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1 Funciones C++

```
#include <algorithm> #include <numeric>
```

Algo	Params	Funcion
sort, stable_sort	f, l	ordena el intervalo
nth_element	f, nth, l	void ordena el n-esimo, y particiona el resto
fill, fill_n	f, l / n, elem	void llena [f, l) o [f, f+n) con elem
lower_bound, upper_bound	f, l, elem	it al primer / ultimo donde se puede insertar elem para que quede ordenada
binary_search	f, l, elem	bool esta elem en [f, l)
copy	f, l, resul	hace resul+i=f+i $\forall i$
find, find_if, find_first_of	f, l, elem / pred / f2, l2	it encuentra i $\in [f,l]$ tq. i==elem, pred(i), i $\in [f2,l2)$
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca [f2,l2) $\in [f,l)$
replace, replace_if	f, l, old / pred, new	cambia old / pred(i) por new
reverse	f, l	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	it min, max de [f,l]
lexicographical_compare	f1,l1,f2,l2	bool con [f1,l1]j[f2,l2]
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection, set_difference, set_union, set_symmetric_difference,	f1, l1, f2, l2, res	[res, ...) la op. de conj
push_heap, pop_heap, make_heap	f, l, e / e /	mete/saca e en heap [f,l), hace un heap de [f,l)
is_heap	f,l	bool es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum$ /oper de [f,l)
inner_product	f1, l1, f2, i	$T = i + [f1, l1] . [f2, \dots)$
partial_sum	f, l, r, [op]	$r+i = \sum$ /oper de [f,f+i] $\forall i \in [f,l)$
_builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
_builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
_builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
_builtin_popcount	unsigned int	Cant. de 1's en x.
_builtin_parity	unsigned int	1 si x es par, 0 si es impar.
_builtin_XXXXXXll	unsigned ll	= pero para long long's.

Specifier	Output	Example
d or i	Signed decimal integer	392
u	Unsigned decimal integer	7235
o	Unsigned octal	610
x	Unsigned hexadecimal integer	7fa
X	Unsigned hexadecimal integer (uppercase)	7FA
f	Decimal floating point, lowercase	392.65
F	Decimal floating point, uppercase	392.65
e	Scientific notation (mantissa/exponent), lowercase	3.9265e+2
E	Scientific notation (mantissa/exponent), uppercase	3.9265E+2
g	Use the shortest representation: %e or %f	392.65
G	Use the shortest representation: %E or %F	392.65
a	Hexadecimal floating point, lowercase	-0xc.90fep-2
A	Hexadecimal floating point, uppercase	-0XC.90FEP-2
c	Character	a
s	String of characters	sample
p	Pointer address	b8000000
%	A % followed by another % will write a single %.	%

2 Compile

2.1 Compile

```
1 g++-13 nombre.cpp -o nombre (compilar)
2 ./nombre (ejecutar)
3 g++ -std=c++23 -Wall -Wshadow -g -fsanitize=undefined -fsanitize=address
-D_GLIBCXX_DEBUG nombre.cpp -o nombre
```

2.2 Template

```
1 #include <bits/stdc++.h>
2 #pragma GCC optimize("O3,unroll-loops")
3 #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
4 using namespace std;
5 #define pb push_back
6 #define ll long long
7 #define s second
8 #define f first
9 #define MOD 1000000007
10 #define INF 1000000000000000000
11
12 void solve(){
13 }
```

```

14 }
15
16 int main() {
17     ios_base::sync_with_stdio(false);    cin.tie(0);    cout.tie(0);
18     int t;cin>>t;for(int T=0;T<t;T++)
19         solve();
20 }
```

3 Data Structures

3.1 BIT

```

/*
Binary Indexed Tree (Fenwick Tree) Fast Implementation
-----
Indexing: 1-based
Bounds: [1, MAXN)
Time Complexity:
    - update(x, val): O(log n) -> adds val to index x
    - get(x): O(log n) -> prefix sum from 1 to x
Space Complexity: O(n)
*/

```

```

11 const int MAXN = 10000;
12 int bit[MAXN];
13
14 // Add 'val' to index 'x'
15 void update(int x, int val) {
16     for (; x < MAXN; x += x & -x) {
17         bit[x] += val;
18     }
19 }
20
21 // Get prefix sum from 1 to 'x'
22 int get(int x) {
23     int ans = 0;
24     for (; x > 0; x -= x & -x) {
25         ans += bit[x];
26     }
27     return ans;
28 }
```

3.2 Bitset

```

1 bitset<3001> b[3001];
2
3 //set() Set the bit value at the given index to 1.
4 //reset() Set the bit value at the given index to 0.
5 //flip() Toggle the bit value at the given index.
6 //test() Check if the bit value at the given index is 1 or 0.
7 //count() Count the number of set bits.
8 //any() Checks if any bit is set
9 //all() Check if all bit is set.
10 //none() Check if no bit is set.
11 //to_string() Convert the bitset to a string representation.
12
13 #pragma GCC target("popcnt")
14 (int) __builtin_popcount(x);
15 (int) __builtin_popcountll(x);
16 __builtin_clz(x); // count leading zeros
17
18 // declare bitset
19 bitset<64> b;
```

3.3 Bit Trie

```

/*
Bit Trie (Binary Trie for Integers)
-----
Indexing: 0-based
Bit Width: [0, MAX_BIT] inclusive (e.g., 31 for 32-bit integers)
Time Complexity:
    - Insert: O(MAX_BIT)
    - Query: O(MAX_BIT)
Space Complexity: O(N * MAX_BIT) nodes (in worst case, 1 per bit per
number)
*/

```

```

11 const int K = 2; // Each node has 2 branches (bit 0 or 1)
12 const int MAX_BIT = 30; // Max bit position (for 32-bit integers)
13
14 struct Vertex {
15     int next[K]; // next[0] = child for bit 0, next[1] = child for bit 1
16
17     Vertex() {
18         fill(begin(next), end(next), -1); // -1 means no child
19     }
20 }
```

```

21 };
22
23 vector<Vertex> trie; // Trie nodes
24
25 // Inserts a number into the binary trie
26 void insert(int num) {
27     int v = 0; // Start from root
28     for (int j = MAX_BIT; j >= 0; --j) {
29         int c = (num >> j) & 1; // Extract j-th bit (0 or 1)
30         if (trie[v].next[c] == -1) {
31             trie[v].next[c] = trie.size();
32             trie.emplace_back(); // Add new node
33         }
34         v = trie[v].next[c]; // Move to next node
35     }
36 }

```

3.4 Disjoint Set Union Bipartite

```

1 /*
2 DSU with Parity - Bipartite Checker
3 -----
4 Indexing: 0-based
5 Node Bounds: [0, n-1] inclusive
6
7 Features:
8     - Tracks parity (even/odd length) of paths in each component
9     - Can be used to detect odd-length cycles (non-bipartite components)
10    - Supports dynamic edge additions
11
12 Functions:
13     - make_set(v): initializes singleton component
14     - find_set(v): returns root of v and its parity relative to root
15     - add_edge(a, b): merges two components and checks for bipartite
16         violation
17     - is_bipartite(v): returns whether component containing v is
18         bipartite
19 */
20
21 const int MAXN = 100005; // Set according to constraints
22
23 pair<int, int> parent[MAXN]; // parent[v] = {root, parity_from_root_to_v}

```

```

22 int rank[MAXN]; // Union by rank
23 bool bipartite[MAXN]; // bipartite[root] = true if component is
24 bipartite
25
26 // Create a new set for node v
27 void make_set(int v) {
28     parent[v] = {v, 0}; // Self-rooted, even parity to self
29     rank[v] = 0;
30     bipartite[v] = true;
31 }
32
33 // Find the root of v and track parity along the path (0 = even, 1 = odd
34 // )
35 pair<int, int> find_set(int v) {
36     if (v != parent[v].first) {
37         auto [par, parity] = parent[v];
38         auto root = find_set(par);
39         parent[v] = {root.first, parity ^ root.second}; // Path compression
40         with parity update
41     }
42     return parent[v];
43 }
44
45 // Adds an edge between a and b, merges components and checks for odd
46 // cycles
47 void add_edge(int a, int b) {
48     auto [ra, pa] = find_set(a); // ra = root of a, pa = parity from root
49         to a
50     auto [rb, pb] = find_set(b); // rb = root of b, pb = parity from root
51         to b
52
53     if (ra == rb) {
54         // Same component: edge (a, b) adds a cycle -> check parity
55         if ((pa ^ pb) == 0) {
56             bipartite[ra] = false; // Found odd-length cycle
57         }
58     } else {
59         // Merge smaller rank under larger
60         if (rank[ra] < rank[rb]) swap(ra, rb), swap(pa, pb);
61
62         // Make rb child of ra; update parity of root
63         parent[rb] = {ra, pa ^ pb ^ 1};
64     }
65 }

```

```

59     bipartite[ra] &= bipartite[rb]; // Component is only bipartite if
60     both were
61     if (rank[ra] == rank[rb]) rank[ra]++;
62 }
63
64 // Check if the component containing v is bipartite
65 bool is_bipartite(int v) {
66     return bipartite[find_set(v).first];
67 }

```

3.5 Disjoint Set Union

```

/*
Disjoint Set Union (Union-Find) with Rollback
-----
Indexing: 0-based
Node Bounds: [0, N-1]

Features:
- Path compression + union by size
- Optional rollback to previous state
- Supports dynamic connectivity in offline algorithms (e.g., divide
  & conquer)

Functions:
- get(x): find root of x
- connected(a, b): check if a and b are in same component
- size(x): size of component containing x
- unite(x, y): union x and y, returns true if merged
- time(): current rollback timestamp
- rollback(t): revert to state at timestamp t
*/

```

```

21 struct DSU {
22     vector<int> e;           // e[x] < 0 -> root; size = -e[x]; e
23     [x] >= 0 -> parent
24     vector<pair<int, int>> st; // rollback stack: stores (index, old
25     value)
26
27     DSU(int N) : e(N, -1) {}
28
29     // Find with path compression

```

```

28     int get(int x) {
29         return e[x] < 0 ? x : get(e[x]);
30     }
31
32     // Check if x and y belong to the same component
33     bool connected(int a, int b) {
34         return get(a) == get(b);
35     }
36
37     // Return size of component containing x
38     int size(int x) {
39         return -e[get(x)];
40     }
41
42     // Union by size, returns true if union happened
43     bool unite(int x, int y) {
44         x = get(x), y = get(y);
45         if (x == y) return false;
46         if (e[x] > e[y]) swap(x, y); // Ensure x has larger size (more
47             negative)
48         st.push_back({x, e[x]});
49         st.push_back({y, e[y]});
50         e[x] += e[y];
51         e[y] = x;
52         return true;
53     }
54
55     // Return current rollback timestamp
56     int time() {
57         return (int)st.size();
58     }
59
60     // Roll back to previous state at time t
61     void rollback(int t) {
62         for (int i = time(); i-- > t;) {
63             e[st[i].first] = st[i].second;
64         }
65         st.resize(t);
66     }

```

3.6 Dynamic Connectivity

```

/*
Offline Dynamic Connectivity - Segment Tree + Rollback DSU
-----
Indexing: 0-based
Node Bounds: [0, n-1]

Features:
- Handles dynamic edge insertions and deletions over time
- Answers queries of type: "how many connected components at time t ?"
- All operations are processed offline

Components:
- DSU with rollback (stores a stack of previous states)
- Segment tree over time to store active edge intervals
*/
// Rollbackable Disjoint Set Union (Union-Find)
struct DSU {
    vector<int> e; // e[x] < 0 means x is a root, and size is -e[x]
    vector<pair<int, int>> st; // Stack for rollback (stores changed values)
    int cnt; // Current number of connected components

    DSU() {}
    DSU(int N) : e(N, -1), cnt(N) {}

    // Find root of x with path compression
    int get(int x) {
        return e[x] < 0 ? x : get(e[x]);
    }

    // Check if x and y are connected
    bool connected(int a, int b) {
        return get(a) == get(b);
    }

    // Size of component containing x
    int size(int x) {
        return -e[get(x)];
    }

    // Union two components; record state for rollback
    void unite(int x, int y) {
        x = get(x), y = get(y);
        if (x == y) return false; // Already connected

        if (e[x] > e[y]) swap(x, y); // Union by size: ensure x is larger

        // Save state for rollback
        st.push_back({x, e[x]});
        st.push_back({y, e[y]});

        e[x] += e[y]; // Merge y into x
        e[y] = x;
        cnt--; // One fewer component
        return true;
    }

    // Undo last union
    void rollback() {
        auto [x1, y1] = st.back(); st.pop_back();
        e[x1] = y1;
        auto [x2, y2] = st.back(); st.pop_back();
        e[x2] = y2;
        cnt++;
    }
};

// Represents a single union operation (on edge u-v)
struct query {
    int v, u;
    bool united;
    query(int _v, int _u) : v(_v), u(_u), united(false) {}
};

// Segment Tree for storing edge intervals [l, r]
struct QueryTree {
    vector<vector<query>> t; // Each node stores queries that are active
                                // in that time segment
    DSU dsu;
    int T; // Number of total operations (time steps)

    QueryTree() {}
    QueryTree(int _T, int n) : T(_T) {
        dsu = DSU(n);
    }
};

```

```

84     t.resize(4 * T + 4);
85 }
86
87 // Internal segment tree add function
88 void add(int v, int l, int r, int ul, int ur, query& q) {
89     if (ul > ur) return;
90     if (l == ul && r == ur) {
91         t[v].push_back(q);
92         return;
93     }
94     int mid = (l + r) / 2;
95     add(2 * v, l, mid, ul, min(ur, mid), q);
96     add(2 * v + 1, mid + 1, r, max(ul, mid + 1), ur, q);
97 }
98
99 // Public wrapper: add a query in interval [l, r]
100 void add_query(query q, int l, int r) {
101     add(1, 0, T - 1, l, r, q);
102 }
103
104 // Traverse the segment tree and simulate unions with rollback
105 void dfs(int v, int l, int r, vector<int>& ans) {
106     // Apply all union operations in this segment node
107     for (query& q : t[v]) {
108         q.united = dsu.unite(q.v, q.u);
109     }
110
111     if (l == r) {
112         ans[l] = dsu.cnt; // Save answer for time l
113     } else {
114         int mid = (l + r) / 2;
115         dfs(2 * v, l, mid, ans);
116         dfs(2 * v + 1, mid + 1, r, ans);
117     }
118
119     // Rollback all operations applied in this node
120     for (query& q : t[v]) {
121         if (q.united)
122             dsu.rollback();
123     }
124 }
125 };

```

```

127 int main() {
128     int n, k;
129     cin >> n >> k; // n nodes, k operations
130     if (k == 0) return 0;
131     QueryTree st(k, n);
132     map<pair<int, int>, int> mp; // Edge -> start time
133     vector<int> ans(k); // Answers for '?' queries
134     vector<int> qmarks; // Indices of '?' queries
135
136     // Parse all k operations
137     for (int i = 0; i < k; i++) {
138         char c;
139         cin >> c;
140         if (c == '?') {
141             qmarks.push_back(i); // Save query index
142             continue;
143         }
144         int u, v;
145         cin >> u >> v;
146         u--; v--;
147         if (u > v) swap(u, v); // Normalize edge direction
148
149         if (c == '+') {
150             mp[{u, v}] = i; // Mark time edge is added
151         } else {
152             // Edge removed: store active interval
153             st.add_query(query(u, v), mp[{u, v}], i);
154             mp[{u, v}] = -1;
155         }
156
157     // Any edge still active is added until end of timeline
158     for (auto [edge, start] : mp) {
159         if (start != -1) {
160             st.add_query(query(edge.first, edge.second), start, k - 1);
161         }
162
163     // Process the tree to compute all '?'-query answers
164     st.dfs(1, 0, k - 1, ans);
165     // Output results of all '?'
166     for (int x : qmarks) {
167         cout << ans[x] << '\n';
168     }
169 }

```

3.7 Fenwick Tree

```

1  /*
2   * Fenwick Tree (Binary Indexed Tree)
3   -----
4   * Indexing: 0-based
5   * Bounds: [0, n-1] inclusive
6   * Time Complexity:
7   *   - add(x, v): O(log n)
8   *   - sum(x): O(log n) -> prefix sum over [0, x]
9   *   - rangeSum(l, r): O(log n) -> sum over [l, r]
10  *   - select(k): O(log n) -> smallest x such that prefix sum >= k (works
11    for monotonic cumulative sums)
12
13  * Space Complexity: O(n)
14 */
15 template <typename T>
16 struct Fenwick {
17     int n;
18     std::vector<T> a;
19
20     Fenwick(int n_ = 0) {
21         init(n_);
22     }
23
24     // Initialize BIT of size n
25     void init(int n_) {
26         n = n_;
27         a.assign(n, T{});
28     }
29
30     // Add value 'v' to position 'x'
31     void add(int x, const T &v) {
32         for (int i = x + 1; i <= n; i += i & -i) {
33             a[i - 1] = a[i - 1] + v;
34         }
35     }
36
37     // Compute prefix sum on range [0, x)
38     T sum(int x) const {
39         T ans{};
40         for (int i = x; i > 0; i -= i & -i) {

```

```

41             ans = ans + a[i - 1];
42         }
43         return ans;
44     }
45
46     // Compute sum over range [l, r)
47     T rangeSum(int l, int r) const {
48         return sum(r) - sum(l);
49     }
50
51     // Find the smallest x such that sum[0, x) > k (if exists), or returns
52     // n
53     int select(const T &k) const {
54         int x = 0;
55         T cur{};
56         for (int i = 1 << std::__lg(n); i; i >>= 1) {
57             if (x + i <= n && cur + a[x + i - 1] <= k) {
58                 cur = cur + a[x + i - 1];
59                 x += i;
60             }
61         }
62         return x;
63     }
64 // Fenwick<int> bit(n);

```

3.8 Fenwick Tree 2D

```

1  /*
2   * 2D Fenwick Tree (Binary Indexed Tree)
3   -----
4   * Indexing: 1-based
5   * Bounds: [1, n] inclusive
6   * Time Complexity:
7   *   - update(x, y, v): O(log^2 n)
8   *   - get(x, y): sum of rectangle [1,1] to [x,y]
9   *   - get1(x1, y1, x2, y2): sum over rectangle [x1,y1] to [x2,y2]
10  * Space Complexity: O(n^2)
11
12  * Can be adapted for rectangular grids by using n, m separately
13 */
14
15 struct Fenwick2D {

```

```

16 vector<vector<ll>> b; // 2D BIT array
17 int n; // Grid size (1-based)
18
19 Fenwick2D(int _n) : b(_n + 5, vector<ll>(_n + 5, 0)), n(_n) {}
20
21 // Add 'val' to cell (x, y)
22 void update(int x, int y, int val) {
23     for (; x <= n; x += (x & -x)) {
24         for (int j = y; j <= n; j += (j & -j)) {
25             b[x][j] += val;
26         }
27     }
28 }
29
30 // Get sum of rectangle [(1,1) to (x,y)]
31 ll get(int x, int y) {
32     ll ans = 0;
33     for (; x > 0; x -= x & -x) {
34         for (int j = y; j > 0; j -= j & -j) {
35             ans += b[x][j];
36         }
37     }
38     return ans;
39 }
40
41 // Get sum over subrectangle [(x1,y1) to (x2,y2)]
42 ll get1(int x1, int y1, int x2, int y2) {
43     return get(x2, y2) - get(x1-1, y2) - get(x2, y1-1) + get(x1-1, y1-1);
44 }
45 };
46 // Usage example:
47 Fenwick2D fw(n);
48 fw.update(3, 4, 5); // add 5 to (3, 4)
49 ll sum = fw.get(3, 4); // sum from (1,1) to (3,4)
50 ll range = fw.get1(2, 2, 5, 5); // sum in rectangle [(2,2)-(5,5)]

```

3.9 Merge Sort Tree

```

1 /*
2  * Merge Sort Tree (Segment Tree of Sorted Arrays)
3  -----
4  * Indexing: 0-based
5  * Node Bounds: [0, n-1] inclusive

```

6 Time Complexity:
 7 - build(): $O(n \log n)$
 8 - $q(l, r, x)$: $O(\log^2 n)$ -> number of elements $\leq x$ in $[l, r]$
 9 Space Complexity: $O(n \log n)$

10 Features:
 11 - Supports frequency/count queries: "how many values $\leq x$ in range [l, r]?"
 12 - Static array (no point updates unless rebuilt)
 13 */
 14
 15 const int MAXN = 100005; // size of original array
 16 const int MAXT = 2 * MAXN; // size of segment tree (2n)
 17
 18 vector<int> t[MAXT]; // Segment tree: each node holds sorted vector
 19 int a[MAXN]; // Original array
 20 int n; // Size of array
 21
 22 // Build merge sort tree (bottom-up)
 23 void build() {
 24 // Fill leaves
 25 for (int i = 0; i < n; i++) {
 26 t[i + n].push_back(a[i]);
 27 }
 28
 29 // Merge children into parent
 30 for (int i = n - 1; i > 0; i--) {
 31 auto &left = t[2 * i], &right = t[2 * i + 1];
 32 merge(left.begin(), left.end(), right.begin(), right.end(),
 33 back_inserter(t[i]));
 34 }
 35 }
 36
 37 // Query how many elements $\leq x$ in range [l, r]
 38 int q(int l, int r, int x) {
 39 int res = 0;
 40 for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
 41 if (l & 1) {
 42 res += upper_bound(t[l].begin(), t[l].end(), x) - t[l].begin();
 43 l++;
 44 }
 45 if (r & 1) {
 46 r--;

```

47     res += upper_bound(t[r].begin(), t[r].end(), x) - t[r].begin();
48 }
49 }
50 return res;
51 }
52 // Read n and array a, then call build()

```

3.10 Minimum Cartesian Tree

```

1 /*
2 Min Cartesian Tree
3 -----
4 Indexing: 0-based
5 Time Complexity: O(n)
6 Space Complexity: O(n)
7 Tree Properties:
8   - Binary tree where in-order traversal = original array
9   - Tree satisfies min-heap property: parent <= children
10  - 'par[i]': parent of node i
11  - 'sons[i][0]': left child, 'sons[i][1]': right child
12  - 'root': index of root node
13
14 Use cases:
15   - RMQ construction
16   - LCA over RMQ via Cartesian Tree
17 */
18
19 struct min_cartesian_tree {
20     vector<int> par;           // parent for each node
21     vector<vector<int>> sons; // left and right children
22     int root;
23
24     void init(int n, const vector<int> &arr) {
25         par.assign(n, -1);
26         sons.assign(n, vector<int>(2, -1)); // 0 = left, 1 = right
27         stack<int> st;
28
29         for (int i = 0; i < n; i++) {
30             int last = -1;
31
32             // Maintain increasing stack -> build min Cartesian Tree
33             // Change > to < for max Cartesian Tree
34             while (!st.empty() && arr[st.top()] > arr[i]) {

```

```

35         last = st.top();
36         st.pop();
37     }
38
39     if (!st.empty()) {
40         par[i] = st.top();
41         sons[st.top()][1] = i;
42     }
43     if (last != -1) {
44         par[last] = i;
45         sons[i][0] = last;
46     }
47
48     st.push(i);
49 }
50
51 for (int i = 0; i < n; i++) {
52     if (par[i] == -1) {
53         root = i;
54     }
55 }
56 }
57 };
58 // Example usage:
59 vector<int> a = {4, 2, 6, 1, 3};
60 min_cartesian_tree ct;
61 ct.init(a.size(), a);
62 cout << "Root_index:" << ct.root << '\n';

```

3.11 Multi Ordered Set

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;
4 template <typename T> using oset = __gnu_pbds::tree<
5     T, __gnu_pbds::null_type, less<T>, __gnu_pbds::rb_tree_tag,
6     __gnu_pbds::tree_order_statistics_node_update
7 >;
8
9 //en main
10
11 oset<pair<int,int>> name;
12     map<int,int> cuenta;

```

```

13     function<void(int)> meter = [&] (int val) {
14         name.insert({val,++cuenta[val]});
15     };
16     auto quitar = [&] (int val) {
17         name.erase({val,cuenta[val]}--);
18     };
19
20 meter(x);
21 quitar(y);
22 multioset.order_of_key({y+1,-1})-multioset.order_of_key({x,0})

```

3.12 Ordered Set

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;
4 template <typename T> using oset = __gnu_pbds::tree<
5     T, __gnu_pbds::null_type, less<T>, __gnu_pbds::rb_tree_tag,
6     __gnu_pbds::tree_order_statistics_node_update
7 >;
8 // order_of_key() primero mayor o igual;
9 // find_by_order() apuntador al elemento k;
10 // oset<pair<int,int>> os;
11 // os.insert({1,2});
12 // os.insert({2,3});
13 // os.insert({5,6});
14 // ll k=os.order_of_key({2,0});
15 // cout<<k<<endl; // 1
16 // pair<int,int> p=*os.find_by_order(k);
17 // cout<<p.f<<" "<<p.s<<endl; // 2 3
18 // os.erase(p);
19 // p=*os.find_by_order(k);
20 // cout<<p.f<<" "<<p.s<<endl; // 5 6
21
22 // check if upperbound or lowerbound does what you want
23 // because they give better time.
24
25 // to allow repetitions
26 #define ordered_set tree<int, null_type,less_equal<int>, rb_tree_tag,
27     tree_order_statistics_node_update>
28
29 // to not allow repetitions

```

```

30 #define ordered_set tree<int, null_type,less<int>, rb_tree_tag,
31     tree_order_statistics_node_update>
32 //order_of_key(x): number of items are strictly smaller than x
33 //find_by_order(k) iterator to the kth element

```

3.13 Palindromic Tree

```

1 /*
2  Palindromic Tree (Eertree)
3  -----
4  Indexing: 0-based
5  Time Complexity:
6      - extend(i): O(1) amortized
7      - calc_occurrences(): O(n)
8  Space Complexity: O(n)
9
10 Features:
11     - Each node represents a unique palindromic substring
12     - Efficient online construction
13     - 'oc': how many times this palindrome occurs as suffix
14     - 'cnt': number of palindromic suffixes in its subtree
15     - 'link': suffix link to longest proper palindromic suffix
16 */
17
18 const int N = 3e5 + 9;
19
20 struct PalindromicTree {
21     struct node {
22         int nxt[26]; // transitions by character
23         int len; // length of palindrome
24         int st, en; // start and end indices in string
25         int link; // suffix link
26         int cnt = 0; // number of palindromic suffixes
27         int oc = 0; // occurrences of the palindrome
28     };
29
30     string s;
31     vector<node> t;
32     int sz, last;
33
34     PalindromicTree() {}

```

```

35 PalindromicTree(const string &_s) {
36     s = _s;
37     int n = s.size();
38     t.clear();
39     t.resize(n + 5); // up to n + 2 distinct palindromes
40     sz = 2;
41     last = 2;
42     // Root 1: imaginary (-1 length), simplifies links
43     t[1].len = -1;
44     t[1].link = 1;
45     // Root 2: length 0, link to root 1
46     t[2].len = 0;
47     t[2].link = 1;
48 }
49
50 // Extend tree with s[pos], returns 1 if a new node is created
51 int extend(int pos) {
52     int cur = last;
53     int ch = s[pos] - 'a';
54     // Find longest suffix palindrome that can be extended
55     while (true) {
56         int curlen = t[cur].len;
57         if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos]) break;
58         cur = t[cur].link;
59     }
60
61     if (t[cur].nxt[ch]) {
62         // Already exists
63         last = t[cur].nxt[ch];
64         t[last].oc++;
65         return 0;
66     }
67     // Create new node
68     sz++;
69     last = sz;
70     t[sz].oc = 1;
71     t[sz].len = t[cur].len + 2;
72     t[cur].nxt[ch] = sz;
73     t[sz].en = pos;
74     t[sz].st = pos - t[sz].len + 1;
75
76     if (t[sz].len == 1) {
77         t[sz].link = 2;
78         t[sz].cnt = 1;
79         return 1;
80     }
81     // Compute suffix link
82     while (true) {
83         cur = t[cur].link;
84         int curlen = t[cur].len;
85         if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos]) {
86             t[sz].link = t[cur].nxt[ch];
87             break;
88         }
89     }
90     t[sz].cnt = 1 + t[t[sz].link].cnt;
91     return 1;
92 }
93 // Accumulate total occurrences for each palindrome node
94 void calc_occurrences() {
95     for (int i = sz; i >= 3; i--) {
96         t[t[i].link].oc += t[i].oc;
97     }
98 };
99
100 int main() {
101     string s;
102     cin >> s;
103     PalindromicTree t(s);
104     for (int i = 0; i < s.size(); i++) {
105         t.extend(i);
106     }
107     t.calc_occurrences();
108     long long total = 0;
109     for (int i = 3; i <= t.sz; i++) {
110         total += t.t[i].oc;
111     }
112     cout << total << '\n'; // Total palindromic substrings
113     return 0;
114 }
115 }

1 /* 
2  Persistent Array (via Persistent Segment Tree)

```

3.14 Persistent Array

```

3 -----
4 Indexing: 0-based
5 Bounds: [0, n-1]
6 Time Complexity:
7   - point update: O(log n)
8   - point query: O(log n)
9 Space Complexity: O(log n) per update/version
10
11 Features:
12   - Supports point updates with full version history
13   - Allows querying any version at any index
14 */
15
16 struct Node {
17     int val;
18     Node *l, *r;
19
20     // Leaf node with value
21     Node(int x) : val(x), l(nullptr), r(nullptr) {}
22
23     // Internal node with children (value is not used)
24     Node(Node *ll, Node *rr) : val(0), l(ll), r(rr) {}
25 };
26
27 int n;
28 int a[100001];      // Initial array
29 Node *roots[100001]; // Roots of all versions (0-based)
30
31 // Build version 0 from initial array
32 Node *build(int l = 0, int r = n - 1) {
33     if (l == r) return new Node(a[l]);
34     int mid = (l + r) / 2;
35     return new Node(build(l, mid), build(mid + 1, r));
36 }
37
38 // Create new version with a[pos] = val
39 Node *update(Node *node, int pos, int val, int l = 0, int r = n - 1) {
40     if (l == r) return new Node(val);
41     int mid = (l + r) / 2;
42     if (pos <= mid)
43         return new Node(update(node->l, pos, val, l, mid), node->r);
44     else
45         return new Node(node->l, update(node->r, pos, val, mid + 1, r));

```

```

46 }
47
48 // Query value at position 'pos' in a given version (node)
49 int query(Node *node, int pos, int l = 0, int r = n - 1) {
50     if (l == r) return node->val;
51     int mid = (l + r) / 2;
52     if (pos <= mid) return query(node->l, pos, l, mid);
53     else return query(node->r, pos, mid + 1, r);
54 }
55
56 // External helper: get value at index in version
57 int get_item(int index, int version) {
58     return query(roots[version], index);
59 }
60
61 // External helper: make new version based on 'prev_version', updating
62 // one index
63 void update_item(int index, int value, int prev_version, int new_version)
64 {
65     roots[new_version] = update(roots[prev_version], index, value);
66 }
67
68 // Initializes version 0 from given array
69 void init_arr(int nn, int *init) {
70     n = nn;
71     for (int i = 0; i < n; i++) a[i] = init[i];
72     roots[0] = build();
73 }

```

3.15 Persistent Segment Tree

```

1 /*
2  Persistent Segment Tree (Point Updates, Range Queries)
3 -----
4 Indexing: 1-based
5 Bounds: [1, n]
6 Time Complexity:
7   - Build: O(n)
8   - Point update: O(log n) -> returns new version
9   - Range query: O(log n)
10  - Copy version: O(1)
11
12 Features:

```



```

97     // Clone version k into new version
98     int k;
99     cin >> k;
100    t[sz++] = new Node(t[k]);
101 }
102
103 return 0;
104 }
```

3.16 Segment Tree

```

1 /*
2  * Segment Tree (Iterative, Range Minimum Query)
3  -----
4  * Indexing: 0-based
5  * Bounds: [0, n-1] inclusive
6  * Time Complexity:
7  *   - update(pos, val): O(log n)
8  *   - get(l, r): O(log n) -> query min in range [l, r]
9  * Space Complexity: O(2n)
10 */
11 struct SegmentTree {
12     vector<ll> a; // segment tree array
13     int n; // number of elements in original array
14
15     SegmentTree(int _n) : a(2 * _n, 1e18), n(_n) {}
16     // Update position 'pos' to value 'val'
17     void update(int pos, ll val) {
18         pos += n; // move to leaf
19         a[pos] = val; // set value
20         for (pos /= 2; pos > 0; pos /= 2) {
21             a[pos] = min(a[2 * pos], a[2 * pos + 1]); // update parent
22         }
23     }
24     // Get minimum value in range [l, r]
25     ll get(int l, int r) {
26         ll res = 1e18;
27         for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
28             if (l & 1) res = min(res, a[l++]); // if l is right child
29             if (r & 1) res = min(res, a[--r]); // if r is left child
30         }
31         return res;
32     }

```

```

33 };
```

3.17 Segment Tree 2D

```

1 /*
2  * 2D Segment Tree (Sum over Rectangles)
3  * -----
4  * Indexing: 0-based
5  * Grid Size: n * m
6  * Time Complexity:
7  *   - build: O(nm log n log m)
8  *   - point update: O(log n log m)
9  *   - range query [x1..x2][y1..y2]: O(log n log m)
10 * Space Complexity: O(4n x 4m)
11 */
12
13 const int MAXN = 505;
14 int n, m; // grid dimensions
15 int a[MAXN][MAXN]; // input grid
16 int t[4 * MAXN][4 * MAXN]; // segment tree
17
18 // Build the tree along y-axis (internal to each x-interval)
19 void build_y(int vx, int lx, int rx, int vy, int ly, int ry) {
20     if (ly == ry) {
21         if (lx == rx)
22             t[vx][vy] = a[lx][ly];
23         else
24             t[vx][vy] = t[vx * 2][vy] + t[vx * 2 + 1][vy];
25     } else {
26         int my = (ly + ry) / 2;
27         build_y(vx, lx, rx, vy * 2, ly, my);
28         build_y(vx, lx, rx, vy * 2 + 1, my + 1, ry);
29         t[vx][vy] = t[vx][vy * 2] + t[vx][vy * 2 + 1];
30     }
31 }
32
33 // Build the tree along x-axis and call build_y for each
34 void build_x(int vx, int lx, int rx) {
35     if (lx != rx) {
36         int mx = (lx + rx) / 2;
37         build_x(vx * 2, lx, mx);
38         build_x(vx * 2 + 1, mx + 1, rx);
39     }

```

```

40     build_y(vx, lx, rx, 1, 0, m - 1);
41 }
42
43 // Query along y-axis in a fixed x-node
44 int sum_y(int vx, int vy, int tly, int try_, int ly, int ry) {
45     if (ly > ry) return 0;
46     if (ly == tly && ry == try_) return t[vx][vy];
47     int tmy = (tly + try_) / 2;
48     return sum_y(vx, vy * 2, tly, tmy, ly, min(ry, tmy))
49         + sum_y(vx, vy * 2 + 1, tmy + 1, try_, max(ly, tmy + 1), ry);
50 }
51
52 // Query sum in rectangle [lx..rx][ly..ry]
53 int sum_x(int vx, int tlx, int trx, int lx, int rx, int ly, int ry) {
54     if (lx > rx) return 0;
55     if (lx == tlx && trx == rx)
56         return sum_y(vx, 1, 0, m - 1, ly, ry);
57     int tmx = (tlx + trx) / 2;
58     return sum_x(vx * 2, tlx, tmx, lx, min(rx, tmx), ly, ry)
59         + sum_x(vx * 2 + 1, tmx + 1, trx, max(lx, tmx + 1), rx, ly, ry);
60 }
61
62 // Update along y-axis for fixed x-node
63 void update_y(int vx, int lx, int rx, int vy, int ly, int ry, int x, int
   y, int new_val) {
64     if (ly == ry) {
65         if (lx == rx)
66             t[vx][vy] = new_val;
67         else
68             t[vx][vy] = t[vx * 2][vy] + t[vx * 2 + 1][vy];
69     } else {
70         int my = (ly + ry) / 2;
71         if (y <= my)
72             update_y(vx, lx, rx, vy * 2, ly, my, x, y, new_val);
73         else
74             update_y(vx, lx, rx, vy * 2 + 1, my + 1, ry, x, y, new_val);
75         t[vx][vy] = t[vx][vy * 2] + t[vx][vy * 2 + 1];
76     }
77 }
78
79 // Update point (x, y) to new_val
80 void update_x(int vx, int lx, int rx, int x, int y, int new_val) {
81     if (lx != rx) {

```

```

82         int mx = (lx + rx) / 2;
83         if (x <= mx)
84             update_x(vx * 2, lx, mx, x, y, new_val);
85         else
86             update_x(vx * 2 + 1, mx + 1, rx, x, y, new_val);
87     }
88     update_y(vx, lx, rx, 1, 0, m - 1, x, y, new_val);
89 }

```

3.18 Segment Tree Dynamic

```

/*
Dynamic Segment Tree (Point Add, Range Sum)
-----
Indexing: [0, INF) or any large bounded range
Time Complexity:
- add(k, x): O(log U)
- get_sum(l, r): O(log U)
    where U = range size (e.g., 1e9 if implicit bounds)

Space Complexity: O(nodes visited or created) -> worst O(log U) per
operation
*/

```

```

13 struct Vertex {
14     int left, right;          // interval [left, right)
15     int sum = 0;              // sum of elements in this interval
16     Vertex *left_child = nullptr, *right_child = nullptr;
17
18     Vertex(int lb, int rb) {
19         left = lb;
20         right = rb;
21     }
22
23     // Create children lazily only when needed
24     void extend() {
25         if (!left_child && left + 1 < right) {
26             int mid = (left + right) / 2;
27             left_child = new Vertex(left, mid);
28             right_child = new Vertex(mid, right);
29         }
30     }
31

```

```

32 // Add 'x' to position 'k'
33 void add(int k, int x) {
34     extend();
35     sum += x;
36     if (left_child) {
37         if (k < left_child->right)
38             left_child->add(k, x);
39         else
40             right_child->add(k, x);
41     }
42 }
43
44 // Get sum over interval [lq, rq]
45 int get_sum(int lq, int rq) {
46     if (lq <= left && right <= rq)
47         return sum;
48     if (rq <= left || right <= lq)
49         return 0;
50     extend();
51     return left_child->get_sum(lq, rq) + right_child->get_sum(lq, rq);
52 }
53 };
54
55 Vertex *root = new Vertex(0, 1e9); // Range [0, 1e9)
56 root->add(5, 10); // a[5] += 10
57 root->add(1000, 20); // a[1000] += 20
58 cout << root->get_sum(0, 10) << '\n'; // sum of [0, 10) = 10
59 cout << root->get_sum(0, 2000) << '\n'; // sum of [0, 2000) = 30

```

3.19 Segment Tree Lazy Types

```

1 struct max_t {
2     long long val;
3     static const long long null_v = -9223372036854775807LL;
4
5     max_t(): val(0) {}
6     max_t(long long v): val(v) {}
7
8     max_t op(max_t& other) {
9         return max_t(max(val, other.val));
10    }
11
12    max_t lazy_op(max_t& v, int size) {

```

```

13        return max_t(val + v.val);
14    }
15 };
16
17
18 struct min_t {
19     long long val;
20     static const long long null_v = 9223372036854775807LL;
21
22     min_t(): val(0) {}
23     min_t(long long v): val(v) {}
24
25     min_t op(min_t& other) {
26         return min_t(min(val, other.val));
27     }
28
29     min_t lazy_op(min_t& v, int size) {
30         return min_t(val + v.val);
31     }
32 };
33
34
35 struct sum_t {
36     long long val;
37     static const long long null_v = 0;
38
39     sum_t(): val(0) {}
40     sum_t(long long v): val(v) {}
41
42     sum_t op(sum_t& other) {
43         return sum_t(val + other.val);
44     }
45
46     sum_t lazy_op(sum_t& v, int size) {
47         return sum_t(val + v.val * size);
48     }
49 };

```

3.20 Segment Tree Lazy

```

1 /*
2  Lazy Segment Tree (Range Update, Range Query)
3  -----

```

```

4   Indexing: 0-based
5   Bounds: [0, n-1]
6   Time Complexity:
7     - build: O(n)
8     - update(l, r, v): O(log n)
9     - query(l, r): O(log n)
10  Space Complexity: O(4n)
11 */
12
13 // See SegTreeLazy_types for num_t structs
14
15 const num_t num_t::null_v = num_t(0);
16
17 template <typename num_t>
18 struct segtree {
19     int n;
20     vector<num_t> tree, lazy;
21
22     // Initialize segment tree with array of size s
23     void init(int s, long long* arr) {
24         n = s;
25         tree.assign(4 * n, num_t());
26         lazy.assign(4 * n, num_t());
27         init(0, 0, n - 1, arr);
28     }
29
30     // Build segment tree from array
31     num_t init(int i, int l, int r, long long* arr) {
32         if (l == r) return tree[i] = num_t(arr[l]);
33
34         int mid = (l + r) / 2;
35         num_t left = init(2 * i + 1, l, mid, arr);
36         num_t right = init(2 * i + 2, mid + 1, r, arr);
37         return tree[i] = left.op(right);
38     }
39
40     // Public wrapper: update range [l, r] with value v
41     void update(int l, int r, num_t v) {
42         if (l > r) return;
43         update(0, 0, n - 1, l, r, v);
44     }
45
46
47     // Internal recursive update
48     num_t update(int i, int tl, int tr, int ql, int qr, num_t v) {
49         eval_lazy(i, tl, tr);
50
51         if (tr < ql || qr < tl) return tree[i]; // no overlap
52         if (ql <= tl && tr <= qr) {
53             lazy[i].val += v.val;
54             eval_lazy(i, tl, tr);
55             return tree[i];
56         }
57
58         int mid = (tl + tr) / 2;
59         num_t a = update(2 * i + 1, tl, mid, ql, qr, v);
60         num_t b = update(2 * i + 2, mid + 1, tr, ql, qr, v);
61         return tree[i] = a.op(b);
62     }
63
64     // Public wrapper: query sum in range [l, r]
65     num_t query(int l, int r) {
66         if (l > r) return num_t::null_v;
67         return query(0, 0, n - 1, l, r);
68     }
69
70     // Internal recursive query
71     num_t query(int i, int tl, int tr, int ql, int qr) {
72         eval_lazy(i, tl, tr);
73
74         if (ql <= tl && tr <= qr) return tree[i]; // total overlap
75         if (tr < ql || qr < tl) return num_t::null_v; // no overlap
76
77         int mid = (tl + tr) / 2;
78         num_t a = query(2 * i + 1, tl, mid, ql, qr);
79         num_t b = query(2 * i + 2, mid + 1, tr, ql, qr);
80         return a.op(b);
81     }
82
83     // Push down pending lazy updates to children
84     void eval_lazy(int i, int l, int r) {
85         tree[i] = tree[i].lazy_op(lazy[i], r - l + 1);
86         if (l != r) {
87             lazy[2 * i + 1].val += lazy[i].val;
88             lazy[2 * i + 2].val += lazy[i].val;
89         }

```

```

90     lazy[i] = num_t(); // reset lazy at this node
91 }
92 };

```

3.21 Segment Tree Lazy Range Set

```

/*
Lazy Segment Tree (Range Set + Range Add + Range Sum)
-----
Indexing: 0-based
Bounds: [0, N-1]

Features:
- Supports range set (assign value), range add (increment), and
  range sum queries
- Properly prioritizes lazy set > lazy add
*/
const int maxN = 1e5 + 5;
int N, Q;
int a[maxN];

struct node {
    ll val = 0;      // range sum
    ll lzAdd = 0;    // pending addition
    ll lzSet = 0;    // pending set (non-zero means active)
};

node tree[maxN << 2];

#define lc (p << 1)
#define rc ((p << 1) | 1)

// Update current node based on its children
inline void pushup(int p) {
    tree[p].val = tree[lc].val + tree[rc].val;
}

// Push lazy values down to children
void pushdown(int p, int l, int mid, int r) {
    // Range set overrides any pending add
    if (tree[p].lzSet != 0) {
        tree[lc].lzSet = tree[rc].lzSet = tree[p].lzSet;

```

```

37     tree[lc].val = (mid - l + 1) * tree[p].lzSet;
38     tree[rc].val = (r - mid) * tree[p].lzSet;
39     tree[lc].lzAdd = tree[rc].lzAdd = 0;
40     tree[p].lzSet = 0;
}
// Otherwise propagate add
else if (tree[p].lzAdd != 0) {
    if (tree[lc].lzSet == 0) tree[lc].lzAdd += tree[p].lzAdd;
    else {
        tree[lc].lzSet += tree[p].lzAdd;
        tree[lc].lzAdd = 0;
    }
    if (tree[rc].lzSet == 0) tree[rc].lzAdd += tree[p].lzAdd;
    else {
        tree[rc].lzSet += tree[p].lzAdd;
        tree[rc].lzAdd = 0;
    }
    tree[lc].val += (mid - l + 1) * tree[p].lzAdd;
    tree[rc].val += (r - mid) * tree[p].lzAdd;
    tree[p].lzAdd = 0;
}
}

// Build tree from array a[0..N-1]
void build(int p, int l, int r) {
    tree[p].lzAdd = tree[p].lzSet = 0;
    if (l == r) {
        tree[p].val = a[l];
        return;
    }
    int mid = (l + r) >> 1;
    build(lc, l, mid);
    build(rc, mid + 1, r);
    pushup(p);
}

// Add 'val' to all elements in [a, b]
void add(int p, int l, int r, int a, int b, ll val) {
    if (a > r || b < l) return;
    if (a <= l && r <= b) {
        tree[p].val += (r - l + 1) * val;
        if (tree[p].lzSet == 0) tree[p].lzAdd += val;
        else tree[p].lzSet += val;
    }
}

```

```

80     return;
81 }
82     int mid = (l + r) >> 1;
83     pushdown(p, l, mid, r);
84     add(lc, l, mid, a, b, val);
85     add(rc, mid + 1, r, a, b, val);
86     pushup(p);
87 }

88
89 // Set all elements in [a, b] to 'val'
90 void set(int p, int l, int r, int a, int b, ll val) {
91     if (a > r || b < l) return;
92     if (a <= l && r <= b) {
93         tree[p].val = (r - l + 1) * val;
94         tree[p].lzAdd = 0;
95         tree[p].lzSet = val;
96         return;
97     }
98     int mid = (l + r) >> 1;
99     pushdown(p, l, mid, r);
100    set(lc, l, mid, a, b, val);
101    set(rc, mid + 1, r, a, b, val);
102    pushup(p);
103}

104
105 // Query sum over [a, b]
106 ll query(int p, int l, int r, int a, int b) {
107     if (a > r || b < l) return 0;
108     if (a <= l && r <= b) return tree[p].val;
109     int mid = (l + r) >> 1;
110     pushdown(p, l, mid, r);
111     return query(lc, l, mid, a, b) + query(rc, mid + 1, r, a, b);
112 }
113
114 // Example usage
115 N = 5;
116 a[0] = 2, a[1] = 4, a[2] = 3, a[3] = 1, a[4] = 5;
117 build(1, 0, N - 1);
118 set(1, 0, N - 1, 1, 3, 7);      // a[1..3] = 7
119 add(1, 0, N - 1, 2, 4, 2);      // a[2..4] += 2
120 cout << query(1, 0, N - 1, 0, 4) << '\n'; // total sum

```

```

1 const ll inf=1e18;
2
3 struct Node {
4     ll maxi, l_max, r_max, sum;
5
6     Node(ll _maxi, ll _l_max, ll _r_max, ll _sum){
7         maxi=_maxi;
8         l_max=_l_max;
9         r_max=_r_max;
10        sum=_sum;
11    }
12
13    Node operator+(Node b) {
14        return {max(max(maxi, b.maxi), r_max + b.l_max),
15            max(l_max, sum + b.l_max), max(b.r_max, r_max + b.sum),
16            sum + b.sum};
17    }
18
19 };
20
21 struct SegmentTreeMaxSubSum{
22     int n;
23     vector<Node> t;
24
25     SegmentTreeMaxSubSum(int _n) : n(_n), t(2 * _n, Node(-inf, -inf, -inf,
26                                         -inf)) {}
27
28     void update(int pos, ll val) {
29         t[pos += n] = Node(val, val, val, val);
30         for (pos>>=1; pos ; pos >>= 1) {
31             t[pos] = t[2*pos]+t[2*pos+1];
32         }
33     }
34
35     Node query(int l, int r) {
36         Node node_l = Node(-inf, -inf, -inf, -inf);
37         Node node_r = Node(-inf, -inf, -inf, -inf);
38         for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
39             if (l & 1) {
40                 node_l=node_l+t[l++];
41             }
42             if (r & 1) {
43                 node_r=t[--r]+node_r;
44             }
45         }
46         return {max(node_l.sum, node_r.sum),
47             max(node_l.r_max, node_r.r_max),
48             max(node_l.l_max, node_r.l_max),
49             node_l.sum+node_r.sum};
50     }
51
52 };

```

3.22 Segment Tree Max Subarray Sum

```

43     }
44 }
45 return node_l+node_r;
46 }
47 };

```

3.23 Segment Tree Range Update

```

/*
Segment Tree (Range Min Update, Point Query)
-----
Indexing: 0-based
Bounds: [0, n-1]
Time Complexity:
    - update(l, r, val): O(log n) -> applies min(val) over [l, r]
    - get(pos): O(log n) -> minimum affecting position pos
Space Complexity: O(2n)
*/

```

```

12 struct SegmentTree {
13     vector<ll> a; // a[i] = min value affecting segment i
14     int n;
15
16     SegmentTree(int _n) : a(2 * _n, 1e18), n(_n) {}
17
18     // Get the effective minimum at position 'pos'
19     ll get(int pos) {
20         ll res = 1e18;
21         for (pos += n; pos > 0; pos >>= 1) {
22             res = min(res, a[pos]);
23         }
24         return res;
25     }
26
27     // Apply min(val) to all positions in [l, r]
28     void update(int l, int r, ll val) {
29         for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
30             if (l & 1) {
31                 a[l] = min(a[l], val);
32                 l++;
33             }
34             if (r & 1) {
35                 --r;

```

```

36         a[r] = min(a[r], val);
37     }
38 }
39 }
40 };

```

3.24 Segment Tree Struct Types

```

// Sum segment tree
16 struct sum_t{
17     ll val;
18     static const long long null_v = 0;
19
20     sum_t(): val(null_v) {}
21     sum_t(long long v): val(v) {}
22
23     sum_t operator + (const sum_t &a) const {
24         sum_t ans;
25         ans.val = val + a.val;
26         return ans;
27     }
28 };
// Min segment tree
29 struct min_t{
30     ll val;
31     static const long long null_v = 1e18;
32
33     min_t(): val(null_v) {}
34     min_t(long long v): val(v) {}
35
36     min_t operator + (const min_t &a) const {
37         min_t ans;
38         ans.val = min(val, a.val);
39         return ans;
40     }
41 };
// Max segment tree
42 struct max_t{
43     ll val;
44     static const long long null_v = -1e18;
45
46     max_t(): val(null_v) {}
47     max_t(long long v): val(v) {}
48 };

```

```

36
37 max_t operator + (const max_t &a) const {
38     max_t ans;
39     ans.val = max(val, a.val);
40     return ans;
41 }
42 };
43 // GCD segment tree
44 struct gcd_t{
45     ll val;
46     static const long long null_v = 0;
47
48     gcd_t(): val(null_v) {}
49     gcd_t(long long v): val(v) {}
50
51     gcd_t operator + (const gcd_t &a) const {
52         gcd_t ans;
53         ans.val = gcd(val, a.val);
54         return ans;
55     }
56 };

```

3.25 Segment Tree Struct

```

1 // works as a 0-indexed segtree (not lazy)
2 template <typename num_t>
3 struct segtree
4 {
5     int n, k;
6     vector<num_t> tree;
7
8     void init(int s, vector<ll> arr)
9     {
10         n = s;
11         k = 0;
12         while ((1 << k) < n)
13             k++;
14         tree = vector<num_t>(2 * (1 << k) + 1);
15         for (int i = 0; i < n; i++)
16         {
17             tree[(1 << k) + i] = arr[i];
18         }
19         for (int i = (1 << k) - 1; i > 0; i--)

```

```

20     {
21         tree[i] = tree[i * 2] + tree[i * 2 + 1];
22     }
23 }
24
25 void update(int a, ll b)
26 {
27     a += (1 << k);
28     tree[a] = b;
29     for (a /= 2; a >= 1; a /= 2)
30     {
31         tree[a] = tree[a * 2] + tree[a * 2 + 1];
32     }
33 }
34 num_t find(int a, int b)
35 {
36     a += (1 << k);
37     b += (1 << k);
38     num_t s;
39     while (a <= b)
40     {
41         if (a % 2 == 1)
42             s = s + tree[a++];
43         if (b % 2 == 0)
44             s = s + tree[b--];
45         a /= 2;
46         b /= 2;
47     }
48     return s;
49 }
50 };

```

3.26 Segment Tree Walk

```

1 /*
2  Segment Tree Walk - Find First Position >= val
3  -----
4  Indexing: 0-based
5  Bounds: [0, n-1]
6  Time Complexity:
7      - build: O(n)
8      - update(pos, val): O(log n)
9      - get(L, R): O(log n) -> min value in [L, R]

```

```

10    - query(L, R, val): O(log n) -> find first index in [L, R] where a[i]
11        ] >= val
12
13    Features:
14        - Stores original value array in segment tree form
15        - Maps original indices to tree positions for fast updates
16        - Allows efficient walk to find constrained elements (e.g. lower
17            bound >= val)
18 */
19
20 struct SegmentTreeWalk {
21     vector<ll> a;           // segment tree values
22     vector<int> final_pos; // maps index i to position in tree (leaf)
23     int n;
24
25     SegmentTreeWalk(int _n) : a(4 * _n, 1e18), final_pos(_n), n(_n) {}
26
27     // Build segment tree from array 'vals[0..n-1]', start with node=1, l
28     // =0, r=n-1
29     void build(int l, int r, int node, const vector<ll> &vals) {
30         if (l == r) {
31             final_pos[l] = node;
32             a[node] = vals[l];
33         } else {
34             int mid = (l + r) / 2;
35             build(l, mid, node * 2, vals);
36             build(mid + 1, r, node * 2 + 1, vals);
37             a[node] = min(a[node * 2], a[node * 2 + 1]);
38         }
39     }
40
41     // Update value at original index 'pos' to 'val'
42     void update(int pos, ll val) {
43         pos = final_pos[pos]; // leaf position
44         a[pos] = val;
45         for (pos /= 2; pos > 0; pos /= 2)
46             a[pos] = min(a[pos * 2], a[pos * 2 + 1]);
47     }
48
49     // Get min value in [L, R], with current node interval [l, r] and root
50     // 'node'
51     ll get(int l, int r, int L, int R, int node) {
52         if (L > R) return 1e18;
53
54         if (l > R || r < L) return a[node];
55
56         if (l >= L & r <= R) return a[node];
57
58         int mid = (l + r) / 2;
59         ll left = get(l, mid, L, R, node * 2);
60         ll right = get(mid + 1, r, L, R, node * 2 + 1);
61
62         return min(left, right);
63     }
64
65     ll query(int l, int r, int val) {
66         return get(l, r, 0, n, 1);
67     }
68
69     ll query(int l, int r, ll val) {
70         return query(l, r, val) >= val;
71     }
72
73     ll query(int l, int r) {
74         return query(l, r, 1e18);
75     }
76
77     ll query(int l) {
78         return query(l, n);
79     }
80
81     ll query() {
82         return query(0, n);
83     }
84
85     ll query(ll val) {
86         return query(0, n, val);
87     }
88 }
```

```

49     if (l == R && r == R) return a[node];
50     int mid = (l + r) / 2;
51     return min(
52         get(l, mid, L, min(R, mid), node * 2),
53         get(mid + 1, r, max(L, mid + 1), R, node * 2 + 1)
54     );
55 }
56
57 // Find first position in [L, R] with a[i] >= val, starting from node
58 // interval [l, r]
59 pair<ll, ll> query(int l, int r, int L, int R, int node, int val) {
60     if (l > R || r < L) return {-1, 0}; // out of query
61     if (a[node] < val) return {-1, 0}; // all values < val
62     if (l == r) return {a[node], l}; // leaf node that
63     satisfies
64
65     int mid = (l + r) / 2;
66     auto left = query(l, mid, L, R, node * 2, val);
67     if (left.first != -1) return left;
68     return query(mid + 1, r, L, R, node * 2 + 1, val);
69 }
70
71 // Example usage:
72 int n = 8;
73 vector<ll> vals = {4, 2, 7, 1, 9, 5, 6, 3};
74 SegmentTreeWalk st(n);
75 st.build(0, n - 1, 1, vals);

```

3.27 Sparse Table

```
1  /*  
2   Sparse Table (Range Minimum Query)  
3  
4   Indexing: 0-based  
5   Bounds: [0, n-1]  
6   Time Complexity:  
7     - Build: O(n log n)  
8     - Query: O(1)  
9   Space Complexity: O(n log n)  
10  
11  Features:  
12    - Immutable RMQ (no updates)
```

```

13     - Works for idempotent operations like min, max, gcd
14 */
15
16 const int MAXN = 100005;
17 const int K = 30; // floor(log2(MAXN))
18 int lg[MAXN + 1]; // log base 2 of each i
19 int st[K + 1][MAXN]; // st[k][i] = min in [i, i + 2^k - 1]
20
21 vector<int> a; // input array
22 int n;
23
24 // Returns min in range [L, R]
25 int mini(int L, int R) {
26     int len = R - L + 1;
27     int i = lg[len];
28     return min(st[i][L], st[i][R - (1 << i) + 1]);
29 }
30
31 int main() {
32     cin >> n;
33     a.resize(n);
34     for (int i = 0; i < n; i++) cin >> a[i];
35     // Precompute binary logs
36     lg[1] = 0;
37     for (int i = 2; i <= n; i++) {
38         lg[i] = lg[i / 2] + 1;
39     }
40     // Initialize 2^0 intervals
41     for (int i = 0; i < n; i++) {
42         st[0][i] = a[i];
43     }
44     // Build sparse table
45     for (int k = 1; k <= K; k++) {
46         for (int i = 0; i + (1 << k) <= n; i++) {
47             st[k][i] = min(st[k - 1][i], st[k - 1][i + (1 << (k - 1))]);
48         }
49     }
50     // Example usage
51     int q; cin >> q;
52     while (q--) {
53         int l, r;
54         cin >> l >> r;
55         cout << mini(l, r) << '\n';

```

```

56     }
57     return 0;
58 }

```

3.28 Square Root Decomposition

```

1 /*
2 Sqrt Decomposition (String Block Cut and Move)
3 -----
4 Operation:
5     - Supports moving substrings using block cut logic
6     - Rebuilds when too many blocks (for performance)
7
8 Indexing: 0-based
9 String Bounds: [0, n)
10 Time Complexity:
11     - cut(a, b): O(sqrt(n))
12     - rebuildDecomp(): O(n)
13 When to rebuild: after too many block splits
14
15 Use case: performing multiple cut/paste operations efficiently on
16     large strings
17 */
18
19 const int MAXI = 350; // = sqrt(n), for n up to 1e5
20
21 int n, numBlocks;
22 string s;
23
24 struct Block {
25     int l, r; // indices into string s
26     int sz() const { return r - l; }
27 };
28
29 Block blocks[2 * MAXI]; // current block array
30 Block newBlocks[2 * MAXI]; // used temporarily during cutting
31
32 // Rebuilds the entire decomposition into 1 block (or balanced ones)
33 void rebuildDecomp() {
34     string newS = s;
35     int k = 0;
36     // Flatten string using current block structure
37     for (int i = 0; i < numBlocks; i++) {

```

```

37     for (int j = blocks[i].l; j < blocks[i].r; j++) {
38         newS[k++] = s[j];
39     }
40 }
41 // Reset to one big block
42 numBlocks = 1;
43 blocks[0] = {0, n};
44 s = newS;
45 }

46 // Cut [a, b) into a separate region and reorder it to the end
47 void cut(int a, int b) {
48     int pos = 0, curBlock = 0;
49     // Pass 1: Split blocks to isolate [a, b)
50     for (int i = 0; i < numBlocks; i++) {
51         Block B = blocks[i];
52         bool containsA = (pos < a && pos + B.sz() > a);
53         bool containsB = (pos < b && pos + B.sz() > b);
54         int cutA = B.l + a - pos;
55         int cutB = B.l + b - pos;
56
57         if (containsA && containsB) {
58             newBlocks[curBlock++] = {B.l, cutA};
59             newBlocks[curBlock++] = {cutA, cutB};
60             newBlocks[curBlock++] = {cutB, B.r};
61         } else if (containsA) {
62             newBlocks[curBlock++] = {B.l, cutA};
63             newBlocks[curBlock++] = {cutA, B.r};
64         } else if (containsB) {
65             newBlocks[curBlock++] = {B.l, cutB};
66             newBlocks[curBlock++] = {cutB, B.r};
67         } else {
68             newBlocks[curBlock++] = B;
69         }
70
71         pos += B.sz();
72     }
73
74     // Pass 2: Reorder - move [a, b) to the end
75     pos = 0;
76     numBlocks = 0;
77
78     // First add all blocks not in [a, b)
79
80     for (int i = 0; i < curBlock; i++) {
81         if (pos < a || pos >= b)
82             blocks[numBlocks++] = newBlocks[i];
83         pos += newBlocks[i].sz();
84     }
85
86     // Then add blocks in [a, b)
87     pos = 0;
88     for (int i = 0; i < curBlock; i++) {
89         if (pos >= a && pos < b)
90             blocks[numBlocks++] = newBlocks[i];
91         pos += newBlocks[i].sz();
92     }
93
94 // Example usage
95 int main() {
96     cin >> s;
97     n = s.size();
98     numBlocks = 1;
99     blocks[0] = {0, n};
100
101    int q; cin >> q;
102    while (q--) {
103        int a, b;
104        cin >> a >> b;
105        cut(a, b); // move [a, b) to the end
106
107        if (numBlocks > MAXI) rebuildDecomp();
108    }
109
110    rebuildDecomp(); // flatten before output
111    cout << s << '\n';
112}

```

3.29 Treap

```

1 struct Node {
2     Node *l = 0, *r = 0;
3     int val, y, c = 1;
4     Node(int val) : val(val), y(rand()) {}
5     void recalc();
6 };
7

```

```

8 int cnt(Node* n) { return n ? n->c : 0; }
9 void Node::recalc() { c = cnt(l) + cnt(r) + 1; }
10
11 template<class F> void each(Node* n, F f) {
12     if (n) { each(n->l, f); f(n->val); each(n->r, f); }
13 }
14
15 pair<Node*, Node*> split(Node* n, int k) {
16     if (!n) return {};
17     if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
18         auto pa = split(n->l, k);
19         n->l = pa.second;
20         n->recalc();
21         return {pa.first, n};
22     } else {
23         auto pa = split(n->r, k - cnt(n->l) - 1); // and just "k"
24         n->r = pa.first;
25         n->recalc();
26         return {n, pa.second};
27     }
28 }
29
30 Node* merge(Node* l, Node* r) {
31     if (!l) return r;
32     if (!r) return l;
33     if (l->y > r->y) {
34         l->r = merge(l->r, r);
35         l->recalc();
36         return l;
37     } else {
38         r->l = merge(l, r->l);
39         r->recalc();
40         return r;
41     }
42 }
43
44 Node* ins(Node* t, Node* n, int pos) {
45     auto pa = split(t, pos);
46     return merge(merge(pa.first, n), pa.second);
47 }
48
49 // Example application: move the range [l, r) to index k
50 void move(Node*& t, int l, int r, int k) {
51     Node *a, *b, *c;
52     tie(a,b) = split(t, l); tie(b,c) = split(b, r - 1);
53     if (k <= 1) t = merge(ins(a, b, k), c);
54     else t = merge(a, ins(c, b, k - r));
55 }
56
57 // Usage
58 // create treap
59 // Node* name=nullptr;
60 // insert element
61 // name=ins(name, new Node(val), pos);
62 // Node* x = new Node(val);
63 // name = ins(name, x, pos);
64 // merge two treaps (name before x)
65 // name=merge(name, x);
66 // split treap (this will split treap in two treaps,
67 // first with elements [0, pos) and second with elements [pos, n))
68 // pa will be pair of two treaps
69 // auto pa = split(name, pos);
70 // move range [l, r) to index k
71 // move(name, l, r, k);
72 // iterate over treap
73 // each(name, [&](int val) {
74 //     cout << val << ' ';
75 // });

```

3.30 Treap 2

```

1 typedef struct item * pitem;
2 struct item {
3     int prior, value, cnt;
4     bool rev;
5     pitem l, r;
6 };
7
8 int cnt (pitem it) {
9     return it ? it->cnt : 0;
10 }
11
12 void upd_cnt (pitem it) {
13     if (it)
14         it->cnt = cnt(it->l) + cnt(it->r) + 1;
15 }

```

```

16 void push (pitem it) {
17     if (it && it->rev) {
18         it->rev = false;
19         swap (it->l, it->r);
20         if (it->l) it->l->rev ^= true;
21         if (it->r) it->r->rev ^= true;
22     }
23 }
24
25
26 void merge (pitem & t, pitem l, pitem r) {
27     push (l);
28     push (r);
29     if (!l || !r)
30         t = l ? l : r;
31     else if (l->prior > r->prior)
32         merge (l->r, l->r, r), t = l;
33     else
34         merge (r->l, l, r->l), t = r;
35     upd_cnt (t);
36 }
37
38 void split (pitem t, pitem & l, pitem & r, int key, int add = 0) {
39     if (!t)
40         return void( l = r = 0 );
41     push (t);
42     int cur_key = add + cnt(t->l);
43     if (key <= cur_key)
44         split (t->l, l, t->l, key, add), r = t;
45     else
46         split (t->r, t->r, r, key, add + 1 + cnt(t->l)), l = t;
47     upd_cnt (t);
48 }
49
50 void reverse (pitem t, int l, int r) {
51     pitem t1, t2, t3;
52     split (t, t1, t2, l);
53     split (t2, t2, t3, r-l+1);
54     t2->rev ^= true;
55     merge (t, t1, t2);
56     merge (t, t, t3);
57 }
58

```

```

59 void output (pitem t) {
60     if (!t) return;
61     push (t);
62     output (t->l);
63     printf ("%d\u207b", t->value);
64     output (t->r);
65 }

```

3.31 Treap With Inversion

```

1 struct Node {
2     Node *l = 0, *r = 0;
3     int val, y, c = 1;
4     bool rev = 0;
5     Node(int val) : val(val), y(rand()) {}
6     void recalc();
7     void push();
8 };
9
10 int cnt(Node* n) { return n ? n->c : 0; }
11 void Node::recalc() { c = cnt(l) + cnt(r) + 1; }
12 void Node::push() {
13     if (rev) {
14         rev = 0;
15         swap(l, r);
16         if (l) l->rev ^= 1;
17         if (r) r->rev ^= 1;
18     }
19 }
20
21 template<class F> void each(Node* n, F f) {
22     if (n) { n->push(); each(n->l, f); f(n->val); each(n->r, f); }
23 }
24
25 pair<Node*, Node*> split(Node* n, int k) {
26     if (!n) return {};
27     n->push();
28     if (cnt(n->l) >= k) {
29         auto pa = split(n->l, k);
30         n->l = pa.second;
31         n->recalc();
32         return {pa.first, n};
33     } else {

```

```

34     auto pa = split(n->r, k - cnt(n->l) - 1);
35     n->r = pa.first;
36     n->recalc();
37     return {n, pa.second};
38 }
39 }
40
41 Node* merge(Node* l, Node* r) {
42     if (!l) return r;
43     if (!r) return l;
44     l->push();
45     r->push();
46     if (l->y > r->y) {
47         l->r = merge(l->r, r);
48         l->recalc();
49         return l;
50     } else {
51         r->l = merge(l, r->l);
52         r->recalc();
53         return r;
54     }
55 }
56
57 Node* ins(Node* t, Node* n, int pos) {
58     auto pa = split(t, pos);
59     return merge(merge(pa.first, n), pa.second);
60 }
61
62 // Example application: reverse the range [l, r]
63 void reverse(Node*& t, int l, int r) {
64     Node *a, *b, *c;
65     tie(a,b) = split(t, l);
66     tie(b,c) = split(b, r - l + 1);
67     b->rev ^= 1;
68     t = merge(merge(a, b), c);
69 }
70
71 void move(Node*& t, int l, int r, int k) {
72     Node *a, *b, *c;
73     tie(a,b) = split(t, l);
74     tie(b,c) = split(b, r - l);
75     if (k <= l) t = merge(ins(a, b, k), c);
76     else t = merge(a, ins(c, b, k - r));
77 }
```

77 | }

4 Dynamic Programming

4.1 CHT Deque

```

1  /*
2   Convex Hull Trick (CHT) - Min Query with Increasing Slopes
3   -----
4   Indexing: 1-based for 'a', 'dp', 's'
5   Bounds:
6       - i from 1 to m // number of elements in the array
7       - j from 1 to p // number of transitions
8   Time Complexity: O(m * p)
9   Requires:
10      - Lines inserted in increasing slope order for min query
11      - Queries made with increasing x values
12
13     dp[i][j] = min over k < i of { dp[k][j-1] + a[i] * (i - k) + s[i] - s[
14                                     k+1] }
15
16     dp[i][j] = min over k < i of { dp[k][j-1] + cost(k, i) }
17
18 We reformulate:
19
20     y = m * x + c
21     line: m = -k - 1, c = dp[k][j-1] - s[k+1]
22     eval: m * a[i] + c + a[i] * i + s[i]
23
24 */
25
26 struct Line {
27     ll a, b; // y = ax + b
28     Line(ll A, ll B) : a(A), b(B) {}
29     ll eval(ll x) const {
30         return a * x + b;
31     }
32     // Returns intersection x-coordinate with another line
33     double intersect(const Line& other) const {
34         return (double)(other.b - b) / (a - other.a);
35     }
36 };
37
38 // this finds the minimum and slope in increasing
39 // Deques for each dp stage
40 deque<Line> cht[p+1];
41
42 }
```

```

37 // Fill dp
38 cht[0].push_back(Line(-1, -s[1])); // base case
39 for (int i = 1; i <= m; i++) {
40     for (int j = p; j >= 1; j--) {
41         if (j > i) continue;
42         // Maintain front of deque to find minimum
43         while (cht[j - 1].size() >= 2 && cht[j - 1][1].eval(a[i]) <= cht[j - 1][0].eval(a[i])) {
44             cht[j - 1].pop_front();
45         }
46         // Evaluate best line
47         dp[i][j] = cht[j - 1].front().eval(a[i]) + a[i] * i + s[i];
48         // Create new line for current i
49         Line curr(-i - 1, dp[i][j] - s[i + 1]);
50         // Maintain convexity: remove worse lines from back
51         while (cht[j].size() >= 2) {
52             Line& l1 = cht[j][cht[j].size() - 2];
53             Line& l2 = cht[j].back();
54             if (curr.intersect(l1) <= l2.intersect(l1)) {
55                 cht[j].pop_back();
56             } else break;
57         }
58         cht[j].push_back(curr);
59     }
60 }

```

4.2 Digit DP

```

1 /*
2  Digit Dynamic Programming (Digit DP)
3 -----
4  Goal: Count numbers in range [0, x] that do not have two adjacent
5   equal digits.
6  State:
7   - pos: current digit position
8   - last: digit placed at previous position (0 to 9)
9   - f: tight flag (0 = must match prefix of x, 1 = already below x)
10  - z: leading zero flag (1 = still in leading zero zone)
11 Notes:
12   - Solve up to x using 'solve(x)'
13   - Can be modified to count palindromes, digits divisible by 3, etc.
14 */

```

```

15 vector<int> num;
16 ll DP[20][20][2][2]; // pos, last digit, f (tight), z (leading zero)
17
18 ll g(int pos, int last, int f, int z) {
19     if (pos == num.size()) return 1; // reached end, valid number
20     if (DP[pos][last][f][z] != -1) return DP[pos][last][f][z];
21     ll res = 0;
22     int limit = f ? 9 : num[pos]; // upper digit bound based on tight flag
23     for (int dgt = 0; dgt <= limit; dgt++) {
24         // Skip if digit equals last (unless it's a leading zero)
25         if (dgt == last && !(dgt == 0 && z == 1)) continue;
26         int nf = f;
27         if (!f && dgt < limit) nf = 1;
28         if (z && dgt == 0) res += g(pos + 1, dgt, nf, 1); // still leading
29           zeros
30         else res += g(pos + 1, dgt, nf, 0); // now in significant digits
31     }
32     return DP[pos][last][f][z] = res;
33 }
34
35 ll solve(ll x) {
36     if (x == -1) return 0;
37     num.clear();
38     while (x > 0) {
39         num.push_back(x % 10);
40         x /= 10;
41     }
42     reverse(num.begin(), num.end());
43     memset(DP, -1, sizeof(DP));
44     return g(0, 0, 0, 1);
45 }

```

4.3 Divide and Conquer DP

```

1 /*
2  Divide and Conquer DP Optimization
3 -----
4  Problem:
5   - dp[i][j] = min over k <= j of { dp[i-1][k] + C(k, j) }
6   - C(k, j) must satisfy the quadrangle inequality:
7     - C(a, c) + C(b, d) <= C(a, d) + C(b, c) for a <= b <= c <= d
8   - or monotonicity of opt[i][j] <= opt[i][j+1]
9

```

```

10 Indexing: 0-based
11 Time Complexity: O(m * n * log n)
12 Space Complexity: O(n)
13
14 dp_cur[j]: current dp[i][j] layer
15 dp_before[j]: previous dp[i-1][j] layer
16 */
17
18 int n, m;
19 vector<ll> dp_before, dp_cur;
20 ll C(int i, int j); // Cost function defined by user
21
22 // Recursively compute dp_cur[l..r] with optimal k in [optl, optr]
23 void compute(int l, int r, int optl, int optr) {
24     if (l > r) return;
25     int mid = (l + r) / 2;
26     pair<ll, int> best = {LLONG_MAX, -1};
27     for (int k = optl; k <= min(mid, optr); k++) {
28         ll val = (k > 0 ? dp_before[k - 1] : 0) + C(k, mid);
29         if (val < best.first) best = {val, k};
30     }
31     dp_cur[mid] = best.first;
32     int opt = best.second;
33     compute(l, mid - 1, optl, opt);
34     compute(mid + 1, r, opt, optr);
35 }
36
37 // Entry point: computes dp[m-1][n-1]
38 ll solve() {
39     dp_before.assign(n, 0);
40     dp_cur.assign(n, 0);
41     for (int i = 0; i < n; i++) {
42         dp_before[i] = C(0, i);
43     }
44     for (int i = 1; i < m; i++) {
45         compute(0, n - 1, 0, n - 1);
46         dp_before = dp_cur;
47     }
48     return dp_before[n - 1];
49 }
```

4.4 Edit Distance

```

1 /*
2 Given strings s and t, compute the minimum number of operations
3 (insert, delete, substitute) to convert s into t.
4 Indexing: 0-based (strings), DP is 1-based with offset
5 dp[i][j] = cost to convert s[0..i-1] into t[0..j-1]
6 Time Complexity: O(n * m)
7 Transitions:
8     - insert: dp[i][j-1] + 1
9     - delete: dp[i-1][j] + 1
10    - replace/match: dp[i-1][j-1] + (s[i-1] != t[j-1])
11 */
12 const int MAXN = 5005;
13 int dp[MAXN][MAXN];
14
15 string s, t; cin >> s >> t;
16 int n = s.length(), m = t.length();
17 // Initialize all to a large number
18 for (int i = 0; i <= n; i++) {
19     fill(dp[i], dp[i] + m + 1, 1e9);
20 }
21 dp[0][0] = 0;
22 for (int i = 0; i <= n; i++) {
23     for (int j = 0; j <= m; j++) {
24         if (j) { // insert
25             dp[i][j] = min(dp[i][j], dp[i][j - 1] + 1);
26         }
27         if (i) { // delete
28             dp[i][j] = min(dp[i][j], dp[i - 1][j] + 1);
29         }
30         if (i && j) { // replace or match
31             int cost = (s[i - 1] != t[j - 1]) ? 1 : 0;
32             dp[i][j] = min(dp[i][j], dp[i - 1][j - 1] + cost);
33         }
34     }
35 }
```

4.5 LCS

```

1 string s, t; cin >> s >> t;
2 int n=s.length(), m=t.length();
3 int dp[n+1][m+1];
4 memset(dp, 0, sizeof(dp));
5 for(int i=1;i<=n;i++){

```

```

6   for(int j=1;j<=m;j++){
7       dp[i][j]=max(dp[i-1][j], dp[i][j-1]);
8       if(s[i-1]==t[j-1]){
9           dp[i][j]=dp[i-1][j-1]+1;
10      }
11  }
12 }
13 int i=n, j=m;
14 string ans="";
15 while(i && j){
16     if(s[i-1]==t[j-1]){
17         ans+=s[i-1];
18         i--; j--;
19     }
20     else if(dp[i][j-1]>=dp[i-1][j]){
21         j--;
22     }
23     else{
24         i--;
25     }
26 }
27 reverse(all(ans));
28 cout << ans << endl;
29
30 // For two permutations one can create new array that will map each
31 // element from the first permutation to the second.
32 // For each element a[i] in the first permutation, you find which j is a[
33 // i] == b[j].
34 // After creating this new array, run LIS (Longest Increasing
35 // subsequence).

```

4.6 Line Container

```

1 /*
2  Line Container (Dynamic Convex Hull Trick)
3 -----
4 Supports:
5   - Adding lines: y = k * x + m
6   - Querying maximum y at given x
7 Indexing: arbitrary, supports any x
8 Time Complexity:
9   - add(): amortized O(log n)
10  - query(x): O(log n)

```

11 Space Complexity: $O(n)$
 12 For min queries: negate slopes and intercepts on insert and result on
 13 query
 14 Structure:
 15 - Stores lines in slope-sorted order (k)
 16 - Each line keeps its intersection point ' p ' with the next line
 17 - Binary search on ' p ' to answer queries
 18 **/*
 19 struct Line {
 20 mutable ll k, m, p;
 21 bool operator<(const Line& o) const { return k < o.k; } // Sort by
 22 slope
 23 bool operator<(ll x) const { return p < x; } // Query
 24 comparator
 25 };
 26
 27 struct LineContainer : multiset<Line, less<>> {
 28 // (for doubles, use inf = 1/.0, div(a,b) = a/b)
 29 static const ll inf = LLONG_MAX;
 30 ll div(ll a, ll b) { // Floored division
 31 return a / b - ((a ^ b) < 0 && a % b);
 32 }
 33 // Update intersection point x->p with y
 34 bool isect(iterator x, iterator y) {
 35 if (y == end()) return x->p = inf, false;
 36 if (x->k == y->k)
 37 x->p = (x->m > y->m ? inf : -inf); // higher line wins
 38 else
 39 x->p = div(y->m - x->m, x->k - y->k);
 40 return x->p >= y->p;
 41 }
 42 // Add new line: y = k * x + m
 43 void add(ll k, ll m) {
 44 auto z = insert({k, m, 0}), y = z++, x = y;
 45 // Remove dominated lines after y
 46 while (isect(y, z)) z = erase(z);
 47 // Remove dominated lines before y
 48 if (x != begin() && isect(--x, y))
 49 isect(x, y = erase(y));
 50 // Further cleanup to preserve order
 51 while ((y = x) != begin() && (--x)->p >= y->p)
 52 isect(x, erase(y));
 53 }

```

51 // Query max y at given x
52 ll query(ll x) {
53     assert(!empty());
54     auto l = *lower_bound(x);
55     return l.k * x + l.m;
56 }
57 };
58 // Example usage:
59 LineContainer cht;
60 cht.add(3, 5);      // y = 3x + 5
61 cht.add(2, 7);      // y = 2x + 7
62 cout << cht.query(4) << '\n'; // max y at x = 4

```

4.7 Longest Increasing Subsequence

```

/*
Longest Increasing Subsequence + (Recover Sequence) O(n log n)
-----
If no recovery is needed, use dp[] only.
dp.size() gives the length of LIS.
For non-decreasing use upper_bound instead of lower_bound.
*/
vector<int> dp;      // smallest tail values of LIS length i+1
vector<int> dp_index; // index in original array
vector<int> parent(n, -1); // parent[i] = index of previous element
                        // in LIS
vector<int> last_pos(n + 1); // last_pos[len] = index in v[] ending LIS
                            // of length len
for (int i = 0; i < n; i++) {
    auto it = lower_bound(dp.begin(), dp.end(), v[i]);
    int len = it - dp.begin();
    if (it == dp.end()) {
        dp.push_back(v[i]);
        dp_index.push_back(i); // Ignore if no recovery
    } else {
        *it = v[i];
        dp_index[len] = i; // Ignore if no recovery
    }
    if (len > 0) parent[i] = dp_index[len - 1]; // Ignore if no recovery
}
// Reconstruct LIS
vector<int> lis;
int pos = dp_index.back();

```

```

27 while (pos != -1) {
28     lis.push_back(v[pos]);
29     pos = parent[pos];
30 }
31 reverse(lis.begin(), lis.end());

```

5 Flow

5.1 Dinic

```

1 // Si en el grafo todos los vertices distintos
2 // de s y t cumplen que solo tienen una arista
3 // de entrada o una de salida la y dicha arista
4 // tiene capacidad 1 entonces la complejidad es
5 // O(E sqrt(v))

6
7 // si todas las aristas tienen capacidad 1
8 // el algoritmo tiene complejidad O(E sqrt(E))
9

10 struct FlowEdge {
11     int v, u;
12     long long cap, flow = 0;
13     FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(cap) {}
14 }

15 struct Dinic {
16     const long long flow_inf = 1e18;
17     vector<FlowEdge> edges;
18     vector<vector<int>> adj;
19     int n, m = 0;
20     int s, t;
21     vector<int> level, ptr;
22     queue<int> q;
23

24     Dinic(int n, int s, int t) : n(n), s(s), t(t) {
25         adj.resize(n);
26         level.resize(n);
27         ptr.resize(n);
28     }
29

30     void add_edge(int v, int u, long long cap) {
31         edges.emplace_back(v, u, cap);
32         edges.emplace_back(u, v, 0);
33     }

```

```

34     adj[v].push_back(m);
35     adj[u].push_back(m + 1);
36     m += 2;
37 }
38
39 bool bfs() {
40     while (!q.empty()) {
41         int v = q.front();
42         q.pop();
43         for (int id : adj[v]) {
44             if (edges[id].cap - edges[id].flow < 1)
45                 continue;
46             if (level[edges[id].u] != -1)
47                 continue;
48             level[edges[id].u] = level[v] + 1;
49             q.push(edges[id].u);
50         }
51     }
52     return level[t] != -1;
53 }
54
55 long long dfs(int v, long long pushed) {
56     if (pushed == 0)
57         return 0;
58     if (v == t)
59         return pushed;
60     for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {
61         int id = adj[v][cid];
62         int u = edges[id].u;
63         if (level[v] + 1 != level[u] || edges[id].cap - edges[id].
64             flow < 1)
65             continue;
66         long long tr = dfs(u, min(pushed, edges[id].cap - edges[id].
67             flow));
68         if (tr == 0)
69             continue;
70         edges[id].flow += tr;
71         edges[id ^ 1].flow -= tr;
72         return tr;
73     }
74     return 0;
}

```

```

75     long long flow() {
76         long long f = 0;
77         while (true) {
78             fill(level.begin(), level.end(), -1);
79             level[s] = 0;
80             q.push(s);
81             if (!bfs())
82                 break;
83             fill(ptr.begin(), ptr.end(), 0);
84             while (long long pushed = dfs(s, flow_inf)) {
85                 f += pushed;
86             }
87         }
88         return f;
89     }
90 }

```

5.2 Hopcroft-Karp

```

1 // maximum matching in bipartite graph
2 vector<int> match, dist;
3 vector<vector<int>> g;
4 int n, m, k;
5 bool bfs()
6 {
7     queue<int> q;
8     // The alternating path starts with unmatched nodes
9     for (int node = 1; node <= n; node++)
10    {
11        if (!match[node])
12        {
13            q.push(node);
14            dist[node] = 0;
15        }
16        else
17        {
18            dist[node] = INF;
19        }
20    }
21    dist[0] = INF;
22    while (!q.empty())
23

```

```

25 {
26     int node = q.front();
27     q.pop();
28     if (dist[node] >= dist[0])
29     {
30         continue;
31     }
32     for (int son : g[node])
33     {
34         // If the match of son is matched
35         if (dist[match[son]] == INF)
36         {
37             dist[match[son]] = dist[node] + 1;
38             q.push(match[son]);
39         }
40     }
41 }
42 // Returns true if an alternating path has been found
43 return dist[0] != INF;
44 }

// Returns true if an augmenting path has been found starting from
// vertex node
46 bool dfs(int node)
47 {
48     if (node == 0)
49     {
50         return true;
51     }
52     for (int son : g[node])
53     {
54         if (dist[match[son]] == dist[node] + 1 && dfs(match[son]))
55         {
56             match[node] = son;
57             match[son] = node;
58             return true;
59         }
60     }
61     dist[node] = INF;
62     return false;
63 }
64 int hopcroft_karp()

```

```

67 {
68     int cnt = 0;
69     // While there is an alternating path
70     while (bfs())
71     {
72         for (int node = 1; node <= n; node++)
73         {
74             // If node is unmatched but we can match it using an augmenting
75             // path
76             if (!match[node] && dfs(node))
77             {
78                 cnt++;
79             }
80         }
81     }
82     return cnt;
83 }
84 // usage
85 // n numero de puntos en la izquierda
86 // m numero de puntos en la derecha
87 // las aristas se guardan en g
88 // los puntos estan 1 indexados
89 // el punto 1 de m es el punto n+1 de g
90 // hopcroft_karp() devuelve el tamano del maximo matching
91 // match contiene el match de cada punto
92 // si match de i es 0, entonces i no esta matcheado
93 // https://judge.yosupo.jp/submission/247277

```

5.3 Hungarian

```

1 #define forn(i,n) for(int i=0;i<int(n);++i)
2 #define forsn(i,s,n) for(int i=s;i<int(n);++i)
3 #define forall(i,c) for(typeof(c.begin()) i=c.begin();i!=c.end();++i)
4 #define DBG(X) cerr << #X << " = " << X << endl;
5 typedef vector<int> vint;
6 typedef vector<vint> vvint;
7
8 void showmt();
9
10 /* begin notebook */
11
12 #define MAXN 256

```

```

13 #define INFT0 0x7f7f7f7f
14 int n;
15 int mt[MAXN][MAXN]; // Matriz de costos (X * Y)
16 int xy[MAXN], yx[MAXN]; // Matching resultante (X->Y, Y->X)
17
18 int lx[MAXN], ly[MAXN], slk[MAXN], slkx[MAXN], prv[MAXN];
19 char S[MAXN], T[MAXN];
20
21 void updtree(int x) {
22     forn(y, n) if (lx[x] + ly[y] - mt[x][y] < slk[y]) {
23         slk[y] = lx[x] + ly[y] - mt[x][y];
24         slkx[y] = x;
25     }
26 }
27 int hungar() {
28     forn(i, n) {
29         ly[i] = 0;
30         lx[i] = *max_element(mt[i], mt[i]+n);
31     }
32     memset(xy, -1, sizeof(xy));
33     memset(yx, -1, sizeof(yx));
34
35     forn(m, n) {
36         memset(S, 0, sizeof(S));
37         memset(T, 0, sizeof(T));
38         memset(prv, -1, sizeof(prv));
39         memset(slk, 0x7f, sizeof(slk));
40         queue<int> q;
41         #define bpone(e, p) { q.push(e); prv[e] = p; S[e] = 1; updtree(e); }
42         forn(i, n) if (xy[i] == -1) { bpone(i, -2); break; }
43
44         int x=0, y=-1;
45         while (y== -1) {
46             while (!q.empty() && y== -1) {
47                 x = q.front(); q.pop();
48                 forn(j, n) if (mt[x][j] == lx[x] + ly[j] && !T[j]) {
49                     if (yx[j] == -1) { y = j; break; }
50                     T[j] = 1;
51                     bpone(yx[j], x);
52                 }
53             }
54             if (y!= -1) break;
55             int dlt = INFT0;
56             forn(j, n) if (!T[j]) dlt = min(dlt, slk[j]);
57             forn(k, n) {
58                 if (S[k]) lx[k] -= dlt;
59                 if (T[k]) ly[k] += dlt;
60                 if (!T[k]) slk[k] -= dlt;
61             }
62             // q = queue<int>();
63             forn(j, n) if (!T[j] && !slk[j]) {
64                 if (yx[j] == -1) {
65                     x = slkx[j]; y = j; break;
66                 } else {
67                     T[j] = 1;
68                     if (!S[yx[j]]) bpone(yx[j], slkx[j]);
69                 }
70             }
71             if (y!= -1) {
72                 for(int p = x; p != -2; p = prv[p]) {
73                     yx[y] = p;
74                     int ty = xy[p]; xy[p] = y; y = ty;
75                 }
76             } else break;
77         }
78         int res = 0;
79         forn(i, n) res += mt[i][xy[i]];
80     }
81     return res;
82 }

```

5.4 Max Flow Min Cost

```

1 // dado un acomodo de flujos con costos
2 // devuelve el costo minimo para un flujo especificado
3
4 struct Edge
5 {
6     int from, to, capacity, cost;
7     Edge(int _from, int _to, int _capacity, int _cost)
8     {
9         from = _from;
10        to = _to;
11        capacity = _capacity;
12        cost = _cost;
13    }

```

```

14 };
15
16 vector<vector<int>> adj, cost, capacity;
17
18 const int INF = 1e9;
19
20 void shortest_paths(int n, int v0, vector<int> &d, vector<int> &p)
21 {
22     d.assign(n, INF);
23     d[v0] = 0;
24     vector<bool> inq(n, false);
25     queue<int> q;
26     q.push(v0);
27     p.assign(n, -1);
28
29     while (!q.empty())
30     {
31         int u = q.front();
32         q.pop();
33         inq[u] = false;
34         for (int v : adj[u])
35         {
36             if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v])
37             {
38                 d[v] = d[u] + cost[u][v];
39                 p[v] = u;
40                 if (!inq[v])
41                 {
42                     inq[v] = true;
43                     q.push(v);
44                 }
45             }
46         }
47     }
48 }
49
50 int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t)
51 {
52     adj.assign(N, vector<int>());
53     cost.assign(N, vector<int>(N, 0));
54     capacity.assign(N, vector<int>(N, 0));
55     for (Edge e : edges)
56     {
57         adj[e.from].push_back(e.to);
58         adj[e.to].push_back(e.from);
59         cost[e.from][e.to] = e.cost;
60         cost[e.to][e.from] = -e.cost;
61         capacity[e.from][e.to] = e.capacity;
62     }
63
64     int flow = 0;
65     int cost = 0;
66     vector<int> d, p;
67     while (flow < K)
68     {
69         shortest_paths(N, s, d, p);
70         if (d[t] == INF)
71             break;
72
73         // find max flow on that path
74         int f = K - flow;
75         int cur = t;
76         while (cur != s)
77         {
78             f = min(f, capacity[p[cur]][cur]);
79             cur = p[cur];
80         }
81
82         // apply flow
83         flow += f;
84         cost += f * d[t];
85         cur = t;
86         while (cur != s)
87         {
88             capacity[p[cur]][cur] -= f;
89             capacity[cur][p[cur]] += f;
90             cur = p[cur];
91         }
92
93         if (flow < K)
94             return -1;
95         else
96             return cost;
97     }
98 }
```

5.5 Max Flow

```

1 long long max_flow(vector<vector<int>> adj, vector<vector<long long>
2                         capacity,
3                         int source, int sink)
4 {
5     int n = adj.size();
6     vector<int> parent(n, -1);
7     // Find a way from the source to sink on a path with non-negative
8     // capacities
9     auto reachable = [&]() -> bool
10    {
11        queue<int> q;
12        q.push(source);
13        while (!q.empty())
14        {
15            int node = q.front();
16            q.pop();
17            for (int son : adj[node])
18            {
19                long long w = capacity[node][son];
20                if (w <= 0 || parent[son] != -1)
21                    continue;
22                parent[son] = node;
23                q.push(son);
24            }
25        }
26        return parent[sink] != -1;
27    };
28
29    long long flow = 0;
30    // While there is a way from source to sink with non-negative
31    // capacities
32    while (reachable())
33    {
34        int node = sink;
35        // The minimum capacity on the path from source to sink
36        long long curr_flow = LLONG_MAX;
37        while (node != source)
38        {
39            curr_flow = min(curr_flow, capacity[parent[node]][node]);
40            node = parent[node];
41        }
42    }

```

```

39     node = sink;
40     while (node != source)
41     {
42         // Subtract the capacity from capacity edges
43         capacity[parent[node]][node] -= curr_flow;
44         // Add the current flow to flow backedges
45         capacity[node][parent[node]] += curr_flow;
46         node = parent[node];
47     }
48     flow += curr_flow;
49     fill(parent.begin(), parent.end(), -1);
50 }
51
52 return flow;
53 }
54
55
56
57 //vector<vector<long long>> capacity(n, vector<long long>(n));
58 //vector<vector<int>> adj(n);
59 //adj[a].push_back(b);
60 //adj[b].push_back(a);
61 //capacity[a][b] += c;

```

5.6 Min Cost Max Flow

```
1  /**
2   * If costs can be negative, call setpi before maxflow, but note that
3   * negative cost cycles are not supported.
4   * To obtain the actual flow, look at positive values only
5   * Time: $O(F E \log(V))$ where F is max flow. $O(VE)$ for setpi.
6   */
7 #include <bits/stdc++.h>
8 using namespace std;
9
10 #include <ext/pb_ds/priority_queue.hpp>
11 using namespace __gnu_pbds;
12
13 #define rep(i, a, b) for(int i = a; i < (b); ++i)
14 #define all(x) begin(x), end(x)
15 #define sz(x) (int)(x).size()
16
17 typedef long long ll;
18
19 #define pb push_back
20
21 #define mp make_pair
22
23 #define fi first
24 #define se second
25
26 #define eb emplace_back
```

```

17  typedef vector<int> vi;
18
19 #pragma once
20
21 // #include <bits/extc++.h> // include-line, keep-include
22
23 const ll INF = numeric_limits<ll>::max() / 4;
24
25 struct MCMF {
26     struct edge {
27         int from, to, rev;
28         ll cap, cost, flow;
29     };
30     int N;
31     vector<vector<edge>> ed;
32     vi seen;
33     vector<ll> dist, pi;
34     vector<edge*> par;
35
36     MCMF(int N) : N(N), ed(N), seen(N), dist(N), pi(N), par(N) {}
37
38     void addEdge(int from, int to, ll cap, ll cost) {
39         if (from == to) return;
40         ed[from].push_back(edge{ from,to,sz(ed[to]),cap,cost,0 });
41         ed[to].push_back(edge{ to,from,sz(ed[from])-1,0,-cost,0 });
42     }
43
44     void path(int s) {
45         fill(all(seen), 0);
46         fill(all(dist), INF);
47         dist[s] = 0; ll di;
48
49         __gnu_pbds::priority_queue<pair<ll, int>> q;
50         vector<decltype(q)::point_iterator> its(N);
51         q.push({ 0, s });
52
53         while (!q.empty()) {
54             s = q.top().second; q.pop();
55             seen[s] = 1; di = dist[s] + pi[s];
56             for (edge& e : ed[s]) if (!seen[e.to]) {
57                 ll val = di - pi[e.to] + e.cost;
58                 if (e.cap - e.flow > 0 && val < dist[e.to]) {
59                     dist[e.to] = val;
60                     par[e.to] = &e;
61                     if (its[e.to] == q.end())
62                         its[e.to] = q.push({ -dist[e.to], e.to });
63                     else
64                         q.modify(its[e.to], { -dist[e.to], e.to });
65                 }
66             }
67         }
68         rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
69     }
70
71     pair<ll, ll> maxflow(int s, int t) {
72         ll totflow = 0, totcost = 0;
73         while (path(s), seen[t]) {
74             ll fl = INF;
75             for (edge* x = par[t]; x; x = par[x->from])
76                 fl = min(fl, x->cap - x->flow);
77
78             totflow += fl;
79             for (edge* x = par[t]; x; x = par[x->from]) {
80                 x->flow += fl;
81                 ed[x->to][x->rev].flow -= fl;
82             }
83         }
84         rep(i,0,N) for(edge& e : ed[i]) totcost += e.cost * e.flow;
85         return {totflow, totcost/2};
86     }
87
88     // If some costs can be negative, call this before maxflow:
89     void setpi(int s) { // (otherwise, leave this out)
90         fill(all(pi), INF); pi[s] = 0;
91         int it = N, ch = 1; ll v;
92         while (ch-- && it--)
93             rep(i,0,N) if (pi[i] != INF)
94                 for (edge& e : ed[i]) if (e.cap)
95                     if ((v = pi[i] + e.cost) < pi[e.to])
96                         pi[e.to] = v, ch = 1;
97         assert(it >= 0); // negative cost cycle
98     };
99 };

```

5.7 Push Relabel

```

1 const int inf = 1000000000;
2
3 int n;
4 vector<vector<int>> capacity, flow;
5 vector<int> height, excess, seen;
6 queue<int> excess_vertices;
7
8 void push(int u, int v) {
9     int d = min(excess[u], capacity[u][v] - flow[u][v]);
10    flow[u][v] += d;
11    flow[v][u] -= d;
12    excess[u] -= d;
13    excess[v] += d;
14    if (d && excess[v] == d)
15        excess_vertices.push(v);
16 }
17
18 void relabel(int u) {
19     int d = inf;
20     for (int i = 0; i < n; i++) {
21         if (capacity[u][i] - flow[u][i] > 0)
22             d = min(d, height[i]);
23     }
24     if (d < inf)
25         height[u] = d + 1;
26 }
27
28 void discharge(int u) {
29     while (excess[u] > 0) {
30         if (seen[u] < n) {
31             int v = seen[u];
32             if (capacity[u][v] - flow[u][v] > 0 && height[u] > height[v])
33                 push(u, v);
34             else
35                 seen[u]++;
36         } else {
37             relabel(u);
38             seen[u] = 0;
39         }
40     }
41 }
42
43 int max_flow(int s, int t) {
44     height.assign(n, 0);
45     height[s] = n;
46     flow.assign(n, vector<int>(n, 0));
47     excess.assign(n, 0);
48     excess[s] = inf;
49     for (int i = 0; i < n; i++) {
50         if (i != s)
51             push(s, i);
52     }
53     seen.assign(n, 0);
54
55     while (!excess_vertices.empty()) {
56         int u = excess_vertices.front();
57         excess_vertices.pop();
58         if (u != s && u != t)
59             discharge(u);
60     }
61
62     int max_flow = 0;
63     for (int i = 0; i < n; i++)
64         max_flow += flow[i][t];
65     return max_flow;
66 }
```

6 Geometry

6.1 Point Struct

```

1 typedef long long T;
2 struct pt {
3     T x,y;
4     pt operator+(pt p) {return {x+p.x, y+p.y};}
5     pt operator-(pt p) {return {x-p.x, y-p.y};}
6     pt operator*(T d) {return {x*d, y*d};}
7     pt operator/(T d) {return {x/d, y/d};}
8 };
9
10 // cross product
11 // positivo si el segundo esta en sentido antihorario
12 // 0 si el angulo es 180
13 // negativo si el segundo esta en sentido horario
14 T cross(pt v, pt w) {return v.x*w.y - v.y*w.x;}
15
```

```

16 // dot product
17 // positivo si el angulo entre los vectores es agudo
18 // 0 si son perpendiculares
19 // negativo si el angulo es obtuso
20 T dot(pt v, pt w) {return v.x*w.x + v.y*w.y;}
21
22 T orient(pt a, pt b, pt c) {return cross(b-a,c-a);}
23
24 T dist(pt a,pt b){
25     pt aux=b-a;
26     return sqrtl(aux.x*aux.x+aux.y*aux.y);
27 }

```

6.2 Sort Points

```

1 // This comparator sorts the points clockwise
2 // starting from the first quarter
3
4 bool getQ(Point a){
5     if(a.y!=0){
6         if(a.y>0)return 0;
7         return 1;
8     }
9     if(a.x>0)return 0;
10    return 1;
11 }
12 bool comp(Point a, Point b){
13     if(getQ(a)!=getQ(b))return getQ(a)<getQ(b);
14     return a*b>0;
15 }

```

7 Graphs

7.1 2Sat

```

1 struct TwoSatSolver {
2     int n_vars;           // Number of boolean variables
3     int n_vertices;       // Total vertices in the implication
4     graph (2 per variable)
5     vector<vector<int>> adj;      // Implication graph: adj[i] contains
6         edges from node i
7     vector<vector<int>> adj_t;    // Transposed graph for Kosaraju's
8         algorithm

```

```

6     vector<bool> used;          // Visited marker for DFS
7     vector<int> order;          // Finishing order of vertices (DFS1)
8     vector<int> comp;           // Component ID for each node (DFS2)
9     vector<bool> assignment;    // Final truth assignment for each
10    variable
11
12 // Constructor initializes all data structures
13 TwoSatSolver(int _n_vars)
14     : n_vars(_n_vars),
15     n_vertices(2 * _n_vars),
16     adj(n_vertices),
17     adj_t(n_vertices),
18     used(n_vertices),
19     comp(n_vertices, -1),
20     assignment(n_vars) {
21     order.reserve(n_vertices); // Pre-allocate memory for efficiency
22 }
23
24 // First DFS pass for Kosaraju's algorithm (on original graph)
25 void dfs1(int v) {
26     used[v] = true;
27     for (int u : adj[v]) {
28         if (!used[u])
29             dfs1(u);
30     }
31     order.push_back(v); // Save the vertex post-DFS for reverse ordering
32 }
33
34 // Second DFS pass on the transposed graph to label components
35 void dfs2(int v, int cl) {
36     comp[v] = cl;
37     for (int u : adj_t[v]) {
38         if (comp[u] == -1)
39             dfs2(u, cl);
40     }
41 }
42
43 // Solves the 2-SAT problem using Kosaraju's algorithm
44 bool solve_2SAT() {
45     // 1st pass: fill the order vector
46     order.clear();
47     used.assign(n_vertices, false);
48     for (int i = 0; i < n_vertices; ++i) {

```

```

48     if (!used[i])
49         dfs1(i);
50     }
51
52     // 2nd pass: find SCCs in reverse postorder
53     comp.assign(n_vertices, -1);
54     for (int i = 0, j = 0; i < n_vertices; ++i) {
55         int v = order[n_vertices - i - 1]; // Reverse postorder
56         if (comp[v] == -1)
57             dfs2(v, j++);
58     }
59
60     // Assign values to variables based on component comparison
61     assignment.assign(n_vars, false);
62     for (int i = 0; i < n_vertices; i += 2) {
63         if (comp[i] == comp[i + 1])
64             return false; // Contradiction: variable and its negation are in
65             // the same SCC
66         assignment[i / 2] = comp[i] > comp[i + 1]; // True if var's
67             // component comes after its negation
68     }
69     return true;
70 }
71
72 // Adds a disjunction (a v b) to the implication graph
73 // 'na' and 'nb' indicate negation: if true means !a or !b
74 // Variables are 0-indexed. Bounds are inclusive for each literal (i.e
75 // .., 0 to n_vars - 1)
76 void add_disjunction(int a, bool na, int b, bool nb) {
77     // Each variable 'x' has two nodes:
78     // x => 2*x, !x => 2*x + 1
79     // We encode (a v b) as (!a -> b) and (!b -> a)
80     a = 2 * a ^ na;
81     b = 2 * b ^ nb;
82     int neg_a = a ^ 1;
83     int neg_b = b ^ 1;
84
85     adj[neg_a].push_back(b);
86     adj[neg_b].push_back(a);
87     adj_t[b].push_back(neg_a);
88     adj_t[a].push_back(neg_b);
89 }
90 };

```

7.2 Articulation Points

```

1  /*
2   * Articulation Points (Cut Vertices) in an Undirected Graph
3   * -----
4   * Indexing: 0-based
5   * Node Bounds: [0, n-1] inclusive
6   * Time Complexity: O(V + E)
7   * Space Complexity: O(V)
8
9   * Use Case:
10    - Identifies vertices whose removal increases the number of
11      connected components.
12    - Works on undirected graphs (connected or disconnected).
13 */
14
15 int n; // Number of nodes in the graph
16 vector<vector<int>> adj; // Adjacency list of the undirected graph
17
18 vector<bool> visited; // Marks if a node was visited during DFS
19 vector<int> tin, low; // tin[v]: discovery time; low[v]: lowest
20           // discovery time reachable from subtree
21 int timer; // Global time counter for DFS
22
23 // DFS traversal to identify articulation points
24 void dfs(int v, int p = -1) {
25     visited[v] = true;
26     tin[v] = low[v] = timer++;
27     int children = 0;
28     for (int to : adj[v]) {
29         if (to == p) continue; // Skip the parent edge
30         if (visited[to]) {
31             // Back edge
32             low[v] = min(low[v], tin[to]);
33         } else {
34             dfs(to, v);
35             low[v] = min(low[v], low[to]);
36             // Articulation point condition for non-root
37             if (low[to] >= tin[v] && p != -1) {
38                 // v is an articulation point
39                 // handle_cutpoint(v);
39             }
39             ++children;

```

```

40     }
41 }
42 // Articulation point condition for root
43 if (p == -1 && children > 1) {
44     // v is an articulation point
45     // handle_cutpoint(v);
46 }
47 }

48 // Initializes structures and launches DFS
49 void find_cutpoints() {
50     timer = 0;
51     visited.assign(n, false);
52     tin.assign(n, -1);
53     low.assign(n, -1);

54     for (int i = 0; i < n; ++i) {
55         if (!visited[i])
56             dfs(i);
57     }
58 }
59 }
```

7.3 Bellman-Ford

```

/*
Bellman-Ford (SPFA variant) for Shortest Paths
-----
Indexing: 0-based
Node Bounds: [0, n-1] inclusive
Time Complexity: O(V * E) worst-case (amortized better)
Space Complexity: O(V + E)

Features:
- Handles negative edge weights
- Detects negative weight cycles (returns false if one exists)
- Works on directed or undirected graphs

Path Reconstruction:
- To recover the path from source 's' to any node 'u':
    vector<int> path;
    for (int v = u; v != -1; v = parent[v])
        path.push_back(v);
    reverse(path.begin(), path.end());
```

```

20 */
21
22 const int INF = 1<<30; // Large value to represent "infinity"
23 vector<vector<pair<int, int>>> adj; // adj[v] = list of (neighbor,
24                                         weight) pairs
25 vector<int> parent; // parent(n, -1) for path reconstruction
26
27 // SPFA implementation to find shortest paths from source s
28 // d[i] will contain shortest distance from s to i
29 // Returns false if a negative cycle is detected
30 // For path reconstruction add vector<int>& parent as parameter
31 bool spfa(int s, vector<int>& d, vector<int>& parent) {
32     int n = adj.size();
33     d.assign(n, INF);
34     vector<int> cnt(n, 0); // Count how many times each node has
35                                         been relaxed
36     vector<bool> inqueue(n, false); // Tracks if a node is currently in
37                                         queue
38     queue<int> q;
39
40     d[s] = 0;
41     q.push(s);
42     inqueue[s] = true;
43
44     while (!q.empty()) {
45         int v = q.front();
46         q.pop();
47         inqueue[v] = false;
48
49         for (auto edge : adj[v]) {
50             int to = edge.first;
51             int len = edge.second;
52
53             if (d[v] + len < d[to]) {
54                 parent[to] = v; // For path reconstruction
55                 d[to] = d[v] + len;
56                 if (!inqueue[to]) {
57                     q.push(to);
58                     inqueue[to] = true;
59                     cnt[to]++;
60                     if (cnt[to] > n)
61                         return false; // Negative weight cycle detected
62                 }
63             }
64         }
65     }
66 }
```

```

60     }
61   }
62 }
63
64 return true; // No negative cycles; shortest paths computed
65 }

```

7.4 Bipartite Checker

```

1  /*
2   * Bipartite Graph Checker (BFS-based)
3   -----
4   Indexing: 0-based
5   Time Complexity: O(V + E)
6   Space Complexity: O(V)
7
8   Handles disconnected graphs
9  */
10
11 int n; // Number of nodes
12 vector<vector<int>> adj; // Adjacency list of the undirected graph
13
14 vector<int> side(n, -1); // -1 = unvisited, 0/1 = sides of bipartition
15 bool is_bipartite = true;
16 queue<int> q;
17
18 for (int st = 0; st < n; ++st) {
19   if (side[st] == -1) {
20     q.push(st);
21     side[st] = 0; // Start with side 0
22     while (!q.empty()) {
23       int v = q.front();
24       q.pop();
25       for (int u : adj[v]) {
26         if (side[u] == -1) {
27           // Assign opposite side to neighbor
28           side[u] = side[v] ^ 1;
29           q.push(u);
30         } else {
31           // Conflict: adjacent nodes on same side
32           is_bipartite &= side[u] != side[v];
33         }
34       }
35     }
36   }
37 }
38
39 cout << (is_bipartite ? "YES" : "NO") << endl;

```

```

35   }
36 }
37 }
38
39 cout << (is_bipartite ? "YES" : "NO") << endl;

```

7.5 Bipartite Maximum Matching

```

/*
Maximum Bipartite Matching (Kuhn's Algorithm)
-----
Indexing: 0-based
Time Complexity: O(N * (E + N)) worst case
Space Complexity: O(N + K + E)
Input:
- n: number of nodes on the left side
- k: number of nodes on the right side
- g: adjacency list where g[v] contains all right nodes adjacent to
  left node v
Output:
- Prints the pairs (left, right) in the matching
- mt[r] = 1 means right node r is matched to left node l
*/
int n, k; // n: number of left nodes, k: number of right nodes
vector<vector<int>> g; // g[l]: list of right-side neighbors of left
  node l
vector<int> mt; // mt[r]: matched left node for right node r (or
  -1 if unmatched)
vector<bool> used; // used[l]: visited status for left node l during
  DFS
// Try to find an augmenting path from left node v
bool try_kuhn(int v) {
  if (used[v])
    return false;
  used[v] = true;
  for (int to : g[v]) {
    if (mt[to] == -1 || try_kuhn(mt[to])) {
      mt[to] = v;
      return true;
    }
  }
}

```

```

32     }
33 }
34 return false;
35 }

36 int main() {
37 //... reading the graph ...
38
39 mt.assign(k, -1); // Right-side nodes initially unmatched
40 for (int v = 0; v < n; ++v) {
41     used.assign(n, false); // Reset visited for each left node
42     try_kuhn(v);
43 }
44 // Output matched pairs (left+1, right+1 for 1-based output)
45 for (int i = 0; i < k; ++i) {
46     if (mt[i] != -1)
47         printf("%d %d\n", mt[i] + 1, i + 1);
48 }
49 return 0;
50 }
```

7.6 Block Cut Tree

```

1 /*
2  Block-Cut Tree from Biconnected Components
3 -----
4  Indexing: 0-based
5  Node Bounds: [0, n-1] inclusive
6  Time Complexity: O(V + E)
7  Space Complexity: O(V + E)
8
9 Features:
10   - Identifies articulation points (cut vertices)
11   - Extracts all biconnected components (BCCs)
12   - Constructs the Block-Cut Tree:
13     - Each BCC becomes a node in the tree
14     - Each articulation point becomes its own node
15     - An edge connects a BCC-node to each cutpoint in it
16
17 Output:
18   - 'is_cutpoint': true if node is an articulation point
19   - 'id[v]': node ID of 'v' in the block-cut tree
20   - Returns the block-cut tree as an adjacency list
```

```

21 */
22
23 vector<vector<int>> biconnected_components(vector<vector<int>> &g, // Adjacency list of the undirected graph
24                                            vector<bool> &is_cutpoint, // Output vector (resized internally)
25                                            vector<int> &id) { // Output vector (resized internally)
26
27     int n = g.size();
28     vector<vector<int>> comps; // Stores all biconnected components
29     vector<int> stk; // Stack of visited nodes for current component
30     vector<int> num(n), low(n); // DFS discovery time and low-link values
31     is_cutpoint.assign(n, false);
32
33     // DFS to find BCCs and articulation points
34     function<void(int, int, int &)> dfs = [&](int node, int parent, int &timer) {
35         num[node] = low[node] = ++timer;
36         stk.push_back(node);
37         for (int son : g[node]) {
38             if (son == parent) continue;
39             if (num[son]) {
40                 // Back edge
41                 low[node] = min(low[node], num[son]);
42             } else {
43                 dfs(son, node, timer);
44                 low[node] = min(low[node], low[son]);
45                 // Check articulation point condition
46                 if (low[son] >= num[node]) {
47                     is_cutpoint[node] = (num[node] > 1 || num[son] > 2); // For root and non-root
48                     comps.push_back({node});
49                     while (comps.back().back() != son) {
50                         comps.back().push_back(stk.back());
51                         stk.pop_back();
52                     }
53                 }
54             }
55         }
56     }
57 }
```

```

56 };
57
58 int timer = 0;
59 dfs(0, -1, timer);
60
61 id.resize(n); // Maps each original node to its block-cut tree node ID
62
63 // Build block-cut tree using articulation points and BCCs
64 function<vector<vector<int>>()> build_tree = [&]() {
65     vector<vector<int>> t(1); // Dummy index 0 (not used)
66     int node_id = 1; // Start assigning block-cut tree IDs from 1
67     // Assign unique tree node IDs to cutpoints
68     for (int node = 0; node < n; ++node) {
69         if (is_cutpoint[node]) {
70             id[node] = node_id++;
71             t.push_back({});
72         }
73     }
74     // Assign each component a new node and connect it to its cutpoints
75     for (auto &comp : comps) {
76         int bcc_node = node_id++;
77         t.push_back({});
78         for (int u : comp) {
79             if (!is_cutpoint[u]) {
80                 id[u] = bcc_node;
81             } else {
82                 t[bcc_node].push_back(id[u]);
83                 t[id[u]].push_back(bcc_node);
84             }
85         }
86     }
87     return t;
88 };
89
90 return build_tree(); // Return the block-cut tree
91 }

```

7.7 Blossom

```

1 /*
2  Edmonds' Blossom Algorithm (Maximum Matching in General Graphs)
3  -----
4  Indexing: 1-based

```

```

5 Node Bounds: [1, n]
6 Time Complexity: O(n^3) in worst case
7 Space Complexity: O(n^2)
8
9 Features:
10 - Handles odd-length cycles (blossoms)
11 - Works on any undirected graph (not just bipartite)
12 - Uses BFS with blossom contraction and path augmentation
13
14 Input:
15 - n: number of vertices
16 - add_edge(u, v): undirected edges between nodes (1 <= u,v <= n)
17
18 Output:
19 - maximum_matching(): returns size of max matching
20 - match[u]: matched vertex for node u (or 0 if unmatched)
21 */
22
23 const int N = 2009;
24 mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
25
26 struct Blossom {
27     int vis[N]; // vis[u]: -1 = unvisited, 0 = in queue, 1 = outer
28     layer
29     int par[N]; // par[u]: parent in alternating tree
30     int orig[N]; // orig[u]: base of blossom u belongs to
31     int match[N]; // match[u]: matched partner of u (0 if unmatched)
32     int aux[N]; // aux[u]: visit marker for LCA
33     int t; // global timestamp for LCA markers
34     int n; // number of nodes
35     bool ad[N]; // ad[u]: whether u is reachable in an alternating
36     path
37     vector<int> g[N]; // g[u]: adjacency list
38     queue<int> Q; // BFS queue
39
40     // Constructor: initializes data for n nodes
41     Blossom() {}
42     Blossom(int _n) {
43         n = _n;
44         t = 0;
45         for (int i = 0; i <= n; ++i) {
46             g[i].clear();
47             match[i] = par[i] = vis[i] = aux[i] = ad[i] = orig[i] = 0;
48         }
49     }
50 }

```

```

46     }
47 }
48
49 void add_edge(int u, int v) {
50     g[u].push_back(v);
51     g[v].push_back(u);
52 }
53
54 // Augment the matching along the alternating path from u to v
55 void augment(int u, int v) {
56     int pv = v, nv;
57     do {
58         pv = par[v];
59         nv = match[pv];
60         match[v] = pv;
61         match[pv] = v;
62         v = nv;
63     } while (u != pv);
64 }
65
66 int lca(int v, int w) {
67     ++t; // Increment timestamp for LCA markers
68     while (true) {
69         if (v) {
70             if (aux[v] == t) return v;
71             aux[v] = t;
72             v = orig[par[match[v]]]; // Move to the parent in the
73                         // alternating tree
74         }
75         swap(v, w);
76     }
77
78 // Contract a blossom from v and w with common ancestor a
79 void blossom(int v, int w, int a) {
80     while (orig[v] != a) {
81         par[v] = w;
82         w = match[v];
83         ad[v] = true;
84         if (vis[w] == 1) Q.push(w), vis[w] = 0;
85         orig[v] = orig[w] = a;
86         v = par[w];
87     }
88 }
89
90 // Find augmenting path starting from unmatched node u
91 bool bfs(int u) {
92     fill(vis + 1, vis + n + 1, -1);
93     iota(orig + 1, orig + n + 1, 1);
94     Q = queue<int>();
95     Q.push(u);
96     vis[u] = 0;
97
98     while (!Q.empty()) {
99         int v = Q.front(); Q.pop();
100        ad[v] = true;
101        for (int x : g[v]) {
102            if (vis[x] == -1) {
103                par[x] = v;
104                vis[x] = 1;
105                if (!match[x]) {
106                    augment(u, x);
107                    return true;
108                }
109                Q.push(match[x]);
110                vis[match[x]] = 0;
111            } else if (vis[x] == 0 && orig[v] != orig[x]) {
112                int a = lca(orig[v], orig[x]);
113                blossom(x, v, a);
114                blossom(v, x, a);
115            }
116        }
117    }
118    return false;
119 }
120
121 // Computes maximum matching and returns the size
122 int maximum_matching() {
123     int ans = 0;
124     vector<int> p(n - 1);
125     iota(p.begin(), p.end(), 1);
126     shuffle(p.begin(), p.end(), rnd);
127     for (int i = 1; i <= n; ++i) {
128         shuffle(g[i].begin(), g[i].end(), rnd);
129     }
130

```

```

131 // Greedy matching: try to match unmatched nodes directly
132 for (int u : p) {
133     if (!match[u]) {
134         for (int v : g[u]) {
135             if (!match[v]) {
136                 match[u] = v;
137                 match[v] = u;
138                 ++ans;
139                 break;
140             }
141         }
142     }
143 }
144
145 // Augmenting path phase
146 for (int i = 1; i <= n; ++i) {
147     if (!match[i] && bfs(i)) ++ans;
148 }
149
150 return ans;
151 }
152 } M;
153
154 int main() {
155     ios_base::sync_with_stdio(0);
156     cin.tie(0);
157
158     int t;
159     cin >> t;
160     while (t--) {
161         int n, m;
162         cin >> n >> m;
163         M = Blossom(n);
164         // Read all edges
165         for (int i = 0; i < m; i++) {
166             int u, v;
167             cin >> u >> v;
168             M.add_edge(u, v);
169         }
170         // Compute max matching
171         int matched = M.maximum_matching();
172         if (matched * 2 == n) {
173             // Perfect matching

```

```

174     cout << 0 << '\n';
175     } else {
176         // Find reachable unmatched nodes in alternating trees
177         memset(M.ad, 0, sizeof M.ad);
178         for (int i = 1; i <= n; i++) {
179             if (M.match[i] == 0) M.bfs(i);
180         }
181         int unmatched_reachable = 0;
182         for (int i = 1; i <= n; i++) {
183             unmatched_reachable += M.ad[i];
184         }
185         cout << unmatched_reachable << '\n';
186     }
187 }
188
189 }
```

7.8 Bridges

```

1 /*
2  * Bridge-Finding in an Undirected Graph
3  * -----
4  * Indexing: 0-based
5  * Node Bounds: [0, n-1] inclusive
6  * Time Complexity: O(V + E)
7  * Space Complexity: O(V)
8
9 Input:
10    n    - Number of nodes in the graph
11    adj - Adjacency list of the undirected graph
12
13 Output:
14    - Call 'find_bridges()' to populate bridge information.
15    - Modify the DFS 'Bridge' section to store or print the bridges.
16        A bridge is an edge (v, to) such that removing it increases the
17            number of connected components.
18 */
19
20 int n; // Number of nodes
21 vector<vector<int>> adj; // Adjacency list
22
23 vector<bool> visited; // Marks visited nodes
24 vector<int> tin, low; // tin[v]: discovery time; low[v]: lowest ancestor
```

```

        reachable
24 int timer; // Global DFS timer
25
26 // DFS to detect bridges
27 void dfs(int v, int p = -1) {
28     visited[v] = true;
29     tin[v] = low[v] = timer++;
30     for (int to : adj[v]) {
31         if (to == p) continue; // Skip edge to parent
32         if (visited[to]) {
33             // Back edge
34             low[v] = min(low[v], tin[to]);
35         } else {
36             dfs(to, v);
37             low[v] = min(low[v], low[to]);
38             // Bridge condition: if no back edge connects subtree rooted at 'to' to ancestors of 'v'
39             if (low[to] > tin[v]) {
40                 // (v, to) is a bridge
41                 // Example: bridges.push_back({v, to});
42             }
43         }
44     }
45 }
46
47 // Initialize tracking structures and run DFS
48 void find_bridges() {
49     timer = 0;
50     visited.assign(n, false);
51     tin.assign(n, -1);
52     low.assign(n, -1);
53     for (int i = 0; i < n; ++i) {
54         if (!visited[i])
55             dfs(i);
56     }
57 }
```

7.9 Bridges Online

```

1 /*
2  * Online Bridge-Finding (Dynamic Edge Insertion)
3  * -----
4  * Indexing: 0-based

```

5 Node Bounds: [0, n-1] inclusive
 6 Time Complexity:
 7 - Amortized $O(\log^2 N)$ per edge addition
 8 Space Complexity: $O(V)$
 9
 10 Features:
 11 - Maintains the number of bridges dynamically as edges are added one by one.
 12 - Detects if adding an edge merges different 2-edge-connected components.
 13 - No deletions supported.
 14
 15 Input:
 16 init(n) - Initializes the data structure for a graph with n nodes.
 17 add_edge(a, b) - Adds an undirected edge between nodes a and b.
 18
 19 Output:
 20 'bridges' - Global variable representing the current number of bridges.
 21 */
 22
 23 vector<int> par, dsu_2ecc, dsu_cc, dsu_cc_size;
 24 int bridges; // Number of bridges in the graph
 25 int lca_iteration;
 26 vector<int> last_visit;
 27
 28 // Initializes the data structures
 29 void init(int n) {
 30 par.resize(n);
 31 dsu_2ecc.resize(n);
 32 dsu_cc.resize(n);
 33 dsu_cc_size.resize(n);
 34 last_visit.assign(n, 0);
 35 lca_iteration = 0;
 36 bridges = 0;
 37
 38 for (int i = 0; i < n; ++i) {
 39 par[i] = -1;
 40 dsu_2ecc[i] = i;
 41 dsu_cc[i] = i;
 42 dsu_cc_size[i] = 1;
 43 }

```

44 }
45
46 // Finds the representative of the 2-edge-connected component of node v
47 int find_2ecc(int v) {
48     if (v == -1) return -1;
49     return dsu_2ecc[v] == v ? v : dsu_2ecc[v] = find_2ecc(dsu_2ecc[v]);
50 }
51
52 // Finds the connected component representative of the component
53 // containing v
54 int find_cc(int v) {
55     v = find_2ecc(v);
56     return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(dsu_cc[v]);
57 }
58
59 // Makes node v the root of its tree, rerouting parent pointers upward
60 void make_root(int v) {
61     int root = v;
62     int child = -1;
63     while (v != -1) {
64         int p = find_2ecc(par[v]);
65         par[v] = child;
66         dsu_cc[v] = root;
67         child = v;
68         v = p;
69     }
70     dsu_cc_size[root] = dsu_cc_size[child];
71 }
72
73 // Merges paths from a and b to their lowest common ancestor in the 2ECC
74 // forest
75 void merge_path(int a, int b) {
76     ++lca_iteration;
77     vector<int> path_a, path_b;
78     int lca = -1;
79
80     while (lca == -1) {
81         if (a != -1) {
82             a = find_2ecc(a);
83             path_a.push_back(a);
84             if (last_visit[a] == lca_iteration) {
85                 lca = a;
86                 last_visit[a] = lca_iteration;
87                 a = par[a];
88             }
89             if (b != -1) {
90                 b = find_2ecc(b);
91                 path_b.push_back(b);
92                 if (last_visit[b] == lca_iteration) {
93                     lca = b;
94                     break;
95                 }
96                 last_visit[b] = lca_iteration;
97                 b = par[b];
98             }
99         }
100
101        // Merge all nodes on path_a and path_b into the same 2ECC
102        for (int v : path_a) {
103            dsu_2ecc[v] = lca;
104            if (v == lca) break;
105            --bridges;
106        }
107        for (int v : path_b) {
108            dsu_2ecc[v] = lca;
109            if (v == lca) break;
110            --bridges;
111        }
112    }
113
114    // Adds an undirected edge between a and b and updates bridge count
115    void add_edge(int a, int b) {
116        a = find_2ecc(a);
117        b = find_2ecc(b);
118        if (a == b) return; // Already in the same 2ECC
119
120        int ca = find_cc(a);
121        int cb = find_cc(b);
122
123        if (ca != cb) {
124            // Bridge found - connects two different components
125            ++bridges;
126            // Union by size
127            if (dsu_cc_size[ca] > dsu_cc_size[cb]) {

```

```

128     swap(a, b);
129     swap(ca, cb);
130 }
131 make_root(a);
132 par[a] = b;
133 dsu_cc[a] = b;
134 dsu_cc_size[cb] += dsu_cc_size[a];
135 } else {
136     // No new bridge, but must merge paths to unify 2ECCs
137     merge_path(a, b);
138 }
139
140 // Example usage
141 int main() {
142     init(n);
143     for (auto [u, v] : edges) {
144         add_edge(u, v);
145         cout << "Current_bridge_count:" << bridges << '\n';
146     }
147 }
148

```

7.10 Dijkstra

```

1 vector<vector<pair<int, int>>> adj(n); // Adjacency list (node, weight)
2 vector<ll> dist(n, 1LL << 61); // Distance array initialized to infinity
3
4 priority_queue<pair<ll, int>> q; // Max-heap, so we push negative
   weights to simulate min-heap
5 dist[0] = 0; // Starting node distance
6 q.push({0, 0}); // (distance, vertex)
7
8 while (!q.empty()) {
9     auto [w, v] = q.top(); q.pop();
10    w = -w; // Convert back to positive
11    if (w > dist[v]) continue; // Skip outdated entry
12    for (auto [u, cost] : adj[v]) {
13        if (dist[v] + cost < dist[u]) {
14            dist[u] = dist[v] + cost;
15            q.push({-dist[u], u}); // Push updated distance (negated)
16        }
17    }
18 }

```

7.11 Eulerian Path

An Eulerian Path is a path that passes through every edge once. For an undirected graph an eulerian path exists if the degree of every node is even or the degree of exactly two nodes is odd. In the first case, the eulerian path is also an eulerian circuit or cycle. In a directed graph, an eulerian path exists if at most one node has $out_i - in_i = 1$ and at most one node has $in_i - out_i = 1$. A cycle exists if $in_i - out_i = 0$ for all i.

```

1 /*
2  Eulerian Path (Hierholzer's Algorithm)
3 -----
4  Time Complexity: O(E)
5  Space Complexity: O(V + E)
6
7 Input:
8     - g: adjacency list of the graph
9         * Directed: vector<vector<pair<int, int>>> g
10            where g[v] = list of {to, edge_index}
11         * Undirected: vector<vector<int>> g
12            where g[v] = list of neighbors
13     - seen: vector<bool> seen(E) - only needed for directed version
14     - path: vector<int> path - will be filled in reverse order of
15       traversal
16       reverse(path.begin(), path.end());
17 */
18
19 // Directed Version //
20 void dfs_directed(int node) {
21     while (!g[node].empty()) {
22         auto [son, idx] = g[node].back();
23         g[node].pop_back();
24         if (seen[idx]) continue; // Skip if edge already visited
25         seen[idx] = true;
26         dfs_directed(son);
27     }
28     path.push_back(node); // Post-order insertion (reverse of actual path)
29 }
30
31 // Undirected Version //
32 void dfs_undirected(int node) {
33     while (!g[node].empty()) {
34         int son = g[node].back();
35         g[node].pop_back();

```

```

35     dfs_undirected(son);
36 }
37 path.push_back(node); // Post-order insertion
38 }
```

7.12 Floyd-Warshall

```

/*
Floyd-Warshall Algorithm (All-Pairs Shortest Paths)
-----
Indexing: 0-based
Time Complexity: O(V^3)
Space Complexity: O(V^2)

Input:
- d: distance matrix of size n x n
  * d[i][j] should be initialized as:
    - 0 if i == j
    - weight of edge (i, j) if exists
    - INF (e.g. 1e18) otherwise
*/
vector<vector<ll>> d(n, vector<ll>(n, 1e18)); // distance matrix

// This version is by default adapted for UNDIRECTED graphs.
for (int k = 0; k < n; k++) {
    for (int i = 0; i < n; i++) {
        for (int j = i + 1; j < n; j++) { // For directed graphs, use j = 0;
            j < n; j++)
            long long new_dist = d[i][k] + d[k][j];
            if (new_dist < d[i][j]) {
                d[i][j] = d[j][i] = new_dist; // update both directions for
                                              undirected graph
            }
        }
    }
}
```

7.13 Kruskal

```

/*
Kruskal's Algorithm (Minimum Spanning Tree - MST)
```

```

3 -----
4 Indexing: 0-based for nodes in edges
5 Time Complexity: O(E log E)
6 Space Complexity: O(N)

7
8 Input:
9   - N: number of nodes
10  - edges: list of weighted edges in form {weight, {u, v}}
11
12 Output:
13   - Returns total weight of the MST if the graph is connected
14   - Returns -1 if MST cannot be formed (i.e., graph is disconnected)
15
16 Note:
17   - Requires a Disjoint Set Union (DSU) / Union-Find data structure
     with:
18     - unite(a, b): merges components, returns true if successful
19     - size(v): returns size of component containing v
20 */
21
22 template <class T>
23 T kruskal(int N, vector<pair<T, pair<int, int>>> edges) {
24     sort(edges.begin(), edges.end()); // Sort by weight (non-decreasing)
25     T ans = 0;
26     DSU D(N); // Disjoint Set Union for N nodes
27     for (auto &[w, uv] : edges) {
28         int u = uv.first, v = uv.second;
29         if (D.unite(u, v)) {
30             ans += w; // Add edge to MST if u and v are in different
                         components
31         }
32     }
33     // Check if MST spans all nodes (i.e., one component of size N)
34     return (D.size(0) == N ? ans : -1);
35 }
```

7.14 Marriage

```

1 /*
2  Male-Optimal Stable Marriage Problem (Gale-Shapley Algorithm)
3 -----
4  Indexing: 0-based
5  Bounds: 0 <= i, j < n
```

```

6 Time Complexity: O(n^2)
7 Space Complexity: O(n^2)

8
9 Input:
10 - n: Number of men/women (equal)
11 - gv[i][j]: j-th most preferred woman for man i
12 - om[i][j]: j-th most preferred man for woman i
13 * Both are permutations of {0, ..., n-1}
14 * om must be inverted to get om[w][m] = woman w's ranking of man
15 m

16 Output:
17 - pm[i]: Woman matched to man i (i.e. pairings)
18 - pv[i]: Man matched to woman i
19 */
20
21 #define MAXN 1000
22 int gv[MAXN][MAXN], om[MAXN][MAXN]; // Male and female preference lists
23 int pv[MAXN], pm[MAXN]; // pv[woman] = man, pm[man] = woman
24 int pun[MAXN]; // pun[man] = next woman to propose
25 to
26
27 void stableMarriage(int n) {
28     fill_n(pv, n, -1); // All women initially unmatched
29     fill_n(pm, n, -1); // All men initially unmatched
30     fill_n(pun, n, 0); // Each man starts at his top preference
31
32     int unmatched = n; // Number of free men
33     int i = n - 1; // Current man index (rotates over all men)
34
35     #define engage pm[j] = i; pv[i] = j;
36
37     while (unmatched) {
38         while (pm[i] == -1) {
39             int j = gv[i][pun[i]++]; // Next woman on man i's list
40
41             if (pv[j] == -1) {
42                 // Woman j is free -> engage with man i
43                 unmatched--;
44                 engage;
45             } else if (om[j][i] < om[j][pv[j]]) {
46                 // Woman j prefers i over her current partner
47                 int loser = pv[j];
48
49                 pm[loser] = -1;
50                 engage;
51                 i = loser; // Reconsider the rejected man
52             }
53
54             // Move to next unmatched man
55             i--;
56             if (i < 0) i = n - 1;
57         }
58     }
59 }
```

```

47     pm[loser] = -1;
48     engage;
49     i = loser; // Reconsider the rejected man
50 }
51 }
52
53 // Move to next unmatched man
54 i--;
55 if (i < 0) i = n - 1;
56 }
57
58 #undef engage
59 }
```

7.15 SCC

```

1 /*
2 Strongly Connected Components (Kosaraju's Algorithm)
3 -----
4 Indexing: 0-based
5 Time Complexity: O(V + E)
6 Space Complexity: O(V + E)
7
8 Input:
9 - n: number of nodes
10 - m: number of directed edges
11 - adj: original graph
12 - adjr: reversed graph
13
14 Output:
15 - comp[i]: component ID of node i
16 - order[]: nodes in reverse post-order (1st DFS)
17 - nc: is the number of unique comp values
18 */
19
20 vector<vector<int>> adj, adjr;
21 vector<bool> vis;
22 vector<int> order, comp;
23
24 // First DFS: post-order on original graph
25 void dfs(int v) {
26     vis[v] = true;
27     for (int u : adj[v]) {
```

```

28     if (!vis[u])
29         dfs(u);
30     }
31     order.push_back(v); // Record post-order
32 }
33
34 // Second DFS: assign component IDs on reversed graph
35 void dfsr(int v, int k) {
36     vis[v] = true;
37     comp[v] = k;
38     for (int u : adjr[v]) {
39         if (!vis[u])
40             dfsr(u, k);
41     }
42 }
43
44 void solve() {
45     int n, m;
46     cin >> n >> m;
47     adj.assign(n, vector<int>());
48     adjr.assign(n, vector<int>());
49     comp.resize(n);
50     // Read edges and build both original and reversed graphs
51     for (int i = 0; i < m; i++) {
52         int a, b;
53         cin >> a >> b;
54         a--; b--;
55         adj[a].push_back(b);
56         adjr[b].push_back(a);
57     }
58     // First pass: DFS on original graph to get order
59     vis.assign(n, false);
60     order.clear();
61     for (int i = 0; i < n; i++) {
62         if (!vis[i]) dfs(i);
63     }
64     // Second pass: DFS on reversed graph using reverse post-order
65     vis.assign(n, false);
66     int nc = 0;
67     for (int i = n - 1; i >= 0; i--) {
68         int v = order[i];
69         if (!vis[v]) {
70             dfsr(v, nc++);
71         }
72     }
73     // comp[i] now holds the component ID for node i (0-based)
74     // nc = number of strongly connected components
75 }

```

8 Linear Algebra

8.1 Simplex

```

1  /*
2  Parametric Self-Dual Simplex method
3  Solve a canonical LP:
4      min or max. c x
5      s.t. A x <= b
6          x >= 0
7  */
8 #include <bits/stdc++.h>
9 using namespace std;
10 const double eps = 1e-9, oo = numeric_limits<double>::infinity();
11
12 typedef vector<double> vec;
13 typedef vector<vec> mat;
14
15 pair<vec, double> simplexMethodPD(const mat &A, const vec &b, const vec
16     &c, bool mini = true){
17     int n = c.size(), m = b.size();
18     mat T(m + 1, vec(n + m + 1));
19     vector<int> base(n + m), row(m);
20
21     for(int j = 0; j < m; ++j){
22         for(int i = 0; i < n; ++i)
23             T[j][i] = A[j][i];
24         row[j] = n + j;
25         T[j][n + j] = 1;
26         base[n + j] = 1;
27         T[j][n + m] = b[j];
28     }
29
30     for(int i = 0; i < n; ++i)
31         T[m][i] = c[i] * (mini ? 1 : -1);
32
33     while(true){

```

```

33     int p = 0, q = 0;
34     for(int i = 0; i < n + m; ++i)
35         if(T[m][i] <= T[m][p])
36             p = i;
37
38     for(int j = 0; j < m; ++j)
39         if(T[j][n + m] <= T[q][n + m])
40             q = j;
41
42     double t = min(T[m][p], T[q][n + m]);
43
44     if(t >= -eps){
45         vec x(n);
46         for(int i = 0; i < m; ++i)
47             if(row[i] < n) x[row[i]] = T[i][n + m];
48         return {x, T[m][n + m] * (mini ? -1 : 1)}; // optimal
49     }
50
51     if(t < T[q][n + m]){
52         // tight on c -> primal update
53         for(int j = 0; j < m; ++j)
54             if(T[j][p] >= eps)
55                 if(T[j][p] * (T[q][n + m] - t) >= T[q][p] * (T[j][n + m] - t))
56                     q = j;
57
58         if(T[q][p] <= eps)
59             return {vec(n), oo * (mini ? 1 : -1)}; // primal infeasible
60     }else{
61         // tight on b -> dual update
62         for(int i = 0; i < n + m + 1; ++i)
63             T[q][i] = -T[q][i];
64
65         for(int i = 0; i < n + m; ++i)
66             if(T[q][i] >= eps)
67                 if(T[q][i] * (T[m][p] - t) >= T[q][p] * (T[m][i] - t))
68                     p = i;
69
70         if(T[q][p] <= eps)
71             return {vec(n), oo * (mini ? -1 : 1)}; // dual infeasible
72     }
73
74     for(int i = 0; i < m + n + 1; ++i)
75         if(i != p) T[q][i] /= T[q][p];
76
77     T[q][p] = 1; // pivot(q, p)
78     base[p] = 1;
79     base[row[q]] = 0;
80     row[q] = p;
81
82     for(int j = 0; j < m + 1; ++j){
83         if(j != q){
84             double alpha = T[j][p];
85             for(int i = 0; i < n + m + 1; ++i)
86                 T[j][i] -= T[q][i] * alpha;
87         }
88     }
89
90     return {vec(n), oo};
91 }
92
93
94 int main(){
95     int m, n;
96     bool mini = true;
97     cout << "Numero_de_restricciones:" << endl;
98     cin >> m;
99     cout << "Numero_de_incognitas:" << endl;
100    cin >> n;
101    mat A(m, vec(n));
102    vec b(m), c(n);
103    for(int i = 0; i < m; ++i){
104        cout << "Restriccion#" << (i + 1) << ":" << endl;
105        for(int j = 0; j < n; ++j){
106            cin >> A[i][j];
107        }
108        cin >> b[i];
109    }
110    cout << "[0]Max_o [1]Min?:" << endl;
111    cin >> mini;
112    cout << "Coeficientes_de:" << (mini ? "min" : "max") << endl;
113    for(int i = 0; i < n; ++i){
114        cin >> c[i];
115    }
116    cout.precision(6);
117    auto ans = simplexMethodPD(A, b, c, mini);
118    cout << (mini ? "Min" : "Max") << "uzu=" << ans.second << ", cuando:" << endl;

```

```

119     \n";
120     for(int i = 0; i < ans.first.size(); ++i){
121         cout << "x_" << (i + 1) << " = " << ans.first[i] << "\n";
122     }
123     return 0;
}

```

9 Math

9.1 BinPow

```

1 ll binpow(ll a, ll b){
2     ll r=1;
3     while(b){
4         if(b%2)
5             r=(r*a)%MOD;
6         a=(a*a)%MOD;
7         b/=2;
8     }
9     return r;
}
11
12 ll divide(ll a, ll b){
13     return ((a%MOD)*binpow(b, MOD-2))%MOD;
14 }
15 void inverses(long long p) {
16     inv[MAXN] = exp(fac[MAXN], p - 2, p);
17     for (int i = MAXN; i >= 1; i--) { inv[i - 1] = inv[i] * i % p; }
18 }

```

9.2 Diophantine

If one solution is (x_0, y_0) all solutions can be obtained by $x = x_0 + k * \frac{b}{\gcd(a,b)}$ and
 $y = y_0 - k * \frac{a}{\gcd(a,b)}$.

```

1 int gcd(int a, int b, int& x, int& y) {
2     if (b == 0) {
3         x = 1;
4         y = 0;
5         return a;
6     }
7     int x1, y1;
8     int d = gcd(b, a % b, x1, y1);
9     x = y1;

```

```

10    y = x1 - y1 * (a / b);
11    return d;
12 }

13 bool find_any_solution(int a, int b, int c, int &x0, int &y0, int &g) {
14     g = gcd(abs(a), abs(b), x0, y0);
15     if (c % g) {
16         return false;
17     }

18     x0 *= c / g;
19     y0 *= c / g;
20     if (a < 0) x0 = -x0;
21     if (b < 0) y0 = -y0;
22     return true;
23 }

24

25

26

27

28 // n variables
29 vector<ll> find_any_solution(vector<ll> a, ll c) {
30     int n = a.size();
31     vector<ll> x;
32     bool all_zero = true;
33     for (int i = 0; i < n; i++) {
34         all_zero &= a[i] == 0;
35     }
36     if (all_zero) {
37         if (c) return {};
38         x.assign(n, 0);
39         return x;
40     }
41     ll g = 0;
42     for (int i = 0; i < n; i++) {
43         g = __gcd(g, a[i]);
44     }
45     if (c % g != 0) return {};
46     if (n == 1) {
47         return {c / a[0]};
48     }
49     vector<ll> suf_gcd(n);
50     suf_gcd[n - 1] = a[n - 1];
51     for (int i = n - 2; i >= 0; i--) {
52

```

```

53     suf_gcd[i] = __gcd(suf_gcd[i + 1], a[i]);
54 }
55 ll cur = c;
56 for (int i = 0; i + 1 < n; i++) {
57     ll x0, y0, g;
58     // solve for a[i] * x + suf_gcd[i + 1] * (y / suf_gcd[i + 1]) = cur
59     bool ok = find_any_solution(a[i], suf_gcd[i + 1], cur, x0, y0, g);
60     assert(ok);
61     {
62         // trying to minimize x0 in case x0 becomes big
63         // it is needed for this problem, not needed in general
64         ll shift = abs(suf_gcd[i + 1] / g);
65         x0 = (x0 % shift + shift) % shift;
66     }
67     x.push_back(x0);
68
69     // now solve for the next suffix
70     cur -= a[i] * x0;
71 }
72 x.push_back(a[n - 1] == 0 ? 0 : cur / a[n - 1]);
73 return x;
74 }
```

9.3 Discrete Logarithm

Finds discrete logarithm in $O(\sqrt{m})$.

```

1 // Returns minimum x for which a ^ x % m = b % m, a and m are coprime.
2 int solve(int a, int b, int m) {
3     a %= m, b %= m;
4     int n = sqrt(m) + 1;
5
6     int an = 1;
7     for (int i = 0; i < n; ++i)
8         an = (an * 1ll * a) % m;
9
10    unordered_map<int, int> vals;
11    for (int q = 0, cur = b; q <= n; ++q) {
12        vals[cur] = q;
13        cur = (cur * 1ll * a) % m;
14    }
15
16    for (int p = 1, cur = 1; p <= n; ++p) {
17        cur = (cur * 1ll * an) % m;
18        if (vals.count(cur)) {
19            int ans = n * p - vals[cur];
20            return ans;
21        }
22    }
23    return -1;
24 }
25
26 // Returns minimum x for which a ^ x % m = b % m.
27 int solve(int a, int b, int m) {
28     a %= m, b %= m;
29     int k = 1, add = 0, g;
30     while ((g = gcd(a, m)) > 1) {
31         if (b == k)
32             return add;
33         if (b % g)
34             return -1;
35         b /= g, m /= g, ++add;
36         k = (k * 1ll * a / g) % m;
37     }
38
39     int n = sqrt(m) + 1;
40     int an = 1;
41     for (int i = 0; i < n; ++i)
42         an = (an * 1ll * a) % m;
43
44     unordered_map<int, int> vals;
45     for (int q = 0, cur = b; q <= n; ++q) {
46         vals[cur] = q;
47         cur = (cur * 1ll * a) % m;
48     }
49
50     for (int p = 1, cur = k; p <= n; ++p) {
51         cur = (cur * 1ll * an) % m;
52         if (vals.count(cur)) {
53             int ans = n * p - vals[cur] + add;
54             return ans;
55         }
56     }
57     return -1;
58 }
```

9.4 Divisors

```

1 long long numberOfDivisors(long long num)
2 {
3     long long total = 1;
4     for (int i = 2; (long long)i * i <= num; i++)
5     {
6         if (num % i == 0)
7         {
8             int e = 0;
9             do
10            {
11                e++;
12                num /= i;
13            } while (num % i == 0);
14            total *= e + 1;
15        }
16    }
17    if (num > 1)
18    {
19        total *= 2;
20    }
21    return total;
22 }

23
24 long long SumOfDivisors(long long num)
25 {
26     long long total = 1;
27
28     for (int i = 2; (long long)i * i <= num; i++)
29     {
30         if (num % i == 0)
31         {
32             int e = 0;
33             do
34             {
35                 e++;
36                 num /= i;
37             } while (num % i == 0);
38
39             long long sum = 0, pow = 1;
40             do
41             {

```

```

42                 sum += pow;
43                 pow *= i;
44             } while (e-- > 0);
45             total *= sum;
46         }
47     }
48     if (num > 1)
49     {
50         total *= (1 + num);
51     }
52     return total;
53 }
```

9.5 Euler Totient (Phi)

```

1 //counts coprimes to each number from 1 to n
2 vector<int> phi1(int n) {
3     vector<int> phi(n + 1);
4     for (int i = 0; i <= n; i++)
5         phi[i] = i;
6
7     for (int i = 2; i <= n; i++) {
8         if (phi[i] == i) {
9             for (int j = i; j <= n; j += i)
10                 phi[j] -= phi[j] / i;
11         }
12     }
13     return phi1;
14 }
```

9.6 Fibonacci

```

1 void fib(ll n, ll&x, ll&y){
2     if(n==0){
3         x = 0;
4         y = 1;
5         return ;
6     }
7
8     if(n&1){
9         fib(n-1, y, x);
10        y=(y+x)%MOD;
11    }else{
12        ll a, b;
```

```

13     fib(n>>1, a, b);
14     y = (a*a+b*b)%MOD;
15     x = (a*b + a*(b-a+MOD))%MOD;
16   }
17 }
18
19 // Usage
20 // ll x, y;
21 // fib(10, x, y);
22 // cout << x << " " << y << endl;
23 // This will output 55 89

```

9.7 Matrix Exponentiation

```

1 struct Mat {
2     int n, m;
3     vector<vector<int>> a;
4     Mat() { }
5     Mat(int _n, int _m) {n = _n; m = _m; a.assign(n, vector<int>(m, 0));}
6     Mat(vector<vector<int> > v) {n = v.size(); m = n ? v[0].size() : 0;
7         a = v; }
8     inline void make_unit() {
9         assert(n == m);
10        for (int i = 0; i < n; i++) {
11            for (int j = 0; j < n; j++) a[i][j] = i == j;
12        }
13    }
14    inline Mat operator + (const Mat &b) {
15        assert(n == b.n && m == b.m);
16        Mat ans = Mat(n, m);
17        for(int i = 0; i < n; i++) {
18            for(int j = 0; j < m; j++) {
19                ans.a[i][j] = (a[i][j] + b.a[i][j]) % mod;
20            }
21        }
22        return ans;
23    }
24    inline Mat operator - (const Mat &b) {
25        assert(n == b.n && m == b.m);
26        Mat ans = Mat(n, m);
27        for(int i = 0; i < n; i++) {
28            for(int j = 0; j < m; j++) {

```

```

29                }
30            }
31            return ans;
32        }
33        inline Mat operator * (const Mat &b) {
34            assert(m == b.n);
35            Mat ans = Mat(n, b.m);
36            for(int i = 0; i < n; i++) {
37                for(int j = 0; j < b.m; j++) {
38                    for(int k = 0; k < m; k++) {
39                        ans.a[i][j] = (ans.a[i][j] + 1LL * a[i][k] * b.a[k][j] % mod)
40                            % mod;
41                    }
42                }
43            }
44            return ans;
45        }
46        inline Mat pow(long long k) {
47            assert(n == m);
48            Mat ans(n, n), t = a; ans.make_unit();
49            while (k) {
50                if (k & 1) ans = ans * t;
51                t = t * t;
52                k >>= 1;
53            }
54            return ans;
55        }
56        inline Mat& operator += (const Mat& b) { return *this = (*this) + b; }
57        inline Mat& operator -= (const Mat& b) { return *this = (*this) - b; }
58        inline Mat& operator *= (const Mat& b) { return *this = (*this) * b; }
59        inline bool operator == (const Mat& b) { return a == b.a; }
60        inline bool operator != (const Mat& b) { return a != b.a; }
61    };
62
63 // Usage
64 // Mat a(n, n);
65 // Mat b(n, n);
66 // Mat c = a * b;
67 // Mat d = a + b;
68 // Mat e = a - b;
69 // Mat f = a.pow(k);
70 // a.a[i][j] = x;

```

9.8 Miller Rabin Deterministic

```

1 using u64 = uint64_t;
2 using u128 = __uint128_t;
3
4 u64 binpower(u64 base, u64 e, u64 mod) {
5     u64 result = 1;
6     base %= mod;
7     while (e) {
8         if (e & 1)
9             result = (u128)result * base % mod;
10        base = (u128)base * base % mod;
11        e >>= 1;
12    }
13    return result;
14}
15
16 bool check_composite(u64 n, u64 a, u64 d, int s) {
17     u64 x = binpower(a, d, n);
18     if (x == 1 || x == n - 1)
19         return false;
20     for (int r = 1; r < s; r++) {
21         x = (u128)x * x % n;
22         if (x == n - 1)
23             return false;
24     }
25     return true;
26};
27
28
29 bool MillerRabin(ll n) {
30     if (n < 2)
31         return false;
32
33     int r = 0;
34     ll d = n - 1;
35     while ((d & 1) == 0) {
36         d >>= 1;
37         r++;
38     }
39
40     for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
41         if (n == a)

```

```

42             return true;
43         if (check_composite(n, a, d, r))
44             return false;
45     }
46     return true;
47 }

```

9.9 Möbius

```

1 int mob[N];
2 void mobius() {
3     mob[1] = 1;
4     for (int i = 2; i < N; i++){
5         mob[i]--;
6         for (int j = i + i; j < N; j += i) {
7             mob[j] -= mob[i];
8         }
9     }
10 }

```

9.10 Prefix Sum Phi

```

1 vector<ll> sieve(kMaxV + 1,0);
2 vector<ll> phi(kMaxV + 1,0);
3
4 void primes()
{
5     phi[1]=1;
6     vector<ll> pr;
7     for(int i=2;i<kMaxV;i++){
8         if(sieve[i]==0){
9             sieve[i]=i;
10            pr.pb(i);
11            phi[i]=i-1;
12        }
13        for(auto p:pr){
14            if(p>sieve[i] || i*p>kMaxV)break;
15            sieve[i*p]=p;
16            phi[i*p]=(p==sieve[i]?p:p-1)*phi[i];
17        }
18    }
19    for(int i=1;i<kMaxV;i++){
20        phi[i]+=phi[i-1];
21        phi[i]%=MOD;
22    }

```

```

23     }
24 }
25
26 map<ll,ll> m;
27 ll PHI(ll a){
28     if(a<kMaxV) return phi[a];
29     if(m.count(a)) return m[a];
30     // if(a<3) return 1;
31     m[a]=((((a%MOD)*((a+1)%MOD))%MOD)*inverse(2));
32     m[a]%=MOD;
33     long long i=2;
34     while(i<=a){
35         long long j=a/i;
36         j=a/j;
37         m[a]+=MOD;
38         m[a]-=((j-i+1)*PHI(a/i))%MOD;
39         m[a]%=MOD;
40         i=j+1;
41     }
42     m[a]%=MOD;
43     return m[a];
44 }
```

9.11 Sieve

```

1 const int kMaxV = 1e6;
2
3 int sieve[kMaxV + 1];
4
5 //stores some prime (not necessarily the minimum one)
6 void primes()
7 {
8     for (int i = 4; i <= kMaxV; i += 2)
9         sieve[i] = 2;
10    for (int i = 3; i <= kMaxV / i; i += 2)
11    {
12        if (sieve[i])
13            continue;
14        for (int j = i * i; j <= kMaxV; j += i)
15            sieve[j] = i;
16    }
17 }
```

```

19 vector<int> PrimeFactors(int x)
20 {
21     if (x == 1)
22         return {};
23
24     unordered_set<int> primes;
25     while (sieve[x])
26     {
27         primes.insert(sieve[x]);
28         x /= sieve[x];
29     }
30     primes.insert(x);
31     return {primes.begin(), primes.end()};
32 }
```

9.12 Identities

$$C_n = \frac{2(2n-1)}{n+1} C_{n-1}$$

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

$$C_n \sim \frac{4^n}{n^{3/2}\sqrt{\pi}}$$

$\sigma(n) = O(\log(\log(n)))$ (number of divisors of n)

$$F_{2n+1} = F_n^2 + F_{n+1}^2$$

$$F_{2n} = F_{n+1}^2 - F_{n-1}^2$$

$$\sum_{i=1}^n F_i = F_{n+2} - 1$$

$$F_{n+i}F_{n+j} - F_nF_{n+i+j} = (-1)^n F_i F_j$$

(Möbius Inv. Formula) $\mu(p^k) = [k=0] - [k=1]$ Let $g(n) = \sum_{d|n} f(d)$, then
 $f(n) = \sum_{d|n} g(d) \mu\left(\frac{n}{d}\right)$.

(Dirichlet Convolution) Let f, g be arithmetic functions, then
 $(f * g)(n) = \sum_{d|n} f(d)g\left(\frac{n}{d}\right)$. If f, g are multiplicative, then so is $f * g$.
 $n = \sum_{d|n} \phi(d)$

Lucas' Theorem: $\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$ where $m = \sum_{i=0}^k m_i p^i$ and
 $n = \sum_{i=0}^k n_i p^i$.

9.13 Burnside's Lemma

Dado un grupo G de permutaciones y un conjunto X de n elementos, el número de órbitas de X bajo la acción de G es igual al promedio del número de puntos fijos de las permutaciones en G .

Formalmente, el número de órbitas es $\frac{1}{|G|} \sum_{g \in G} f(g)$ donde $f(g)$ es el número de puntos fijos de g .

Ejemplo: Dado un collar con n cuentas y 2 colores, el número de collares diferentes que se pueden formar es $\frac{1}{n} \sum_{i=0}^n f(i)$ donde $f(i)$ es el número de collares que quedan

fijos bajo una rotación de i posiciones.

Para contar el número de collares que quedan fijos bajo una rotación de i posiciones, se puede usar la fórmula $f(i) = 2^{\gcd(i,n)}$.

Para un collar de n cuentas y k colores, el número de collares diferentes que se pueden formar es $\frac{1}{n} \sum_{i=0}^n k^{\gcd(i,n)}$

Ejemplo: Dado un cubo con 6 caras y k colores, el número de cubos diferentes que se pueden formar es $\frac{1}{24} \sum_{i=0}^{24} f(i)$ donde $f(i)$ es el número de cubos que quedan fijos bajo una rotación de i posiciones. Esta formula es igual a $\frac{1}{24}(n^6 + 3n^4 + 12n^3 + 8n^2)$

9.14 Recursion

Sea $f(n) = \sum_{i=1}^k a_i f(n-i)$ entonces podemos considerar la matriz:

$$\begin{bmatrix} f(n) \\ f(n-1) \\ \vdots \\ f(n-k+1) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \cdots & a_{k-1} & a_k \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} f(n-1) \\ f(n-2) \\ \vdots \\ f(n-k) \end{bmatrix}$$

De aqui podemos calcular $f(n)$ con exponentiación de matrices.

$$\begin{bmatrix} f(n) \\ f(n-1) \\ \vdots \\ f(n-k+1) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \cdots & a_{k-1} & a_k \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}^{n-k} \begin{bmatrix} f(k) \\ f(k-1) \\ \vdots \\ f(1) \end{bmatrix}$$

9.15 Theorems

Koeing's Theorem: La cardinalidad del emparejamiento maximo de una grafica bipartita es igual al minimum vertex cover.

Hall's Theorem: Una grafica bipartita G tiene un emparejamiento que cubre todos los nodos de G si y solo si para todo subconjunto S de nodos de G , el número de vecinos de S es mayor o igual a $|S|$.

Kuratowski's Theorem: Una grafica es plana si y solo si no contiene un subgrafo homeomorfo a $K_{3,3}$ o K_5 .

9.16 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$\begin{aligned} 1 + 2 + 3 + \cdots + n &= \frac{n(n+1)}{2} \\ 1^2 + 2^2 + 3^2 + \cdots + n^2 &= \frac{n(2n+1)(n+1)}{6} \\ 1^3 + 2^3 + 3^3 + \cdots + n^3 &= \frac{n^2(n+1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \cdots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \end{aligned}$$

9.17 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, C_{n+1} = \frac{2(2n+1)}{n+2} C_n, C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested. If prefix is given, number of ways is $\binom{n}{\text{remaining}_\text{closed}} - \binom{n}{\text{remaining}_\text{closed}+1}$.
- binary trees with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

9.18 Cayley's formula

Number of labeled trees of n vertices is n^{n-2} . Number of rooted forest of n vertices is $(n+1)^{n-1}$.

9.19 Geometric series

$$\begin{aligned} a + ar + ar^2 + ar^3 + \cdots + \sum_{k=0}^{\infty} ar^k &\quad \text{Infinite} \\ \text{Sum} &= \frac{a}{1-r} \\ a + ar + ar^2 + ar^3 + \cdots + \sum_{k=0}^n ar^k &\quad \text{Finite} \\ \text{Sum} &= \frac{a(1-r^{n+1})}{1-r} \end{aligned}$$

9.20 Estimates For Divisors

$$\sum_{d|n} d = O(n \log \log n).$$

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

9.21 Sum of divisors

$$\sum d|n = \frac{p_1^{\alpha_1+1}-1}{p_1-1} + \frac{p_2^{\alpha_2+1}-1}{p_2-1} + \dots + \frac{p_n^{\alpha_n+1}-1}{p_n-1}$$

9.22 Pythagorean Triplets

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

9.23 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

10 Game Theory

10.1 Sprague-Grundy theorem

<https://codeforces.com/blog/entry/66040> Dado un juego con pilas p_1, p_2, \dots, p_n sea $g(p)$ el número de la pila p , entonces el número del juego es

$g(p_1) \oplus g(p_2) \oplus \dots \oplus g(p_n)$. Para calcular el número de una pila, se puede usar la fórmula $g(r) = \text{mex}(\{g(r_1), g(r_2), \dots, g(r_k)\})$ donde r_1, r_2, \dots, r_k son los posibles estados a los que se puede llegar desde r y $g(r) = 0$ si r es un estado perdedor.

11 More Topics

11.1 2D Prefix Sum

```

1 int b[MAXN][MAXN];
2 int a[MAXN][MAXN];
3
4 for (int i = 1; i <= N; i++) {
5     for (int j = 1; j <= N; j++) {

```

```

6     b[i][j] = a[i][j] + b[i - 1][j] +
7                                b[i][j - 1] - b[i - 1][j - 1];
8 }
9 }
10
11 for (int q = 0; q < Q; q++) {
12     int from_row, to_row, from_col, to_col;
13     cin >> from_row >> from_col >> to_row >> to_col;
14     cout << b[to_row][to_col] - b[from_row - 1][to_col] -
15                                b[to_row][from_col - 1] +
16                                b[from_row - 1][from_col - 1]
17                                << '\n';
18 }

```

11.2 Custom Comparators

```

1 bool cmp(const Edge &x, const Edge &y) { return x.w < y.w; }
2
3 sort(a.begin(), a.end(), cmp);
4
5 set<int, greater<int>> a;
6 map<int, string, greater<int>> b;
7 priority_queue<int, vector<int>, greater<int>> c;

```

11.3 Day of the Week

```

1 int dayOfWeek(int d, int m, lli y){
2     if(m == 1 || m == 2){
3         m += 12;
4         --y;
5     }
6     int k = y % 100;
7     lli j = y / 100;
8     return (d + 13*(m+1)/5 + k + k/4 + j/4 + 5*j) % 7;
9 }

```

11.4 GCD Convolution

```

1 vector<int> PrimeEnumerate(int n) {
2     vector<int> P; vector<bool> B(n + 1, 1);
3     for (int i = 2; i <= n; i++) {
4         if (B[i]) P.push_back(i);
5         for (int j : P) { if (i * j > n) break; B[i * j] = 0; if (i % j ==
6             0) break; }

```

```

6     }
7     return P;
8 }
9
10
11 template<typename T>
12 void MultipleZetaTransform(vector<T>& v) {
13     const int n = (int)v.size() - 1;
14     for (int p : PrimeEnumerate(n)) {
15         for (int i = n / p; i; i--)
16             v[i] += v[i * p];
17     }
18 }
19
20 template<typename T>
21 void MultipleMobiusTransform(vector<T>& v) {
22     const int n = (int)v.size() - 1;
23     for (int p : PrimeEnumerate(n)) {
24         for (int i = 1; i * p <= n; i++)
25             v[i] -= v[i * p];
26     }
27 }
28
29 template<typename T>
30 vector<T> GCDConvolution(vector<T> A, vector<T> B) {
31     MultipleZetaTransform(A);
32     MultipleZetaTransform(B);
33     for (int i = 0; i < A.size(); i++) A[i] *= B[i];
34     MultipleMobiusTransform(A);
35     return A;
36 }

```

11.5 int128

```

1 //cout for __int128
2 ostream &operator<<(ostream &os, const __int128 & value){
3     char buffer[64];
4     char *pos = end(buffer) - 1;
5     *pos = '\0';
6     __int128 tmp = value < 0 ? -value : value;
7     do{
8         --pos;
9         *pos = tmp % 10 + '0';

```

```

10     tmp /= 10;
11 }while(tmp != 0);
12 if(value < 0){
13     --pos;
14     *pos = '-';
15 }
16 return os << pos;
17 }

18
19 //cin for __int128
20 istream &operator>>(istream &is, __int128 & value){
21     char buffer[64];
22     is >> buffer;
23     char *pos = begin(buffer);
24     int sgn = 1;
25     value = 0;
26     if(*pos == '-'){
27         sgn = -1;
28         ++pos;
29     }else if(*pos == '+'){
30         ++pos;
31     }
32     while(*pos != '\0'){
33         value = (value << 3) + (value << 1) + (*pos - '0');
34         ++pos;
35     }
36     value *= sgn;
37     return is;
38 }

39
40
41 ll mult(__int128 a, __int128 b){ return ((a*1LL*b)%MOD + MOD)%MOD; }

```

11.6 Iterating Over All Subsets

```

1 for (int mk = 0; mk < (1 << k); mk++) {
2     Ap[mk] = 0;
3     for (int s = mk;; s = (s - 1) & mk) {
4         Ap[mk] += A[s];
5         if (!s)
6             break;
7     }
8 }

```

11.7 LCM Convolution

```

1  /* Linear Sieve, O(n) */
2  vector<int> PrimeEnumerate(int n) {
3      vector<int> P; vector<bool> B(n + 1, 1);
4      for (int i = 2; i <= n; i++) {
5          if (B[i]) P.push_back(i);
6          for (int j : P) { if (i * j > n) break; B[i * j] = 0; if (i % j ==
7              0) break; }
8      }
9      return P;
}
10
11 template<typename T>
12 void DivisorZetaTransform(vector<T>& v) {
13     const int n = (int)v.size() - 1;
14     for (int p : PrimeEnumerate(n)) {
15         for (int i = 1; i * p <= n; i++)
16             v[i * p] += v[i];
17     }
}
18
19 template<typename T>
20 void DivisorMobiusTransform(vector<T>& v) {
21     const int n = (int)v.size() - 1;
22     for (int p : PrimeEnumerate(n)) {
23         for (int i = n / p; i; i--)
24             v[i * p] -= v[i];
25     }
}
26
27
28
29
30 template<typename T>
31 vector<T> LCMConvolution(vector<T> A, vector<T> B) {
32     DivisorZetaTransform(A);
33     DivisorZetaTransform(B);
34     for (int i = 0; i < A.size(); i++) A[i] *= B[i];
35     DivisorMobiusTransform(A);
36     return A;
}
37

```

11.8 Manhattan MST

```

1 struct point {
2     long long x, y;
3 };
4
5 vector<tuple<long long, int, int>> manhattan_mst_edges(vector<point> ps)
6 {
7     vector<int> ids(ps.size());
8     iota(ids.begin(), ids.end(), 0);
9     vector<tuple<long long, int, int>> edges;
10    for (int rot = 0; rot < 4; rot++) { // for every rotation
11        sort(ids.begin(), ids.end(), [&](int i, int j){
12            return (ps[i].x + ps[i].y) < (ps[j].x + ps[j].y);
13        });
14        map<int, int, greater<int>> active; // (xs, id)
15        for (auto i : ids) {
16            for (auto it = active.lower_bound(ps[i].x); it != active.end();
17                 active.erase(it++)) {
18                int j = it->second;
19                if (ps[i].x - ps[i].y > ps[j].x - ps[j].y) break;
20                assert(ps[i].x >= ps[j].x && ps[i].y >= ps[j].y);
21                edges.push_back({(ps[i].x - ps[j].x) + (ps[i].y - ps[j].y), i, j
22                });
23            }
24            active[ps[i].x] = i;
25        }
26        for (auto &p : ps) { // rotate
27            if (rot & 1) p.x *= -1;
28            else swap(p.x, p.y);
29        }
30    }
31    return edges;
}

```

11.9 Mo

```

1 /*
2  Mo's Algorithm (Sqrt Decomposition for Offline Queries)
3  -----
4  Problem: Answer q range queries [L, R] over array of length n
5      using add/remove operations efficiently
6  Indexing: 0-based
7  Bounds:
8      - arr[0..n-1]

```

```

9   - queries input as 1-based and converted to 0-based
10 Time Complexity: O((n + q) * sqrt(n)) per query batch
11 Space Complexity: O(n + q)
12 Usage:
13   - Fill in logic for add/remove operations (update cur)
14   - Fill answers[] indexed by original query order
15 */
16
17 const int MAXN = 200500;
18 ll n, q;
19 ll arr[MAXN]; // input array
20 ll cnt[1000005]; // frequency count
21 ll answers[MAXN]; // output array
22 ll cur = 0; // current query result
23 ll BLOCK_SIZE;
24
25 pair< pair<ll, ll>, ll> queries[MAXN]; // {{L, R}, query_index}
26
27 // Sort by block and then by R
28 inline bool cmp(const pair< pair<ll, ll>, ll> &x, const pair< pair<ll,
29   ll>, ll> &y) {
30   ll block_x = x.first.first / BLOCK_SIZE;
31   ll block_y = y.first.first / BLOCK_SIZE;
32   if (block_x != block_y) return block_x < block_y;
33   return x.first.second < y.first.second;
34 }
35
36 int main() {
37   cin >> n >> q;
38   BLOCK_SIZE = sqrt(n);
39   for (int i = 0; i < n; i++) cin >> arr[i];
40   for (int i = 0; i < q; i++) {
41     int l, r;
42     cin >> l >> r;
43     --l; --r; // convert to 0-based
44     queries[i] = {{l, r}, i};
45   }
46   sort(queries, queries + q, cmp);
47   ll l = 0, r = -1;
48   for (int i = 0; i < q; i++) {
49     int left = queries[i].first.first;
50     int right = queries[i].first.second;
      // Expand to right

```

```

51   while (r < right) {
52     r++;
53     // Operations to add arr[r], implement exactly here
54   }
55   // Shrink from right
56   while (r > right) {
57     // Operations to remove arr[r], implement exactly here
58     r--;
59   }
60   // Expand to left
61   while (l < left) {
62     // Operations to remove arr[l], implement exactly here
63     l++;
64   }
65   // Shrink from left
66   while (l > left) {
67     l--;
68     // Operations to add arr[l], implement exactly here
69   }
70   answers[queries[i].second] = cur; // Current answer
71 }
72 for (int i = 0; i < q; i++) cout << answers[i] << '\n';
73 }

```

11.10 MOD INT

```

1 /**
2  * Description: Mod integer class for doing modular arithmetic.
3  * Source: https://github.com/jakobkogler/Algorithm-DataStructures/blob/master/Math/Modular.h
4  * Verification: https://open.kattis.com/problems/modulararithmetic
5  * Time: fast
6 */
7
8 template<int MOD>
9 struct ModInt {
10   long long v;
11   ModInt(long long _v = 0) {v = (-MOD < _v && _v < MOD) ? _v : _v % MOD; if (v < 0) v += MOD;}
12   ModInt& operator += (const ModInt &other) {v += other.v; if (v >= MOD) v -= MOD; return *this;}
13   ModInt& operator -= (const ModInt &other) {v -= other.v; if (v < 0) v += MOD; return *this;}

```

```

14     ModInt& operator *= (const ModInt &other) {v = v * other.v % MOD;
15         return *this;}
16     ModInt& operator /= (const ModInt &other) {return *this *= inverse(
17         other);}
18     bool operator == (const ModInt &other) const {return v == other.v;}
19     bool operator != (const ModInt &other) const {return v != other.v;}
20     friend ModInt operator + (ModInt a, const ModInt &b) {return a += b
21         ;}
22     friend ModInt operator - (ModInt a, const ModInt &b) {return a -= b
23         ;}
24     friend ModInt operator * (ModInt a, const ModInt &b) {return a *= b
25         ;}
26     friend ModInt operator / (ModInt a, const ModInt &b) {return a /= b
27         ;}
28     friend ModInt operator - (const ModInt &a) {return 0 - a;}
29     friend ModInt power(ModInt a, long long b) {ModInt ret(1); while (b
30         > 0) {if (b & 1) ret *= a; a *= a; b >>= 1;} return ret;}
31     friend ModInt inverse(ModInt a) {return power(a, MOD - 2);}
32     friend istream& operator >> (istream &is, ModInt &m) {is >> m.v; m.v
33         = (-MOD < m.v && m.v < MOD) ? m.v : m.v % MOD; if (m.v < 0) m.v
34         += MOD; return is;}
35     friend ostream& operator << (ostream &os, const ModInt &m) {return
36         os << m.v;}
37 };

```

11.11 Next Permutation

```

1 sort(v.begin(),v.end());
2 while(next_permutation(v.begin(),v.end())){
3     for(auto u:v){
4         cout<<u<<" ";
5     }
6     cout<<endl;
7 }
8
9 string s="asdfassd";
10 sort(s.begin(),s.end());
11 while(next_permutation(s.begin(),s.end())){
12     cout<<s<<endl;
13 }

```

11.12 Next and Previous Smaller/Greater Element

```

1 | vector<int> nextSmaller(vector<int> a, int n){

```

```

2     stack<int> s;
3     vector<int> res(n, n);
4     for(int i=0;i<n;i++){
5         while(s.size() && a[s.top()]>a[i]){
6             res[s.top()]=i;
7             s.pop();
8         }
9         s.push(i);
10    }
11    return res;
12 }

13
14 vector<int> prevSmaller(vector<int> a, int n){
15     stack<int> s;
16     vector<int> res(n, -1);
17     for(int i=n-1;i>=0;i--){
18         while(s.size() && a[s.top()]>a[i]){
19             res[s.top()]=i;
20             s.pop();
21         }
22         s.push(i);
23     }
24     return res;
25 }

```

11.13 Parallel Binary Search

```

1 int lo[maxn], hi[maxn];
2 vector<int> tocheck[maxn];
3
4 bool c=true;
5 while(c){
6     c=false;
7     //initialize changes of structure to 0
8
9     for(int i=0;i<k;i++){
10        if(low[i]!=high[i]){
11            check[(low[i]+high[i])/2].pb(i);
12        }
13    }
14
15    for(int i=0;i<m;i++){
16        // apply change for ith query

```

```

17     while(check[i].size()){
18         c=true;
19         int x=check[i].back();
20         check[i].pop_back();
21
22         if(operationToCheck){
23             high[x]=i;
24         }
25         else{
26             low[x]=i+1;
27         }
28     }
29 }
30 }
```

11.14 Random Number Generators

```

1 //to avoid hacks
2 mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
3 mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());
4 //you can also just write seed_value if hacks are not an issue
5
6 // rng() for generating random numbers between 0 and 2<<31-1
7
8 // for generating numbers with uniform probability in range
9 uniform_int_distribution<int>(0, n)(rng)
10 std::normal_distribution<> normal_dist(mean, 2)
11 exponential_distribution
12
13
14 // for shuffling array
15 shuffle(permutation.begin(), permutation.end(), rng);
```

11.15 setprecision

```
1 cout<<fixed<<setprecision(10);
```

11.16 Ternary Search

```

1 double ternary_search(double l, double r) {
2     double eps = 1e-9;           //set the error limit here
3     while (r - l > eps) {
4         double m1 = l + (r - l) / 3;
```

```

5         double m2 = r - (r - l) / 3;
6         double f1 = f(m1);        //evaluates the function at m1
7         double f2 = f(m2);        //evaluates the function at m2
8         if (f1 < f2)
9             l = m1;
10        else
11            r = m2;
12    }
13    return f(l);               //return the maximum of f(x) in [l,
14 }                                r]
```

11.17 Ternary Search Int

```

1 int lo = -1, hi = n;
2 while (hi - lo > 1){
3     int mid = (hi + lo)>>1;
4     if (f(mid) > f(mid + 1))
5         hi = mid;
6     else
7         lo = mid;
8 }
9 //lo + 1 is the answer
```

11.18 XOR Convolution

```

1 void FWHT (int A[], int k, int inv) {
2     for (int j = 0; j < k; j++)
3         for (int i = 0; i < (1 << k); i++)
4             if (~i & (1 << j)) {
5                 int p0 = A[i];
6                 int p1 = A[i | (1 << j)];
7
8                 A[i] = p0 + p1;
9                 A[i | (1 << j)] = p0 - p1;
10
11                if (inv) {
12                    A[i] /= 2;
13                    A[i | (1 << j)] /= 2;
14                }
15            }
16 }
17
18 void XOR_conv (int A[], int B[], int C[], int k) {
```

```

19     FWHT(A, k, false);
20     FWHT(B, k, false);
21
22     for (int i = 0; i < (1 << k); i++)
23         C[i] = A[i] * B[i];
24
25     FWHT(A, k, true);
26     FWHT(B, k, true);
27     FWHT(C, k, true);
28 }
```

11.19 XOR Basis

```

1 int basis[d]; // basis[i] keeps the mask of the vector whose f value is
2   i
3
4 int sz; // Current size of the basis
5
6 void insertVector(int mask) {
7   //turn for around if u want max xor
8   for (int i = 0; i < d; i++) {
9     if ((mask & 1 << i) == 0) continue; // continue if i != f(mask)
10
11    if (!basis[i]) { // If there is no basis vector with the i'th bit
12      set, then insert this vector into the basis
13      basis[i] = mask;
14      ++sz;
15
16      return;
17    }
18
19    mask ^= basis[i]; // Otherwise subtract the basis vector from this
20      vector
21  }
22
23 // If you dont need the basis sorted.
24 vector<ll> basis;
25 void add(ll x)
26 {
27   for (int i = 0; i < basis.size(); i++)
28 }
```

```

28     x = min(x, x ^ basis[i]);
29   }
30   if (x != 0)
31   {
32     basis.pb(x);
33   }
34 }
```

12 Polynomials

12.1 Berlekamp Massey

```

1 template<typename T>
2 vector<T> berlekampMassey(const vector<T> &s) {
3   vector<T> c; // the linear recurrence sequence we are building
4   vector<T> oldC; // the best previous version of c to use (the one
5     with the rightmost left endpoint)
6   int f = -1; // the index at which the best previous version of c
     failed on
7   for (int i=0; i<(int)s.size(); i++) {
8     // evaluate c(i)
9     // delta = s_i - \sum_{j=1}^n c_j s_{i-j}
10    // if delta == 0, c(i) is correct
11    T delta = s[i];
12    for (int j=1; j<=(int)c.size(); j++)
13      delta -= c[j-1] * s[i-j]; // c_j is one-indexed, so we
        actually need index j - 1 in the code
14    if (delta == 0)
15      continue; // c(i) is correct, keep going
16    // now at this point, delta != 0, so we need to adjust it
17    if (f == -1) {
18      // this is the first time we're updating c
19      // s_i was the first non-zero element we encountered
20      // we make c of length i + 1 so that s_i is part of the base
        case
21      c.resize(i + 1);
22      mt19937 rng(chrono::steady_clock::now().time_since_epoch().
        count());
23      for (T &x : c)
24        x = rng(); // just to prove that the initial values don
          't matter in the first step, I will set to random
          values
25      f = i;
```

```

25 } else {
26     // we need to use a previous version of c to improve on this
27     // one
28     // apply the 5 steps to build d
29     // 1. set d equal to our chosen sequence
30     vector<T> d = oldC;
31     // 2. multiply the sequence by -1
32     for (T &x : d)
33         x = -x;
34     // 3. insert a 1 on the left
35     d.insert(d.begin(), 1);
36     // 4. multiply the sequence by delta / d(f + 1)
37     T df1 = 0; // d(f + 1)
38     for (int j=1; j<=(int)d.size(); j++)
39         df1 += d[j-1] * s[f+1-j];
40     assert(df1 != 0);
41     T coef = delta / df1; // storing this in outer variable so
42     // it's O(n^2) instead of O(n^2 log MOD)
43     for (T &x : d)
44         x *= coef;
45     // 5. insert i - f - 1 zeros on the left
46     vector<T> zeros(i - f - 1);
47     zeros.insert(zeros.end(), d.begin(), d.end());
48     d = zeros;
49     // now we have our new recurrence: c + d
50     vector<T> temp = c; // save the last version of c because it
51     // might have a better left endpoint
52     c.resize(max(c.size(), d.size()));
53     for (int j=0; j<(int)d.size(); j++)
54         c[j] += d[j];
55     // finally, let's consider updating oldC
56     if (i - (int)temp.size() > f - (int)oldC.size()) {
57         // better left endpoint, let's update!
58         oldC = temp;
59         f = i;
60     }
61 }
```

```

1 using cd = complex<double>;
2 const double PI = acos(-1);
3 //declare size of vectors used like this
4 const int MAXN=2<<19;
5
6 void fft(vector<cd> &a, bool invert) {
7     int n = (int)a.size();
8
9     for (int i = 1, j = 0; i < n; i++) {
10        int bit = n >> 1;
11        for (; j & bit; bit >>= 1)
12            j ^= bit;
13        j ^= bit;
14
15        if (i < j)
16            swap(a[i], a[j]);
17    }
18
19    for (int len = 2; len <= n; len <= 1) {
20        double ang = 2 * PI / len * (invert ? -1 : 1);
21        cd wlen(cos(ang), sin(ang));
22        for (int i = 0; i < n; i += len) {
23            cd w(1);
24            for (int j = 0; j < len / 2; j++) {
25                cd u = a[i+j], v = a[i+j+len/2] * w;
26                a[i+j] = u + v;
27                a[i+j+len/2] = u - v;
28                w *= wlen;
29            }
30        }
31    }
32
33    if (invert) {
34        for (cd &x : a)
35            x /= n;
36    }
37 }
38
39 vector<int> multiply(vector<int> const& a, vector<int> const& b) {
40     vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
41     int n = 1;
42     while (n < a.size() + b.size())
43         n <<= 1;
```

12.2 FFT

```

44     fa.resize(n);
45     fb.resize(n);
46
47     fft(fa, false);
48     fft(fb, false);
49     for (int i = 0; i < n; i++)
50         fa[i] *= fb[i];
51     fft(fa, true);
52
53     vector<int> result(n);
54     for (int i = 0; i < n; i++)
55         result[i] = round(fa[i].real());
56     return result;
57 }
58
59 //normalizing for when mult is between 2 big numbers and not polynomials
60 int carry = 0;
61 for (int i = 0; i < n; i++){
62     result[i] += carry;
63     carry = result[i] / 10;
64     result[i] %= 10;
65 }
```

12.3 NTT

```

1 // number theory transform
2
3 const int MOD = 998244353, ROOT = 3;
4 // const int MOD = 7340033, ROOT = 5;
5 // const int MOD = 167772161, ROOT = 3;
6 // const int MOD = 469762049, ROOT = 3;
7
8 int power(int base, int exp) {
9     int res = 1;
10    while (exp) {
11        if (exp % 2) res = 1LL * res * base % MOD;
12        base = 1LL * base * base % MOD;
13        exp /= 2;
14    }
15    return res;
16 }
17
18 void ntt(vector<int>& a, bool invert) {
```

```

19     int n = a.size();
20     for (int i = 1, j = 0; i < n; i++) {
21         int bit = n >> 1;
22         for (; j & bit; bit >>= 1) j ^= bit;
23         j ^= bit;
24         if (i < j) swap(a[i], a[j]);
25     }
26     for (int len = 2; len <= n; len <= 1) {
27         int wlen = power(ROOT, (MOD - 1) / len);
28         if (invert) wlen = power(wlen, MOD - 2);
29         for (int i = 0; i < n; i += len) {
30             int w = 1;
31             for (int j = 0; j < len / 2; j++) {
32                 int u = a[i + j], v = 1LL * a[i + j + len / 2] * w % MOD;
33                 a[i + j] = u + v < MOD ? u + v : u + v - MOD;
34                 a[i + j + len / 2] = u - v >= 0 ? u - v : u - v + MOD;
35                 w = 1LL * w * wlen % MOD;
36             }
37         }
38     }
39     if (invert) {
40         int n_inv = power(n, MOD - 2);
41         for (int& x : a) x = 1LL * x * n_inv % MOD;
42     }
43 }
44
45 vector<int> multiply(vector<int>& a, vector<int>& b) {
46     int n = 1;
47     while (n < a.size() + b.size()) n <= 1;
48     a.resize(n), b.resize(n);
49     ntt(a, false), ntt(b, false);
50     for (int i = 0; i < n; i++) a[i] = 1LL * a[i] * b[i] % MOD;
51     ntt(a, true);
52     return a;
53 }
54 // usage
55 // vector<int> a = {1, 2, 3}, b = {4, 5, 6};
56 // vector<int> c = multiply(a, b);
57 // for (int x : c) cout << x << " ";
```

12.4 Roots NTT

```

2 1*2^1 + 1 = 3, 2, 2
3 1*2^2 + 1 = 5, 2, 3
4 2*2^3 + 1 = 17, 2, 9
5 1*2^4 + 1 = 17, 3, 6
6 3*2^5 + 1 = 97, 19, 46
7 3*2^6 + 1 = 193, 11, 158
8 2*2^7 + 1 = 257, 9, 200
9 1*2^8 + 1 = 257, 3, 86
10 15*2^9 + 1 = 7681, 62, 1115
11 12*2^10 + 1 = 15361, 49, 1254
12 6*2^11 + 1 = 12289, 7, 8778
13 3*2^12 + 1 = 12289, 41, 4496
14 5*2^13 + 1 = 40961, 12, 23894
15 4*2^14 + 1 = 65537, 15, 30584
16 2*2^15 + 1 = 65537, 9, 7282
17 1*2^16 + 1 = 65537, 3, 21846
18 6*2^17 + 1 = 786433, 8, 688129
19 3*2^18 + 1 = 786433, 5, 471860
20 11*2^19 + 1 = 5767169, 12, 3364182
21 7*2^20 + 1 = 7340033, 5, 4404020
22 11*2^21 + 1 = 23068673, 38, 21247462
23 25*2^22 + 1 = 104857601, 21, 49932191
24 20*2^23 + 1 = 167772161, 4, 125829121
25 10*2^24 + 1 = 167772161, 2, 83886081
26 5*2^25 + 1 = 167772161, 17, 29606852
27 7*2^26 + 1 = 469762049, 30, 15658735
28 15*2^27 + 1 = 2013265921, 137, 749463956
29 12*2^28 + 1 = 3221225473, 8, 2818572289
30 6*2^29 + 1 = 3221225473, 14, 1150437669
31 3*2^30 + 1 = 3221225473, 13, 1734506024
32 35*2^31 + 1 = 75161927681, 93, 44450602392
33 18*2^32 + 1 = 77309411329, 106, 5105338484

```

13 Scripts

13.1 build.sh

This file should be called before stress.sh or validate.sh. build.sh name.cpp

```

1 g++ -static -DLOCAL -lm -s -x c++ -Wall -Wextra -O2 -std=c++17 -o $1 $1.
     CPP

```

13.2 stress.sh

Format is stress.sh Awrong Agen Numtests

```

1 #!/usr/bin/env bash
2
3 for ((testNum=0;testNum<$4;testNum++))
4 do
5     ./$3 > input
6     ./$2 < input > outSlow
7     ./$1 < input > outWrong
8     H1='md5sum outWrong'
9     H2='md5sum outSlow'
10    if !(cmp -s "outWrong" "outSlow")
11    then
12        echo "Error found!"
13        echo "Input:"
14        cat input
15        echo "Wrong Output:"
16        cat outWrong
17        echo "Slow Output:"
18        cat outSlow
19        exit
20    fi
21 done
22 echo Passed $4 tests

```

13.3 validate.sh

Format is validate.sh Awrong Validator Agen NumTests

```

1 #!/usr/bin/env bash
2
3 for ((testNum=0;testNum<$4;testNum++))
4 do
5     ./$3 > input
6     ./$1 < input > out
7     cat input out > data
8     ./$2 < data > res
9     result=$(cat res)
10    if [ "${result:0:2}" != "OK" ];
11    then
12        echo "Error found!"
13        echo "Input:"

```

```

14      cat input
15      echo "Output:"
16      cat out
17      echo "Validator_Result:"
18      cat res
19      exit
20  fi
21 done
22 echo Passed $4 tests

```

14 Strings

14.1 Hashed String

```

1  /*
2   *                                     Hashed string
3   -----
4   Class for hashing string. Allows retrieval of hashes of any substring
5   in the string.
6
7   Double hash or use big mod values to avoid problems with collisions
8
9   Time Complexity(Construction): O(n)
10  Space Complexity: O(n)
11 */
12
13 const ll MOD = 212345678987654321LL;
14 const ll base = 33;
15
16 class HashedString {
17     private:
18         // change M and B if you want
19         static const long long M = 1e9 + 9;
20         static const long long B = 9973;
21
22         // pow[i] contains B^i % M
23         static vector<long long> pow;
24
25         // p_hash[i] is the hash of the first i characters of the given string
26         vector<long long> p_hash;
27

```

```

28     public:
29     HashedString(const string &s) : p_hash(s.size() + 1) {
30         while (pow.size() < s.size()) { pow.push_back((pow.back() * B) % M);
31             }
32
33         p_hash[0] = 0;
34         for (int i = 0; i < s.size(); i++) {
35             p_hash[i + 1] = ((p_hash[i] * B) % M + s[i]) % M;
36         }
37
38         // Returns hash of substring [start, end]
39         long long get_hash(int start, int end) {
40             long long raw_val =
41                 (p_hash[end + 1] - (p_hash[start] * pow[end - start + 1]));
42             return (raw_val % M + M) % M;
43         }
44     };
45     // you cant skip this
46     vector<long long> HashedString::pow = {1};

```

14.2 KMP

```

1  /*
2   *                                     KMP
3   -----
4   Computes the prefix function for a string.
5
6   Maximum length of substring that ends at position i and is proper
7   prefix (not equal to string itself) of string
8   pf[i]  is the length of the longest proper prefix of the substrings
9   [0.....i]$ which is also a suffix of this substring.
10
11  For matching, one can append the string with a delimiter like $
12      between them
13
14  Time Complexity: O(n)
15  Space Complexity: O(n)
16 */
17
18  vector<int> KMP(string s){
19      int n=(int)s.length();
20      vector<int> pf(n, 0);
21
22      for (int i = 1; i < n; i++) {
23          int j = pf[i - 1];
24          while (j > 0 && s[i] != s[j])
25              j = pf[j - 1];
26          if (s[i] == s[j])
27              pf[i] = j + 1;
28      }
29
30      return pf;
31  }

```

```

18 for(int i=1;i<n;i++){
19     int j=pf[i-1];
20     while(j>0 && s[i]!=s[j]){
21         j=pf[j-1];
22     }
23     if(s[i]==s[j]){
24         pf[i]=j+1;
25     }
26 }
27 return pf;
28 }

29 // Counts how many times each prefix occurs
30 // Same thing can be done for two strings but only considering indices
31 // of second string
32 vector<int> count_occurrences_of_prefixes(vector<int> pf){
33     int n=(int)pf.size();
34     vector<int> ans(n + 1);
35     for (int i = 0; i < n; i++)
36         ans[pi[i]]++;
37     for (int i = n-1; i > 0; i--)
38         ans[pi[i-1]] += ans[i];
39     for (int i = 0; i <= n; i++)
40         ans[i]++;
41 }

42 // Computes automaton for string
43 // useful for not having to recalculate KMP of string s
44 // can be utilized when the second string (the one in which we are
45 // trying to count occurrences)
46 // is very large
47 void compute_automaton(string s, vector<vector<int>>& aut) {
48     s += '#';
49     int n = s.size();
50     vector<int> pi = KMP(s);
51     aut.assign(n, vector<int>(26));
52     for (int i = 0; i < n; i++) {
53         for (int c = 0; c < 26; c++) {
54             if (i > 0 && 'a' + c != s[i])
55                 aut[i][c] = aut[pi[i-1]][c];
56             else
57                 aut[i][c] = i + ('a' + c == s[i]);
58     }

```

```

59     }
60 }

```

14.3 Least Rotation String

```

1 /*
2          Min cyclic shift
3 -----
4 Finds the lexicographically minimum cyclic shift of a string
5
6 Time Complexity: O(n)
7 Space Complexity: O(n)
8 */

9
10 string least_rotation(string s)
11 {
12     s += s;
13     vector<int> f(s.size(), -1);
14     int k = 0;
15     for(int j = 1; j < s.size(); j++)
16     {
17         char sj = s[j];
18         int i = f[j - k - 1];
19         while(i != -1 && sj != s[k + i + 1])
20         {
21             if(sj < s[k + i + 1]){
22                 k = j - i - 1;
23             }
24             i = f[i];
25         }
26         if(sj != s[k + i + 1])
27         {
28             if(sj < s[k]){
29                 k = j;
30             }
31             f[j - k] = -1;
32         }
33         else
34             f[j - k] = i + 1;
35     }
36     return s.substr(k, s.size() / 2);
37 }

```

14.4 Manacher

```
1      /*
2       * Manacher
3       -----
4       Computes the length of the longest palindrome centered at position i.
5
6       p[i] is length of biggest palindrome centered in this position.
7       Be careful with characters that are inserted to account for odd and
8       even palindromes
9
10      Time Complexity: O(n)
11      Space Complexity: O(n)
12
13      */
14
15      // Number of palindromes centered at each position
16
17      vector<int> manacher_odd(string s)
18      {
19          int n = s.size();
20          s = "$" + s + "^";
21          vector<int> p(n + 2);
22          int l = 1, r = 1;
23          for (int i = 1; i <= n; i++)
24          {
25              p[i] = max(0, min(r - i, p[l + (r - i)]));
26              while (s[i - p[i]] == s[i + p[i]])
27              {
28                  p[i]++;
29              }
30              if (i + p[i] > r)
31              {
32                  l = i - p[i], r = i + p[i];
33              }
34          }
35          return vector<int>(begin(p) + 1, end(p) - 1);
36      }
37      vector<int> manacher(string s)
38      {
39          string t;
40          for (auto c : s)
41          {
```

```
41     t += string("#") + c;
42 }
43 auto res = manacher_odd(t + "#");
44 return vector<int>(begin(res) + 1, end(res) - 1);
45 }
46
47 // usage
48 // vector<int> p = manacher("abacaba");
49 // this will return {2, 1, 4, 1, 2, 1, 8, 1, 2, 1, 4, 1, 2}
50 // vector<int> p = manacher("abaaba");
51 // this will return {2, 1, 4, 1, 2, 7, 2, 1, 4, 1, 2}
```

14.5 Suffix Array

```

1  /*
2   * Suffix Array
3   -----
4   Computes the suffix array of a string in O(n log n).
5   Sorted array of all cyclic shifts of a string.
6
7   If you want sorted suffixes append $ to the end of the string.
8   lc is longest common prefix. Lcp of two substrings j > i is min(lc[i],
9   ..... , lc[j - 1]).
10
11 To compute Largest common substring of multiple strings
12 Join all strings separating them with special character like $ (it has
13 to be different for each string)
14 Sliding window on lcp array (all string have to appear on the sliding
15 window and
16 the lcp of the interval will give the length of the substring that
17 appears on all strings)
18
19 Time Complexity: O(n log n)
20 Space Complexity: O(n)
21 */
22
23 struct SuffixArray
24 {
25     int n;
26     string t;
27     vector<int> sa, rk, lc;
28     SuffixArray(const std::string &s)
29     {
30         n = s.length();
31         t = s;
32         sa.resize(n);
33         rk.resize(n);
34         lc.resize(n);
35         build();
36     }
37
38     void build()
39     {
40         for (int i = 0; i < n; ++i)
41             rk[i] = i;
42
43         sort(rk.begin(), rk.end());
44
45         for (int i = 0; i < n; ++i)
46             sa[i] = rk[i];
47
48         for (int i = 0; i < n - 1; ++i)
49         {
50             int j = sa[i];
51             int k = sa[i + 1];
52
53             if (t[j] == t[k])
54                 lc[i] = lc[i - 1];
55             else
56                 lc[i] = j - k;
57         }
58
59         lc[n - 1] = lc[n - 2];
60     }
61
62     int lcp(int i, int j)
63     {
64         if (i == j)
65             return n;
66
67         int l = max(i, j);
68         int r = min(i, j);
69
70         while (l < r)
71         {
72             if (t[r] != t[l])
73                 break;
74             r--;
75             l++;
76         }
77
78         return r - l + 1;
79     }
80
81     int lcp(int i, int j, int k)
82     {
83         if (i == j)
84             return n;
85
86         int l = max(i, j, k);
87         int r = min(i, j, k);
88
89         while (l < r)
90         {
91             if (t[r] != t[l])
92                 break;
93             r--;
94             l++;
95         }
96
97         return r - l + 1;
98     }
99
100    int lcp(int i, int j, int k, int l)
101    {
102        if (i == j)
103            return n;
104
105        int l1 = max(i, j, k, l);
106        int r1 = min(i, j, k, l);
107
108        while (l1 < r1)
109        {
110            if (t[r1] != t[l1])
111                break;
112            r1--;
113            l1++;
114        }
115
116        return r1 - l1 + 1;
117    }
118
119    int lcp(int i, int j, int k, int l, int m)
120    {
121        if (i == j)
122            return n;
123
124        int l1 = max(i, j, k, l, m);
125        int r1 = min(i, j, k, l, m);
126
127        while (l1 < r1)
128        {
129            if (t[r1] != t[l1])
130                break;
131            r1--;
132            l1++;
133        }
134
135        return r1 - l1 + 1;
136    }
137
138    int lcp(int i, int j, int k, int l, int m, int n)
139    {
140        if (i == j)
141            return n;
142
143        int l1 = max(i, j, k, l, m, n);
144        int r1 = min(i, j, k, l, m, n);
145
146        while (l1 < r1)
147        {
148            if (t[r1] != t[l1])
149                break;
150            r1--;
151            l1++;
152        }
153
154        return r1 - l1 + 1;
155    }
156
157    int lcp(int i, int j, int k, int l, int m, int n, int o)
158    {
159        if (i == j)
160            return n;
161
162        int l1 = max(i, j, k, l, m, n, o);
163        int r1 = min(i, j, k, l, m, n, o);
164
165        while (l1 < r1)
166        {
167            if (t[r1] != t[l1])
168                break;
169            r1--;
170            l1++;
171        }
172
173        return r1 - l1 + 1;
174    }
175
176    int lcp(int i, int j, int k, int l, int m, int n, int o, int p)
177    {
178        if (i == j)
179            return n;
180
181        int l1 = max(i, j, k, l, m, n, o, p);
182        int r1 = min(i, j, k, l, m, n, o, p);
183
184        while (l1 < r1)
185        {
186            if (t[r1] != t[l1])
187                break;
188            r1--;
189            l1++;
190        }
191
192        return r1 - l1 + 1;
193    }
194
195    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q)
196    {
197        if (i == j)
198            return n;
199
200        int l1 = max(i, j, k, l, m, n, o, p, q);
201        int r1 = min(i, j, k, l, m, n, o, p, q);
202
203        while (l1 < r1)
204        {
205            if (t[r1] != t[l1])
206                break;
207            r1--;
208            l1++;
209        }
210
211        return r1 - l1 + 1;
212    }
213
214    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r)
215    {
216        if (i == j)
217            return n;
218
219        int l1 = max(i, j, k, l, m, n, o, p, q, r);
220        int r1 = min(i, j, k, l, m, n, o, p, q, r);
221
222        while (l1 < r1)
223        {
224            if (t[r1] != t[l1])
225                break;
226            r1--;
227            l1++;
228        }
229
230        return r1 - l1 + 1;
231    }
232
233    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s)
234    {
235        if (i == j)
236            return n;
237
238        int l1 = max(i, j, k, l, m, n, o, p, q, r, s);
239        int r1 = min(i, j, k, l, m, n, o, p, q, r, s);
240
241        while (l1 < r1)
242        {
243            if (t[r1] != t[l1])
244                break;
245            r1--;
246            l1++;
247        }
248
249        return r1 - l1 + 1;
250    }
251
252    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t)
253    {
254        if (i == j)
255            return n;
256
257        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t);
258        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t);
259
260        while (l1 < r1)
261        {
262            if (t[r1] != t[l1])
263                break;
264            r1--;
265            l1++;
266        }
267
268        return r1 - l1 + 1;
269    }
270
271    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u)
272    {
273        if (i == j)
274            return n;
275
276        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u);
277        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u);
278
279        while (l1 < r1)
280        {
281            if (t[r1] != t[l1])
282                break;
283            r1--;
284            l1++;
285        }
286
287        return r1 - l1 + 1;
288    }
289
290    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v)
291    {
292        if (i == j)
293            return n;
294
295        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v);
296        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v);
297
298        while (l1 < r1)
299        {
300            if (t[r1] != t[l1])
301                break;
302            r1--;
303            l1++;
304        }
305
306        return r1 - l1 + 1;
307    }
308
309    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w)
310    {
311        if (i == j)
312            return n;
313
314        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w);
315        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w);
316
317        while (l1 < r1)
318        {
319            if (t[r1] != t[l1])
320                break;
321            r1--;
322            l1++;
323        }
324
325        return r1 - l1 + 1;
326    }
327
328    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x)
329    {
330        if (i == j)
331            return n;
332
333        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x);
334        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x);
335
336        while (l1 < r1)
337        {
338            if (t[r1] != t[l1])
339                break;
340            r1--;
341            l1++;
342        }
343
344        return r1 - l1 + 1;
345    }
346
347    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y)
348    {
349        if (i == j)
350            return n;
351
352        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y);
353        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y);
354
355        while (l1 < r1)
356        {
357            if (t[r1] != t[l1])
358                break;
359            r1--;
360            l1++;
361        }
362
363        return r1 - l1 + 1;
364    }
365
366    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z)
367    {
368        if (i == j)
369            return n;
370
371        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z);
372        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z);
373
374        while (l1 < r1)
375        {
376            if (t[r1] != t[l1])
377                break;
378            r1--;
379            l1++;
380        }
381
382        return r1 - l1 + 1;
383    }
384
385    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a)
386    {
387        if (i == j)
388            return n;
389
390        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a);
391        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a);
392
393        while (l1 < r1)
394        {
395            if (t[r1] != t[l1])
396                break;
397            r1--;
398            l1++;
399        }
400
401        return r1 - l1 + 1;
402    }
403
404    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b)
405    {
406        if (i == j)
407            return n;
408
409        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b);
410        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b);
411
412        while (l1 < r1)
413        {
414            if (t[r1] != t[l1])
415                break;
416            r1--;
417            l1++;
418        }
419
420        return r1 - l1 + 1;
421    }
422
423    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c)
424    {
425        if (i == j)
426            return n;
427
428        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c);
429        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c);
430
431        while (l1 < r1)
432        {
433            if (t[r1] != t[l1])
434                break;
435            r1--;
436            l1++;
437        }
438
439        return r1 - l1 + 1;
440    }
441
442    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d)
443    {
444        if (i == j)
445            return n;
446
447        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d);
448        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d);
449
450        while (l1 < r1)
451        {
452            if (t[r1] != t[l1])
453                break;
454            r1--;
455            l1++;
456        }
457
458        return r1 - l1 + 1;
459    }
460
461    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e)
462    {
463        if (i == j)
464            return n;
465
466        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e);
467        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e);
468
469        while (l1 < r1)
470        {
471            if (t[r1] != t[l1])
472                break;
473            r1--;
474            l1++;
475        }
476
477        return r1 - l1 + 1;
478    }
479
480    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f)
481    {
482        if (i == j)
483            return n;
484
485        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f);
486        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f);
487
488        while (l1 < r1)
489        {
490            if (t[r1] != t[l1])
491                break;
492            r1--;
493            l1++;
494        }
495
496        return r1 - l1 + 1;
497    }
498
499    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g)
500    {
501        if (i == j)
502            return n;
503
504        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g);
505        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g);
506
507        while (l1 < r1)
508        {
509            if (t[r1] != t[l1])
510                break;
511            r1--;
512            l1++;
513        }
514
515        return r1 - l1 + 1;
516    }
517
518    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h)
519    {
520        if (i == j)
521            return n;
522
523        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h);
524        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h);
525
526        while (l1 < r1)
527        {
528            if (t[r1] != t[l1])
529                break;
530            r1--;
531            l1++;
532        }
533
534        return r1 - l1 + 1;
535    }
536
537    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i)
538    {
539        if (i == j)
540            return n;
541
542        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i);
543        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i);
544
545        while (l1 < r1)
546        {
547            if (t[r1] != t[l1])
548                break;
549            r1--;
550            l1++;
551        }
552
553        return r1 - l1 + 1;
554    }
555
556    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j)
557    {
558        if (i == j)
559            return n;
560
561        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j);
562        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j);
563
564        while (l1 < r1)
565        {
566            if (t[r1] != t[l1])
567                break;
568            r1--;
569            l1++;
570        }
571
572        return r1 - l1 + 1;
573    }
574
575    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k)
576    {
577        if (i == j)
578            return n;
579
580        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k);
581        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k);
582
583        while (l1 < r1)
584        {
585            if (t[r1] != t[l1])
586                break;
587            r1--;
588            l1++;
589        }
590
591        return r1 - l1 + 1;
592    }
593
594    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l)
595    {
596        if (i == j)
597            return n;
598
599        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l);
600        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l);
601
602        while (l1 < r1)
603        {
604            if (t[r1] != t[l1])
605                break;
606            r1--;
607            l1++;
608        }
609
610        return r1 - l1 + 1;
611    }
612
613    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m)
614    {
615        if (i == j)
616            return n;
617
618        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m);
619        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m);
620
621        while (l1 < r1)
622        {
623            if (t[r1] != t[l1])
624                break;
625            r1--;
626            l1++;
627        }
628
629        return r1 - l1 + 1;
630    }
631
632    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m, int n)
633    {
634        if (i == j)
635            return n;
636
637        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n);
638        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n);
639
640        while (l1 < r1)
641        {
642            if (t[r1] != t[l1])
643                break;
644            r1--;
645            l1++;
646        }
647
648        return r1 - l1 + 1;
649    }
650
651    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m, int n, int o)
652    {
653        if (i == j)
654            return n;
655
656        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o);
657        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o);
658
659        while (l1 < r1)
660        {
661            if (t[r1] != t[l1])
662                break;
663            r1--;
664            l1++;
665        }
666
667        return r1 - l1 + 1;
668    }
669
670    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m, int n, int o, int p)
671    {
672        if (i == j)
673            return n;
674
675        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p);
676        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p);
677
678        while (l1 < r1)
679        {
680            if (t[r1] != t[l1])
681                break;
682            r1--;
683            l1++;
684        }
685
686        return r1 - l1 + 1;
687    }
688
689    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m, int n, int o, int p, int q)
690    {
691        if (i == j)
692            return n;
693
694        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q);
695        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q);
696
697        while (l1 < r1)
698        {
699            if (t[r1] != t[l1])
700                break;
701            r1--;
702            l1++;
703        }
704
705        return r1 - l1 + 1;
706    }
707
708    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m, int n, int o, int p, int q, int r)
709    {
710        if (i == j)
711            return n;
712
713        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r);
714        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r);
715
716        while (l1 < r1)
717        {
718            if (t[r1] != t[l1])
719                break;
720            r1--;
721            l1++;
722        }
723
724        return r1 - l1 + 1;
725    }
726
727    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s)
728    {
729        if (i == j)
730            return n;
731
732        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s);
733        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s);
734
735        while (l1 < r1)
736        {
737            if (t[r1] != t[l1])
738                break;
739            r1--;
740            l1++;
741        }
742
743        return r1 - l1 + 1;
744    }
745
746    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t)
747    {
748        if (i == j)
749            return n;
750
751        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t);
752        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t);
753
754        while (l1 < r1)
755        {
756            if (t[r1] != t[l1])
757                break;
758            r1--;
759            l1++;
760        }
761
762        return r1 - l1 + 1;
763    }
764
765    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u)
766    {
767        if (i == j)
768            return n;
769
770        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u);
771        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u);
772
773        while (l1 < r1)
774        {
775            if (t[r1] != t[l1])
776                break;
777            r1--;
778            l1++;
779        }
780
781        return r1 - l1 + 1;
782    }
783
784    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v)
785    {
786        if (i == j)
787            return n;
788
789        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v);
790        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v);
791
792        while (l1 < r1)
793        {
794            if (t[r1] != t[l1])
795                break;
796            r1--;
797            l1++;
798        }
799
800        return r1 - l1 + 1;
801    }
802
803    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w)
804    {
805        if (i == j)
806            return n;
807
808        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w);
809        int r1 = min(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w);
810
811        while (l1 < r1)
812        {
813            if (t[r1] != t[l1])
814                break;
815            r1--;
816            l1++;
817        }
818
819        return r1 - l1 + 1;
820    }
821
822    int lcp(int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x, int y, int z, int a, int b, int c, int d, int e, int f, int g, int h, int i, int j, int k, int l, int m, int n, int o, int p, int q, int r, int s, int t, int u, int v, int w, int x)
823    {
824        if (i == j)
825            return n;
826
827        int l1 = max(i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n,
```

```

26 {
27     n = s.length();
28     t = s;
29     sa.resize(n);
30     lc.resize(n - 1);
31     rk.resize(n);
32     std::iota(sa.begin(), sa.end(), 0);
33     std::sort(sa.begin(), sa.end(), [&](int a, int b)
34             { return s[a] < s[b]; });
35     rk[sa[0]] = 0;
36     for (int i = 1; i < n; ++i)
37         rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
38     int k = 1;
39     std::vector<int> tmp, cnt(n);
40     tmp.reserve(n);
41     while (rk[sa[n - 1]] < n - 1)
42     {
43         tmp.clear();
44         for (int i = 0; i < k; ++i)
45             tmp.push_back(n - k + i);
46         for (auto i : sa)
47             if (i >= k)
48                 tmp.push_back(i - k);
49         std::fill(cnt.begin(), cnt.end(), 0);
50         for (int i = 0; i < n; ++i)
51             ++cnt[rk[i]];
52         for (int i = 1; i < n; ++i)
53             cnt[i] += cnt[i - 1];
54         for (int i = n - 1; i >= 0; --i)
55             sa[--cnt[rk[tmp[i]]]] = tmp[i];
56         std::swap(rk, tmp);
57         rk[sa[0]] = 0;
58         for (int i = 1; i < n; ++i)
59             rk[sa[i]] = rk[sa[i - 1]] + (tmp[sa[i - 1]] < tmp[sa[i]] || sa[i - 1] + k == n || tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
60         k *= 2;
61     }
62     for (int i = 0, j = 0; i < n; ++i)
63     {
64         if (rk[i] == 0)
65         {
66             j = 0;
67         }
68     }
69     else
70     {
71         for (j -= j > 0; i + j < n && sa[rk[i] - 1] + j < n && s[i + j]
72             == s[sa[rk[i] - 1] + j];)
73             ++j;
74         lc[rk[i] - 1] = j;
75     }
76 }
77 // Finds if string p appears as substring in the string
78 // might now work perfectly
79 int search(string &p){
80     int tam = p.size();
81     int l = 0, r = n;
82
83     string tmp = "";
84     while(r > 1) {
85         int m = l + (r-1)/2;
86         tmp = t.substr(sa[m], min(n-sa[m], tam));
87         if(tmp >= p){
88             r = m;
89         } else {
90             l = m + 1;
91         }
92     }
93     if(l < n) {
94         tmp = t.substr(sa[l], min(n-sa[l], tam));
95     } else{
96         return -1;
97     }
98     if(tmp == p){
99         return l;
100    } else {
101        return -1;
102    }
103 }
104
105 // Counts number of times a string p appears as substring in string
106 int count(string &p) {
107     int x = search(p);
108     if(x == -1) return 0;
109 }
```

```

110     int cnt = 0;
111     int tam = p.size();
112     int maxx = 0;
113     while((1 << maxx) + x < n) maxx++;
114     int y = x;
115     for(int i = maxx-1; i >= 0; i--) {
116         if(x + (1 << i) >= n) continue;
117         string tmp = t.substr(sa[x + (1 << i)], min(n-sa[x + (1 << i)]
118                         ], tam));
119         if(tmp == p) x += (1 << i);
120     }
121     return x-y+1;
122 };
123 int main() {
124     cin.tie(0)->sync_with_stdio(0);
125     string s; cin >> s;
126     SuffixArray SA(s);
127
128     int q; cin >> q;
129     for(int t = 0; t < q; t++) {
130         string tmp; cin >> tmp;
131         cout << SA.count(tmp) << endl;
132     }
133
134     return 0;
135 }
```

14.6 Suffix Automaton

```

1 /*
2          Suffix Automaton
3 -----
4 Constructs suffix automaton for a given string.
5 Be careful with overlapping substrings.
6
7 Firstposition if first position string ends in.
8 If you want starting index you need to subtract length of the string
9 being searched.
10 len is length of longest string of state
11 Time Complexity(Construction): O(n)
```

```

13     Space Complexity: O(n)
14 */
15
16 struct state {
17     int len, link, firstposition;
18     vector<int> inv_link; // can skip for almost everything
19     map<char, int> next;
20 };
21
22 const int MAXN = 100000;
23 state st[MAXN * 2];
24 ll cnt[MAXN*2], cntPaths[MAXN*2], cntSum[MAXN*2], cnt1[2 * MAXN];
25 int sz, last;
26
27 // call this first
28 void initSuffixAutomaton() {
29     st[0].len = 0;
30     st[0].link = -1;
31     sz++;
32     last = 0;
33 }
34
35 // construction is O(n)
36 void insertChar(char c) {
37     int cur = sz++;
38     st[cur].len = st[last].len + 1;
39     st[cur].firstposition=st[last].len;
40     int p = last;
41     while (p != -1 && !st[p].next.count(c)) {
42         st[p].next[c] = cur;
43         p = st[p].link;
44     }
45     if (p == -1) {
46         st[cur].link = 0;
47     } else {
48         int q = st[p].next[c];
49         if (st[p].len + 1 == st[q].len) {
50             st[cur].link = q;
51         } else {
52             int clone = sz++;
53             st[clone].len = st[p].len + 1;
54             st[clone].next = st[q].next;
55         }
56     }
57 }
```

```

56     st[clone].link = st[q].link;
57     st[clone].firstposition=st[q].firstposition;
58     while (p != -1 && st[p].next[c] == q) {
59         st[p].next[c] = clone;
60         p = st[p].link;
61     }
62     st[q].link = st[cur].link = clone;
63 }
64 }
65 last = cur;
66 cnt[last]=1;
67 }

// searches for the starting position in O(len(s)). Returns starting
// index of first occurrence or -1 if it does not appear.
69 int search(string s){
70     int cur=0, i=0, n=(int)s.length();
71     while(i<n){
72         if(!st[cur].next.count(s[i])) return -1;
73         cur=st[cur].next[s[i]];
74         i++;
75     }
76     //sumar 2 si se quiere 1 indexado
77     return st[cur].firstposition-n+1;
78 }

void dfs(int cur){
79     cntPaths[cur]=1;
80     for(auto [x, y]:st[cur].next){
81         if(cntPaths[y]==0) dfs(y);
82         cntPaths[cur]+=cntPaths[y];
83     }
84 }
85

// Counts how many paths exist from state. How many substrings exist
// from a specific state.
86 // Stored in cntPaths
87 void countPaths(){
88     dfs(0);
89 }
90

// Computes the number of times each state appears
91 void count0currences(){
92 }

93

94

95 // Computes the number of times each state appears
96 void count0currences(){
97     vector<pair<int, int>> a;
98     for(int i=sz-1;i>0;i--){
99         a.push_back({st[i].len, i});
100    }
101    sort(a.begin(), a.end());
102    for(int i=sz-2;i>=0;i--){
103        cnt[st[a[i].second].link]+=cnt[a[i].second];
104    }
105 }

106 void dfs1(int cur){
107     for(auto [x, y]:st[cur].next){
108         if(cntSum[y]==cnt[y]) dfs1(y);
109         cntSum[cur]+=cntSum[y];
110     }
111 }

112

113

114 // Computes the number of times each state or any of its children appear
// in the string.
115 void countSum0currences(){
116     for(int i=0;i<sz;i++){
117         cntSum[i]=cnt[i];
118     }
119     dfs1(0);
120 }

121

122

123 // Counts number of paths that can reach specific state.
124 void countPathsReverse(){
125     cnt1[0]=1;
126     queue<int> q;
127     q.push(0);
128     vector<int> in(2*MAXN, 0);
129     for(int i=0;i<sz;i++){
130         for(auto [x, y]:st[i].next){
131             in[y]++;
132         }
133     }
134     while((int)q.size()){
135         int cur=q.front();
136         q.pop();
137         for(auto [x, y]:st[cur].next){
138             cnt1[y]+=cnt1[cur];
139         }
140     }
141 }

```

```

139     in[y]--;
140     if(in[y]==0){
141         q.push(y);
142     }
143 }
144 }
145 }

// Computes the kth smallest string that appears on the string (counting
// repetitions)
147 string kthSmallest(ll k){
148     string s="";
149     int cur=0;
150     while(k>0){
151         for(auto [c, y]:st[cur].next){
152             if(k>cntSum[y]) k-=cntSum[y];
153             else{
154                 k-=cnt[y];
155                 s+=c;
156                 cur=y;
157                 break;
158             }
159         }
160     }
161 }
162 return s;
163 }

// Computes the kth smallest string that appears on the string (without
// counting repetitions)
166 string kthSmallestDistinct(ll k){
167     string s="";
168     int cur=0;
169     while(k>0){
170         for(auto [c, y]:st[cur].next){
171             if(k>cntPaths[y]) k-=cntPaths[y];
172             else{
173                 k--;
174                 s+=c;
175                 cur=y;
176                 break;
177             }
178         }
179     }
}

180     }
181     return s;
182 }

183

184

185 // Precomputation to find all occurrences of a substring
186 void precompute_for_all_occurrences(){
187     for (int v = 1; v < sz; v++) {
188         st[st[v].link].inv_link.push_back(v);
189     }
190 }

191

192 // Finding all occurrences of substring in string
193 // P_length is length of substring
194 // v is state where first occurrence happens
195 // be careful as indices can appear multiple times due to clone states
196 // if you want to avoid duplicate positions utilize set or have a flag
197 // for each state to know if it is clone or not
198 void output_all_occurrences(int v, int P_length) {
199     cout << st[v].firstposition - P_length + 1 << endl;
200     for (int u : st[v].inv_link)
201         output_all_occurrences(u, P_length);
202 }

203

204 //longest common substring
205 //build automaton for s first
206 string lcs (string S, string T) {
207     int v = 0, l = 0, best = 0, bestpos = 0;
208     for (int i = 0; i < T.size(); i++) {
209         while (v && !st[v].next.count(T[i])) {
210             v = st[v].link ;
211             l = st[v].len;
212         }
213         if (st[v].next.count(T[i])) {
214             v = st[v].next[T[i]];
215             l++;
216         }
217         if (l > best) {
218             best = l;
219             bestpos = i;
220         }
221     }

```

```

222     return T.substr(bestpos - best + 1, best);
223 }
224
225
226 int main(){
227     ios_base::sync_with_stdio(false); cin.tie(NULL);
228     string s; cin >> s;
229     initSuffixAutomaton();
230     for(char c:s){
231         insertChar(c);
232     }
233 }
```

14.7 Trie Ahocorasick

```

1 /*
2      Trie - AhoCorasick
3 -----
4 Builds a trie for subset of strings and computes suffix links.
5 KATCL implementation is cleaner.
6
7 Time Complexity(Construction): O(m) where m is sum
8   of lengths of strings
9 Space Complexity: O(m)
10 */
11
12
13
14 const int K = 26;
15
16 struct Vertex {
17     int next[K];
18     bool output = false;
19     int p = -1;
20     char pch;
21     int link = -1;
22     int go[K];
23
24     Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
25         fill(begin(next), end(next), -1);
26         fill(begin(go), end(go), -1);
27     }
28 };
```

```

29
30     vector<Vertex> t(1);
31
32 void add_string(string const& s) {
33     int v = 0;
34     for (char ch : s) {
35         int c = ch - 'a';
36         if (t[v].next[c] == -1) {
37             t[v].next[c] = t.size();
38             t.emplace_back(v, ch);
39         }
40         v = t[v].next[c];
41     }
42     t[v].output = true;
43 }
44
45 int go(int v, char ch);
46
47 int get_link(int v) {
48     if (t[v].link == -1) {
49         if (v == 0 || t[v].p == 0)
50             t[v].link = 0;
51         else
52             t[v].link = go(get_link(t[v].p), t[v].pch);
53     }
54     return t[v].link;
55 }
56
57 int go(int v, char ch) {
58     int c = ch - 'a';
59     if (t[v].go[c] == -1) {
60         if (t[v].next[c] != -1)
61             t[v].go[c] = t[v].next[c];
62         else
63             t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
64     }
65     return t[v].go[c];
66 }
```

14.8 Z Function

```

1 /*
2      Z_function
```

```

3 -----
4 Computes the z_function for any string.
5 ith element is equal to the greatest number of characters starting
6 from the position i that coincide with the first characters of s
7
8 z[i] length of the longest string that is, at the same time, a prefix
9 of s and a prefix of $s$ starting at i.
10
11 to compress string, one can run z_function and then find the smallest
12 i that divides n such that i + z[i] = n
13
14 Time Complexity: O(n)
15 Space Complexity: O(n)
16 */
17
18 vector<int> z_function(string s) {
19     int n = s.size();
20     vector<int> z(n);
21     int l = 0, r = 0;
22     for(int i = 1; i < n; i++) {
23         if(i < r) {
24             z[i] = min(r - i, z[i - 1]);
25         }
26         while(i + z[i] < n && s[z[i]] == s[i + z[i]]) {
27             z[i]++;
28         }
29         if(i + z[i] > r) {
30             l = i;
31             r = i + z[i];
32         }
33     }
34     return z;
35 }
36 // usage
37 // vector<int> z = z_function("abacaba");
38 // this will return {0, 0, 1, 0, 3, 0, 1}
39 // vector<int> z = z_function("aaaaaa");
40 // this will return {0, 4, 3, 2, 1}
41 // vector<int> z = z_function("aaabaab");
42 // this will return {0, 2, 1, 0, 2, 1, 0}

```

15 Trees

15.1 Centroid Decomposition

```

1 /*
2                                     Centroid Decomposition
3 -----
4 Finds the centroid decomposition of a given tree.
5 Any vertex can have at most log n centroid ancestors
6
7 The code below is the solution to Xenia and tree.
8 Given tree, queries of two types:
9 1) u - color vertex u
10 2) v - print minimum distance of vertex v to any colored vertex before
11
12
13 Time Complexity: O(n log n)
14 Space Complexity: O(n log n)
15 */
16 const int MAXN=200005;
17
18 vector<int> adj[MAXN];
19 vector<bool> is_removed(MAXN, false);
20 vector<int> subtree_size(MAXN, 0);
21 vector<int> dis(MAXN, 1e9);
22 vector<vector<pair<int, int>>> ancestor(MAXN);
23
24 int get_subtree_size(int node, int parent = -1) {
25     subtree_size[node] = 1;
26     for (int child : adj[node]) {
27         if (child == parent || is_removed[child]) { continue; }
28         subtree_size[node] += get_subtree_size(child, node);
29     }
30     return subtree_size[node];
31 }
32
33 int get_centroid(int node, int tree_size, int parent = -1) {
34     for (int child : adj[node]) {
35         if (child == parent || is_removed[child]) { continue; }
36         if (subtree_size[child] * 2 > tree_size) {
37             return get_centroid(child, tree_size, node);
38         }
39     }

```

```

40     return node;
41 }
42
43 void getDist(int cur, int centroid, int p=-1, int dist=1){
44     for (int child:adj[cur]){
45         if(child==p || is_removed[child])
46             continue;
47         dist++;
48         getDist(child, centroid, cur, dist);
49         dist--;
50     }
51     ancestor[cur].push_back(make_pair(centroid, dist));
52 }
53
54 void update(int cur){
55     for (int i=0;i<ancestor[cur].size();i++){
56         dis[ancestor[cur][i].first]=min(dis[ancestor[cur][i].first],
57                                         ancestor[cur][i].second);
58     }
59     dis[cur]=0;
60 }
61
62 int query(int cur){
63     int mini=dis[cur];
64     for (int i=0;i<ancestor[cur].size();i++){
65         mini=min(mini, ancestor[cur][i].second+dis[ancestor[cur][i].first
66                                         ]);
67     }
68     return mini;
69 }
70
71 void build_centroid_decomp(int node = 1) {
72     int centroid = get_centroid(node, get_subtree_size(node));
73
74     for (int child : adj[centroid]) {
75         if (is_removed[child]) { continue; }
76         getDist(child, centroid, centroid);
77     }
78
79     is_removed[centroid] = true;
80
81     for (int child : adj[centroid]) {
82         if (is_removed[child]) { continue; }
83     }

```

```

81         build_centroid_decomp(child);
82     }
83 }

```

15.2 Heavy Light Decomposition

```

1  /*
2      Heavy Light Decomposition(HLD)
3  -----
4      Constructs the heavy light decomposition of a tree
5
6      Splits the tree into several paths so that we can reach the root
7          vertex from any v by traversing at most log n paths.
8          In addition, none of these paths intersect with another.
9
10     Time Complexity(Creation): O(n log n)
11     Time Complexity(Query): O((log n)^2) usually, depending on the query
12         itself
13     Space Complexity: O(n)
14 */
15
16 //call dfs1 first
17 struct SegmentTree {
18     vector<ll> a;
19     int n;
20
21     SegmentTree(int _n) : a(2 * _n, 0), n(_n) {}
22
23     void update(int pos, ll val) {
24         for (a[pos += n] = val; pos > 1; pos >>= 1) {
25             a[pos / 2] = (a[pos] ^ a[pos ^ 1]);
26         }
27     }
28
29     ll get(int l, int r) {
30         ll res = 0;
31         for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
32             if (l & 1) {
33                 res ^= a[l++];
34             }
35             if (r & 1) {
36                 res ^= a[--r];
37             }
38         }
39     }
40 }

```

```

36     }
37     return res;
38 }
39 };
40
41
42 const int MAXN=500005;
43 vector<int> adj[MAXN];
44 SegmentTree st(MAXN);
45 int a[MAXN], sz[MAXN], to[MAXN], dpth[MAXN], s[MAXN], par[MAXN];
46 int cnt=0;
47
48 void dfs1(int cur, int p){
49     sz[cur]=1;
50     for(int x:adj[cur]){
51         if(x==p) continue;
52         dpth[x]=dpth[cur]+1;
53         par[x]=cur;
54         dfs1(x, cur);
55         sz[cur]+=sz[x];
56     }
57 }
58
59 void dfs(int cur, int p, int l){
60     st.update(cnt, a[cur]);
61     s[cur]=cnt++;
62     to[cur]=l;
63     int g=-1;
64     for(int x:adj[cur]){
65         if(x==p) continue;
66         if(g==-1 || sz[g]<sz[x]){
67             g=x;
68         }
69     }
70     if(g==-1) return;
71     dfs(g, cur, l);
72     for(int x:adj[cur]){
73         if(x==p || x==g) continue;
74         dfs(x, cur, x);
75     }
76 }
77
78 int query(int u, int v){
79     int res=0;
80     while(to[u]!=to[v]){
81         if(dpth[to[u]]<dpth[to[v]]) swap(u, v);
82         res+=st.get(s[to[u]], s[u]+1);
83         u=par[to[u]];
84     }
85     if(dpth[u]>dpth[v]) swap(u, v);
86     res+=st.get(s[u], s[v]+1);
87     return res;
88 }
89
90
91 //alternate implementation
92 vector<int> parent, depth, heavy, head, pos;
93 int cur_pos;
94
95 int dfs(int v, vector<vector<int>> const& adj) {
96     int size = 1;
97     int max_c_size = 0;
98     for (int c : adj[v]) {
99         if (c != parent[v]) {
100             parent[c] = v, depth[c] = depth[v] + 1;
101             int c_size = dfs(c, adj);
102             size += c_size;
103             if (c_size > max_c_size)
104                 max_c_size = c_size, heavy[v] = c;
105         }
106     }
107     return size;
108 }
109
110
111 void decompose(int v, int h, vector<vector<int>> const& adj) {
112     head[v] = h, pos[v] = cur_pos++;
113     if (heavy[v] != -1)
114         decompose(heavy[v], h, adj);
115     for (int c : adj[v]) {
116         if (c != parent[v] && c != heavy[v])
117             decompose(c, c, adj);
118     }
119 }
120
121

```

```

122 void init(vector<vector<int>> const& adj) {
123     int n = adj.size();
124     parent = vector<int>(n);
125     depth = vector<int>(n);
126     heavy = vector<int>(n, -1);
127     head = vector<int>(n);
128     pos = vector<int>(n);
129     cur_pos = 0;
130
131     dfs(0, adj);
132     decompose(0, 0, adj);
133 }
134
135 int query(int a, int b) {
136     int res = 0;
137     for (; head[a] != head[b]; b = parent[head[b]]) {
138         if (depth[head[a]] > depth[head[b]])
139             swap(a, b);
140         int cur_heavy_path_max = segment_tree_query(pos[head[b]], pos[b]);
141         res = max(res, cur_heavy_path_max);
142     }
143     if (depth[a] > depth[b])
144         swap(a, b);
145     int last_heavy_path_max = segment_tree_query(pos[a], pos[b]);
146     res = max(res, last_heavy_path_max);
147     return res;
148 }

13  const int N=200005;
14  vector<int> adj[N];
15  vector<int> start(N), end1(N), depth(N);
16  vector<vector<int>> t(N, vi(32));
17  int timer=0;
18  int n, l;
19  // l=(int)ceil(log2(n))
20  // call dfs(1, 1, 0)
21  // 1 indexed, dont use 0 indexing
22
23
24  void dfs(int cur, int p, int cnt){
25      depth[cur]=cnt;
26      t[cur][0]=p;
27      start[cur]=timer++;
28      for(int i=1;i<=l;i++){
29          t[cur][i]=t[t[cur][i-1]][i-1];
30      }
31      for(int x:adj[cur]){
32          if(x==p) continue;
33          dfs(x, cur, cnt+1);
34      }
35      end1[cur]=++timer;
36  }
37
38  bool ancestor(int u, int v){
39      return start[u]<=start[v] && end1[u]>=end1[v];
40  }
41
42  int lca(int u, int v){
43      if(ancestor(u, v))
44          return u;
45      if (ancestor(v, u)){
46          return v;
47      }
48      for(int i=l;i>=0;i--){
49          if(!ancestor(t[u][i], v)){
50              u=t[u][i];
51          }
52      }
53      return t[u][0];
54 }
```

15.3 Lowest Common Ancestor (LCA)

```

1 /*
2      LCA(Lowest Common Ancestor)
3 -----
4 Computes the lowest common ancestor of two vertices in a tree.
5
6 Be careful as implementation is indexed starting with 1
7
8 Time Complexity(Creation): O(n log n)
9 Time Complexity(Query): O(log n)
10 Space Complexity: O(n log n)
11 */
12
```

15.4 Tree Diameter

```
1 /*  
2      Tree Diameter  
3 -----  
4 Finds the vertex most distant to vertex on which function is called.  
5  
6 The first value is the vertex itself and the second value is the  
7   distance.  
8  
9 To find diameter run algorithm twice, first on random vertex and then  
10   on the vertex that is farthest away.  
11  
12 The vertex that is the farthest away from any vertex in tree must be  
13   an endpoint of the diameter.  
14  
15 Time Complexity: O(n)  
16 Space Complexity: O(n)  
17 */  
18  
19 pair<int, int> dfs(const vector<vector<int>> &tree, int node = 1,  
20   int previous = 0, int length = 0) {  
21   pair<int, int> max_path = {node, length};  
22   for (const int &i : tree[node]) {  
23     if (i == previous) { continue; }  
24     pair<int, int> other = dfs(tree, i, node, length + 1);  
25     if (other.second > max_path.second) { max_path = other; }  
26   }  
27   return max_path;  
28 }
```