] > x[i] && y[j] > x[i] &&
    z[j] > z[i]

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;

struct Pt {
  int x, y, z, i;
};

const int N = (int)1e5 + 5;

struct BIT {
  int b[N], n;

  void init(int _n) {
    n = _n;
    for(int i = 0 ; i <= n ; ++i) b[i] = 0;
  }
  inline int lowbit(int x) { return x & (-x); }
  void update(int x, int v) {
    for(int i = x ; i <= n ; i += lowbit(i)) b[i] += v;
  }
  int query(int x) {
    int ans = 0;
    for(int i = x ; i > 0 ; i -= lowbit(i)) ans += b[i];
    return ans;
  }
} bit;
```

```
31
32   vector<Pt> v;
33   int n, ans[N];
34
35   void cdq(int l, int r) {
36     if(l + 1 == r) return;
37     int m = (l + r) >> 1;
38     cdq(l, m); cdq(m, r);
39     int a = l, b = m, sum = 0;
40     // need to record the modifications on BIT in order to reset it. The
            complexity will be $O(N^2)$ if we resetting it brute-forcely.
41     vector<int> record;
42     // temporary array for merge sort
43     vector<Pt> tmp;
44     while(a < m && b < r) {
45       if(v[a].y > v[b].y) bit.update(v[a].z, 1), sum++, record.push_back(v
              [a].z), tmp.push_back(v[a++]);
46       else ans[v[b].i] += sum - bit.query(v[b].z), tmp.push_back(v[b++]);
47     }
48     while(a < m) tmp.push_back(v[a++]);
49     while(b < r) ans[v[b].i] += sum - bit.query(v[b].z), tmp.push_back(v[b
            ++]);
50     for(int i = l ; i < r ; ++i) v[i] = tmp[i - l];
51     // reset BIT
52     for(auto i : record) bit.update(i, -1);
53     // release used memory
54     vector<int> ().swap(record);
55     vector<Pt> ().swap(tmp);
56   }
57
58   void init() {
59     cin >> n;
60     for(int i = 0 ; i < n ; ++i) {
61       int a, b, c; cin >> a >> b >> c;
62       v.push_back({a, b, c, i});
63     }
64     bit.init(n);
65   }
66   void solve() {
67     // As we require > not >=
68     sort(v.begin(), v.end(), [&](auto a, auto b) {
69         return (a.x == b.x ? (a.y == b.y ? a.z < b.z : a.y < b.y) : a.x >
              b.x);
70     });
71     cdq(0, n);
72     for(int i = 0 ; i < n ; ++i) cout << ans[i] << '\n';
73   }
74
75   int main() {
76     ios_base::sync_with_stdio(0), cin.tie(0);
77     init();
78     solve();
79   }
80
81
82
83
84
85
86   #include <iostream>
87   #include <vector>
88   #define F first
89   #define S second
90   using namespace std;
91   typedef long long ll;
92
93   const int N = (int)2e6 + 5;
94
95   namespace bit {
96     ll b[N], n;
97
98     void init(int _n) {
99       n = _n;
100      fill(b, b + n + 1, 0);
101    }
102    inline int lowbit(int x) { return x & (-x); }
103    void update(int x, int v) {
104      if(x == 0) return;
105      for(int i = x ; i <= n ; i += lowbit(i)) b[i] += v;
106    }
107    ll query(int x) {
108      ll ans = 0;
109      for(int i = x ; i > 0 ; i -= lowbit(i)) ans += b[i];
110      return ans;
111    }
112  }
```

```
113   struct Node {
114     int t, x, y, v, i, sgn;
115   };
116
117   vector<Node> v;
118   vector<ll> ans;
119   int s, w;
120
121   void cdq(int l, int r) {
122     if(l + 1 == r) return;
123     int m = (l + r) >> 1;
124     cdq(l, m); cdq(m, r);
125     vector<Node> tmp;
126     vector<pair<int, ll> > his;
127     int a = l, b = m;
128     while(a < m && b < r) {
129       if(v[a].x <= v[b].x) {
130         bit::update(v[a].y, v[a].v);
131         his.push_back({v[a].y, -v[a].v});
132         tmp.push_back(v[a++]);
133       }
134       else {
135         ans[v[b].i] += v[b].sgn * (bit::query(v[b].y) + (ll)s * v[b].x * v
                [b].y);
136         tmp.push_back(v[b++]);
137       }
138     }
139     while(a < m) tmp.push_back(v[a++]);
140     while(b < r) {
141       ans[v[b].i] += v[b].sgn * (bit::query(v[b].y) + (ll)s * v[b].x * v[b
              ].y);
142       tmp.push_back(v[b++]);
143     }
144     for(int i = l ; i < r ; ++i) v[i] = tmp[i - l];
145     for(int i = 0 ; i < (int)his.size() ; ++i) bit::update(his[i].F, his[i
              ].S);
146     vector<Node> ().swap(tmp);
147     vector<pair<int ,ll> > ().swap(his);
148   }
149
150   void init() {
151     cin >> s >> w;
152     bit::init(w);
```

```
153     v.clear(); ans.clear();
154     ans.push_back(0);
155     int op, i = 0; cin >> op;
156     while(op != 3) {
157       if(op == 1) {
158         int x, y, a; cin >> x >> y >> a;
159         v.push_back({i, x, y, a, 0, 0});
160       }
161       else {
162         int x1, y1, x2, y2; cin >> x1 >> y1 >> x2 >> y2;
163         v.push_back({i, x1 - 1, y1 - 1, 0, (int)ans.size(), 1});
164         v.push_back({i, x1 - 1, y2, 0, (int)ans.size(), -1});
165         v.push_back({i, x2, y1 - 1, 0, (int)ans.size(), -1});
166         v.push_back({i, x2, y2, 0, (int)ans.size(), 1});
167         ans.push_back(0);
168       }
169       cin >> op, ++i;
170     }
171   }
172   void solve() {
173     cdq(0, (int)v.size());
174     for(int i = 1 ; i < (int)ans.size() ; ++i) cout << ans[i] << '\n';
175   }
176
177
178   int main() {
179     ios_base::sync_with_stdio(0), cin.tie(0);
180     init();
181     solve();
182   }
```

12.4   Count Segments Around Index

```
1   // count of segments that include i with at most size k, at most l to
        the left and at most r to the right
2
3   auto calc = [&](ll val, ll mxx, ll left, ll right) -> ll{
4       left = min(left, mxx - 1);
5       right = min(right, mxx - 1);
6       ll mn = min(left, right);
7       ll mx = max(left, right);
8       mn++;
9       ll res = (mn * (mn + 1)) / 2;
```

```
10      mn--;
11      ll dif = mx - mn;
12      res += dif * (mn + 1);
13      mxx = min(mxx, left + right + 1);
14      ll dif1 = mxx - mx - 1;
15      res += (mn * (mn + 1)) / 2;
16      dif1 = mn - dif1;
17      res -= (dif1 * (dif1 + 1)) / 2;
18      return res;
19    };
```

12.5   Custom Comparators

```
1  bool cmp(const Edge &x, const Edge &y) { return x.w < y.w; }
2
3  sort(a.begin(), a.end(), cmp);
4
5  set<int, greater<int>> a;
6  map<int, string, greater<int>> b;
7  priority_queue<int, vector<int>, greater<int>> c;
```

12.6   Day of the Week

```
1  int dayOfWeek(int d, int m, lli y){
2    if(m == 1 || m == 2){
3      m += 12;
4      --y;
5    }
6    int k = y % 100;
7    lli j = y / 100;
8    return (d + 13*(m+1)/5 + k + k/4 + j/4 + 5*j) % 7;
9  }
```

12.7   Directed MST

```
1  #include<bits/stdc++.h>
2  using namespace std;
3
4  const int N = 3e5 + 9;
5
6  const long long inf = 1e18;
7
8  template<typename T> struct PQ {
9    long long sum = 0;
10   priority_queue<T, vector<T>, greater<T>> Q;
11   void push(T x) { x.w -= sum; Q.push(x); }
12   T pop() { auto ans = Q.top(); Q.pop(); ans.w += sum; return ans; }
13   int size() { return Q.size(); }
14   void add(long long x) { sum += x; }
15   void merge(PQ &&x) {
16     if (size() < x.size()) swap(sum, x.sum), swap(Q, x.Q);
17     while (x.size()) {
18       auto tmp = x.pop();
19       tmp.w -= sum;
20       Q.push(tmp);
21     }
22   }
23 };
24 struct edge {
25   int u, v; long long w;
26   bool operator > (const edge &rhs) const { return w > rhs.w; }
27 };
28 struct DSU {
29   vector<int> par;
30   DSU (int n) : par(n, -1) {}
31   int root(int i) { return par[i] < 0 ? i : par[i] = root(par[i]); }
32   void set_par(int c, int p) { par[c] = p; }
33 };
34 // returns parents of each vertex
35 // each edge should be distinct
36 // it assumes that a solution exists (all vertices are reachable from
       root)
37 // 0 indexed
38 // Takes ~300ms for n = 2e5
39 vector<int> DMST(int n, int root, const vector<edge> &edges) {
40   vector<int> u(2 * n - 1, -1), par(2 * n - 1, -1);
41   edge par_edge[2 * n - 1];
42   vector<int> child[2 * n - 1];
43
44   PQ<edge> Q[2 * n - 1];
45   for (auto e : edges) Q[e.v].push(e);
46   for (int i = 0; i < n; i++) Q[(i + 1) % n].push({i, (i + 1) % n, inf})
       ;
47
48   int super = n;
49   DSU dsu(2 * n - 1);
50   int head = 0;
```

```cpp
51    while (Q[head].size()) {
52      auto x = Q[head].pop();
53      int nxt_root = dsu.root(x.u);
54      if (nxt_root == head) continue;
55      u[head] = nxt_root;
56      par_edge[head] = x;
57      if (u[nxt_root] == -1) head = nxt_root;
58      else {
59        int j = nxt_root;
60        do {
61          Q[j].add(-par_edge[j].w);
62          Q[super].merge(move(Q[j]));
63          assert(u[j] != -1);
64          child[super].push_back(j);
65          j = dsu.root(u[j]);
66        } while (j != nxt_root);
67        for (auto u : child[super]) par[u] = super, dsu.set_par(u, super);
68        head = super++;
69      }
70    }
71    vector<int> res(2 * n - 1, -1);
72    queue<int> q; q.push(root);
73    while (q.size()) {
74      int u = q.front();
75      q.pop();
76      while (par[u] != -1) {
77        for (auto v : child[par[u]]) {
78          if (v != u) {
79            res[par_edge[v].v] = par_edge[v].u;
80            q.push(par_edge[v].v);
81            par[v] = -1;
82          }
83        }
84        u = par[u];
85      }
86    }
87    res[root] = root; res.resize(n);
88    return res;
89  }
90  int32_t main() {
91    ios_base::sync_with_stdio(0);
92    cin.tie(0);
93    int n, m, root; cin >> n >> m >> root;
94    vector<edge> edges(m);
95    for (auto &e : edges) cin >> e.u >> e.v >> e.w;
96    auto res = DMST(n, root, edges);
97
98    unordered_map<int, int> W[n];
99    for (auto u : edges) W[u.v][u.u] = u.w;
100
101   long long ans = 0;
102   for (int i = 0; i < n; i++) if (i != root) ans += W[i][res[i]];
103   cout << ans << '\n';
104   for (auto x : res) cout << x << '␣'; cout << '\n';
105   return 0;
106 }
107 // https://judge.yosupo.jp/problem/directedmst
108 // http://www.cs.tau.ac.il/~zwick/grad-algo-13/directed-mst.pdf
```

12.8   GCD Convolution

```cpp
1  /*
2    GCD Convolution via Multiple Zeta and Mobius Transforms
3    -------------------------------------------------------
4    Given two sequences A[0..n] and B[0..n], computes C where
5      C[d] = sum_{i, j: gcd(i, j) = d} A[i] * B[j]
6    Indexing: 1-based, vectors of size n+1 (ignore index 0)
7    Bounds:
8      - i, j, d in [1..n]
9    Time Complexity: O(n log log n + n log n) per convolution
10   Space Complexity: O(n)
11   Steps:
12     1. Multiple Zeta Transform: for each prime p, v[i] += v[i*p]
13     2. Pointwise multiply transformed A and B
14     3. Multiple Mobius Transform: for each prime p, v[i] -= v[i*p]
15 */
16 // Enumerate primes up to n in O(n)
17 vector<int> PrimeEnumerate(int n) {
18   vector<bool> isPrime(n+1, true);
19   vector<int> P;
20   for (int i = 2; i <= n; i++) {
21     if (isPrime[i]) P.push_back(i);
22     for (int p : P) {
23       if (i * p > n) break;
24       isPrime[i*p] = false;
25       if (i % p == 0) break;
```

```
26        }
27      }
28      return P;
29 }
30 // Multiple Zeta Transform (over divisibility poset)
31 template<typename T>
32 void MultipleZetaTransform(vector<T>& v) {
33    int n = (int)v.size() - 1;
34    for (int p : PrimeEnumerate(n)) {
35      for (int i = n/p; i >= 1; i--) {
36        v[i] += v[i * p];
37      }
38    }
39 }
40 // Multiple Mobius Transform (inverse of zeta)
41 template<typename T>
42 void MultipleMobiusTransform(vector<T>& v) {
43    int n = (int)v.size() - 1;
44    for (int p : PrimeEnumerate(n)) {
45      for (int i = 1; i * p <= n; i++) {
46        v[i] -= v[i * p];
47      }
48    }
49 }
50 // GCD convolution of A and B, both size n+1
51 template<typename T>
52 vector<T> GCDConvolution(vector<T> A, vector<T> B) {
53    int n = (int)A.size() - 1;
54    MultipleZetaTransform(A);
55    MultipleZetaTransform(B);
56    for (int i = 1; i <= n; i++) {
57      A[i] *= B[i];  // pointwise multiplication
58    }
59    MultipleMobiusTransform(A);
60    return A;  // result C[0..n], where C[d] = sum_{gcd(i,j)=d} A[i]*B[j]
61 }
62 // Example usage
63 vector<long long> A(n+1), B(n+1);
64 for (int i = 1; i <= n; i++) cin >> A[i];
65 for (int i = 1; i <= n; i++) cin >> B[i];
66 auto C = GCDConvolution<long long>(A, B);
67 for (int d = 1; d <= n; d++){
68    cout << C[d] << (d==n?'\n':' ');
69 }
```

12.9   int128

```
1  //cout for __int128
2  ostream &operator<<(ostream &os, const __int128 & value){
3    char buffer[64];
4    char *pos = end(buffer) - 1;
5    *pos = '\0';
6    __int128 tmp = value < 0 ? -value : value;
7    do{
8      --pos;
9      *pos = tmp % 10 + '0';
10     tmp /= 10;
11   }while(tmp != 0);
12   if(value < 0){
13     --pos;
14     *pos = '-';
15   }
16   return os << pos;
17 }
18
19 //cin for __int128
20 istream &operator>>(istream &is, __int128 & value){
21   char buffer[64];
22   is >> buffer;
23   char *pos = begin(buffer);
24   int sgn = 1;
25   value = 0;
26   if(*pos == '-'){
27     sgn = -1;
28     ++pos;
29   }else if(*pos == '+'){
30     ++pos;
31   }
32   while(*pos != '\0'){
33     value = (value << 3) + (value << 1) + (*pos - '0');
34     ++pos;
35   }
36   value *= sgn;
37   return is;
38 }
39
```

```
40  ll mult(__int128 a, __int128 b){ return ((a*1LL*b)%MOD + MOD)%MOD; }
41
```

12.10   Iterating Over All Subsets

```
1  for (int mk = 0; mk < (1 << k); mk++) {
2    Ap[mk] = 0;
3    for (int s = mk;; s = (s - 1) & mk) {
4      Ap[mk] += A[s];
5      if (!s) break;
6    }
7  }
```

12.11   LCM Convolution

```
1  /*
2    LCM Convolution via Divisor Zeta and Mobius Transforms
3    ------------------------------------------------------
4    Given two sequences A[1..n] and B[1..n], computes C where
5      C[d] = sum_{i, j: lcm(i, j) = d} A[i] * B[j]
6    Indexing: 1-based (vectors of size n+1, ignore index 0)
7    Bounds:
8      - i, j, d in [1..n]
9    Time Complexity: O(n log log n + n log n)
10   Space Complexity: O(n)
11   Steps:
12     1. Divisor Zeta Transform: for each prime p, v[i*p] += v[i]
13     2. Pointwise multiply transformed A and B
14     3. Divisor Mobius Transform: for each prime p, v[i*p] -= v[i]
15  */
16  // Sieve primes up to n in O(n)
17  vector<int> PrimeEnumerate(int n) {
18    vector<bool> isPrime(n+1, true);
19    vector<int> P;
20    for (int i = 2; i <= n; i++) {
21      if (isPrime[i]) P.push_back(i);
22      for (int p : P) {
23        if (i * p > n) break;
24        isPrime[i*p] = false;
25        if (i % p == 0) break;
26      }
27    }
28    return P;
29  }
```

```
30  // Divisor Zeta Transform: v[d] = sum_{d|k} original_v[k]
31  template<typename T>
32  void DivisorZetaTransform(vector<T>& v) {
33    int n = (int)v.size() - 1;
34    auto primes = PrimeEnumerate(n);
35    for (int p : primes) {
36      for (int i = 1; i * p <= n; i++) {
37        v[i * p] += v[i];
38      }
39    }
40  }
41  // Divisor Mobius Transform (inverse of zeta)
42  template<typename T>
43  void DivisorMobiusTransform(vector<T>& v) {
44    int n = (int)v.size() - 1;
45    auto primes = PrimeEnumerate(n);
46    for (int p : primes) {
47      for (int i = n / p; i >= 1; i--) {
48        v[i * p] -= v[i];
49      }
50    }
51  }
52  // LCM convolution of A and B (each size n+1)
53  template<typename T>
54  vector<T> LCMConvolution(vector<T> A, vector<T> B) {
55    int n = (int)A.size() - 1;
56    DivisorZetaTransform(A);
57    DivisorZetaTransform(B);
58    for (int d = 1; d <= n; d++) {
59      A[d] *= B[d];   // pointwise multiplication
60    }
61    DivisorMobiusTransform(A);
62    return A;   // result C[1..n], C[d] = sum_{lcm(i,j)=d} A[i]*B[j]
63  }
64  // Example usage
65  vector<long long> A(n+1), B(n+1);
66  for (int i = 1; i <= n; i++) cin >> A[i];
67  for (int i = 1; i <= n; i++) cin >> B[i];
68  auto C = LCMConvolution<long long>(A, B);
69  for (int d = 1; d <= n; d++){
70    cout << C[d] << (d==n?'\n':'␣');
71  }
```

12.12   Manhattan MST

```
/*
  Manhattan MST Edge Generation
  ------------------------------
  Given n points ps[0..n-1], returns a list of candidate edges
    (weight, u, v) for the Manhattan MST, where
    weight = |x[u]-x[v]| + |y[u]-y[v]|.
  Indexing: 0-based for points and returned edges
  Time Complexity: O(n log n)
*/
struct Point {
  ll x, y;
};

vector<tuple<ll,int,int>> manhattan_mst_edges(vector<Point> ps) {
  int n = ps.size();
  vector<int> order(n);
  iota(order.begin(), order.end(), 0);
  vector<tuple<ll,int,int>> edges;
  edges.reserve(n * 4);
  // Repeat for 4 orientations
  for (int r = 0; r < 4; r++) {
    // Sort by x+y ascending
    sort(order.begin(), order.end(), [&](int i, int j) {
      return ps[i].x + ps[i].y < ps[j].x + ps[j].y;
    });
    // Active map: key = x-coordinate, value = point index
    map<ll,int,greater<ll>> active;
    for (int i : order) {
      // For all active points with x >= ps[i].x
      for (auto it = active.lower_bound(ps[i].x); it != active.end();
           active.erase(it++)) {
        int j = it->second;
        if (ps[i].x - ps[i].y > ps[j].x - ps[j].y) break;
        assert(ps[i].x >= ps[j].x && ps[i].y >= ps[j].y);
        edges.emplace_back({(ps[i].x - ps[j].x) + (ps[i].y - ps[j].y), i
             , j});
      }
      active[ps[i].x] = i;
    }
    // Rotate points 90 degrees: (x,y) -> (y,-x)
    for (auto &p : ps) {
      ll tx = p.x;
      p.x = p.y;
      p.y = -tx;
    }
  }
  return edges;
}
```

12.13   Max Manhattan Distance

```
/*
  Max Manhattan distance
  ------------------------------------
  Finds maximum manhattan distance for any two points with d dimensions.
  Generalized code for d dimensions.
  Time Complexity: (n * 2 ^ d * d)
*/


long long ans = 0;
for (int msk = 0; msk < (1 << d); msk++) {
    long long mx = LLONG_MIN, mn = LLONG_MAX;
    for (int i = 0; i < n; i++) {
        long long cur = 0;
        for (int j = 0; j < d; j++) {
            if (msk & (1 << j)) cur += p[i][j];
            else cur -= p[i][j];
        }
        mx = max(mx, cur);
        mn = min(mn, cur);
    }
    ans = max(ans, mx - mn);
}
```

12.14   Mo

```
/*
  Mo's Algorithm (Sqrt Decomposition for Offline Queries)
  -------------------------------------------------------
  Problem: Answer q range queries [L, R] over array of length n
           using add/remove operations efficiently
  Indexing: 0-based
  Bounds:
    - arr[0..n-1]
```

```
 9        - queries input as 1-based and converted to 0-based
10     Time Complexity: O((n + q) * sqrt(n)) per query batch
11     Space Complexity: O(n + q)
12     Usage:
13        - Fill in logic for add/remove operations (update cur)
14        - Fill answers[] indexed by original query order
15  */
16
17  const int MAXN = 200500;
18  ll n, q;
19  ll arr[MAXN];       // input array
20  ll cnt[1000005];    // frequency count
21  ll answers[MAXN];   // output array
22  ll cur = 0;         // current query result
23  ll BLOCK_SIZE;
24
25  pair< pair<ll, ll>, ll> queries[MAXN]; // {{L, R}, query_index}
26
27  // Sort by block and then by R
28  inline bool cmp(const pair< pair<ll, ll>, ll> &x, const pair< pair<ll,
         ll>, ll> &y) {
29    ll block_x = x.first.first / BLOCK_SIZE;
30    ll block_y = y.first.first / BLOCK_SIZE;
31    if (block_x != block_y) return block_x < block_y;
32    return x.first.second < y.first.second;
33  }
34
35  int main() {
36    cin >> n >> q;
37    BLOCK_SIZE = sqrt(n);
38    for (int i = 0; i < n; i++) cin >> arr[i];
39    for (int i = 0; i < q; i++) {
40      int l, r;
41      cin >> l >> r;
42      --l; --r; // convert to 0-based
43      queries[i] = {{l, r}, i};
44    }
45    sort(queries, queries + q, cmp);
46    ll l = 0, r = -1;
47    for (int i = 0; i < q; i++) {
48      int left = queries[i].first.first;
49      int right = queries[i].first.second;
50      // Expand to right
51      while (r < right) {
52        r++;
53        // Operations to add arr[r], implement exactly here
54      }
55      // Shrink from right
56      while (r > right) {
57        // Operations to remove arr[r], implement exactly here
58        r--;
59      }
60      // Expand to left
61      while (l < left) {
62        // Operations to remove arr[l], implement exactly here
63        l++;
64      }
65      // Shrink from left
66      while (l > left) {
67        l--;
68        // Operations to add arr[l], implement exactly here
69      }
70      answers[queries[i].second] = cur; // Current answer
71    }
72    for (int i = 0; i < q; i++) cout << answers[i] << '\n';
73  }
```

12.15   MOD INT

```
 1  template<int MOD>
 2  struct ModInt {
 3    ll v;
 4    ModInt(ll _v = 0) {v = (-MOD < _v && _v < MOD) ? _v : _v % MOD; if (v
          < 0) v += MOD;}
 5    ModInt& operator += (const ModInt &other) {v += other.v; if (v >= MOD)
           v -= MOD; return *this;}
 6    ModInt& operator -= (const ModInt &other) {v -= other.v; if (v < 0) v
          += MOD; return *this;}
 7    ModInt& operator *= (const ModInt &other) {v = v * other.v % MOD;
          return *this;}
 8    ModInt& operator /= (const ModInt &other) {return *this *= inverse(
          other);}
 9    bool operator == (const ModInt &other) const {return v == other.v;}
10    bool operator != (const ModInt &other) const {return v != other.v;}
11    friend ModInt operator + (ModInt a, const ModInt &b) {return a += b;}
12    friend ModInt operator - (ModInt a, const ModInt &b) {return a -= b;}
```

```
13    friend ModInt operator * (ModInt a, const ModInt &b) {return a *= b;}
14    friend ModInt operator / (ModInt a, const ModInt &b) {return a /= b;}
15    friend ModInt operator - (const ModInt &a) {return 0 - a;}
16    friend ModInt power(ModInt a, ll b) {ModInt ret(1); while (b > 0) {if
         (b & 1) ret *= a; a *= a; b >>= 1;} return ret;}
17    friend ModInt inverse(ModInt a) {return power(a, MOD - 2);}
18    friend istream& operator >> (istream &is, ModInt &m) {is >> m.v; m.v =
         (-MOD < m.v && m.v < MOD) ? m.v : m.v % MOD; if (m.v < 0) m.v +=
         MOD; return is;}
19    friend ostream& operator << (ostream &os, const ModInt &m) {return os
         << m.v;}
20  };
```

12.16   Next Permutation

```
1  sort(v.begin(),v.end());
2  while(next_permutation(v.begin(),v.end())){
3    for(auto u:v){
4      cout<<u<<" ";
5    }
6    cout<<endl;
7  }
8
9  string s="asdfassd";
10 sort(s.begin(),s.end());
11 while(next_permutation(s.begin(),s.end())){
12   cout<<s<<endl;
13 }
```

12.17   Next and Previous Smaller/Greater Element

```
1  /*
2    Next and Previous Smaller / Greater Elements
3    --------------------------------
4    Given: array a[0..n-1]
5    nextSmaller[i]: index of next element smaller than a[i], or n if none
6    prevSmaller[i]: index of previous element smaller than a[i], or -1 if
         none
7    For GREATER, replace a[s.top()] < a[i] with a[s.top()] > a[i]
8    Indexing: 0-based
9  */
10 vector<int> nextSmaller(vector<int> a, int n) {
11   stack<int> s;
12   vector<int> res(n, n); // default: no smaller to right
```

```
13   for (int i = 0; i < n; i++) {
14     while (!s.empty() && a[s.top()] > a[i]) {
15       res[s.top()] = i;
16       s.pop();
17     }
18     s.push(i);
19   }
20   return res;
21 }
22 vector<int> prevSmaller(vector<int> a, int n) {
23   stack<int> s;
24   vector<int> res(n, -1); // default: no smaller to left
25   for (int i = n - 1; i >= 0; i--) {
26     while (!s.empty() && a[s.top()] > a[i]) {
27       res[s.top()] = i;
28       s.pop();
29     }
30     s.push(i);
31   }
32   return res;
33 }
```

12.18   Parallel Binary Search

```
1  /*
2    Parallel Binary Search
3    ----------------------
4    Solves: k independent queries where each answer lies in range [lo, hi]
5    - At each iteration, test the midpoint for a batch of queries
6    Indexing: 0-based
7    Bounds:
8      - i in [0, m-1] for the structure version
9      - q in [0, k-1] for each of the k queries
10   Time Complexity: O((m + k) * log M)
11   Space Complexity: O(k + m)
12   You must:
13     - Reset data structures before each iteration
14     - Implement apply_change(i) and check_query(q)
15 */
16 int lo[MAXN], hi[MAXN];      // binary search bounds for each query
17 vector<int> tocheck[MAXN];  // tocheck[mid] = list of queries to check
       at mid
18
```

```
19  bool done = false;
20  while (!done) {
21    done = true;
22     // Reset changes of structure to 0 before each iteration
23    // Assign queries to current midpoint
24    for (int q = 0; q < k; q++) {
25      if (lo[q] < hi[q]) {
26        tocheck[(lo[q] + hi[q]) / 2].push_back(q);
27      }
28    }
29    // Apply changes and evaluate queries
30    for (int i = 0; i < m; i++) {
31      // Apply change for ith query to your data structure
32      apply_change(i);
33      for (int q : tocheck[i]) {
34        done = false; // at least one query is not done
35        // Evaluate query q using the updated structure
36        if (operationToCheck(q)) {
37          hi[q] = i; // condition is true, try earlier
38        } else {
39          lo[q] = i + 1; // condition false, try later
40        }
41      }
42    }
43  }
```

12.19   Random Number Generators

```
1  //to avoid hacks
2  mt19937 mt(chrono::steady_clock::now().time_since_epoch().count());
3  mt19937_64 mt(chrono::steady_clock::now().time_since_epoch().count());
4  //you can also just write seed_value if hacks are not an issue
5
6  // rng() for generating random numbers between 0 and 2<<31-1
7
8  // for generating numbers with uniform probability in range
9  uniform_int_distribution<int> rng(0, numeric_limits<int>::max());
10
11  // then for generating random values just do
12  int random_value = rng(mt);
13
14  // if you do not want to create variable
15  uniform_int_distribution<int>(0, n)(mt)
```

```
16
17  // other distributions
18  // Normal distribution: mean = 0.0, stddev = 1.0
19  normal_distribution<double> normal_dist(0.0, 1.0);
20
21  // Exponential distribution: lambda = 1.5
22  exponential_distribution<double> exp_dist(1.5);
23
24  // for shuffling array
25  shuffle(permutation.begin(), permutation.end(), rng);
```

12.20   setprecision

```
1  cout<<fixed<<setprecision(10);
```

12.21   Ternary Search

```
1  /*
2    Ternary Search (for continuous unimodal functions)
3    ----------------------------------------------------
4    Finds the maximum (or minimum) value of a unimodal function f(x) in [l
        , r]
5    Indexing: continuous domain (double)
6    Bounds:
7      - Search range is [l, r]
8      - Precision controlled by eps
9    Time Complexity: O(log((r - l) / eps))
10   - For minimum, reverse the condition (if f1 > f2 then l = m1)
11  */
12  double ternary_search(double l, double r) {
13    double eps = 1e-9; // precision error
14    while (r - l > eps) {
15      double m1 = l + (r - l) / 3;
16      double m2 = r - (r - l) / 3;
17      double f1 = f(m1); // evaluate function at m1
18      double f2 = f(m2); // evaluate function at m2
19      if (f1 < f2)
20        l = m1;  // move left bound up if increasing
21      else
22        r = m2;  // move right bound down if decreasing
23    }
24    return f(l); // return value at the best point
25  }
```

12.22   Ternary Search Int

```
1  int lo = -1, hi = n;
2  while (hi - lo > 1){
3      int mid = (hi + lo)>>1;
4      if (f(mid) > f(mid + 1)) hi = mid;
5      else lo = mid;
6  }
7  //lo + 1 is the answer
```

12.23   XOR Convolution

```
1  /*
2    Fast Walsh-Hadamard Transform (FWHT) for XOR Convolution
3    --------------------------------------------------------
4    Given two arrays A[0..N-1], B[0..N-1], with N = 1<<k (power of two).
5    Computes C[d] = sum_{i xor j = d} A[i] * B[j].
6    Indexing: 0-based
7    Bounds:
8      - N must be a power of two
9    Time Complexity: O(N * log(N))
10   Steps:
11     1. fwht(A, false); fwht(B, false); // forward transform
12     2. for i in [0..N): C[i] = A[i] * B[i]
13     3. fwht(C, true);                  // inverse transform
14  */
15  void FWHT (int A[], int k, int inv) {
16    for (int j = 0; j < k; j++)
17      for (int i = 0; i < (1 << k); i++)
18        if (~i & (1 << j)) {
19          int p0 = A[i];
20          int p1 = A[i | (1 << j)];
21          A[i] = p0 + p1;
22          A[i | (1 << j)] = p0 - p1;
23          if (inv) {
24            A[i] /= 2;
25            A[i | (1 << j)] /= 2;
26          }
27        }
28  }
29  void XOR_conv (int A[], int B[], int C[], int k) {
30    FWHT(A, k, false);
31    FWHT(B, k, false);
32    for (int i = 0; i < (1 << k); i++)
33      C[i] = A[i] * B[i];
34    FWHT(A, k, true);
35    FWHT(B, k, true);
36    FWHT(C, k, true);
37  }
38  // Example usage:
39  int A[1 << 20], B[1 << 20], C[1 << 20];
40  XOR_conv(A, B, C, 20);
41  for (int d = 0; d < (1<<20); d++){
42    cout << C[d] << (d==(1<<20)?'\n':'␣');
43  }
44
45
46
47
48
49  #include <bits/stdc++.h>
50
51  using namespace std;
52  #define int long long
53
54  const int mod = 998244353;
55
56  int inverse(int x, int mod) {
57    return x == 1 ? 1 : mod - mod / x * inverse(mod % x, mod) % mod;
58  }
59
60  void xormul(vector<int> a, vector<int> b, vector<int> &c) {
61    int m = (int) a.size();
62    c.resize(m);
63    for (int n = m / 2; n > 0; n /= 2)
64      for (int i = 0; i < m; i += 2 * n)
65        for (int j = 0; j < n; j++) {
66          int x = a[i + j], y = a[i + j + n];
67          a[i + j] = (x + y) % mod;
68          a[i + j + n] = (x - y + mod) % mod;
69        }
70    for (int n = m / 2; n > 0; n /= 2)
71      for (int i = 0; i < m; i += 2 * n)
72        for (int j = 0; j < n; j++) {
73          int x = b[i + j], y = b[i + j + n];
74          b[i + j] = (x + y) % mod;
```

```
75        b[i + j + n] = (x - y + mod) % mod;
76      }
77    for (int i = 0; i < m; i++)
78      c[i] = a[i] * b[i] % mod;
79    for (int n = 1; n < m; n *= 2)
80      for (int i = 0; i < m; i += 2 * n)
81        for (int j = 0; j < n; j++) {
82          int x = c[i + j], y = c[i + j + n];
83          c[i + j] = (x + y) % mod;
84          c[i + j + n] = (x - y + mod) % mod;
85        }
86    int mrev = inverse(m, mod);
87    for (int i = 0; i < m; i++)
88      c[i] = c[i] * mrev % mod;
89  }
90
91  int32_t main() {
92    ios::sync_with_stdio(0); cin.tie(0);
93    int n; cin >> n;
94    vector<int> a(1 << n), b(1 << n), c;
95    for (int i = 0; i < (1 << n); i++)
96      cin >> a[i];
97    for (int i = 0; i < (1 << n); i++)
98      cin >> b[i];
99    xormul(a, b, c);
100   for (int i = 0; i < (1 << n); i++)
101     cout << c[i] << '␣';
102 }
```

12.24   XOR Basis

```
1  /*
2     XOR Basis (Linear Basis over GF(2))
3     -----------------------------------
4     Supports insertion of numbers into a basis and querying the maximum
5     XOR of any subset with a given value.
6
7     Bit Width: up to D bits (here D = 60 for 64-bit integers)
8     Indexing: basis[0] is for bit 0 (least significant), basis[D-1] for
           bit D-1
9     Bounds:
10      - Values x inserted must satisfy 0 <= x < 2^D
11     Time Complexity:
12      - insert(x):   O(D)
13      - getMax(x):   O(D)
14     Space Complexity: O(D)
15
16     Notes:
17      - We maintain one basis vector per bit position.
18      - On insert, we reduce x by existing basis vectors, then store it
19        in the highest bit it still has.
20      - To query max XOR, we greedily try to xor with basis vectors
21        from highest bit down.
22  */
23  const int D = 31; // Maximum number of bits in the numbers
24  int basis[D]; // basis[i] keeps the mask of the vector whose f value is
         i
25  int sz; // Current size of the basis
26
27  void insertVector(int mask) {
28    //turn for around if you want max xor
29    for (int i = 0; i < D; i++) {
30      if ((mask & 1 << i) == 0) continue; // continue if i != f(mask)
31      if (!basis[i]) { // If there is no basis vector with the ith bit set
            , then insert this vector into the basis
32        basis[i] = mask;
33        ++sz;
34        return;
35      }
36      mask ^= basis[i]; // Otherwise subtract the basis vector from this
             vector
37    }
38  }
39
40  // V2: If you dont need the basis sorted.
41  vector<ll> basis;
42  void add(ll x)
43  {
44    for (int i = 0; i < basis.size(); i++)
45    {
46      x = min(x, x ^ basis[i]);
47    }
48    if (x != 0)
49    {
50      basis.pb(x);
51    }
```

```
52  }
```

12.25   XOR Basis Online

```
1   int rebuild_and_delete (int id) {
2     int pos = 0, mn = inf, p2 = 0;
3     for (int i = 0; i < basis.size(); i++) {
4         if (basis[i].id == id) {
5             pos = i;
6         }
7     }
8     int bits = 0;
9     for (int i = 0; i < basis.size(); i++) {
10        if (basis[i].mask & (1 << pos)) {
11            if (mn > basis[i].high) {
12                mn = basis[i].high;
13                p2 = i;
14            }
15            bits ^= 1 << basis[i].high;
16        }
17    }
18
19    if (p2 != pos) {
20        swap(basis[p2].id, basis[pos].id);
21        for (auto &i : basis) {
22            swap_bits(i.mask, pos, p2); // just swaps pos-th and p2-th bit
                     in i.mask
23        }
24        pos = p2;
25    }
26    for (int i = 0; i < basis.size(); i++) {
27        if (i != pos) {
28            if (basis[i].mask & (1 << pos)) {
29                basis[i].val ^= basis[pos].val;
30                basis[i].mask ^= basis[pos].mask;
31            }
32        }
33    }
34    int good = (1 << pos) - 1;
35    int other = ((1 << len(basis)) - 1) ^ (good | (1 << pos));
36    basis.erase(basis.begin() + pos);
37    for (auto &i : basis) {
38        i.mask = ((i.mask & other) >> 1) | (i.mask & good);
```

```
39    }
40    return bits;
41  }
42
43  C++
44  41 lines
45  1161 bytes
46
47
48  int get_the_same_high_bit (int bits, vector <int> &val) {
49    for (auto &i : basis) {
50        if (__builtin_popcount(val[i.id] & bits) & 1) {
51            return i.id;
52        }
53    }
54    return -1;
55  }
56
57  C++
58  8 lines
59  200 bytes
60
61  Thus, we have learned to delete in O(m2).
62
63  Here is full version of the code:
64  Code
65
66  //#pragma GCC optimize("Ofast", "unroll-loops")
67  //#pragma GCC target("sse", "sse2", "sse3", "ssse3", "sse4")
68
69  #include <bits/stdc++.h>
70
71  #define all(a) a.begin(),a.end()
72  #define len(a) (int)(a.size())
73  #define mp make_pair
74  #define pb push_back
75  #define fir first
76  #define sec second
77  #define fi first
78  #define se second
79
80  using namespace std;
81
```

```cpp
 82  typedef pair<int, int> pii;
 83  typedef long long ll;
 84  typedef long double ld;
 85
 86  template<typename T>
 87  bool umin(T &a, T b) {
 88    if (b < a) {
 89        a = b;
 90        return true;
 91    }
 92    return false;
 93  }
 94  template<typename T>
 95  bool umax(T &a, T b) {
 96    if (a < b) {
 97        a = b;
 98        return true;
 99    }
100    return false;
101  }
102
103  #ifdef KIVI
104  #define DEBUG for (bool _FLAG = true; _FLAG; _FLAG = false)
105  #define LOG(...) print(#__VA_ARGS__" ::", __VA_ARGS__) << endl
106  template <class ...Ts> auto &print(Ts ...ts) { return ((cerr << ts << "␣
         "), ...); }
107  #else
108  #define DEBUG while (false)
109  #define LOG(...)
110  #endif
111
112  const int max_n = -1, inf = 1000111222;
113
114
115  const int C = 20;
116
117  struct node {
118    int val, id, mask, high;
119  };
120
121
122  inline int get_high (int x) {
123    if (x == 0) {
124        return -1;
125    }
126    return 31 - __builtin_clz(x);
127  }
128
129  inline void swap_bits (int &x, int a, int b) {
130    int x1 = bool(x & (1 << a));
131    int x2 = bool(x & (1 << b));
132    x ^= (x1 ^ x2) << a;
133    x ^= (x1 ^ x2) << b;
134  }
135
136  struct xor_basis {
137    vector <node> basis;
138
139    inline bool add (int x, int id) {
140        int mask = 0;
141        for (auto &i : basis) {
142            if (umin(x, x ^ i.val)) {
143                mask ^= i.mask;
144            }
145        }
146        if (x) {
147            mask |= 1 << len(basis);
148            for (auto &i : basis) {
149                if (umin(i.val, i.val ^ x)) {
150                    i.mask ^= mask;
151                }
152            }
153            basis.pb(node{val: x, id: id, mask: mask, high: get_high(x)});
154            return true;
155        }
156        return false;
157    }
158
159
160
161    inline int get_the_same_high_bit (int bits, const vector <int> &val) {
162        for (auto &i : basis) {
163            if (__builtin_popcount(val[i.id] & bits) & 1) {
164                return i.id;
165            }
166        }
```

```
167        return -1;
168    }
169
170
171    inline int rebuild_and_delete (int id) {
172        int pos = 0, mn = inf, p2 = 0;
173        for (int i = 0; i < len(basis); i++) {
174            if (basis[i].id == id) {
175                pos = i;
176            }
177        }
178        int bits = 0;
179        for (int i = 0; i < len(basis); i++) {
180            if (basis[i].mask & (1 << pos)) {
181                if (umin(mn, basis[i].high)) {
182                    p2 = i;
183                }
184                bits ^= 1 << basis[i].high;
185            }
186        }
187
188        if (p2 != pos) {
189            swap(basis[p2].id, basis[pos].id);
190            for (auto &i : basis) {
191                swap_bits(i.mask, pos, p2);
192            }
193            pos = p2;
194        }
195        for (int i = 0; i < len(basis); i++) {
196            if (i != pos) {
197                if (basis[i].mask & (1 << pos)) {
198                    basis[i].val ^= basis[pos].val;
199                    basis[i].mask ^= basis[pos].mask;
200                }
201            }
202        }
203        int good = (1 << pos) - 1;
204        int other = ((1 << len(basis)) - 1) ^ (good | (1 << pos));
205        basis.erase(basis.begin() + pos);
206        for (auto &i : basis) {
207            i.mask = ((i.mask & other) >> 1) | (i.mask & good);
208        }
209        return bits;
210    }
211
212 };
213
214 template<int max_bit> // not inclusive
215 struct xor_basis_online {
216
217   vector <xor_basis> basises[max_bit + 1];
218
219   int mx;
220
221   vector <pii> where;
222   vector <int> val;
223
224   xor_basis_online() : mx(0), cur_id(0) {}
225
226   int cur_id;
227
228   inline int add (int x) {
229       val.pb(x);
230       where.pb(make_pair(-1, -1));
231       int id = cur_id++;
232       if (x == 0) {
233           return id;
234       }
235       for (int i = mx; i >= 0; i--) {
236           if (basises[i].empty()) {
237               continue;
238           }
239           if (basises[i].back().add(x, id)) {
240               basises[i + 1].pb(basises[i].back());
241               basises[i].pop_back();
242               umax(mx, i + 1);
243               for (auto &j : basises[i + 1].back().basis) {
244                   where[j.id] = make_pair(i + 1, len(basises[i + 1]) -
                           1);
245               }
246               return id;
247           }
248       }
249       basises[1].pb(xor_basis());
250       basises[1].back().add(x, id);
251       where.back() = make_pair(1, len(basises[1]) - 1);
```

```
252        umax(mx, 1);
253        return id;
254    }
255
256    inline int move_back (int sz, int pos) {
257        int to = len(basises[sz]) - 1;
258        if (to == pos) {
259            return pos;
260        }
261        for (auto &i : basises[sz][pos].basis) {
262            where[i.id].second = to;
263        }
264        for (auto &i : basises[sz][to].basis) {
265            where[i.id].second = pos;
266        }
267        swap(basises[sz][pos], basises[sz][to]);
268        return to;
269    }
270
271    inline void del (int id) {
272        if (val[id] == 0) {
273            return;
274        }
275        int sz, pos;
276        tie(sz, pos) = where[id];
277        pos = move_back(sz, pos);
278        while (true) {
279            int bits = basises[sz].back().rebuild_and_delete(id);
280            int i = sz - 1;
281            while (i > 0 && basises[i].empty()) {
282                --i;
283            }
284            int new_id = -1;
285            if (i > 0) {
286                new_id = basises[i].back().get_the_same_high_bit(bits, val
                    );
287            }
288            if (new_id == -1) {
289                if (sz > 1) {
290                    basises[sz - 1].pb(basises[sz].back());
291                    for (auto &j : basises[sz - 1].back().basis) {
292                        where[j.id] = make_pair(sz - 1, len(basises[sz -
                            1]) - 1);
```

```
293                    }
294                }
295                basises[sz].pop_back();
296                if (mx > 0 && basises[mx].empty()) {
297                    --mx;
298                }
299                return;
300            }
301            int cur = val[new_id];
302            assert(basises[sz].back().add(cur, new_id));
303            int new_sz = i;
304            int new_pos = len(basises[i]) - 1;
305            where[new_id] = make_pair(sz, pos);
306            id = new_id;
307            sz = new_sz;
308            pos = new_pos;
309        }
310    }
311
312    inline int size () {
313        return mx;
314    }
315 };
316
317 int main() {
318 //    freopen("input.txt", "r", stdin);
319 //    freopen("output.txt", "w", stdout);
320
321    ios_base::sync_with_stdio(0);
322    cin.tie(0);
323
324
325    // solution to https://basecamp.eolymp.com/uk/problems/11732
326    int n, q, add;
327    cin >> n >> add;
328    vector <int> p(n);
329    xor_basis_online<19> t;
330    vector <int> now(n);
331    for (int i = 0; i < n; i++) {
332        cin >> p[i];
333        now[i] = t.add(i ^ p[i]);
334    }
335    cin >> q;
```

```
336    int ans = t.size();
337    cout << ans << '\n';
338    for (int i = 0, l, r; i < q; i++) {
339        cin >> l >> r;
340        l = (l + ans * add) % n;
341        r = (r + ans * add) % n;
342        if (l != r) {
343            t.del(now[l]);
344            t.del(now[r]);
345            swap(p[l], p[r]);
346            now[l] = t.add(l ^ p[l]);
347            now[r] = t.add(r ^ p[r]);
348        }
349        ans = t.size();
350        cout << ans << '\n';
351    }
352 }
```

13   Polynomials

13.1   Berlekamp Massey

```
1  template<typename T>
2  vector<T> berlekampMassey(const vector<T> &s) {
3      vector<T> c;      // the linear recurrence sequence we are building
4      vector<T> oldC; // the best previous version of c to use (the one
            with the rightmost left endpoint)
5      int f = -1;       // the index at which the best previous version of c
             failed on
6      for (int i=0; i<(int)s.size(); i++) {
7          // evaluate c(i)
8          // delta = s_i - \sum_{j=1}^n c_j s_{i-j}
9          // if delta == 0, c(i) is correct
10         T delta = s[i];
11         for (int j=1; j<=(int)c.size(); j++)
12             delta -= c[j-1] * s[i-j];   // c_j is one-indexed, so we
                   actually need index j - 1 in the code
13         if (delta == 0)
14             continue;   // c(i) is correct, keep going
15         // now at this point, delta != 0, so we need to adjust it
16         if (f == -1) {
17             // this is the first time we're updating c
18             // s_i was the first non-zero element we encountered
19             // we make c of length i + 1 so that s_i is part of the base
                   case
20             c.resize(i + 1);
21             mt19937 rng(chrono::steady_clock::now().time_since_epoch().
                   count());
22             for (T &x : c)
23                 x = rng();   // just to prove that the initial values don
                       't matter in the first step, I will set to random
                       values
24             f = i;
25         } else {
26             // we need to use a previous version of c to improve on this
                   one
27             // apply the 5 steps to build d
28             // 1. set d equal to our chosen sequence
29             vector<T> d = oldC;
30             // 2. multiply the sequence by -1
31             for (T &x : d)
32                 x = -x;
33             // 3. insert a 1 on the left
34             d.insert(d.begin(), 1);
35             // 4. multiply the sequence by delta / d(f + 1)
36             T df1 = 0;   // d(f + 1)
37             for (int j=1; j<=(int)d.size(); j++)
38                 df1 += d[j-1] * s[f+1-j];
39             assert(df1 != 0);
40             T coef = delta / df1;   // storing this in outer variable so
                   it's O(n^2) instead of O(n^2 log MOD)
41             for (T &x : d)
42                 x *= coef;
43             // 5. insert i - f - 1 zeros on the left
44             vector<T> zeros(i - f - 1);
45             zeros.insert(zeros.end(), d.begin(), d.end());
46             d = zeros;
47             // now we have our new recurrence: c + d
48             vector<T> temp = c; // save the last version of c because it
                   might have a better left endpoint
49             c.resize(max(c.size(), d.size()));
50             for (int j=0; j<(int)d.size(); j++)
51                 c[j] += d[j];
52             // finally, let's consider updating oldC
53             if (i - (int) temp.size() > f - (int) oldC.size()) {
54                 // better left endpoint, let's update!
```

```
55            oldC = temp;
56            f = i;
57         }
58      }
59   }
60   return c;
61 }
```

13.2   Binary Exp FFT

```
1    #include <bits/stdc++.h>
2    using namespace std;
3
4    #define rep(i, a, b) for(int i = a; i < (b); ++i)
5    #define all(x) begin(x), end(x)
6    #define sz(x) (int)(x).size()
7    typedef long long ll;
8    typedef pair<int, int> pii;
9    typedef vector<int> vi;
10
11
12   typedef complex<double> C;
13   typedef vector<double> vd;
14   void fft(vector<C>& a) {
15     int n = sz(a), L = 31 - __builtin_clz(n);
16     static vector<complex<long double>> R(2, 1);
17     static vector<C> rt(2, 1);   // (^ 10% faster if double)
18     for (static int k = 2; k < n; k *= 2) {
19       R.resize(n); rt.resize(n);
20       auto x = polar(1.0L, acos(-1.0L) / k);
21       rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
22     }
23     vi rev(n);
24     rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
25     rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
26     for (int k = 1; k < n; k *= 2)
27       for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
28         // C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
                /// include-line
29         auto x = (double *)&rt[j+k], y = (double *)&a[i+j+k];
                /// exclude-line
30         C z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);
                /// exclude-line
31         a[i + j + k] = a[i + j] - z;
32         a[i + j] += z;
33       }
34   }
35   vd conv(const vd& a, const vd& b) {
36     if (a.empty() || b.empty()) return {};
37     vd res(sz(a) + sz(b) - 1);
38     int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
39     vector<C> in(n), out(n);
40     copy(all(a), begin(in));
41     rep(i,0,sz(b)) in[i].imag(b[i]);
42     fft(in);
43     for (C& x : in) x *= x;
44     rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
45     fft(out);
46     rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
47     return res;
48   }
49
50
51   const int maxn = 1 << 20;
52
53
54   int main(){
55     ios_base::sync_with_stdio(false); cin.tie(NULL);
56     int n, k; cin >> n >> k;
57     vd a(1 << 10, 0);
58     for(int i = 0; i < n; i++){
59       int x; cin >> x;
60       a[x] = 1;
61     }
62     vd res(1 << 10, 0);
63     res[0] = 1;
64     auto fix_sizes = [&]() -> void{
65       while(__builtin_popcount(sz(a)) != 1){
66         a.push_back(0);
67       }
68       while(__builtin_popcount(sz(res)) != 1){
69         res.push_back(0);
70       }
71       while(sz(a) > sz(res)) res.push_back(0);
72       while(sz(res) > sz(a)) a.push_back(0);
73     };
```

```
74        while(k){
75          if(k % 2){
76            res = conv(res, a);
77          }
78          a = conv(a, a);
79          fix_sizes();
80          for(int i = 0; i < sz(a); i++){
81            if(llround(a[i]) > 0) a[i] = 1;
82            else a[i] = 0;
83            if(llround(res[i]) > 0) res[i] = 1;
84            else res[i] = 0;
85          }
86          k /= 2;
87        }
88        for(int i = 0; i < sz(res); i++){
89          ll x = llround(res[i]);
90          if(x > 0){
91            cout << i << "␣";
92          }
93        }
94        cout << endl;
95      }
```

13.3   FFT

```
1  using cd = complex<double>;
2  const double PI = acos(-1);
3  //declare size of vectors used like this
4  const int MAXN=2<<19;
5
6  void fft(vector<cd> & a, bool invert) {
7    int n = (int)a.size();
8
9    for (int i = 1, j = 0; i < n; i++) {
10      int bit = n >> 1;
11      for (; j & bit; bit >>= 1)
12        j ^= bit;
13      j ^= bit;
14
15      if (i < j)
16        swap(a[i], a[j]);
17    }
18
```

```
19    for (int len = 2; len <= n; len <<= 1) {
20      double ang = 2 * PI / len * (invert ? -1 : 1);
21      cd wlen(cos(ang), sin(ang));
22      for (int i = 0; i < n; i += len) {
23        cd w(1);
24        for (int j = 0; j < len / 2; j++) {
25          cd u = a[i+j], v = a[i+j+len/2] * w;
26          a[i+j] = u + v;
27          a[i+j+len/2] = u - v;
28          w *= wlen;
29        }
30      }
31    }
32
33    if (invert) {
34      for (cd & x : a)
35        x /= n;
36    }
37  }
38
39  vector<int> multiply(vector<int> const& a, vector<int> const& b) {
40      vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
41      int n = 1;
42      while (n < a.size() + b.size())
43          n <<= 1;
44      fa.resize(n);
45      fb.resize(n);
46
47      fft(fa, false);
48      fft(fb, false);
49      for (int i = 0; i < n; i++)
50          fa[i] *= fb[i];
51      fft(fa, true);
52
53      vector<int> result(n);
54      for (int i = 0; i < n; i++)
55          result[i] = round(fa[i].real());
56      return result;
57  }
58
59  //normalizing for when mult is between 2 big numbers and not polynomials
60  int carry = 0;
61  for (int i = 0; i < n; i++){
```

```
62        result[i] += carry;
63        carry = result[i] / 10;
64        result[i] %= 10;
65    }
```

13.4   NTT

```
1   // number theory transform
2
3   const int MOD = 998244353, ROOT = 3;
4   // const int MOD = 7340033, ROOT = 5;
5   // const int MOD = 167772161, ROOT = 3;
6   // const int MOD = 469762049, ROOT = 3;
7
8   int power(int base, int exp) {
9       int res = 1;
10      while (exp) {
11          if (exp % 2) res = 1LL * res * base % MOD;
12          base = 1LL * base * base % MOD;
13          exp /= 2;
14      }
15      return res;
16  }
17
18  void ntt(vector<int>& a, bool invert) {
19    int n = a.size();
20    for (int i = 1, j = 0; i < n; i++) {
21      int bit = n >> 1;
22      for (; j & bit; bit >>= 1) j ^= bit;
23      j ^= bit;
24      if (i < j) swap(a[i], a[j]);
25    }
26    for (int len = 2; len <= n; len <<= 1) {
27      int wlen = power(ROOT, (MOD - 1) / len);
28      if (invert) wlen = power(wlen, MOD - 2);
29      for (int i = 0; i < n; i += len) {
30        int w = 1;
31        for (int j = 0; j < len / 2; j++) {
32          int u = a[i + j], v = 1LL * a[i + j + len / 2] * w % MOD;
33          a[i + j] = u + v < MOD ? u + v : u + v - MOD;
34          a[i + j + len / 2] = u - v >= 0 ? u - v : u - v + MOD;
35          w = 1LL * w * wlen % MOD;
36        }
```

```
37      }
38    }
39    if (invert) {
40      int n_inv = power(n, MOD - 2);
41      for (int& x : a) x = 1LL * x * n_inv % MOD;
42    }
43  }
44
45  vector<int> multiply(vector<int>& a, vector<int>& b) {
46    int n = 1;
47    while (n < a.size() + b.size()) n <<= 1;
48    a.resize(n), b.resize(n);
49    ntt(a, false), ntt(b, false);
50    for (int i = 0; i < n; i++) a[i] = 1LL * a[i] * b[i] % MOD;
51    ntt(a, true);
52    return a;
53  }
54  // usage
55  // vector<int> a = {1, 2, 3}, b = {4, 5, 6};
56  // vector<int> c = multiply(a, b);
57  // for (int x : c) cout << x << " ";
```

13.5   Roots NTT

```
1   1*2^0 + 1 = 2, 1, 1
2   1*2^1 + 1 = 3, 2, 2
3   1*2^2 + 1 = 5, 2, 3
4   2*2^3 + 1 = 17, 2, 9
5   1*2^4 + 1 = 17, 3, 6
6   3*2^5 + 1 = 97, 19, 46
7   3*2^6 + 1 = 193, 11, 158
8   2*2^7 + 1 = 257, 9, 200
9   1*2^8 + 1 = 257, 3, 86
10  15*2^9 + 1 = 7681, 62, 1115
11  12*2^10 + 1 = 15361, 49, 1254
12  6*2^11 + 1 = 12289, 7, 8778
13  3*2^12 + 1 = 12289, 41, 4496
14  5*2^13 + 1 = 40961, 12, 23894
15  4*2^14 + 1 = 65537, 15, 30584
16  2*2^15 + 1 = 65537, 9, 7282
17  1*2^16 + 1 = 65537, 3, 21846
18  6*2^17 + 1 = 786433, 8, 688129
19  3*2^18 + 1 = 786433, 5, 471860
```

```
20  11*2^19 + 1 = 5767169, 12, 3364182
21  7*2^20 + 1 = 7340033, 5, 4404020
22  11*2^21 + 1 = 23068673, 38, 21247462
23  25*2^22 + 1 = 104857601, 21, 49932191
24  20*2^23 + 1 = 167772161, 4, 125829121
25  10*2^24 + 1 = 167772161, 2, 83886081
26  5*2^25 + 1 = 167772161, 17, 29606852
27  7*2^26 + 1 = 469762049, 30, 15658735
28  15*2^27 + 1 = 2013265921, 137, 749463956
29  12*2^28 + 1 = 3221225473, 8, 2818572289
30  6*2^29 + 1 = 3221225473, 14, 1150437669
31  3*2^30 + 1 = 3221225473, 13, 1734506024
32  35*2^31 + 1 = 75161927681, 93, 44450602392
33  18*2^32 + 1 = 77309411329, 106, 5105338484
```

14 Strings

14.1 Hashed String

```
1
2  /*
3                              Hashed string
4    ----------------------------------------------------------------
5    Class for hashing string. Allows retrieval of hashes of any substring
        in the string.
6
7    Double hash or use big mod values to avoid problems with collisions
8
9    Important to take into account that the values used for the letters
        are not 0 to 26. it is ascii characters.
10
11   Time Complexity(Construction): O(n)
12   Space Complexity: O(n)
13  */
14
15  const ll MOD = 212345678987654321LL;
16  const ll base = 33;
17
18  class HashedString {
19    private:
20    static const long long M = 1e9 + 9;
21    static const long long B = 9973;
22
```

```
23    static vector<long long> pow;
24
25    vector<long long> p_hash;
26
27  public:
28    HashedString(const string &s) : p_hash(s.size() + 1) {
29      while (pow.size() <= s.size()) { pow.push_back((pow.back() * B) % M)
          ; }
30
31      p_hash[0] = 0;
32      for (int i = 0; i < (int)s.size(); i++) {
33        p_hash[i + 1] = ((p_hash[i] * B) % M + s[i]) % M;
34      }
35    }
36    // substring [start, end]
37    long long get_hash(int start, int end) {
38      long long raw_val =
39          (p_hash[end + 1] - (p_hash[start] * pow[end - start + 1]));
40      return (raw_val % M + M) % M;
41    }
42      // substring [start, end] with l added at the end. Remember it is
          ascii characters, not the number from 1 to 26 for the hash.
43      long long get_hash(int start, int end, int l){
44        if(start > end) return l;
45        long long val = ((p_hash[end + 1] * B) + l - (p_hash[start] * pow[
            end - start + 2]));
46        val = (val % M + M) % M;
47        return val;
48      }
49  };
50  // you cant skip this
51  vector<long long> HashedString::pow = {1};
```

14.2 KMP

```
1  /*
2                              KMP
3    ----------------------------------------------------------------
4    Computes the prefix function for a string.
5
6    Maximum length of substring that ends at position i and is proper
        prefix (not equal to string itself) of string
7    pf[i]  is the length of the longest proper prefix of the substrings
```

```
8          [0......i]$  which is also a suffix of this substring.

9     For matching, one can append the string with a delimiter like $
         between them

10
11    Time Complexity: O(n)
12    Space Complexity: O(n)
13 */
14
15 vector<int> KMP(string s){
16   int n=(int)s.length();
17   vector<int> pf(n, 0);
18   for(int i=1;i<n;i++){
19     int j=pf[i-1];
20     while(j>0 && s[i]!=s[j]){
21       j=pf[j-1];
22     }
23     if(s[i]==s[j]){
24       pf[i]=j+1;
25     }
26   }
27   return pf;
28 }
29
30 // Counts how many times each prefix occurs
31 // Same thing can be done for two strings but only considering indices
         of second string
32 vector<int> count_occurrences_of_prefixes(vector<int> pf){
33   int n=(int)pf.size();
34   vector<int> ans(n + 1);
35   for (int i = 0; i < n; i++)
36     ans[pi[i]]++;
37   for (int i = n-1; i > 0; i--)
38     ans[pi[i-1]] += ans[i];
39   for (int i = 0; i <= n; i++)
40     ans[i]++;
41 }
42
43 // Computes automaton for string
44 // useful for not having to recalculate KMP of string s
45 // can be utilized when the second string (the one in which we are
       trying to count occurences)
46 // is very large
```

```
47 void compute_automaton(string s, vector<vector<int>>& aut) {
48   s += '#';
49   int n = s.size();
50   vector<int> pi = KMP(s);
51   aut.assign(n, vector<int>(26));
52   for (int i = 0; i < n; i++) {
53     for (int c = 0; c < 26; c++) {
54       if (i > 0 && 'a' + c != s[i])
55         aut[i][c] = aut[pi[i-1]][c];
56       else
57         aut[i][c] = i + ('a' + c == s[i]);
58     }
59   }
60 }
```

14.3 Least Rotation String

```
1 /*
2                       Min cyclic shift
3    ------------------------------------------------------------
4    Finds the lexicographically minimum cyclic shift of a string
5
6    Time Complexity: O(n)
7    Space Complexity: O(n)
8 */
9
10 string least_rotation(string s)
11 {
12   s += s;
13   vector<int> f(s.size(), -1);
14   int k = 0;
15   for(int j = 1; j < s.size(); j++)
16   {
17     char sj = s[j];
18     int i = f[j - k - 1];
19     while(i != -1 && sj != s[k + i + 1])
20     {
21       if(sj < s[k + i + 1]){
22         k = j - i - 1;
23       }
24       i = f[i];
25     }
26     if(sj != s[k + i + 1])
```

```
27        {
28          if(sj < s[k]){
29             k = j;
30          }
31          f[j - k] = -1;
32        }
33        else
34          f[j - k] = i + 1;
35      }
36      return s.substr(k, s.size() / 2);
37  }
```

14.4   Manacher

```
1   /*
2                            Manacher
3      ------------------------------------------------------------------
4      Computes the length of the longest palindrome centered at position i.
5
6      p[i] is length of biggest palindrome centered in this position.
7      Be careful with characters that are inserted to account for odd and
           even palindromes
8
9      Time Complexity: O(n)
10     Space Complexity: O(n)
11
12  */
13
14  // Number of palindromes centered at each position
15
16  vector<int> manacher_odd(string s)
17  {
18    int n = s.size();
19    s = "$" + s + "^";
20    vector<int> p(n + 2);
21    int l = 1, r = 1;
22    for (int i = 1; i <= n; i++)
23    {
24      p[i] = max(0, min(r - i, p[l + (r - i)]));
25      while (s[i - p[i]] == s[i + p[i]])
26      {
27        p[i]++;
28      }
```

```
29      if (i + p[i] > r)
30      {
31        l = i - p[i], r = i + p[i];
32      }
33    }
34    return vector<int>(begin(p) + 1, end(p) - 1);
35  }
36  vector<int> manacher(string s)
37  {
38    string t;
39    for (auto c : s)
40    {
41      t += string("#") + c;
42    }
43    auto res = manacher_odd(t + "#");
44    return vector<int>(begin(res) + 1, end(res) - 1);
45  }
46
47  // usage
48  // vector<int> p = manacher("abacaba");
49  // this will return {2, 1, 4, 1, 2, 1, 8, 1, 2, 1, 4, 1, 2}
50  // vector<int> p = manacher("abaaba");
51  // this will return {2, 1, 4, 1, 2, 7, 2, 1, 4, 1, 2}
```

14.5   Suffix Array

```
1   /*
2                            Suffix Array
3      ------------------------------------------------------------------
4      Computes the suffix array of a string in O(n log n).
5      Sorted array of all cyclic shifts of a string.
6
7      If you want sorted suffixes append $ to the end of the string.
8      lc is longest common prefix. Lcp of two substrings j > i is min(lc[i],
           ....., lc[j - 1]).
9
10     To compute Largest common substring of multiple strings
11     Join all strings separating them with special character like $ (it has
           to be different for each string)
12     Sliding window on lcp array (all string have to appear on  the sliding
           window and
13     the lcp of the interval will give the length of the substring that
           appears on all strings)
```

```
14
15      Time Complexity: O(n log n)
16      Space Complexity: O(n)
17
18   */
19
20   struct SuffixArray
21   {
22     int n;
23     string t;
24     vector<int> sa, rk, lc;
25     SuffixArray(const std::string &s)
26     {
27       n = s.length();
28       t = s;
29       sa.resize(n);
30       lc.resize(n - 1);
31       rk.resize(n);
32       std::iota(sa.begin(), sa.end(), 0);
33       std::sort(sa.begin(), sa.end(), [&](int a, int b)
34                 { return s[a] < s[b]; });
35       rk[sa[0]] = 0;
36       for (int i = 1; i < n; ++i)
37         rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
38       int k = 1;
39       std::vector<int> tmp, cnt(n);
40       tmp.reserve(n);
41       while (rk[sa[n - 1]] < n - 1)
42       {
43         tmp.clear();
44         for (int i = 0; i < k; ++i)
45           tmp.push_back(n - k + i);
46         for (auto i : sa)
47           if (i >= k)
48             tmp.push_back(i - k);
49         std::fill(cnt.begin(), cnt.end(), 0);
50         for (int i = 0; i < n; ++i)
51           ++cnt[rk[i]];
52         for (int i = 1; i < n; ++i)
53           cnt[i] += cnt[i - 1];
54         for (int i = n - 1; i >= 0; --i)
55           sa[--cnt[rk[tmp[i]]]] = tmp[i];
56         std::swap(rk, tmp);
57         rk[sa[0]] = 0;
58         for (int i = 1; i < n; ++i)
59           rk[sa[i]] = rk[sa[i - 1]] + (tmp[sa[i - 1]] < tmp[sa[i]] || sa[i
60               - 1] + k == n || tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
61         k *= 2;
62       }
63       for (int i = 0, j = 0; i < n; ++i)
64       {
65         if (rk[i] == 0)
66         {
67           j = 0;
68         }
69         else
70         {
71           for (j -= j > 0; i + j < n && sa[rk[i] - 1] + j < n && s[i + j]
72               == s[sa[rk[i] - 1] + j];)
73             ++j;
74           lc[rk[i] - 1] = j;
75         }
76       }
77     }
78   }

   // Finds if string p appears as substring in the string
   // might now work perfectly
   int search(string &p){
       int tam = p.size();
       int l = 0, r = n;

       string tmp = "";
       while(r > l) {
           int m = l + (r-l)/2;
           tmp = t.substr(sa[m], min(n-sa[m], tam));
           if(tmp >= p){
               r = m;
           } else {
               l = m + 1;
           }
       }
       if(l < n) {
           tmp = t.substr(sa[l], min(n-sa[l], tam));
       } else{
           return -1;
       }
```

```
98          if(tmp == p){
99              return l;
100         } else {
101             return -1;
102         }
103     }
104
105
106     // Counts number of times a string p appears as substring in string
107     int count(string &p) {
108         int x = search(p);
109         if(x == -1) return 0;
110         int cnt = 0;
111         int tam = p.size();
112         int maxx = 0;
113         while((1 << maxx) + x < n) maxx++;
114         int y = x;
115         for(int i = maxx-1; i >= 0; i--) {
116             if(x + (1 << i) >= n) continue;
117             string tmp = t.substr(sa[x + (1 << i)], min(n-sa[x + (1 << i)
                    ], tam));
118             if(tmp == p) x += (1 << i);
119         }
120         return x-y+1;
121     }
122 };
123 int main() {
124     cin.tie(0)->sync_with_stdio(0);
125     string s; cin >> s;
126     SuffixArray SA(s);
127
128     int q; cin >> q;
129     for(int t = 0; t < q; t++) {
130         string tmp; cin >> tmp;
131         cout << SA.count(tmp) << endl;
132     }
133
134     return 0;
135 }
```

14.6   Suffix Automaton

```
1  /*
2                          Suffix Automaton
3     ----------------------------------------------------------------
4     Constructs suffix automaton for a given string.
5     Be careful with overlapping substrings.
6
7     Firstposition if first position string ends in.
8     If you want starting index you need to substract length of the string
           being searched.
9
10    len is length of longest string of state
11
12    Time Complexity(Construction): O(n)
13    Space Complexity: O(n)
14  */
15
16
17 struct state {
18    int len, link, firstposition;
19    vector<int> inv_link; // can skip for almost everything
20    map<char, int> next;
21 };
22
23 const int MAXN = 100000;
24 state st[MAXN * 2];
25 ll cnt[MAXN*2], cntPaths[MAXN*2], cntSum[MAXN*2], cnt1[2 * MAXN];
26 int sz, last;
27
28 // call this first
29 void initSuffixAutomaton() {
30    st[0].len = 0;
31    st[0].link = -1;
32    sz++;
33    last = 0;
34 }
35
36 // construction is O(n)
37 void insertChar(char c) {
38    int cur = sz++;
39    st[cur].len = st[last].len + 1;
40    st[cur].firstposition=st[last].len;
41    int p = last;
42    while (p != -1 && !st[p].next.count(c)) {
43        st[p].next[c] = cur;
```

```
44          p = st[p].link;
45      }
46      if (p == -1) {
47          st[cur].link = 0;
48      } else {
49          int q = st[p].next[c];
50          if (st[p].len + 1 == st[q].len) {
51              st[cur].link = q;
52          } else {
53              int clone = sz++;
54              st[clone].len = st[p].len + 1;
55              st[clone].next = st[q].next;
56              st[clone].link = st[q].link;
57              st[clone].firstposition=st[q].firstposition;
58              while (p != -1 && st[p].next[c] == q) {
59                  st[p].next[c] = clone;
60                  p = st[p].link;
61              }
62              st[q].link = st[cur].link = clone;
63          }
64      }
65      last = cur;
66      cnt[last]=1;
67  }
68
69  // searches for the starting position in O(len(s)). Returns starting
       index of first ocurrence or -1 if it does not appear.
70  int search(string s){
71      int cur=0, i=0, n=(int)s.length();
72      while(i<n){
73          if(!st[cur].next.count(s[i])) return -1;
74          cur=st[cur].next[s[i]];
75          i++;
76      }
77      //sumar 2 si se quiere 1 indexado
78      return st[cur].firstposition-n+1;
79  }
80
81  void dfs(int cur){
82      cntPaths[cur]=1;
83      for(auto [x, y]:st[cur].next){
84          if(cntPaths[y]==0) dfs(y);
85          cntPaths[cur]+=cntPaths[y];
```

```
86      }
87  }
88
89  // Counts how many paths exist from state. How many substrings exist
       from a specific state.
90  // Stored in cntPaths
91  void countPaths(){
92      dfs(0);
93  }
94
95  // Computes the number of times each state appears
96  void countOcurrences(){
97      vector<pair<int, int>> a;
98      for(int i=sz-1;i>0;i--){
99          a.push_back({st[i].len, i});
100     }
101     sort(a.begin(), a.end());
102     for(int i=sz-2;i>=0;i--){
103         cnt[st[a[i].second].link]+=cnt[a[i].second];
104     }
105 }
106
107 void dfs1(int cur){
108     for(auto [x, y]:st[cur].next){
109         if(cntSum[y]==cnt[y]) dfs1(y);
110         cntSum[cur]+=cntSum[y];
111     }
112 }
113
114 // Computes the number of times each state or any of its children appear
       in the string.
115 void countSumOcurrences(){
116     for(int i=0;i<sz;i++){
117         cntSum[i]=cnt[i];
118     }
119     dfs1(0);
120 }
121
122
123 // Counts number of paths that can reach specific state.
124 void countPathsReverse(){
125     cnt1[0]=1;
126     queue<int> q;
```

```
127      q.push(0);
128      vector<int> in(2*MAXN, 0);
129      for(int i=0;i<sz;i++){
130        for(auto [x, y]:st[i].next){
131          in[y]++;
132        }
133      }
134      while((int)q.size()){
135        int cur=q.front();
136        q.pop();
137        for(auto [x, y]:st[cur].next){
138          cnt1[y]+=cnt1[cur];
139          in[y]--;
140          if(in[y]==0){
141            q.push(y);
142          }
143        }
144      }
145    }
146
147    // Computes the kth smallest string that appears on the string (counting
             repetitions)
148    string kthSmallest(ll k){
149      string s="";
150      int cur=0;
151      while(k>0){
152        for(auto [c, y]:st[cur].next){
153          if(k>cntSum[y]) k-=cntSum[y];
154          else{
155            k-=cnt[y];
156            s+=c;
157            cur=y;
158            break;
159          }
160        }
161      }
162      return s;
163    }
164
165
166    // Computes the kth smallest string that appears on the string (without
             counting repetitions)
167    string kthSmallestDistinct(ll k){
168      string s="";
169      int cur=0;
170      while(k>0){
171        for(auto [c, y]:st[cur].next){
172          if(k>cntPaths[y]) k-=cntPaths[y];
173          else{
174            k--;
175            s+=c;
176            cur=y;
177            break;
178          }
179        }
180      }
181      return s;
182    }
183
184
185    // Precomputation to find all occurrences of a substring
186    void precoumpte_for_all_ocurrences(){
187      for (int v = 1; v < sz; v++) {
188        st[st[v].link].inv_link.push_back(v);
189      }
190    }
191
192    // Finding all occurences of substring in string
193    // P_length is length of substring
194    // v is state where first ocurrence happens
195    // be careful as indices can appear multiple times due to clone states
196    // if you want to avoid duplicate positions utilize set or have a flag
             for each state to know if it is clone or not
197    void output_all_occurrences(int v, int P_length) {
198      cout << st[v].firstposition - P_length + 1 << endl;
199      for (int u : st[v].inv_link)
200        output_all_occurrences(u, P_length);
201    }
202
203
204    //longest common substring
205    //build automaton for s first
206    string lcs (string S, string T) {
207      int v = 0, l = 0, best = 0, bestpos = 0;
208      for (int i = 0; i < T.size(); i++) {
209        while (v && !st[v].next.count(T[i])) {
```

```
210            v = st[v].link ;
211            l = st[v].len;
212        }
213        if (st[v].next.count(T[i])) {
214            v = st [v].next[T[i]];
215            l++;
216        }
217        if (l > best) {
218            best = l;
219            bestpos = i;
220        }
221    }
222    return T.substr(bestpos - best + 1, best);
223 }
224
225
226 int main(){
227   ios_base::sync_with_stdio(false); cin.tie(NULL);
228   string s; cin >> s;
229   initSuffixAutomaton();
230   for(char c:s){
231     insertChar(c);
232   }
233 }
```

14.7   Trie Ahocorasick

```
1  /*
2                        Trie - AhoCorasick
3     ----------------------------------------------------------------
4     Builds a trie for subset of strings and computes suffix links.
5     KATCL implementation is cleaner.
6
7     Time Complexity(Construction): O(m) where m is sum
8      of lengths of strings
9     Space Complexity: O(m)
10 */
11
12
13
14 const int K = 26;
15
16 struct Vertex {
17     int next[K];
18     bool output = false;
19     int p = -1;
20     char pch;
21     int link = -1;
22     int go[K];
23
24     Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
25         fill(begin(next), end(next), -1);
26         fill(begin(go), end(go), -1);
27     }
28 };
29
30 vector<Vertex> t(1);
31
32 void add_string(string const& s) {
33     int v = 0;
34     for (char ch : s) {
35         int c = ch - 'a';
36         if (t[v].next[c] == -1) {
37             t[v].next[c] = t.size();
38             t.emplace_back(v, ch);
39         }
40         v = t[v].next[c];
41     }
42     t[v].output = true;
43 }
44
45 int go(int v, char ch);
46
47 int get_link(int v) {
48     if (t[v].link == -1) {
49         if (v == 0 || t[v].p == 0)
50             t[v].link = 0;
51         else
52             t[v].link = go(get_link(t[v].p), t[v].pch);
53     }
54     return t[v].link;
55 }
56
57 int go(int v, char ch) {
58     int c = ch - 'a';
59     if (t[v].go[c] == -1) {
```

```
60        if (t[v].next[c] != -1)
61            t[v].go[c] = t[v].next[c];
62        else
63            t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
64    }
65    return t[v].go[c];
66 }
```

14.8   Z Function

```
1  /*
2                           Z_function
3     ----------------------------------------------------------------
4     Computes the z_function for any string.
5     ith element is equal to the greatest number of characters starting
          from the position i that coincide with the first characters of s
6
7     z[i] length of the longest string that is, at the same time, a prefix
          of s and a prefix of the suffix of $s$ starting at  i.
8
9     to compress string, one can run z_function and then find the smallest
          i that divides n such that i + z[i] = n
10
11    Time Complexity: O(n)
12    Space Complexity: O(n)
13 */
14
15
16 vector<int> z_function(string s) {
17    int n = s.size();
18    vector<int> z(n);
19    int l = 0, r = 0;
20    for(int i = 1; i < n; i++) {
21      if(i < r) {
22        z[i] = min(r - i, z[i - l]);
23      }
24      while(i + z[i] < n && s[z[i]] == s[i + z[i]]) {
25        z[i]++;
26      }
27      if(i + z[i] > r) {
28        l = i;
29        r = i + z[i];
30      }
```

```
31    }
32    return z;
33 }
34
35 // usage
36 // vector<int> z = z_function("abacaba");
37 // this will return {0, 0, 1, 0, 3, 0, 1}
38 // vector<int> z = z_function("aaaaa");
39 // this will return {0, 4, 3, 2, 1}
40 // vector<int> z = z_function("aaabaab");
41 // this will return {0, 2, 1, 0, 2, 1, 0}
```

15   Trees

15.1   Centroid Decomposition

```
1  /*
2                       Centroid Decomposition
3     ----------------------------------------------------------------
4     Finds the centroid decomposition of a given tree.
5     Any vertex can have at most log n centroid ancestors
6
7     The code below is the solution to Xenia and tree.
8     Given tree, queries of two types:
9     1) u - color vertex u
10    2) v - print minimum distance of vertex v to any colored vertex before
11
12
13    Time Complexity: O(n log n)
14    Space Complexity: O(n log n)
15 */
16 const int MAXN=200005;
17
18 vector<int> adj[MAXN];
19 vector<bool> is_removed(MAXN, false);
20 vector<int> subtree_size(MAXN, 0);
21 vector<int> dis(MAXN, 1e9);
22 vector<vector<pair<int, int>>> ancestor(MAXN);
23
24 int get_subtree_size(int node, int parent = -1) {
25    subtree_size[node] = 1;
26    for (int child : adj[node]) {
27      if (child == parent || is_removed[child]) { continue; }
```

```
28        subtree_size[node] += get_subtree_size(child, node);
29      }
30      return subtree_size[node];
31   }
32
33   int get_centroid(int node, int tree_size, int parent = -1) {
34      for (int child : adj[node]) {
35         if (child == parent || is_removed[child]) { continue; }
36         if (subtree_size[child] * 2 > tree_size) {
37            return get_centroid(child, tree_size, node);
38         }
39      }
40      return node;
41   }
42
43   void getDist(int cur, int centroid, int p=-1, int dist=1){
44       for (int child:adj[cur]){
45           if(child==p ||is_removed[child])
46               continue;
47           dist++;
48           getDist(child, centroid, cur, dist);
49           dist--;
50       }
51       ancestor[cur].push_back(make_pair(centroid, dist));
52   }
53
54   void update(int cur){
55       for (int i=0;i<ancestor[cur].size();i++){
56           dis[ancestor[cur][i].first]=min(dis[ancestor[cur][i].first],
57               ancestor[cur][i].second);
58       }
59       dis[cur]=0;
60   }
61
62   int query(int cur){
63       int mini=dis[cur];
64       for (int i=0;i<ancestor[cur].size();i++){
65           mini=min(mini, ancestor[cur][i].second+dis[ancestor[cur][i].first
66               ]);
67       }
68       return mini;
69   }
```

```
69   void build_centroid_decomp(int node = 1) {
70      int centroid = get_centroid(node, get_subtree_size(node));
71
72      for (int child : adj[centroid]) {
73         if (is_removed[child]) { continue; }
74         getDist(child, centroid, centroid);
75      }
76
77      is_removed[centroid] = true;
78
79      for (int child : adj[centroid]) {
80         if (is_removed[child]) { continue; }
81         build_centroid_decomp(child);
82      }
83   }
```

15.2   HLD subtree query

```
1    //hld subtree queries
2    // path is [in[nextv]....in[v]]
3    void dfs_sz(int v = 0) {
4        sz[v] = 1;
5        for(auto &u: g[v]) {
6            dfs_sz(u);
7            sz[v] += sz[u];
8            if(sz[u] > sz[g[v][0]]) {
9                swap(u, g[v][0]);
10           }
11       }
12   }
13
14   void dfs_hld(int v = 0) {
15       in[v] = t++;
16       for(auto u: g[v]) {
17           nxt[u] = (u == g[v][0] ? nxt[v] : u);
18           dfs_hld(u);
19       }
20       out[v] = t;
21   }
```

15.3   Heavy Light Decomposition

```
1    /*
2                    Heavy Light Decomposition(HLD)
```

```cpp
    ----------------------------------------------------------------
    Constructs the heavy light decomposition of a tree

    Splits the tree into several paths so that we can reach the root
        vertex from any v  by traversing at most log n paths.
    In addition, none of these paths intersect with another.

    Time Complexity(Creation): O(n log n)
    Time Complexity(Query): O((log n) ^ 2) usually, depending on the query
        itself
    Space Complexity: O(n)
*/

//call dfs1 first
struct SegmentTree {
  vector<ll> a;
  int n;

  SegmentTree(int _n) : a(2 * _n, 0), n(_n) {}

  void update(int pos, ll val) {
    for (a[pos += n] = val; pos > 1; pos >>= 1) {
      a[pos / 2] = (a[pos]^a[pos ^ 1]);
    }
  }

  ll get(int l, int r) {
    ll res = 0;
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
      if (l & 1) {
        res^=a[l++];
      }
      if (r & 1) {
        res^=a[--r];
      }
    }
    return res;
  }
};

const int MAXN=500005;
vector<int> adj[MAXN];
```

```cpp
SegmentTree st(MAXN);
int a[MAXN], sz[MAXN], to[MAXN], dpth[MAXN], s[MAXN], par[MAXN];
int cnt=0;

void dfs1(int cur, int p){
    sz[cur]=1;
    for(int x:adj[cur]){
        if(x==p) continue;
        dpth[x]=dpth[cur]+1;
        par[x]=cur;
        dfs1(x, cur);
        sz[cur]+=sz[x];
    }
}

void dfs(int cur, int p, int l){
    st.update(cnt, a[cur]);
    s[cur]=cnt++;
    to[cur]=l;
    int g=-1;
    for(int x:adj[cur]){
        if(x==p) continue;
        if(g==-1 || sz[g]<sz[x]){
            g=x;
        }
    }
    if(g==-1) return;
    dfs(g, cur, l);
    for(int x:adj[cur]){
        if(x==p || x==g) continue;
        dfs(x, cur, x);
    }
}

int query(int u, int v){
    int res=0;
    while(to[u]!=to[v]){
        if(dpth[to[u]]<dpth[to[v]]) swap(u, v);
        res^=st.get(s[to[u]], s[u]+1);
        u=par[to[u]];
    }
    if(dpth[u]>dpth[v]) swap(u, v);
    res^=st.get(s[u], s[v]+1);
```

```
87          return res;
88      }
89
90
91
92
93      //alternate implementation
94      vector<int> parent, depth, heavy, head, pos;
95      int cur_pos;
96
97      int dfs(int v, vector<vector<int>> const& adj) {
98          int size = 1;
99          int max_c_size = 0;
100         for (int c : adj[v]) {
101             if (c != parent[v]) {
102                 parent[c] = v, depth[c] = depth[v] + 1;
103                 int c_size = dfs(c, adj);
104                 size += c_size;
105                 if (c_size > max_c_size)
106                     max_c_size = c_size, heavy[v] = c;
107             }
108         }
109         return size;
110     }
111
112     void decompose(int v, int h, vector<vector<int>> const& adj) {
113         head[v] = h, pos[v] = cur_pos++;
114         if (heavy[v] != -1)
115             decompose(heavy[v], h, adj);
116         for (int c : adj[v]) {
117             if (c != parent[v] && c != heavy[v])
118                 decompose(c, c, adj);
119         }
120     }
121
122     void init(vector<vector<int>> const& adj) {
123         int n = adj.size();
124         parent = vector<int>(n);
125         depth = vector<int>(n);
126         heavy = vector<int>(n, -1);
127         head = vector<int>(n);
128         pos = vector<int>(n);
129         cur_pos = 0;
```

```
130
131         dfs(0, adj);
132         decompose(0, 0, adj);
133     }
134
135     int query(int a, int b) {
136         int res = 0;
137         for (; head[a] != head[b]; b = parent[head[b]]) {
138             if (depth[head[a]] > depth[head[b]])
139                 swap(a, b);
140             int cur_heavy_path_max = segment_tree_query(pos[head[b]], pos[b
                    ]);
141             res = max(res, cur_heavy_path_max);
142         }
143         if (depth[a] > depth[b])
144             swap(a, b);
145         int last_heavy_path_max = segment_tree_query(pos[a], pos[b]);
146         res = max(res, last_heavy_path_max);
147         return res;
148     }
```

15.4   Kruskal Reconstruction Tree

```
1    #include <bits/stdc++.h>
2    using namespace std;
3
4    constexpr int MAX_N = 4e5 + 5;
5    constexpr int LG = 20;
6
7    int n, m, q, va[MAX_N], f[MAX_N], nx;
8    int trace(int v) { return f[v] == v ? v : f[v] = trace(f[v]); }
9
10   /** Implements LCA with binary lifting */
11   namespace LCA {
12   int lift[MAX_N][LG], ch[MAX_N][2], t, tin[MAX_N], tout[MAX_N], tour[2 *
         MAX_N];
13
14   bool is_ancestor(int u, int v) { return tin[u] <= tin[v] && tout[u] >=
         tout[v]; }
15
16   int lca(int u, int v) {
17     if (is_ancestor(u, v)) { return u; }
18     if (is_ancestor(v, u)) { return v; }
```

```
19
20    for (int i = LG - 1; i >= 0; --i) {
21      if (!is_ancestor(lift[u][i], v)) { u = lift[u][i]; }
22    }
23
24    return lift[u][0];
25  }
26
27  void dfs(int v) {
28    if (v == -1) { return; }
29    tour[t] = v;
30    tin[v] = t;
31
32    for (int i = 1; i < LG; i++) { lift[v][i] = lift[lift[v][i - 1]][i -
           1]; }
33
34    dfs(ch[v][0]);
35    dfs(ch[v][1]);
36    tout[v] = t++;
37  }
38  }  // namespace LCA
39
40  /** Point Update / Range Min/Max Segment Tree */
41  namespace SGT {
42  int sg_min[1 << LG + 1], sg_max[1 << LG + 1];
43
44  void point_set(int i, int v) {
45    sg_min[i + (1 << LG)] = sg_max[i + (1 << LG)] = v;
46    for (int j = (i + (1 << LG)) / 2; j; j /= 2) {
47      sg_min[j] = min(sg_min[2 * j], sg_min[2 * j + 1]),
48      sg_max[j] = max(sg_max[2 * j], sg_max[2 * j + 1]);
49    }
50  }
51
52  /** @return edge (LCA of nodes w/ lowest and highest traversal time in [
         l, r]) */
53  int query(int l, int r) {
54    int lift = MAX_N, rt = -1;
55    for (l += 1 << LG, r += 1 << LG; l < r; l >>= 1, r >>= 1) {
56      if (l & 1) { lift = min(lift, sg_min[l]), rt = max(rt, sg_max[l++]);
             }
57      if (r & 1) { lift = min(lift, sg_min[--r]), rt = max(rt, sg_max[r]);
             }
```

```
58    }
59    return va[LCA::lca(LCA::tour[lift], LCA::tour[rt])];
60  }
61  }  // namespace SGT
62
63  void solve() {
64    cin >> n >> m >> q;
65
66    iota(f, f + 2 * n, 0);
67    nx = n;
68    memset(LCA::ch, -1, sizeof(LCA::ch));
69
70    for (int i = 0; i < m; i++) {
71      int u, v;
72      cin >> u >> v;
73      // assign u and v to the top of their component
74      u = trace(--u), v = trace(--v);
75
76      // if the edge is within one component, no further work is needed
77      // otherwise, create a new node that is the parent of both
             components
78
79      if (u == v) { continue; }
80      va[nx] = i, LCA::ch[nx][0] = u, LCA::ch[nx][1] = v;
81      f[u] = f[v] = LCA::lift[u][0] = LCA::lift[v][0] = nx++;
82    }
83
84    LCA::lift[2 * n - 2][0] = 2 * n - 2;
85    LCA::dfs(2 * n - 2);
86
87    // update segment tree for range min/max of dfs-order traversal times
88    for (int i = 0; i < 2 * n - 1; i++) { SGT::point_set(i, LCA::tin[i]);
             }
89
90    while (q--) {
91      int l, r;
92      cin >> l >> r;
93      if (l == r) {
94        cout << "0␣";
95      } else {
96        cout << SGT::query(l - 1, r) + 1 << "␣";
97      }
98    }
```

```
99
100    cout << "\n";
101  }
102
103  int main() {
104    int test_num;
105    cin >> test_num;
106    for (int t = 0; t < test_num; t++) { solve(); }
107  }
```

15.5   Lowest Common Ancestor (LCA)

```
1  /*
2                     LCA(Lowest Common Ancestor)
3    ----------------------------------------------------------------------
4    Computes the lowest common ancestor of two vertices in a tree.
5
6    Be careful as implementation is indexed starting with 1
7
8    Time Complexity(Creation): O(n log n)
9    Time Complexity(Query): O(log n)
10   Space Complexity: O(n log n)
11 */
12
13 const int N=200005;
14 vector<int> adj[N];
15 vector<int> start(N), end1(N), depth(N);
16 vector<vector<int>> t(N, vi(32));
17 int timer=0;
18 int n, l;
19 // l=(int)ceil(log2(n))
20 // call dfs(1, 1, 0)
21 // 1 indexed, dont use 0 indexing
22
23
24 void dfs(int cur, int p, int cnt){
25     depth[cur]=cnt;
26     t[cur][0]=p;
27     start[cur]=timer++;
28     for(int i=1;i<=l;i++){
29         t[cur][i]=t[t[cur][i-1]][i-1];
30     }
31     for(int x:adj[cur]){
32         if(x==p) continue;
33         dfs(x, cur, cnt+1);
34     }
35     end1[cur]=++timer;
36 }
37
38 bool ancestor(int u, int v){
39     return start[u]<=start[v] && end1[u]>=end1[v];
40 }
41
42 int lca(int u, int v){
43     if(ancestor(u, v))
44         return u;
45     if (ancestor(v, u)){
46         return v;
47     }
48     for(int i=l;i>=0;i--){
49         if(!ancestor(t[u][i], v)){
50             u=t[u][i];
51         }
52     }
53     return t[u][0];
54 }
```

15.6   Tree Diameter

```
1  /*
2                     Tree Diameter
3    ----------------------------------------------------------------
4    Finds the vertex most distant to vertex on which function is called.
5
6    The first value is the vertex itself and the second value is the
7        distance.
8    To find diameter run algorithm twice, first on random vertex and then
9        on the vertex that is farthest away.
10   The vertex that is the farthest away from any vertex in tree must be
11       an endpoint of the diameter.
12   Time Complexity: O(n)
13   Space Complexity: O(n)
14 */
```

```
15 pair<int, int> dfs(const vector<vector<int>> &tree, int node = 1,
16   int previous = 0, int length = 0) {
17   pair<int, int> max_path = {node, length};
18   for (const int &i : tree[node]) {
19     if (i == previous) { continue; }
20     pair<int, int> other = dfs(tree, i, node, length + 1);
21     if (other.second > max_path.second) { max_path = other; }
22   }
23   return max_path;
24 }
25
```

15.7   Virtual Tree

```
1  #include <bits/stdc++.h>
2  using namespace std;
3
4  const int MAXN = 2e5 + 10;
5  const int LOGN = 20;
6  const int MOD = 998244353;
7
8  vector<int> g[MAXN];        // Adjacency list of the given graph
9  vector<int> ver[MAXN];      // For each color c, ver[c] stores vertices
        with color c
10 vector<int> aux_g[MAXN];    // Adjacency list for the virtual tree
11 int col[MAXN];              // Color of each vertex
12 int tmr, n;                 // Global time counter and number of
      vertices
13 int up[LOGN][MAXN], dep[MAXN], tin[MAXN]; // For computing LCA and entry
         time
14
15 // DP arrays for dynamic programming on the virtual tree
16 int dp[MAXN][2], sum[MAXN];
17
18 // Preprocessing function: DFS to compute tin, up array, and dep
19 void dfs_precalc(int v, int p) {
20     tin[v] = ++tmr;
21     up[0][v] = p;
22     dep[v] = dep[p] + 1;
23     for (int i = 1; i < LOGN; ++i)
24         up[i][v] = up[i - 1][up[i - 1][v]];
25     for (auto to : g[v])
26         if (to != p)
```

```
27             dfs_precalc(to, v);
28 }
29
30 // Function to compute LCA using binary lifting
31 int getlca(int x, int y) {
32     if (dep[x] < dep[y]) swap(x, y);
33     for (int i = LOGN - 1; i >= 0; --i)
34         if (dep[up[i][x]] >= dep[y])
35             x = up[i][x];
36     if (x == y) return x;
37     for (int i = LOGN - 1; i >= 0; --i)
38         if (up[i][x] != up[i][y]) {
39             x = up[i][x];
40             y = up[i][y];
41         }
42     return up[0][x];
43 }
44
45 // DFS on the virtual tree to perform DP.
46 // Parameter c - target color for which counting is performed.
47
48 void dfs_calc(int v, int p, int c, int &ans) {
49     dp[v][0] = dp[v][1] = 0;
50     sum[v] = 0;
51     for(auto to : aux_g[v]) {
52         if(to == p) continue;
53         dfs_calc(to, v, c, ans);
54         // DP transitions: combining current state with result from
              subtree.
55         int nxt0 = (dp[v][0] + sum[to]) % MOD;
56         int nxt1 = ((dp[v][0] + dp[v][1]) * 1ll * sum[to] % MOD + dp[v
             ][1]) % MOD;
57         dp[v][0] = nxt0;
58         dp[v][1] = nxt1;
59     }
60     sum[v] = (dp[v][0] + dp[v][1]) % MOD;
61     if(col[v] == c) {
62         // If the vertex has the target color, it can participate in a
              valid subtree.
63         sum[v] = (sum[v] + 1) % MOD;
64         ans = (ans + sum[v]) % MOD;
65     } else {
66         ans = (ans + dp[v][1]) % MOD;
```

```cpp
67          }
68  }
69
70  // Function to build a virtual tree for color c and perform DP.
71  void calc_aux(int c, int &ans) {
72      auto p = ver[c];
73      if (p.empty()) return;
74      // Sort vertices by entry time (tin) - DFS traversal order.
75      sort(p.begin(), p.end(), [&](const int a, const int b) { return tin[
           a] < tin[b]; });
76      vector<int> verstk = {1}; // Initialize stack with the root of tree
           T (vertex 1).
77      aux_g[1].clear();
78      auto add = [&](int u, int v) {
79          aux_g[u].push_back(v);
80          aux_g[v].push_back(u);
81      };
82      // Process each vertex from set p, maintaining a stack to build the
           virtual tree.
83      for (auto u : p) {
84          if (u == 1) continue;
85          int lca = getlca(u, verstk.back());
86          if (lca != verstk.back()) {
87              while (verstk.size() >= 2 && tin[lca] < tin[verstk[verstk.
                   size() - 2]]) {
88                  add(verstk.back(), verstk[verstk.size() - 2]);
89                  verstk.pop_back();
90              }
91              if (verstk.size() >= 2 && tin[lca] != tin[verstk[verstk.size
                   () - 2]]) {
92                  aux_g[lca].clear();
93                  add(verstk.back(), lca);
94                  verstk.back() = lca;
95              } else {
96                  add(verstk.back(), lca);
97                  verstk.pop_back();
98              }
99          }
100         aux_g[u].clear();
101         verstk.push_back(u);
102     }
103     while (verstk.size() > 1) {
104         add(verstk.back(), verstk[verstk.size() - 2]);
105         verstk.pop_back();
106     }
107     // Perform DP on the virtual tree, starting from root 1.
108     return dfs_calc(1, 0, c, ans);
109 }
110
111 int main() {
112     ios_base::sync_with_stdio(0);
113     cin.tie(0);
114     cin >> n;
115     // Read colors and group vertices by color.
116     for (int i = 1; i <= n; ++i) {
117         cin >> col[i];
118         ver[col[i]].push_back(i);
119     }
120     for (int i = 1; i < n; ++i) {
121         int u, v;
122         cin >> u >> v;
123         g[u].push_back(v);
124         g[v].push_back(u);
125     }
126     dfs_precalc(1, 0);
127     int ans = 0;
128     // Process the corresponding virtual tree for each possible color.
129     for (int i = 1; i <= n; ++i)
130         calc_aux(i, ans);
131     cout << ans;
132     return 0;
133 }
```

16   Scripts

16.1   build.sh

This file should be called before stress.sh or validate.sh. build.sh name.cpp

```
1  g++ -static -DLOCAL -lm -s -x c++ -Wall -Wextra -O2 -std=c++17 -o $1 $1.
       cpp
```

16.2   stress.sh

Format is stress.sh Awrong Aslow Agen Numtests

```
1  #!/usr/bin/env bash
```

```
2
3   for ((testNum=0;testNum<$4;testNum++))
4   do
5       ./$3 > input
6       ./$2 < input > outSlow
7       ./$1 < input > outWrong
8       H1=`md5sum outWrong`
9       H2=`md5sum outSlow`
10      if !(cmp -s "outWrong" "outSlow")
11      then
12          echo "Error found!"
13          echo "Input:"
14          cat input
15          echo "Wrong Output:"
16          cat outWrong
17          echo "Slow Output:"
18          cat outSlow
19          exit
20      fi
21  done
22  echo Passed $4 tests
```

16.3   validate.sh

Format is validate.sh Awrong Avalidator Agen NumTests

```
1   #!/usr/bin/env bash
2
3   for ((testNum=0;testNum<$4;testNum++))
4   do
5       ./$3 > input
6       ./$1 < input > out
7       cat input out > data
8       ./$2 < data > res
9       result=$(cat res)
10      if [ "${result:0:2}" != "OK" ];
11      then
12          echo "Error found!"
13          echo "Input:"
14          cat input
15          echo "Output:"
16          cat out
17          echo "Validator Result:"
```

```
18          cat res
19          exit
20      fi
21  done
22  echo Passed $4 tests
```

specifier	Output	Example
d or i	Signed decimal integer	392
u	Unsigned decimal integer	7235
o	Unsigned octal	610
x	Unsigned hexadecimal integer (lower-case)	7fa
X	Unsigned hexadecimal integer (upper-case)	7FA
f	Decimal floating point, lowercase	392.65
F	Decimal floating point, uppercase	392.65
e	Scientific notation (mantissa/exponent), lowercase	3.9265e+2
E	Scientific notation (mantissa/exponent), uppercase	3.9265E+2
g	Use the shortest representation: %e or %f	392.65
G	Use the shortest representation: %E or %F	392.65
a	Hexadecimal floating point, lowercase	-0xc.90fep-2
A	Hexadecimal floating point, uppercase	-0XC.90FEP-2
c	Character	a
s	String of characters	sample
p	Pointer address	b8000000
n	Nothing printed. The argument must be a pointer to a `signed int`. The number of characters written so far is stored.	
%	A % followed by another % prints a single %	%