TD1 :: Beta-reduction

Functional programming

Question 1. *Decide which of these words are correct lambda-terms*

1. $\lambda x.(xx)$ 2. $(x.\lambda y)$ 3. λ 4. $\lambda x.\lambda y.\lambda z.(xz)(yz)$

Question 2. *Perform one step of β-reduction*

1. $(\lambda x.x)(\lambda y.y)$

2. $(\lambda x.\lambda y.x)(\lambda z.z)$

3. $(\lambda x.(xx))(\lambda x.(xx))$

4. $F(\Delta\Delta)$, *where* $F = \lambda x.\lambda y.y$ *and* $\Delta = \lambda x.(xx)$

5. $\Delta(\mathcal{II})$, *where* $\mathcal{I} = \lambda x.x$

Question 3 (Boolean type)**.** *Let us define three lambda terms representing TRUE, FALSE and CONDITIONAL:*

$$\mathcal{T} = \lambda x.\lambda y.x, \quad \mathcal{F} = \lambda x.\lambda y.y, \quad \text{Cond} = \lambda x.\lambda y.\lambda z.xyz$$

Express the following logical functions in terms of $\mathcal{T}, \mathcal{F}, \text{Cond}$:

1. $\neg$ 2. $\wedge$ 3. $\vee$

Question 4 (Binary trees)**.** *Let us define three lambda terms representing tree constructor, left child, right child:*

$$\text{Cons} = \lambda s_1.\lambda s_2.\lambda f.f s_1 s_2, \qquad fg = \lambda s.s\mathcal{T}, \qquad fd = \lambda s.s\mathcal{F}$$

Simplify the following lambda terms using β-reduction:

1. $\text{Cons}\,A\,B$

2. $\text{Cons}\,C\,\text{Cons}\,A\,B$

3. $fd\,fd\,\text{Cons}\,C\,\text{Cons}\,A\,B$

Question 5 (Barendregt integer encoding)**.** *Let us define two lambda terms corresponding to integer numbers generators:*

$$\overline{0} = \mathcal{I}, \qquad \text{Succ} = \lambda x.\text{Cons}\,\mathcal{F}\,x$$

Evaluate the following expressions using β-reduction:

1. Pred   2. EstZero