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Berilgan kompleks sonni trigonometrik shaklda ifodalang. z^n va $\sqrt[k]{z}$ ni hisoblang.

1.1.1. $z = -3 + i\sqrt{3}$ $\quad n = 6 \quad k = 3$

$z = r(\cos\varphi + i\cdot\sin\varphi) \qquad r = \sqrt{x^2+y^2}. \qquad x = -3 \quad y = \sqrt{3}$

$r = \sqrt{(-3)^2+(\sqrt{3})^2} = \sqrt{12}. \qquad tg\,\varphi = \frac{y}{x} = -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}.$

$\varphi = \pi - arctg\frac{y}{x} \qquad \varphi = \pi - arctg\frac{1}{\sqrt{3}} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$

$z = \sqrt{12}\cdot\left(\cos\frac{5\pi}{6} + i\cdot\sin\frac{5\pi}{6}\right) \qquad z^n = r^n(\cos n\varphi + i\cdot\sin n\varphi)$

$\boxed{z^6 = 12^3(\cos 6\pi + i\cdot\sin 3\pi).}$

$k=0$ da $\quad z_1 = \sqrt[3]{12}\left(\cos\frac{5\pi}{6} + i\cdot\sin\frac{5\pi}{6}\right)$

$k=1$ da $\quad z_2 = \sqrt[3]{12}\left(\cos\frac{5\pi+2\cdot1\cdot\pi}{6} + i\cdot\sin\frac{5\pi+2\cdot1\cdot\pi}{6}\right) =$

$= \sqrt[3]{12}\left(\cos\frac{7\pi}{6} + i\cdot\sin\frac{7\pi}{6}\right).$

$k=2$ da $\quad z_3 = \sqrt[3]{12}\left(\cos\frac{9\pi}{6} + i\cdot\sin\frac{9\pi}{6}\right).$

$k=3$ da $\quad z_4 = \sqrt[3]{12}\left(\cos\frac{11\pi}{6} + i\cdot\sin\frac{11\pi}{6}\right).$

Limitlarni Lopital qoidasidan foydalanmasdan hisoblang.

1.2.1. $\lim\limits_{x\to x_0}\frac{2x^2+x-1}{x^2-3x-4} \qquad$ a) $x_0 = 2 \quad$ b) $x_0 = -1 \quad$ v) $x_0 = \infty.$

a) $x_0 = 2$ da

$\lim\limits_{x\to 2}\frac{2x^2+x-1}{x^2-3x-4} = \lim\limits_{x\to 2}\frac{2\cdot4+2-1}{2^2-6-4} = -\frac{9}{6}.$

b) $x_0 = -1$ da.

$$\lim_{x\to -1} \frac{2x^2+x-1}{x^2-3x-4} = \lim_{x\to -1} \frac{2(-1)^2+(-1)-1}{(-1)^2+3-4} = \frac{0}{0} = \lim_{x\to -1} \frac{(2x-1)\cancel{(x+1)}}{(x-4)\cancel{(x+1)}} =$$

$$= \lim_{x\to -1} \frac{2x-1}{x-4} = \frac{3}{5}.$$

c) $x_0 = \infty$ da.

$$\lim_{x\to\infty} \frac{2x^2+x-1}{x^2-3x-4} = \frac{\infty}{\infty} \quad \lim_{x\to\infty} \frac{\frac{2x^2}{x^2}+\frac{x}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}-\frac{3x}{x^2}-\frac{4}{x^2}} = 2.$$

1- va 2- ajoyib limit formulalaridan foydalanib hisoblang

1.3.1. a) $\lim\limits_{x\to 0} \frac{tg2x}{\sin 3x}$. b) $\lim\limits_{x\to\infty} \left(\frac{x+3}{x-2}\right)^x$.

a) $$\lim_{x\to 0} \frac{tg2x}{\sin 3x} = \lim_{x\to 0} \frac{\frac{\sin 2x}{\cos 2x}}{\sin 3x} = \lim_{x\to 0} \left(\frac{\sin 2x}{\sin 3x}\cdot\frac{1}{\cos 2x}\right) =$$

$$= \lim_{x\to 0} \frac{\sin 2x}{\sin 3x}\cdot \lim_{x\to 0} \frac{1}{\cos 2x} = \frac{2}{3}.$$

b) $$\lim_{x\to\infty} \left(\frac{x+3}{x-2}\right)^x = \lim_{x\to\infty} \left(1+\frac{1}{\frac{x-2}{5}}\right)^{\frac{x-2}{5}\cdot\frac{5}{x-2}\cdot x} = \lim e^{\frac{5x}{x-2}} = e^5;$$

Berilgan funksiyalarni uzluksizlikka tekshiring, uzilish oraliqlarini va uzilish nuqtalarini turini aniqlang.

1.4.1. a) $$f(x) = \begin{cases} x+4 & x < -1 \\ x^2+2 & -1 \le x < 1 \\ 2x & x \ge 1. \end{cases}$$

b) $f(x) = 9^{\frac{1}{2-x}}$ $\quad x_1 = 0$, $x_2 = 2$.

$x = -1$ $\qquad$ $x = 1.$

$f(-1) = 3$ $\qquad$ $f(1) = 2x = 2$

$\lim\limits_{x \to -1-0} (x^2+2) = 3$ $\qquad$ $\lim\limits_{x \to 2-0} (2x) = 4.$

$\lim\limits_{x \to -1+0} (x+4) = -3$ $\qquad$ $\lim\limits_{x \to 2+0} (x+4) = 6.$

I tur sakrash. $\qquad$ $f(a-0) \neq f(a+0)$

b) $f(x) = 9^{\frac{1}{2-x}}$

$f(0) = 9^{\frac{1}{2-0}} = 9^{\frac{1}{2}} = 3$ $\qquad$ $\lim\limits_{x \to 2+0} \left(9^{\frac{1}{2-x}}\right) = 9^{-\infty} = 0.$

$f(2) = 9^{\frac{1}{2-2}} = \emptyset.$ $\qquad$ II tur uzulish.

$\lim\limits_{x \to 2-0} \left(9^{\frac{1}{2-x}}\right) = 9^{\infty} = \infty.$

Berilgan darajali korsatkichli, trigonometrik funksiyalar hosilalarini hisoblang.

1.5.1. a) $y = (3x - 4\sqrt[3]{x} + 2)^4$ $\qquad$ $y' = 4\cdot(3x - 4\sqrt[3]{x} + 2)^3 \cdot \left(3 - \frac{4}{3\sqrt[3]{x^2}}\right)$

b) $y = \frac{4x + 7\operatorname{tg}x}{\sqrt{1+9x^2}}$

$$y' = \frac{(4x+7\operatorname{tg}x)' \sqrt{1+9x^2} - (4x+7\operatorname{tg}x)\left(\sqrt{1+9x^2}\right)'}{1+9x^2} =$$

$$= \frac{\left(4 + \frac{7}{\cos^2 x}\right)\cdot\sqrt{1+9x^2} - (4x+7\operatorname{tg}x)\cdot\frac{9x}{\sqrt{1+9x^2}}}{1+9x^2}.$$

d) $y = 5^{\sin x} \cdot \arccos 2x \cdot$ $\quad$ $y' = 5^{\sin x} \cdot \ln 5 \cdot \cos x \cdot \arccos 2x - \frac{5^{\sin x}}{\sqrt{1-4x^2}}$

Funksiyaning differensiali yordamida 0,01 aniqlikda taqribiy hisoblang va nisbiy xatolikni toping.

1.6.1. a) $\sqrt[3]{27{,}5}$ b) $\operatorname{arctg} 1{,}02$.

a) $y=\sqrt[3]{x}$ $\quad x=x_0+\Delta x$ $\quad \sqrt[3]{27+0{,}5}$ $\quad x_0=27$ $\quad \Delta x=0{,}5$

$y(x_0+\Delta x)\approx y(x_0)+y'(x_0)\cdot\Delta x$ $\qquad y(x_0)=\sqrt[3]{27}=3$

$y'(x_0)=y'(27)=\frac{1}{3}\frac{1}{\sqrt[3]{27^2}}=\frac{1}{27}$ $\qquad y'=\frac{1}{3\cdot\sqrt[3]{x^2}}$

$\sqrt[3]{27{,}5}\approx 3+\frac{1}{2}\cdot\frac{1}{27}=3+\frac{1}{54}=\frac{163}{54}=3{,}0185.$

b) $\operatorname{arctg}(1{,}02)$. $\quad y(x_0+\Delta x)\approx y(x_0)+y'(x_0)\cdot\Delta x$

$y=\operatorname{arctg} x,\quad x_0=1\quad \Delta x=1{,}02-1=0{,}02\quad y(x_0)=\operatorname{arctg}1=\frac{\pi}{4}$

$y'=\frac{1}{1+x^2}\quad y'(1)=\frac{1}{2}$

$\operatorname{arctg}1{,}02\approx\frac{\pi}{4}+\frac{1}{2}\cdot 0{,}02=\frac{3{,}14}{4}+0{,}01=0{,}79$

$\delta=\left|\frac{0{,}79-0{,}80}{0{,}79}\right|\cdot 100\%=1{,}2\%$

a) Funksiyaning berilgan oraliqdagi eng katta va eng kichik qiymatlarini.

b) Qavariqlik, botiqlik oraliqlari va egilish nuqtalarini toping.

1.7.1.

a) $y = 2\cdot\sin x + \cos 2x \quad [0.\ \pi/2]$

$y' = 2\cdot\cos x - 2\cdot\sin 2x \qquad 2\cdot\cos x - 2\cdot\sin 2x = 0 \qquad \cos x = 0 \quad \sin x = \frac{1}{2}$

$x_1 = \frac{\pi}{2} + \pi k \qquad x_2 = \frac{\pi}{6} + 2\pi k.$

$x_1 = 0$ da $y = 2\cdot\sin 0 + \cos 2\cdot 0 = 1.$

$x_2 = \frac{\pi}{6}$ da $y = 2\cdot\sin\frac{\pi}{6} + \cos\frac{\pi}{3} = 1 + \frac{1}{2} = \frac{3}{2}$

$x_3 = \frac{\pi}{2}$ da $y = 2\cdot\sin\frac{\pi}{2} + \cos\pi = 2 + 1 = 3.$

eng katta qiymati 3.

eng kichik qiymati 1.

b) $y'' = (2\cdot\sin x + \cos 2x)''$

$y'' = (2\cdot\cos x - 2\cdot\sin 2x)' = -2\sin x - 4\cos 2x = -2(\sin x + 2\cos 2x)$

a) Funksiyaning berilgan oraliqdagi eng katta va eng kichik qiymatlarini;

b) Qavariqlik, botiqlik oraliqlari va egilish nuqtalarini toping

1.8.1.

$$L(x) = \sum_{i=0}^{n} y_i\, l_i(x). \qquad l_i(x) = \frac{\omega(x)}{(x-x_i)\,\omega'(x_i)}$$

$$x = \frac{1}{6};\ \frac{1}{4};\ \frac{1}{3};\ \frac{1}{2}. \qquad f(x) = \cos\pi x \qquad x = \frac{5}{12}$$

$x_0 = \frac{1}{6}$ da $y_0 = f\left(\frac{1}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\qquad$ $x_2 = \frac{1}{3}$ da $y_2 = f\left(\frac{1}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

$x_1 = \frac{1}{4}$ da $y_1 = f\left(\frac{1}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\qquad$ $x_3 = \frac{1}{2}$ da $y_3 = f\left(\frac{1}{2}\right) = \cos\frac{\pi}{2} = 0.$

$$L(x) = \sum_{i=0}^{n} y_i\, l_i(x)\frac{x-x_i}{x_i-x_j} = y_0\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + y_1\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} +$$

$$+ y_2\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + y_3\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$\cos\pi x = \frac{\sqrt{3}}{2}\cdot\frac{(x-\frac{1}{4})(x-\frac{1}{3})(x-\frac{1}{2})}{(\frac{1}{6}-\frac{1}{4})(\frac{1}{6}-\frac{1}{3})(\frac{1}{6}-\frac{1}{2})} + \frac{\sqrt{2}}{2}\cdot\frac{(x-\frac{1}{6})(x-\frac{1}{3})(x-\frac{1}{2})}{(\frac{1}{4}-\frac{1}{6})(\frac{1}{4}-\frac{1}{3})(\frac{1}{4}-\frac{1}{2})} + \frac{1}{2}\cdot\frac{(x-\frac{1}{6})(x-\frac{1}{4})(x-\frac{1}{2})}{(\frac{1}{3}-\frac{1}{6})(\frac{1}{3}-\frac{1}{4})(\frac{1}{3}-\frac{1}{2})}$$

$$= -\frac{9\cdot\sqrt{3}}{2}(4x-1)(3x-1)(2x-1) - 8\sqrt{2}(6x-1)(3x-1)(2x-1) - \frac{9}{2}(6x-1)(4x-1)(2x-1) =$$

$$\cos\pi\left(\frac{5}{12}\right) = -\frac{9\sqrt{3}}{2}\left(\frac{5}{3}-1\right)\left(\frac{5}{4}-1\right)\left(\frac{5}{6}-1\right) - 8\sqrt{2}\left(\frac{5}{2}-1\right)\left(\frac{5}{4}-1\right)\left(\frac{5}{6}-1\right) - \frac{9}{2}\left(\frac{5}{2}-1\right)\left(\frac{5}{3}-1\right)\left(\frac{5}{6}-1\right) =$$

$$= 0{,}66\cdot 0{,}25\cdot 0{,}16\cdot\frac{9\sqrt{3}}{2} + 8\cdot 1{,}4\cdot 1{,}5\cdot 0{,}25\cdot 0{,}16 + \frac{9}{2}\cdot 1{,}5\cdot 0{,}66\cdot 0{,}16 =$$

$$= 0{,}205 + 0{,}673 + 0{,}71 = 1{,}5898.$$

Quyidagi limitlarni Lopital qoidasi yordamida hisoblang.

1.9.1. a) $\lim\limits_{x\to\pi/4} \dfrac{\frac{1}{\cos^2 x} - 2tgx}{1+\cos 4x}$ $\lim\limits_{x\to x_0} \dfrac{f_1'(x)}{f_2'(x)}$.

$$f_1'(x) = \left(\frac{1}{\cos^2 x} - 2\cdot tgx\right)' = \left(\frac{2}{1+\cos 2x} - 2\cdot tg\right)' =$$

$$= 2\cdot\left(-1\cdot(1+\cos 2x)^{-2}\cdot(-\sin 2x)\cdot 2 - \frac{1}{\cos^2 x}\right) =$$

$$= 2\cdot\left(\frac{2\sin 2x}{(1+\cos 2x)^2} - \frac{1}{\cos^2 x}\right)$$

$$f_2'(x) = (1+\cos 4x)' = -4\sin 4x.$$

$$\lim_{x\to\frac{\pi}{4}} \frac{2\cdot\left(\frac{2\sin\frac{\pi}{2}}{(1+\cos\frac{\pi}{2})^2} - \frac{1}{\cos^2\frac{\pi}{4}}\right)}{-4\cdot\sin\pi} = \frac{0}{0}. \quad \text{aniqmas.}$$

$$f_1'(x) = 2\cdot\left(\frac{2\cdot\sin 2x}{(1+\cos 2x)^2} - \frac{1}{\cos^2 x}\right)' =$$

$$= 2\cdot\left(\frac{(4\cdot\cos 2x(1+\cos 2x)^2 + 8\sin 2x\cdot(1+\cos 2x)\cdot\sin 2x)}{(1+\cos 2x^4}\right) -$$

$$-\frac{2\cdot\sin 2x}{(1+\cos 2x)^2}\Bigg) =$$

$$f_2'(x) = -16\cdot\cos 4x$$

$$\lim_{x\to\frac{\pi}{4}} \frac{f_1'(\frac{\pi}{4})}{f_2'(\frac{\pi}{4})} = \frac{-2}{-16} = \frac{1}{8}.$$