M E 325 Supplemental Homework 1

Group 6

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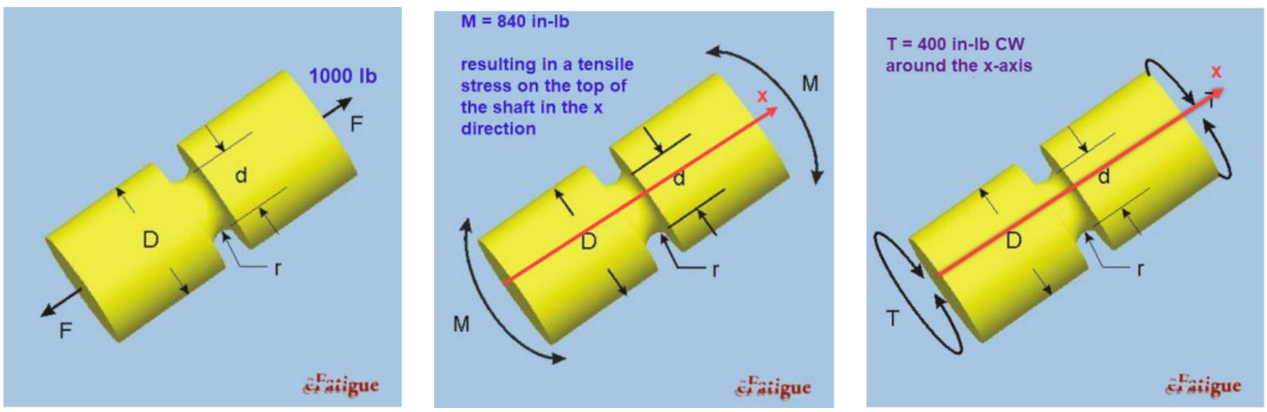
Determine

- The **fatigue strength**, S_f, of a material, at a specified number of stress cycles, N, using correction factors for sizing, temperature, loading, reliability, surface treatment, and miscellaneous effects
- The **fatigue stress concentration factors**, K_f, that should be applied to cyclical stresses
- **Mean and alternating von Mises stresses**, σ'_m, σ'_a
- **Factors of safety**, N, against fatigue and yielding

Specimen Loading and relevant data:




```
Fx1 = 1000
```

```
Fx1 = 1000
```

```
Mx1 = 840
```

```
Mx1 = 840
```

```
Tx1 = 400
```

```
Tx1 = 400
```

The specimen pictured is a cylinder, with a U-shaped groove and the following dimensions:

The major diameter, D, of the shaft is 2.0 inches and the minor diameter, d, is 1.75 inches. The radius of the groove, r, is 0.25 inches.

```
D = 2
```

```
D = 2
```

```
d = 1.75
```

```
d = 1.7500
```

```
r = 0.25
```

```
r = 0.2500
```

Material Properties:

The material from which the shaft is fabricated has an ultimate tensile strength, S_{ut}, of 90 kpsi and the yield strength, S_y, of the material is 75 kpsi.

```
S_ultimate = 90
```

```
S_ultimate = 90
```

```
S_yield = 75
```

```
S_yield = 75
```

Relevant information for Marin endurance strength adjustment factors:

The **surface** of the shaft is machined (Table 6.2), and the shaft is rotating (**if the shaft is not rotating an equivalent diameter replaces the diameter used in the sizing adjustment factor**: $d_e = .370d$)

```
d_e = 0.370 * d
```

```
d_e = 0.6475
```

The **operating temperature** of the shaft is $600\,°F$ and **room temperature** is $72\,°F$.

```
T_op = 600
```

```
T_op = 600
```

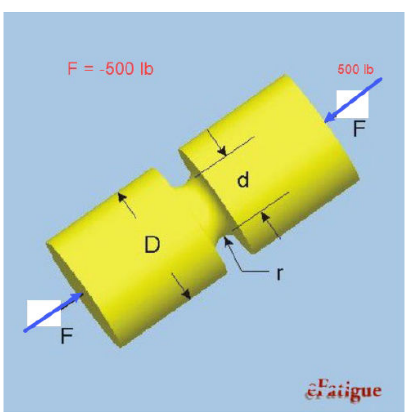
```
T_room = 72
```

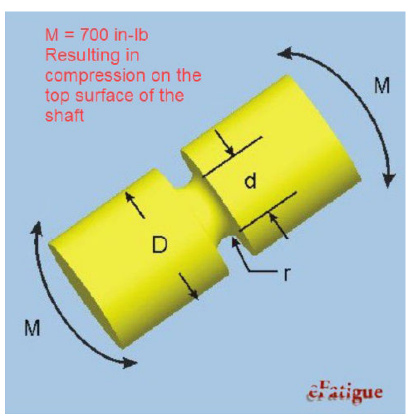
```
T_room = 72
```

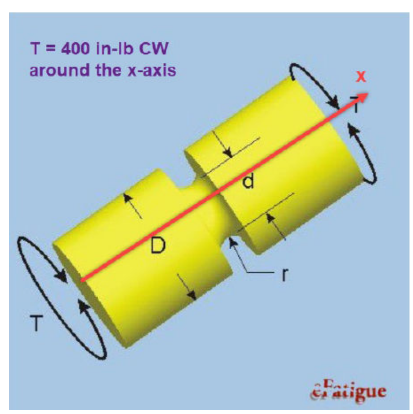
The **material reliability** is 99% (Table 6.4)

There is no impact on the fatigue life of the shaft based on **miscellaneous effects**.

2nd Half of loading cycle








```
Fx2 = -500
```

```
Fx2 = -500
```

```
Mx2 = -700
```

```
Mx2 = -700
```

```
Tx2 = 400
```

```
Tx2 = 400
```

Submit:

- Calculations detailing determination of all parameters.
- Determine the factor of safety against yielding and fatigue for the loadings and conditions given at **50,000 stress cycles**.
- Create Goodman and Langer failure envelopes and add the coordinates for σ'_a and σ'_m. Submit a printout of the failure envelopes and include the load line.

The types of loading experienced by the specimen are axial, bending, and torsion.

Determine stress concentration factors for each type of loading from eFatigue website.

Axial:

```
K_t_axial = 1.94
```

```
K_t_axial = 1.9400
```

Bending:

```
K_t_bending = 1.7
```

```
K_t_bending = 1.7000
```

Torsion:

```
K_t_torsion = 1.35
```

```
K_t_torsion = 1.3500
```

Calculate Marin factors

Surface factor, k_a

For machined surface, a = 2.00, b = -0.217

```
a_surface = 2.00
```

```
a_surface = 2
```

```
b_surface = -0.217
```

```
b_surface = -0.2170
```

```
k_a = a_surface*S_ultimate^b_surface
```

```
k_a = 0.7533
```

Size Factor, k_b:

$$k_b = \left(\frac{d}{0.3}\right)^{-0.107} = 0.879d^{-0.107}$$

```
k_b = 0.879*d^-0.107
```

```
k_b = 0.8279
```

Loading Factor, k_c:

For combined loading:

$$k_c = 1$$

```
k_c = 1
```

```
k_c = 1
```

Temperature Factor, k_d:

$$k_d = S_T/S_{\mathrm{RT}} = 0.98 + 3.5(10^{-4})T_F - 6.3(10^{-7})T_F^2$$

```
k_d = 0.98 + 3.5*(10^-4)*T_op - 6.3*(10^-7)*T_op^2
```

```
k_d = 0.9632
```

Reliability Factor, k_e:

With a reliability percentage of 99% (Table6-4):

$$k_e = 0.814$$

```
k_e = 0.814
```

```
k_e = 0.8140
```

Miscellaneous Factor, k_f:

$$k_f = 1$$

```
k_f = 1
```

```
k_f = 1
```

Calculated combined marin factor

```
K = k_a*k_b*k_c*k_d*k_e*k_f
```

```
K = 0.4890
```

```
% Sf = Se at 1,000,000 cycles
Se = K*1/2*S_ultimate
```

```
Se = 22.0037
```

Calculate Notch sensitivity factors for each type of loading:

First, neubers numbers are needed

```
neuber_axial = 0.246-3.08*(10^-3)*S_ultimate+1.51*(10^-5)*S_ultimate^2-2.67*(10^-8)*S_ultimate
```

```
neuber_axial = 0.0716
```

```
neuber_bending = 0.246-3.08*(10^-3)*S_ultimate+1.51*(10^-5)*S_ultimate^2-2.67*(10^-8)*S_ultimat
```

```
neuber_bending = 0.0716
```

```
neuber_torsion = 0.190-2.51*(10^-3)*S_ultimate + 1.35*(10^-5)*S_ultimate^2 - 2.67*(10^-8)*S_ult
```

```
neuber_torsion = 0.0540
```

Notch sensitivity factors

```
q_axial = 1/(1 + (neuber_axial/sqrt(r)))
```

```
q_axial = 0.8747
```

```
q_bending = 1/(1 + (neuber_bending/sqrt(r)))
```

```
q_bending = 0.8747
```

```
q_torsion = 1/(1 + (neuber_torsion/sqrt(r)))
```

```
q_torsion = 0.9026
```

Calculate fatigue stress concentration factors

```
K_f_axial = q_axial*(K_t_axial-1)+1
```

```
K_f_axial = 1.8222
```

```
K_f_bending = q_bending*(K_t_bending-1)+1
```

```
K_f_bending = 1.6123
```

```
K_f_torsion = q_torsion*(K_t_torsion-1)+1
```

```
K_f_torsion = 1.3159
```

Adjust stresses with stress concentration factors for 1st loading:

```
stress_x_1 = K_f_axial*(Fx1/(pi*d^2/4)) + K_f_bending*(Mx1*(d/2)/(pi*d^4/64))
```

```
stress_x_1 = 3.3315e+03
```

```
shear_xz_1 = K_f_torsion*((Tx1*d/2)/(pi*d^4/32))
```

```
shear_xz_1 = 500.1916
```

Adjust stresses with stress concentration factors for 2nd loading:

```
stress_x_2 = K_f_axial*(Fx2/(pi*d^2/4)) + K_f_bending*(Mx2*(d/2)/(pi*d^4/64))
```

```
stress_x_2 = -2.5238e+03
```

```
shear_xz_2 = K_f_torsion*((Tx2*d/2)/(pi*d^4/32))
```

```
shear_xz_2 = 500.1916
```

Calculate mean and alternating stresses

```
stress_a = (stress_x_1-stress_x_2)/2
```

```
stress_a = 2.9276e+03
```

```
stress_m = (stress_x_1+stress_x_2)/2
```

```
stress_m = 403.8913
```

```
shear_a = (shear_xz_1-shear_xz_2)/2
```

```
shear_a = 0
```

```
shear_m = (shear_xz_1+shear_xz_2)/2
```

```
shear_m = 500.1916
```

Calculate mean and alternating principle stresses:

```
stress_m_p_1 = stress_m/2 + sqrt((stress_m/2)^2+shear_m^2)
```

```
stress_m_p_1 = 741.3654
```

```
stress_m_p_3 = stress_m/2 - sqrt((stress_m/2)^2+shear_m^2)
```

```
stress_m_p_3 = -337.4741
```

```
stress_a_p_1 = stress_a/2 + sqrt((stress_a/2)^2+shear_a^2)
```

```
stress_a_p_1 = 2.9276e+03
```

```
stress_a_p_3 = stress_a/2 - sqrt((stress_a/2)^2+shear_a^2)
```

```
stress_a_p_3 = 0
```

Calculate mean and alternating von Mises stresses:

```
von_mises_m = sqrt(stress_m_p_1^2 + stress_m_p_3^2 - stress_m_p_1*stress_m_p_3)
```

```
von_mises_m = 955.8782
```

```
von_mises_a = sqrt(stress_a_p_1^2 + stress_a_p_3^2 - stress_a_p_1*stress_a_p_3)
```

```
von_mises_a = 2.9276e+03
```

Estimating the fatigue strength at 10^3 cycles

$$f = 1.06 - 2.8\left(10^{-3}\right)S_{ut} + 6.9\left(10^{-6}\right)S_{su}^2 \quad 70 < S_{ut} < 200\ \text{kpsi}$$

$$f = 1.06 - 4.1\ \left(10^{-4}\right)S_{ut} + 1.5\left(10^{-7}\right)S_{ut}^2 \quad 500 < S_{ut} < 1400\ \text{MPa}$$

```
f = 1.06 - 2.8*(10^-3)*S_ultimate + 6.9*(10^-6)*S_ultimate^2
```

```
f = 0.8639
```

```
% Sf at 1,000 cycles
S_f_1000 = f*S_ultimate
```

```
S_f_1000 = 77.7501
```

Calculating basquin parameters:

$$a = \frac{(fS_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right)$$

```
a = (f * S_ultimate)^2 / Se
```

```
a = 274.7295
```

```
b = (-1/3)*log10(f*S_ultimate/Se)
```

```
b = -0.1827
```

Fatigue strength at 50,000 cycles

$$S_f = aN^b$$

```
N = 50000
```

```
N = 50000
```

```
S_f_50000 = a * N^b
```

```
S_f_50000 = 38.0400
```

Factor of safety for fatigue

Failure criterion: $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1$$

Design equation: $$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} \quad \sigma_m \geq 0$$

```
% von mises stresses need to be scaled to kpsi from psi
n_f_50000 = 1/((von_mises_a/1000)/S_f_50000 + (von_mises_m/1000)/S_ultimate)
```

```
n_f_50000 = 11.4177
```

Factor of Safety for yield

$$n_y = \left(\frac{\sigma_a'}{S_y} + \frac{\sigma_m'}{S_y}\right)^{-1} = \left(\frac{S_y}{\sigma_a' + \sigma_m'}\right)$$

```
% von mises stresses need to be scaled to kpsi from psi
n_y_50000 = (S_yield/((von_mises_a+von_mises_m)/1000))
```

```
n_y_50000 = 19.3123
```

Create plot of Goodman and Langer Lines with load line present:

```
figure; hold on

% Goodman Plot
x_goodman = linspace(0,S_ultimate, 2);
y_goodman = (-S_f_50000/S_ultimate)*x_goodman + S_f_50000;
goodman_line = plot(x_goodman, y_goodman, '-o');
goodman_line.MarkerFaceColor = [0, 0, 0];
goodman_line.MarkerSize = 6;


goodman_label = "Goodman Line";
text(0, S_f_50000, 'S_{f}', FontWeight='bold', HorizontalAlignment='right', VerticalAlignment=
text(S_ultimate, 0, ' S_{ut}', 'FontWeight','bold')

% Langer Plot
x_langer = linspace(0,S_yield,2);
y_langer = -x_langer + S_yield;
langer_line = plot(x_langer, y_langer, '-o');
langer_line.MarkerFaceColor = [0, 0, 0];
langer_line.MarkerSize = 6;


langer_label = "Langer Line";
text(0, S_yield, 'S_{y} ','FontWeight', 'bold', 'HorizontalAlignment','right');
text(S_yield, 0, 'S_{y}', FontWeight='bold',VerticalAlignment='top', HorizontalAlignment='left

%Calculate intersection of load and goodman lines
x_intersect_G = S_f_50000*(von_mises_a/von_mises_m + S_f_50000/S_ultimate)^-1
```

```
x_intersect_G = 10.9139
```

```
%Calculate intersection of load and langer lines
x_intersect_L = S_yield*(von_mises_a/von_mises_m + 1)^-1
```

```
x_intersect_L = 18.4602
```

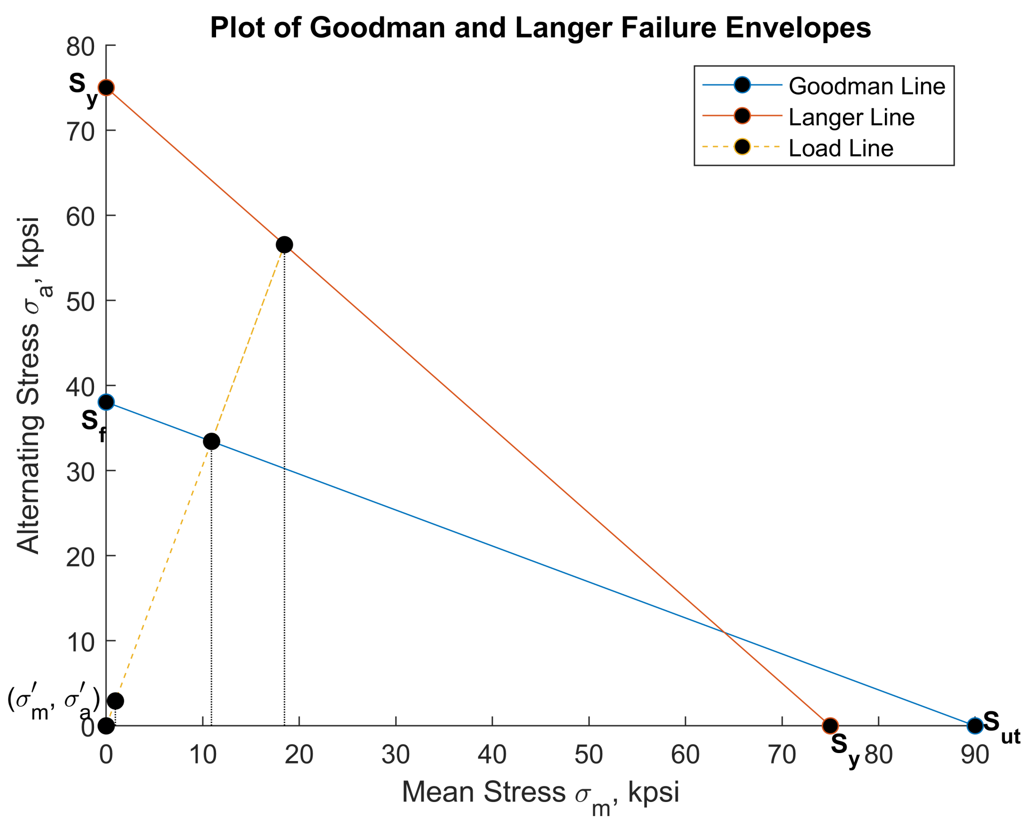
```
% Load line plot (scale von mises to kpsi from psi)
x_load = [0, von_mises_m/1000, x_intersect_L, x_intersect_G];
y_load = (von_mises_a/von_mises_m)*x_load;
load_line = plot(x_load, y_load,'--o');
load_line.MarkerFaceColor = [0, 0, 0];
load_line.MarkerSize = 6;
load_label = "Load Line";
text(von_mises_m/1000,von_mises_a/1000, '(\sigma_{m}^{\prime}, \sigma_{a}^{\prime})  ', Horizon

% Stem for load line
stem(x_load, y_load, ':k')

% Legend for lines
plots = [goodman_line, langer_line, load_line];
labels = [goodman_label, langer_label, load_label];
legend(plots, labels);

% Labels for axes and plot title
xlabel('Mean Stress \sigma_{m}, kpsi')
ylabel('Alternating Stress \sigma_{a}, kpsi');
title('Plot of Goodman and Langer Failure Envelopes')

hold off
```



Determination of factors of safety from failure envelope plot:

```
% von Mises stresses must be scales to kpsi from psi
% fatigue
n_f_50000 = x_intersect_G/(von_mises_m/1000)
```

```
n_f_50000 = 11.4177
```

```
% yield
n_y_50000 = x_intersect_L/(von_mises_m/1000)
```

```
n_y_50000 = 19.3123
```