# StochasticPowerModels

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# Part I

# Home

# Chapter 1

# StochasticPowerModels.jl Documentation

## 1.1 Overview

StochasticPowerModels.jl is a research-grade Julia/JuMP package for experimentation with Steady-State Power Network Optimization under uncertainty, extending PowerModels.jl.

## 1.2 Installation

For now, StochasticPowerModels is unregistered. Nevertheless, you can install it through

```
] add https://github.com/timmyfaraday/StochasticPowerModels.jl.git
```

At least one solver is required for running StochasticPowerModels. The open-source solver Ipopt is recommended, as it is fast, scaleable and can be used to solve a wide variety of the problems and network formulations provided in PowerModels. The Ipopt solver can be installed via the package manager with

```
] add Ipopt
```

Test that the package works by running

```
] test StochasticPowerModels
```

# Part II

# Manual

# Chapter 2

# Getting Started

## 2.1 Quick Start Guide

Once StochasticPowerModels is installed, Ipopt is installed, and a network data file (e.g. `"case5_spm.m"` ) has been acquired, an stochatic AC Optimal Power Flow can be executed with,

```
using StochasticPowerModels
using Ipopt

result = run_ac_sopf("matpower/case5_spm.m", Ipopt.Optimizer)
```

# Chapter 3

# Mathematical Model

## 3.1 The StochasticPowerModels Mathematical Model

### Sets and Parameters

StochasticPowerModels implements a generalized polynomial chaos expansion version of the AC Optimal Power Flow problem from Matpower. The core generalizations of the deterministic OPF problem are,

- Support for multiple load ($S^d_k$) and shunt ($Y^s_k$) components on each bus $i$
- Line charging that supports a conductance and asymmetrical values ($Y^c_{ij}, Y^c_{ji}$)

$$
\begin{aligned}
& \text{sets:} \\
& \quad N \text{ - buses} \\
& \quad R \text{ - reference buses} \\
& \quad E, E^R \text{ - branches, forward and reverse orientation} \\
& \quad G, G_i \text{ - generators and generators at bus } i \\
& \quad L, L_i \text{ - loads and loads at bus } i \\
& \quad S, S_i \text{ - shunts and shunts at bus } i \\
& \text{data:} \\
& \quad S^{gl}_k, S^{gu}_k \;\; \forall k \in G \text{ - generator complex power bounds} \\
& \quad c_{2k}, c_{1k}, c_{0k} \;\; \forall k \in G \text{ - generator cost components} \\
& \quad v^l_i, v^u_i \;\; \forall i \in N \text{ - voltage bounds} \\
& \quad S^d_k \;\; \forall k \in L \text{ - load complex power consumption} \\
& \quad Y^s_k \;\; \forall k \in S \text{ - bus shunt admittance} \\
& \quad Y_{ij}, Y^c_{ij}, Y^c_{ji} \;\; \forall (i,j) \in E \text{ - branch pi-section parameters} \\
& \quad T_{ij} \;\; \forall (i,j) \in E \text{ - branch complex transformation ratio} \\
& \quad s^u_{ij} \;\; \forall (i,j) \in E \text{ - branch apparent power limit} \\
& \quad i^u_{ij} \;\; \forall (i,j) \in E \text{ - branch current limit} \\
& \quad \theta^{\Delta l}_{ij}, \theta^{\Delta u}_{ij} \;\; \forall (i,j) \in E \text{ - branch voltage angle difference bounds}
\end{aligned}
\tag{3.1}
$$

### Stochastic Optimal Power Flow in Current-Voltage Variables

A variable $I^s_{ij}$, representing the current in the direction $i$ to $j$, through the series part of the pi-section, is used. The mathematical structure for a current-voltage formulation is conceived as:

variables:

$I^g_k \;\; \forall k \in G$

$V_i \;\; \forall i \in N$

$I^s_{ij} \;\; \forall (i,j) \in E \cup E^R$ - branch complex (series) current (3.2)

$I_{ij} \;\; \forall (i,j) \in E \cup E^R$ - branch complex (total) current (3.3)

minimize: $\sum_{k \in G} c_{2k} (\Re(S^g_k))^2 + c_{1k} \Re(S^g_k) + c_{0k}$

subject to:

$$\angle V_r = 0 \;\; \forall r \in R$$

$$S^{gl}_k \leq \Re(V_i (I^g_k)^*) + j\Im(V_i (I^g_k)^*) \leq S^{gu}_k \;\; \forall k \in G \tag{3.4}$$

$$v^l_i \leq |V_i| \leq v^u_i \;\; \forall i \in N$$

$$\sum_{k \in G_i} I^g_k - \sum_{k \in L_i} (S^d_k / V_i)^* - \sum_{k \in S_i} Y^s_k V_i = \sum_{(i,j) \in E_i \cup E^R_i} I_{ij} \;\; \forall i \in N \tag{3.5}$$

$$I_{ij} = \frac{I^s_{ij}}{T^*_{ij}} + Y^c_{ij} \frac{V_i}{|T_{ij}|^2} \;\; \forall (i,j) \in E \tag{3.6}$$

$$I_{ji} = -I^s_{ij} + Y^c_{ji} V_j \;\; \forall (i,j) \in E \tag{3.7}$$

$$\frac{V_i}{T_{ij}} = V_j + z_{ij} I^s_{ij} \;\; \forall (i,j) \in E \tag{3.8}$$

$$|S_{ij}| = |V_i||I_{ij}| \leq s^u_{ij} \;\; \forall (i,j) \in E \cup E^R$$

$$|I_{ij}| \leq i^u_{ij} \;\; \forall (i,j) \in E \cup E^R$$

$$\theta^{\Delta l}_{ij} \leq \angle (V_i V^*_j) \leq \theta^{\Delta u}_{ij} \;\; \forall (i,j) \in E$$

### Stochastic Optimal Power Flow in Power-Voltage Variables

A complete mathematical model is as follows, variables:

$$S_k^g \ \ \forall k \in G \text{ - generator complex power dispatch} \tag{3.9}$$

$$V_i \ \ \forall i \in N \text{ - bus complex voltage} \tag{3.10}$$

$$S_{ij} \ \ \forall (i,j) \in E \cup E^R \text{ - branch complex power flow} \tag{3.11}$$

minimize:

$$\sum_{k \in G} c_{2k}(\Re(S_k^g))^2 + c_{1k}\Re(S_k^g) + c_{0k} \tag{3.12}$$

subject to:

$$\angle V_r = 0 \ \ \forall r \in R \tag{3.13}$$

$$S_k^{gl} \leq S_k^g \leq S_k^{gu} \ \ \forall k \in G \tag{3.14}$$

$$v_i^l \leq |V_i| \leq v_i^u \ \ \forall i \in N \tag{3.15}$$

$$\sum_{k \in G_i} S_k^g - \sum_{k \in L_i} S_k^d - \sum_{k \in S_i} (Y_k^s)^* |V_i|^2 = \sum_{(i,j) \in E_i \cup E_i^R} S_{ij} \ \ \forall i \in N \tag{3.16}$$

$$S_{ij} = \left(Y_{ij} + Y_{ij}^c\right)^* \frac{|V_i|^2}{|T_{ij}|^2} - Y_{ij}^* \frac{V_i V_j^*}{T_{ij}} \ \ \forall (i,j) \in E \tag{3.17}$$

$$S_{ji} = \left(Y_{ij} + Y_{ji}^c\right)^* |V_j|^2 - Y_{ij}^* \frac{V_i^* V_j}{T_{ij}^*} \ \ \forall (i,j) \in E \tag{3.18}$$

$$|S_{ij}| \leq s_{ij}^u \ \ \forall (i,j) \in E \cup E^R \tag{3.19}$$

$$|I_{ij}| \leq i_{ij}^u \ \ \forall (i,j) \in E \cup E^R \tag{3.20}$$

$$\theta_{ij}^{\Delta l} \leq \angle(V_i V_j^*) \leq \theta_{ij}^{\Delta u} \ \ \forall (i,j) \in E \tag{3.21}$$

**Part III**

# Library

# Chapter 4

# Network Formulations

## 4.1 Network Formulations

### Type Hierarchy

We begin with the top of the hierarchy, where we can distinguish between current-voltage and power-voltage formulations

```
AbstractACPModel <: AbstractPowerModel
AbstractIVRModel <: AbstractPowerModel
```

### Power Models

Each of these forms can be used as the model parameter for a PowerModel:

```
ACPPowerModel <: AbstractACPForm
IVRPowerModel <: AbstractIVRModel
```