# StochasticPowerModels

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October 23, 2021

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Part I

Home

# StochasticPowerModels.jl Documentation

#### 1.1 Overview

StochasticPowerModels.jl is a research-grade Julia/JuMP package for experimentation with Steady-State Power Network Optimization under uncertainty, extending PowerModels.jl.

#### 1.2 Installation

For now, StochasticPowerModels is unregistered. Nevertheless, you can install it through

] add https://github.com/timmyfaraday/StochasticPowerModels.jl.git

At least one solver is required for running StochasticPowerModels. The open-source solver lpopt is recommended, as it is fast, scaleable and can be used to solve a wide variety of the problems and network formulations provided in PowerModels. The lpopt solver can be installed via the package manager with

] add Ipopt

Test that the package works by running

] test StochasticPowerModels

Part II

Manual

# **Getting Started**

## 2.1 Quick Start Guide

Once StochasticPowerModels is installed, Ipopt is installed, and a network data file (e.g. "case5\_spm.m" ) has been acquired, an stochatic AC Optimal Power Flow can be executed with,

```
using StochasticPowerModels
using Ipopt

result = run_ac_sopf("matpower/case5_spm.m", Ipopt.Optimizer)
```

## **Mathematical Model**

#### 3.1 The StochasticPowerModels Mathematical Model

#### **Sets and Parameters**

StochasticPowerModels implements a generalized polynomial chaos expansion version of the AC Optimal Power Flow problem from Matpower. The core generalizations of the deterministic OPF problem are,

- Support for multiple load ( $S_k^d$ ) and shunt ( $Y_k^s$ ) components on each bus i
- Line charging that supports a conductance and asymmetrical values  $(Y^c_{ij},Y^c_{ji})$

```
sets:
      N - buses
      {\cal R} - reference buses
      E,E^{R} - branches, forward and reverse orientation
      G,G_i - generators and generators at bus i
      L, L_i - loads and loads at bus i
      S, S_i - shunts and shunts at bus i
data:
      S_k^{gl}, S_k^{gu} \;\; \forall k \in G - generator complex power bounds
      c_{2k}, c_{1k}, c_{0k} \;\; \forall k \in G - generator cost components
      v_i^l, v_i^u \ \ \forall i \in N - voltage bounds
      S_k^d \  \, \forall k \in L - load complex power consumption
      Y_k^s \ \ \forall k \in S - bus shunt admittance
      Y_{ij}, Y_{ij}^c, Y_{ji}^c \ \ \forall (i,j) \in E - branch pi-section parameters
      T_{ij} \ \ \forall (i,j) \in E - branch complex transformation ratio
      s_{ij}^u \ \ orall (i,j) \in E - branch apparent power limit
      i^u_{ij} \ \ \forall (i,j) \in E - branch current limit
      \theta_{ij}^{\Delta l}, \theta_{ij}^{\Delta u} \;\; \forall (i,j) \in E - branch voltage angle difference bounds
```

(3.1)

### **Stochastic Optimal Power Flow in Current-Voltage Variables**

A variable  $I_{ij}^s$ , representing the current in the direction i to j, through the series part of the pi-section, is used. The mathematical structure for a current-voltage formulation is conceived as:

variables:

$$I_k^g \ \forall k \in G$$
$$V_i \ \forall i \in N$$

$$I_{ij}^s \ \forall (i,j) \in E \cup E^R$$
 - branch complex (series) current (3.2)

$$I_{ij} \ \ \forall (i,j) \in E \cup E^R$$
 - branch complex (total) current (3.3)

minimize: 
$$\sum_{k \in G} c_{2k} (\Re(S_k^g))^2 + c_{1k} \Re(S_k^g) + c_{0k}$$

subject to:

$$\angle V_r = 0 \ \forall r \in R$$

$$S_k^{gl} \le \Re(V_i(I_k^g)^*) + j\Im(V_i(I_k^g)^*) \le S_k^{gu} \ \forall k \in G$$
 (3.4)

$$v_i^l \le |V_i| \le v_i^u \ \forall i \in N$$

$$\sum_{k \in G_i} I_k^g - \sum_{k \in L_i} (S_k^d / V_i)^* - \sum_{k \in S_i} Y_k^s V_i = \sum_{(i,j) \in E_i \cup E_i^R} I_{ij} \ \forall i \in N$$
 (3.5)

$$I_{ij} = \frac{I_{ij}^s}{T_{ii}^*} + Y_{ij}^c \frac{V_i}{|T_{ij}|^2} \ \forall (i,j) \in E$$
(3.6)

$$I_{ji} = -I_{ij}^s + Y_{ji}^c V_j \ \forall (i,j) \in E$$
 (3.7)

$$\frac{V_i}{T_{ij}} = V_j + z_{ij}I_{ij}^s \ \forall (i,j) \in E$$

$$(3.8)$$

$$|S_{ij}| = |V_i||I_{ij}| \le s_{ij}^u \ \forall (i,j) \in E \cup E^R$$

$$|I_{ij}| \le i_{ij}^u \ \forall (i,j) \in E \cup E^R$$

$$\theta_{ij}^{\Delta l} \le \angle (V_i V_j^*) \le \theta_{ij}^{\Delta u} \ \forall (i,j) \in E$$

### Stochastic Optimal Power Flow in Power-Voltage Variables

A complete mathematical model is as follows,

variables:

$$S_k^g \ \ \forall k \in G$$
 - generator complex power dispatch (3.9)

$$V_i \ \ \forall i \in N$$
 - bus complex voltage (3.10)

$$S_{ij} \ \ \forall (i,j) \in E \cup E^R$$
 - branch complex power flow (3.11)

minimize: 
$$\sum_{k \in G} c_{2k}(\Re(S_k^g))^2 + c_{1k}\Re(S_k^g) + c_{0k} \tag{3.12}$$

subject to:

$$\angle V_r = 0 \ \forall r \in R \tag{3.13}$$

$$S_k^{gl} \le S_k^g \le S_k^{gu} \ \forall k \in G \tag{3.14}$$

$$v_i^l \le |V_i| \le v_i^u \quad \forall i \in N \tag{3.15}$$

$$\sum_{k \in G_i} S_k^g - \sum_{k \in L_i} S_k^d - \sum_{k \in S_i} (Y_k^s)^* |V_i|^2 = \sum_{(i,j) \in E_i \cup E_i^R} S_{ij} \ \forall i \in \mathbb{N}$$
 (3.16)

$$S_{ij} = (Y_{ij} + Y_{ij}^c)^* \frac{|V_i|^2}{|T_{ij}|^2} - Y_{ij}^* \frac{V_i V_j^*}{T_{ij}} \quad \forall (i, j) \in E$$
(3.17)

$$S_{ji} = (Y_{ij} + Y_{ji}^c)^* |V_j|^2 - Y_{ij}^* \frac{V_i^* V_j}{T_{ij}^*} \quad \forall (i, j) \in E$$
(3.18)

$$|S_{ij}| \le s_{ij}^u \ \forall (i,j) \in E \cup E^R$$
(3.19)

$$|I_{ij}| \le i_{ij}^u \ \forall (i,j) \in E \cup E^R \tag{3.20}$$

$$\theta_{ij}^{\Delta l} \le \angle (V_i V_j^*) \le \theta_{ij}^{\Delta u} \ \forall (i,j) \in E$$
 (3.21)

Part III

Library

# **Network Formulations**

### 4.1 Network Formulations

### **Type Hierarchy**

We begin with the top of the hierarchy, where we can distinguish between current-voltage and power-voltage formulations

AbstractACPModel <: AbstractPowerModel AbstractIVRModel <: AbstractPowerModel

### **Power Models**

Each of these forms can be used as the model parameter for a PowerModel:

ACPPowerModel <: AbstractACPForm IVRPowerModel <: AbstractIVRModel