A310 REVIEW
Problem #1

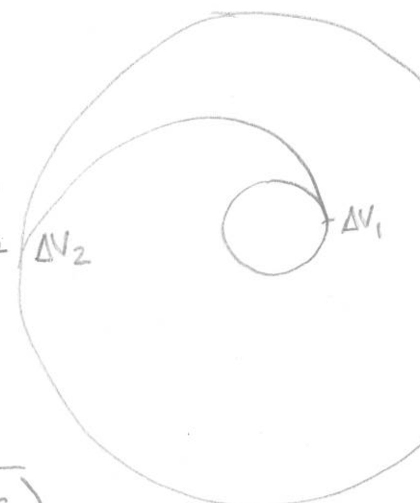
NASA wants to place a communications satellite in a geosynchronous circular orbit with an altitude of 35782 km and inclination of 28.5°. The satellite is currently in a circular orbit with an altitude of 200 km and inclination of 28.5°. What is the ΔV needed and how long will it take?.

$R_1 = 6578.137$ km

$R_2 = 42160.137$ km

$a_T = \dfrac{R_1 + R_2}{2} = 24369.137$ km

$\varepsilon_T = -\dfrac{\mu}{2a_T} = -8.1784 \ km^2/s^2$



$V_1 = \sqrt{\dfrac{\mu}{R_1}} = 7.784$ km/s

$V_2 = \sqrt{\dfrac{\mu}{R_2}} = 3.075$ km/s

$V_{T_1} = \sqrt{2\left(\dfrac{\mu}{R_1} + \varepsilon_T\right)} = 10.239$ km/s

$V_{T_2} = \sqrt{2\left(\dfrac{\mu}{R_2} + \varepsilon_T\right)} = 1.5975$ km/s

$\Delta V_1 = |V_{T_1} - V_1| = 2.455$ km/s

$\Delta V_2 = |V_{T_2} - V_2| = 1.477$ km/s

$\Delta V_{TOT} = \Delta V_1 + \Delta V_2 = 3.932$ km/s

$TOF = \pi\sqrt{\dfrac{a_T^3}{\mu}} = 18929.6$ sec

$= 5.258$ hrs

$\Delta V_1 =$ 2.455 km/s

$\Delta V_2 =$ 1.477 km/s

$\Delta V_{TOTAL} =$ 3.932 km/s

TOF = 18929.6 sec

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Problem #2

Due to a steering malfunction on the launch vehicle, your communications satellite ended up in a circular parking orbit (altitude =200 km) with an inclination of 45°. What is the ΔV needed to change the inclination to 28.5° for the desired parking orbit (altitude =200km)? Where should the maneuver occur?

$R = 6578.137$ km $\quad\quad \Delta i = 16.5°$

$V = \sqrt{\frac{\mu}{R}} = 7.784$ km/s

$\Delta V_s = 2V_i \sin\left(\frac{16.5}{2}\right) = 2.234$ km/s

ΔV = 2.234 km/s

Where? AN OR DN (equator)

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Problem #3

How many burns are required for the most efficient method to maneuver a satellite from its current parking orbit (altitude=200km, i = 45°) to the desired geosynchronous orbit (altitude=35782 km, i=28.5°)?

2 burns

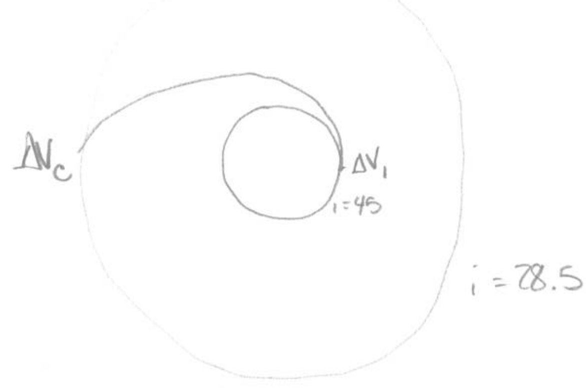
Given: $V_1 = 7.7843$ km/s
$V_2 = 3.0748$ km/s $= V_f$
$\varepsilon_t = -8.1784$ km^2/s^2

$$V_{T_A} = \sqrt{2\left(\frac{\mu}{R_2} + \varepsilon_t\right)}$$
$$= 1.598 \text{ km/s} = V_i$$

2nd burn

What is the ΔV_c for this scenario?

$$\Delta V_c^2 = V_i^2 + V_f^2 - 2V_iV_f\cos(16.5°)$$
$$\Delta V_c = 1.608 \text{ km/s}$$



$\Delta V_C =$ 1.608 km/s

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Problem #4

The communications satellite has made it to the final orbit but is malfunctioning. NASA controllers are going to <u>maneuver the satellite</u> to rendezvous with the Space Shuttle. How long do they need to wait before they begin their maneuver? Draw a picture showing the relative starting positions. Label the interceptor and target.

Space Shuttle	Comm. Satellite
R = 6578 km	R = 42160 km
e = 0	e = 0
u = 90°	u = 180°

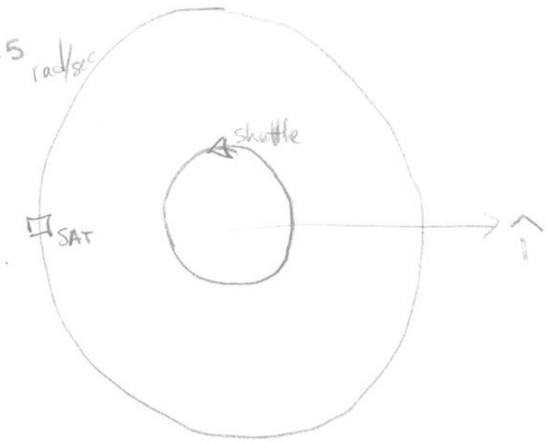
$a_T = \frac{R_1 + R_2}{2} = 24369 \text{ km}$

$\phi_{int} = 90° = \frac{\pi}{2} \text{ rad}$

$\omega_{TGT} = \sqrt{\frac{\mu}{R_{TGT}^3}} = 7.2932\times10^{-5} \text{ rad/sec}$

$TOF = \pi\sqrt{\frac{a_T^3}{\mu}} = 18929.43 \text{ sec}$

$\omega_{INT} = \sqrt{\frac{\mu}{R_{INT}^3}} = 1.1834\times10^{-3} \text{ rad/sec}$



$\alpha_{lead} = \omega_{TGT} \times TOF$

$\alpha_{lead} = 1.3806 \text{ rad}$

$\phi_{final} = \pi - \alpha_{lead} = 1.761 \text{ rad}$

$WT = \underbrace{\frac{\phi_f - \phi_i}{\omega_{TGT} - \omega_{INT}}}_{} \pm \underbrace{\frac{2\pi k}{\omega_{TGT} - \omega_{INT}}}_{}$

$= -171.3108 \pm (-5658.142) = 5486.9 \text{ sec}$

W.T. = 5486.9 sec

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Problem #5

NASA controllers miscalculated their burn and the satellite ended up 10° ahead of the Shuttle in the proper orbit (R=6578 km, i=28.5°, e = 0). What is the semi-major axis of the phasing orbit to complete the rendezvous? Assume the Space Shuttle is the interceptor and the satellite your target.

$$\phi_{init} = 10^\circ \qquad \omega_{TGT} = \sqrt{\frac{\mu}{R^3}} = 1.1834\times10^{-3}\ \text{rad/sec}$$

$$\phi_{travel} = 2\pi - \phi_{init} = 350^\circ = 6.10865\ \text{rad}$$

$$a_{phas} = \sqrt[3]{\mu\left(\frac{\phi_{trav}}{2\pi\,\omega_{TGT}}\right)^2} = 6455.614\ \text{km}$$

↳ too small!

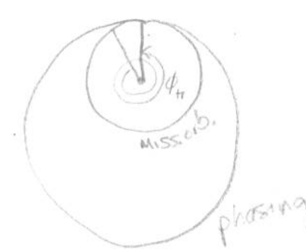
could use big orbit

$$\phi_{travel} = 4\pi - \phi_{init} = 710^\circ = 12.392\ \text{rad}$$

$$a_{phas} = 10344.96\ \text{km}$$

How long will this maneuver take to complete?

$$Per_{phas} = 2\pi\sqrt{\frac{a_{ph}^3}{\mu}} = 10471.4\ \text{sec}$$
$$= 2.909\ \text{hrs}$$



When can this maneuver take place?

Any Time

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Problem #6

You're shopping around for propellant for a rocket engine you've come across. The engine has an inert structural mass of 37kg and can hold enough propellant so that the total mass would be 85kg. If you need to perform a 2.5 Km/sec burn carrying 13kg of payload what would be the minimum Isp you would need from your propellant.

$m_i = 85 + 13 = 98 \text{ kg}$

$m_f = m_i - m_{prop} = m_i - (85-37) = 50 \text{ kg}$

$g_0 = 9.81 \text{ m/s}$

$$\Delta V = I_{sp} \, g_0 \ln\left(\frac{98}{50}\right)$$

$$2500 = I_{sp} \, g_0 \ln\left(\frac{98}{50}\right)$$

$$I_{sp} = 378.7 \text{ sec}$$

Isp: ____________

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Problem #7

Briefly describe the 2 types of orbital perturbations discussed in class as well as which orbital elements they affect. Describe how you might use each of the perturbations for a mission.

Drag : a ↓ e ↓ $\mathcal{E}$ ↓
J2 : ω, Ω change predictably $\mathcal{E}$ stays the same

other COE's unaffected

Drag - makes LEO satellites reenter when mission complete, less space junk
J2 - Molniya at right incl to have ω not move,
Sun-synchronous at right incl. to have Ω move at same rate as earth around sun
- constant shadows

Also, list the advantages and disadvantages to staging your launch vehicle.

↳ complexity

p 595

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Problem #8

The ISS is in a slightly elliptic orbit, e=0.0020, with a semi-major axis of 7500 km. It is currently at perigee. Where will it be in 30 minutes?

$\nu_i = 0$ rad

$E_i = M_i = 0$ rad

$TOF = 30$ min $= 1800$ sec

$n = \sqrt{\frac{\mu}{a^3}} = 9.7202 \times 10^{-4}$ rad/sec

$M_f = M_i + n(TOF)$

$M_f = 1.7496$ rad

guess

$E_f = M_f + e \sin E_f$

$= 1.7496 + .002 \sin 1.7496 = 1.7516$

$= 1.7496 + .002 \sin 1.7516 = 1.7516$

$E_f = 1.7516$ rad

$$\cos \nu_f = \frac{\cos E_f - e}{1 - e \cos E_f}$$

$\nu_f = 1.7535$ rad ✓ plane check: E_f in first ½ plane so is ν_f

$= 100.47°$

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Problem #9

The ISS is in a slightly elliptic orbit, e=0.0020, with semi-major axis of 8000 km. It is currently at apogee. How long will it take to reach a true anomaly of 315°?

$\nu_i = 180° = \pi \text{ rad}$

$\nu_i = E_i = M_i$ (because π)

$M_i = \pi \text{ rad}$

$\nu_f = 315° = 5.4978 \text{ rad}$

$\cos E_f = \dfrac{e + \cos \nu_f}{1 + e \cos \nu_f}$

$E_f = .78398 \text{ rad}$

plane check

$E_f = 2\pi - E_f = 5.4992 \text{ rad}$

$n = \sqrt{\dfrac{\mu}{8000^3}} = 8.8234 \times 10^{-4} \text{ rad/sec}$

$M_f = E_f - e \sin E_f$

$= 5.5006 \text{ rad}$

$M_f - M_i = n(TOF)$

$5.5006 - \pi = 8.8234 \times 10^{-4} (TOF)$

$\boxed{TOF = 2673.608 \text{ sec}}$

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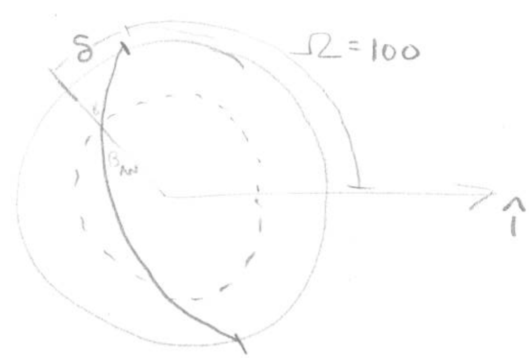
Problem #10

A Russian Progress cargo ship is going to be launched from Baikonur cosmodrome (46 N, 63 E) to resupply the ISS (R=6628 km, i = 51.6°, Ω=100°, e=0). If Baikonur is at the vernal equinox, determine the LWST and launch azimuth for the **ascending** node. How long do we need to wait before we can launch into the ascending node? (HINT: Draw a picture… it might help)

$L_o = 46^\circ$

$LST = 0^\circ$ $\qquad$ $\alpha = 51.6^\circ$



$$\sin\gamma = \frac{\cos\alpha}{\cos L_o}$$

$$\gamma = \boxed{44.975^\circ = \beta_{AN}}$$

$$\cos\delta = \frac{\cos\gamma}{\sin\alpha}$$

$$\delta = 25.489^\circ$$

$$LWST_{AN} = \Omega + \delta = \boxed{125.5^\circ}$$

$$WT = LWST - LST = 125.5^\circ = 8.37 \text{ hrs}$$

$\sin\gamma = \frac{\cos\alpha}{\cos L_0}$

$\gamma = 63.4^\circ$

A310 REVIEW
Problem #11

A Russian Progress cargo ship is going to be launched from Baikonur cosmodrome (46 N, 63 E) to resupply the ISS (R=6628 km, i = 51.6°, Ω=100°, e=0). What is the ΔV_{Needed} to launch the Progress at the descending node opportunity? Your launch azimuth for the descending node opportunity is 116.6°.

$V_{BO} = \sqrt{\frac{\mu}{R_{BO}}} = 7.755$ km/s

$R_{BO} = 6628$ km

$R_L = R_\oplus$

$\beta = \gamma = 63.4^\circ$

$\phi = 0^\circ$

$\bar{V}_{LS} = .4651 \cos L_0 \hat{E} = .3231 \hat{E}$ km/s

$\bar{V}_{LG} = \sqrt{\frac{2\mu (R_{BO} - R_L)}{R_L R_{BO}}} \hat{Z} = 2.1707 \hat{Z}$ km/s

$$\overline{\Delta V}_{needed} = \begin{bmatrix} -V_{BO}\cos\beta \\ V_{BO}\sin\beta - .3231 \\ 0 + 2.1707 \end{bmatrix} \text{km/s} = \begin{bmatrix} -3.472 \\ 6.611 \\ 2.171 \end{bmatrix} \text{km/s}$$

$$\Delta V_{needed} = \sqrt{(-3.472)^2 + (6.611)^2 + (2.171)^2} = 7.776 \text{ km/s}$$

If $V_{\text{Losses other than gravity}}$ is 0.9 km/s, what is ΔV_{Design} for this mission?

$$\Delta V_{design} = \Delta V_{needed} + \Delta V_{LOSS} = 8.676 \text{ km/s}$$

A satellite is in a circular polar orbit with an altitude of 250 km and a RAAN = 100°. Due to mission requirements, you need to change the RAAN to 135°. What is the ΔV required for this maneuver?

$R = R_{\oplus} + 250 = 6628.137$ km

$V = \sqrt{\frac{\mu}{R}} = 7.755$ km/s

$\theta = 135 - 100 = 35°$

$\Delta V_s = 2V \sin\left(\frac{35}{2}\right) = 4.66$ km/s

When should your ΔV occur in your orbit?

over North or South pole