Multi-Objective Electric Vehicle Scheduling Considering Customer and System Objectives

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***Abstract*—Electric vehicle (EV) scheduling is a multi-objective optimization problem with conflicting system and customer interests. They bear the potential to support the grid while providing incentives to the customers through energy transactions, demand response and grid support. Vehicle-to-grid operations provide the customer with attractive avenues for earning revenues but degrade the battery life. Efficient and economical solutions require a balance between customer incurred costs, battery degradation costs and system health. In this paper, the relationships between these objectives have been explored using a multi-objective optimization technique called augmented epsilon-constraint method (AUGMECON). The Pareto optimal solutions will provide day-ahead strategies for coordinating electric vehicles which can then be used for selecting mutually beneficial outcomes.**

***Index Terms*—battery degradation, electric vehicles, multi-objective optimization, vehicle-to-grid**

NOMENCLATURE

η_{ch}	Battery charging efficiency (0.92).
η_{dch}	Battery discharging efficiency (0.90).
λ^t	Electricity rate at time instant t.
v_i^t	Binary optimization variable.
ε	Positive constant of AUGMECON $\in \left[10^{-6}10^{-3}\right]$
$B_{cap,i}$	Battery capacity of vehicle i (kWh).
bat_{life}	Battery lifetime in years = 10 years or 5000 cycles.
C_{bat}	Battery cost in \$ (\$$300/kWh$).
C_{labor}	Labor cost for battery replacement (\$240).
d	Linear battery degradation cost-intercept $= 6.41 \times 10^{-6}$
DOD	Depth of discharge (80%) of battery at end of life.
e_j	Equality constraint parameter.
$E_{\Delta t}^{dch}$	Energy discharged in Δt (kWh).
E_{bat}^t	Energy stored in the battery at time t.
$E_{i,req}$	Energy required for full charge for vehicle i (kWh).
f_j	Objective function j.
$grid_j$	Number of gridpoints of objective.
$iter_j$	Iteration parameter.
m	Linear battery degradation cost-slope parameter $= 1.59 \times 10^{-5}$.
N_{veh}	Total number of vehicles.
P_{avg}	Average load demand $= \sum_{t=1}^{24} P_{res}^t/24$ (kW).
$P_{ch,max}$	Maximum charging power rating $\in \{1.44, 6.66\}$ (kW).
$P_{ch,min}$	Minimum charging power rating $\in \{0\}$ (kW).
$P_{dch,max}$	Maximum discharging power rating $\in \{0\}$ (kW).
$P_{dch,min}$	Minimum discharging power rating $\in \{-1.44, -6.66\}$ (kW).
$P_{i,veh}^t$	Vehicle i load demand at time t (kW).
P_{peak}	Forecasted peak load demand (kW).
P_{res}^t	Residential load demand at time t (kW).
P_{sys}^t	Total system load demand at time t (kW).
$range_j$	Range of objective function.
S	Solution space for variable x.
s_j	Positive slack variable.
$SoC_{i,avg}^t$	Average SOC of battery of vehicle i at time t.
$SoC_{i,max}$	Maximum SOC of battery of vehicle i (100%).
$SoC_{i,min}$	Minimum SOC of battery of vehicle i (20%).
t	Time instant.
$t_{i,avail}$	Time available for charging vehicle i.
ub_j	Upper bound of objective.
x	Optimization variable.
$x_{i,ch}^t$	Charging power of vehicle i at time t.
$x_{i,dch}^t$	Discharging power of vehicle i at time t.

The authors gratefully acknowledge the financial support of the *NSF* under the project ECCS-1068996.

I. INTRODUCTION

Commercialization and widespread adoption of electric vehicles (EVs) faces technical challenges along-with the need for effective transactive energy models, especially to make vehicle-to-grid (V2G) a feasible, lucrative option [1]- [3]. Despite their advantage as a green, higher well-to-wheel efficiency vehicle with grid support capability especially with renewable energy penetration, seamless integration of EVs into the evolving power grid requires further research. Uncontrolled EV charging can result in uneconomical operation due to network congestion, losses due to thermal stresses, assets reinforcement and inadequacy due to higher peak load demand. Therefore, controlled EV scheduling is essential for reliable and efficient grid operation.

Given the advancements in battery technology and power electronics devices, EVs may be used as active sources instead of passive loads in the power system. Participation in the ancillary services market, supporting intermittent renewable energy resources (DERs), reducing ramping requirements of the DERs and active demand response are some of the benefits of EVs in aggregated and/or distributed paradigms [4].

Direct involvement of customer also necessitates provision for individual comfort and incentives for EV utilization. This adds another facet to the control and optimization of EV scheduling. Financial incentives may be leveraged against controlled charging/discharging of EVs in a smart grid environment. The implicit inclusion of customers in the control framework, makes EV scheduling a socio-economic problem with conflicting utility and customer objectives. Thus, there is a need to derive mutually-beneficial alternative solutions that help the customer choose appropriate schemes. This paper explores this idea and seeks to provide multiple optimal solutions through co-optimized system and customer objectives.

Past literature has studied various aspects of multi-objective optimization of electric vehicles from customer vs. system operator perspectives [5]- [12]. Most studies on multi-objective optimization have concentrated on the power management side of EVs. In general, computational methods like genetic algorithms (NSGA-II) or particle swarm optimization (ES-PSO) are used for determination of efficient solutions in multi-objective optimization (MOO) problems. In [6], annual traffic at fast charging stations is maximized while minimizing distribution system energy losses and annual investments. Artificial intelligence based techniques reduce the computational overhead but the global optimality is not necessarily ensured. This paper intends to bridge this gap and proposes to use a mathematical programming technique for implementing multi-objective optimization, thereby extending the work in [6] - [7].

The aim of this study is to find multiple optimal solutions that co-optimize customer's and system operator's (SO) objectives using a centralized EV control, optimization and scheduling (COS) scheme. The algorithm implemented in this paper is based on augmented epsilon-constraint (AUGMECON) method. It addresses the needs of the system operator to control the peak load and that of the customer who is financially motivated while being concerned with battery life during V2G operations. The COS scheme is expected to provide a look ahead into optimal solution choices for the available vehicle set. In real time, this may be used to guide EV scheduling directly or indirectly through change in pricing schemes or by providing charging choices to customers. With the development of transactive business models, such schemes would help in devising effective policies for independent participation of EV aggregators in the electricity markets.

The main contributions of this paper include:

1) Identification of conflicting and in-line objectives.
2) Implementation of multi-objective optimization for EV scheduling for a residential parking lot using AUGMECON method.
3) Co-optimization of customer and utility objectives.

It is important to note that this work has been presented to build the foundation of multi-objective optimization for EV scheduling in a day-ahead scenario. Uncertainty in vehicle availability, electricity prices and real-time implementation will be targeted in the future implementations of this work.

II. Problem Formulation and Methodology

The parking lot controller (PLC) implements the centralized COS scheme. The PLC receives vehicle profile information (VPI) and the vehicle characteristic information (VCI) via the human-machine interface. For each vehicle, the VPI and VCI are expressed as tuples: $\langle B_{i,cap}, SOC_{i,min}, SOC_{i,max}\rangle$, and $\langle t_{i,avail}, SOC^0_{i,avg}\rangle$ respectively. After receiving the information, optimization is run by the control center. The resulting schedules and cost information, and load profiles are sent to each vehicle and system operator respectively. Based on the objectives chosen each customer would receive analytics including cost/revenues information, battery degradation costs and impact on system health. It would also provide the system operator with information pertaining to energy available for transaction, expected hourly load and impact assessment of different schemes (Fig. 1).

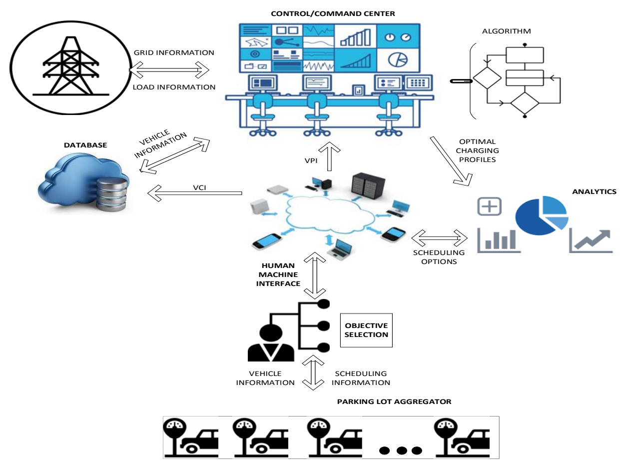

Fig. 1. Control, optimization and scheduling framework (COS)

A. Objective Functions

The objectives considered in this work include:

- Battery degradation costs as a result of cyclic charging and discharging in grid-to-vehicle (G2V) and V2G modes
- Customer costs/revenues as a result of energy transacted with the grid
- Valley filling and peak shaving to flatten the residential load profile

1) Battery Degradation Cost Model (BDCM): V2G implementations require cyclic charging and discharging that may affect the EV battery life adversely, thus incurring costs to the customers [14]. EV battery accounts for a major cost component of the vehicle and thus requires special consideration. Battery aging is associated with power fade and capacity fade that are affected by temperature, open-circuit voltage, C-rate and depth-of-discharge of the battery. Being highly non-linear functions, they impose computational challenges. Thus, a simplified linear lifetime battery degradation cost (BDC) model has been adopted from [7] and [15]. BDC Ψ_i^t for each vehicle i at time instant t is composed of two components: 1) SOC related cost $\Psi_{i,t}^{SOC}$ and 2) DOD related cost $\Psi_{i,t}^{DOD}$, as explained in (1). Equations (2) and 3 define these two cost components for all $x \in X$ where $x = \{\{x_{ch,i}^t, x_{dch,i}^t\} : i \in [1, N_{veh}], t \in [1, t_{avail}]\}$. A capacity fade of 20% at the end of a ten year lifetime of a Li-ion battery has been assumed in this study.

$$\Psi_i^{deg}(x) = \sum_{t=1}^{t_{i,avail}} \Psi_i^t = \sum_{t=1}^{t_{i,avail}} \Psi_{i,t}^{SOC} + \sum_{t=1}^{t_{i,avail}} \Psi_{i,t}^{DOD} \tag{1}$$

$$\Psi_{i,t}^{SOC} = C_{bat} \frac{m \cdot SOC_{avg,t} - d}{CF_{max} \cdot bat_{life} \cdot 8760} \tag{2}$$

$$\Psi_{i,t}^{DOD} = \frac{C_{bat} \cdot B_{cap} + C_{labor}}{bat_{life} \cdot B_{cap} \cdot DOD} E_{\Delta t}^{dch} \tag{3}$$

where,

$$SOC_{i,avg}^{t+1} = SOC_{i,avg}^t + \frac{x_{i,ch}^t + x_{i,dch}^t}{B_{cap}} \tag{4}$$

$$E_{\Delta t}^{dch} = E_{t-1}^{bat} - E_t^{bat} \tag{5}$$

The first objective function is defined as:

$$\underset{x}{\operatorname{argmin}} f_1(x) = \sum_{i=1}^{N_{veh}} \Psi_i(x) \tag{6}$$

2) Customer Charging-Discharging Cost Model (CCDM): A customer can trade off between the costs incurred during charging with revenues earned through V2G capability. In this study, charging and discharging decisions are based on a net-metering policy. The total costs incurred/revenues earned by the customer are represented by Ψ_i^{rev}. The mathematical formulation of the objective is given in (7) and (8).

$$\Psi_i^{rev}(x) = \sum_{t=1}^{t_{i,avail}} \left(\frac{x_{ch,i}^t}{\eta_{ch}} - x_{dch,i}^t \cdot \eta_{dch} \right) \left(\lambda^t \right) \tag{7}$$

$$\underset{x}{\operatorname{argmin}} \; f_2(x) = \sum_{i=1}^{N_{veh}} \Psi_i^{rev}(x) \tag{8}$$

3) Valley-filling Model (VFM): An inadvertent increase in peak load demand due to uncoordinated EV charging results in uneconomical system operation due to 1) increase in network stresses and 2) costly generation. Shifting of load demand to the valley period to create a more level profile, serves the interests of the SO and keeps a check on electricity price inflation during peak hours. Thus, by scheduling the vehicles during valley period, system health is maintained. It is assumed that the initial residential load profile does not violate any system constraints. The valley-filling objective is defined as:

$$\underset{x}{\operatorname{argmin}} \; f_3(x) = \sum_{t=1}^{24} \left(P(x)_{sys}^t - P_{avg} \right)^2 \tag{9}$$

$$P(x)_{sys}^t = P_{res}^t + \sum_{i=1}^{nVeh} P_{i,veh}^t \cdot \Delta t \qquad \forall t \tag{10}$$

where

$$P_{i,veh}^t = \frac{x_{ch,i}^t}{\eta_{ch}} - x_{dch,i}^t \cdot \eta_{dch} \tag{11}$$

Equation (10) defines the system total load at time t and (11) gives the individual vehicle load at any instant. A 1 hour time step Δt has been considered in this study.

B. Constraints

Vehicle scheduling is constrained by system conditions, battery requirements, charger limits and customer convenience. This makes it a constrained optimization problem. The following constraints complete the problem definition.

$$P_{ch,min} \le x_{i,ch}^t \le P_{ch,max} \tag{12}$$

$$P_{dch,min} \le x_{i,dch}^t \le P_{dch,max} \tag{13}$$

$$SoC_{i,min} \le SOC_{i,avg}^t \le SoC_{i,max} \tag{14}$$

$$\sum_{t=1}^{t_{i,avail}} \left(x_{i,ch}^t - x_{i,dch}^t \right) \Delta t = E_{i,req} \tag{15}$$

$$\sum_{i=1}^{nVeh} \left(\frac{x_{ch,i}^t}{\eta_{ch}} - x_{dch,i}^t \cdot \eta_{dch} \right) \le \left(P_{peak} - P_{res}^t \right) \tag{16}$$

The minimum and maximum charging/discharging limits for a charger are given by (12) and (13) respectively. The usable battery capacity is constrained by (14). (15) gives the required battery energy before departure. The total load is maintained below the residential peak load during the day using (16).

C. Augmented ϵ-constraint method for multi-objective optimization

Unlike mathematical programming in single objective, the objectives in multi-objective optimization may not be optimized simultaneously. The concept of optimal solution is thus replaced by the most preferred solution under Pareto optimality or efficiency conditions. A solution is called Pareto optimal if it cannot be improved without deteriorating the performance of at least one of the other objectives. In this study, MOO has been implemented using the augmented ϵ constraint (AUGMECON) method (Fig. 2) which is an improvement on the traditional ϵ-constrain method [16]. It provides a smoother Pareto front in comparison to weighted-sum approach used in [17].

AUGMENCON method for solving MOO for vehicle scheduling is described below:

$$\underset{x,s_2}{\operatorname{argmin}} \left(\underbrace{f_1(x)}_{\text{term-1}} - \underbrace{\varepsilon\left(s_2\right)}_{\text{term-2}} \right) \qquad x \in S \tag{17}$$

subject to :

$$f_2(x) \; + \; s_2 \; = \; e_2 \tag{18}$$

where

$$e_2 = ub_2 - \frac{(iter_2 \times range_2)}{grid_2} \tag{19}$$

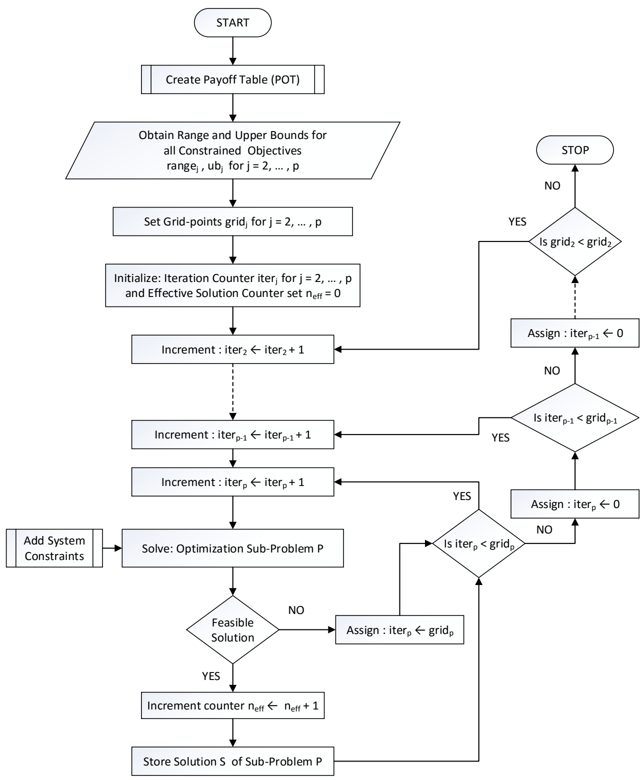

Fig. 2. Flowchart for the AUGMECON method for MOO

AUGMECON optimizes one of the objectives (key objective) while varying the other objectives within a range defined by the payoff table. The payoff table is calculated using a lexicographic method. Each row corresponds to an objective f_j. The optimal function value for objective f_j^* enters column j. The value of the other $j-1$ objectives are calculated at the optimal argument vector (x^*) of objective j and entered into the corresponding columns. The range of an objective is the difference between minimum and maximum values for the row corresponding to the objective. This range is divided into sections called the grid points. e_j increments by a step size of $range_j/grid_j$ at each iteration as the objective moves from its maximum to minimum value. This is identified as the sub-problem solved at each iteration (17-19).

2-way optimization was conducted using 20 grid-points resulting in 20 sub-problems. The value of ϵ was set at 10^{-6}.

Remark: An increase in the number of grid-points is associated with an increase in computation overhead. Here, parallel programming approach could prove beneficial.

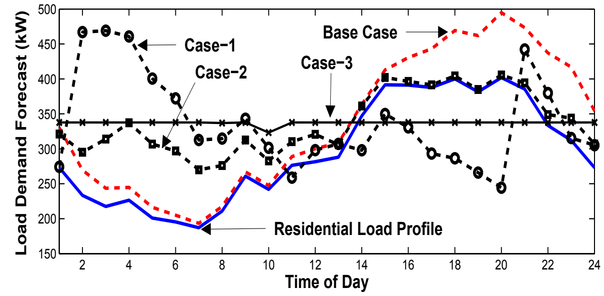

Fig. 3. Optimal load demand forecast under individual objectives

III. Simulation Results and Observations

A typical summer day load profile and 3-tier time-of-use pricing (Table I) were obtained from PGE and subsequently scaled up for simulating the residential parking lot [18]. For better accuracy of TOU scheme, tiers were designed according to the clustering method proposed in [19]. Driving profiles obtained from the NHTS database [20] were used to serve 60% EV penetration in a 245 house residential complex. Battery (5 sizes) and charger assignment (Type I or II) was adopted from [21]. Due to the absence of geo-spatial distribution of the vehicles in the distribution system cumulative effect of vehicles was considered. The optimization model was solved using Gurobi optimization software.

TABLE I
TOU Rate Structure [18]

Rate Type	Energy Charge (¢/kWh)		
TOU 3-tier	13.101	20.779	32.306
	2:00-10:00	11:00-13:00 21:00-1:00	14:00-20:00

The following cases have been discussed in assessing the viability of multi-objective optimization:

- Base Case: Uncoordinated G2V mode of EV charging immediately upon arrival
- Case-1 : Customer level : Minimizing customer charging cost (or maximizing revenues) in V2G/G2V modes (CCDM)
- Case-2 : Customer level : Minimizing battery degradation cost in V2G/G2V modes (BDCM)
- Case-3 : System level: Valley filling in V2G/G2V modes (VFM)
- MOO-1 : Battery degradation vs. Customer cost/revenues
- MOO-2 : Valley filling vs. Customer cost/ revenues
- MOO-3 : Valley filling vs. Battery degradation

Fig. 3 shows the resulting load profiles for cases 1-3 with independent implementation of the three objective functions. The following observations can be made:

- *O1:* Uncontrolled EV charging results in an inadvertent increase in peak load demand and leaves the valley period unused.
- *O2:* Implementation of static time-of-use rates without imposing any system-level constraints lead to creation

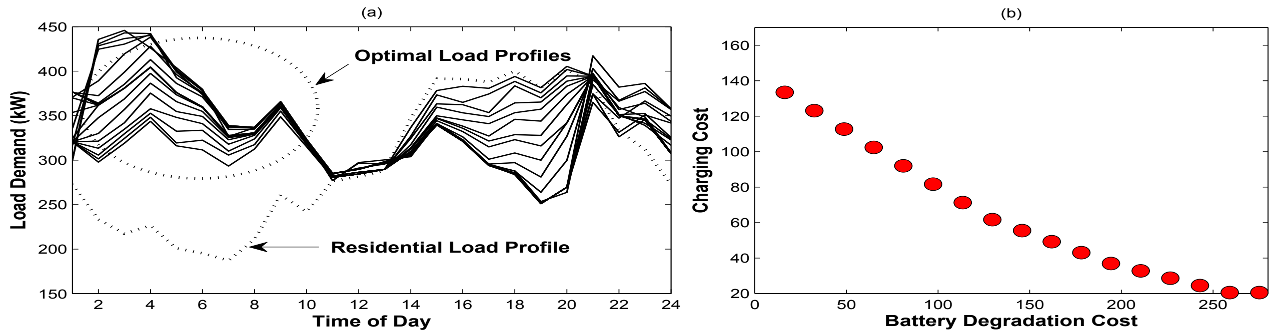

Fig. 4. Results of customer charging cost/revenue vs. battery degradation cost. (a) Load demand profile (b) Pareto front

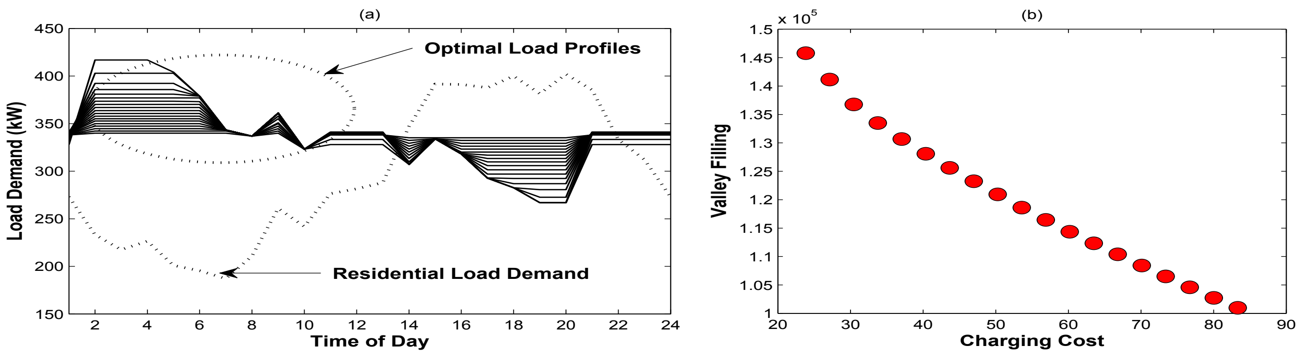

Fig. 5. Results of customer charging cost/revenue vs. valley filling. (a) Load demand profile (b) Pareto front

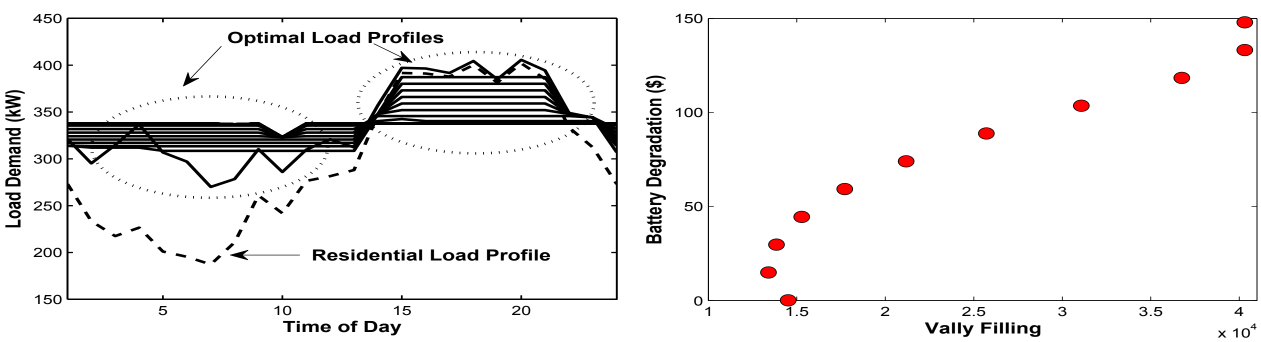

Fig. 6. Results of battery degradation costs vs. valley filling. (a) Load demand profile (b) Pareto front

of two peaks, higher than the peak of the original load profile. This is highly undesirable and defeats the purpose of time-of-use rates that are intended to shift load to the valley period. Even though V2G operation results in decrease in the load during the original peak hours, new peaks offset its purpose.

- *O3:* Minimizing battery degradation costs limits V2G operation and thus results in lower customer revenues. Peak shaving is not observed although, load profile closely follows the residential load demand during peak hours. Charging was observed during the valley period.
- *O4:* V2G and G2V operations in the valley filling mode result in a flat load profile. This is the most desirable load profile and is independent of the effect of pricing structure.

Figs. 4-5 show the resulting load profiles and Pareto fronts for cases MOO-1, MOO-2 and MOO-3. A smooth Pareto front was obtained for each of the conflicting set of objectives. It was observed that battery degradation and valley filling are in-line with each other. The following observations can be made:

- *O5 (MOO-1):* BDCM and CCDM are in conflict with each other (Fig. 4). This is due primarily to the fact that while minimizing charging cost entails more V2G operation (to earn revenues), it also results in increasing the battery degradation costs. Therefore, the Pareto front provides the customer with a choice to operate at an optimal point weighing financial profits against battery degradation.
- *O6 (MOO-2):* In order to minimize charging cost low price tier was used to charge most vehicles while most vehicles discharged during the high price tier. This distorts the load profile as shown in case-3 in Fig. 3. Since

this is undesirable by the system operator, valley filling objective tries to balance this charge/discharge operation. Therefore, customer charging cost/revenues and valley filling conflict each other (Fig. 5). The Pareto front helps the system operator and customer reach a consensus on the point of operation that is beneficial to the customer without being detrimental to the system.
- *O7 (MOO-3):* Valley filling (VFM) and battery degradation (BDCM) both limit V2G operations. While the former levelizes the load profile, the latter follows the initial residential load profile during peak hours. Thus, they are not in conflict with each other and cannot provide a Pareto front. it is important to note that battery degradation costs are higher when valley filling is implemented alone.

Pareto front solutions can provide the system operator with tools to design effective incentives to motivate the customers. Consequently, the system integrity may be preserved while providing financial benefits to the customers through V2G operations. Weighing customer benefits against system requirements can provide better solutions to EV scheduling problem. A MOO approach, as the one discussed above, provides an efficient technique to understand these dynamics and formulate plans accordingly.

IV. Conclusion

This paper explores and analyzes the dynamics between customer and system objectives. Customer and system objectives have been tackled simultaneously in a multi-objective optimization framework using AUGMECON. Co-optimal solutions have been identified for the mutual benefit of the two entities. The proposed day-ahead scheduling scheme would provide valuable information to the system operator about the EV resources. This would also help the customers understand and choose appropriate scheduling according to their needs. Furthermore, Pareto optimal solutions may be used to devise effective policies for customer engagement. Computational improvements and real-time implementations of this MOO based control and scheduling scheme will be investigated in future.

Acknowledgment

The authors gratefully acknowledge the financial support of the National Science Foundation under the project ECCS-1068996.

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