Answer Key

A310 GR2 REVIEW

Problem #1

A satellite has the following COEs:

a = ~~9000~~ 10000 km
e = 0.1
i = 30°
Ω = 45°
ω = 0°

If the satellite is currently at the ascending node, where will the satellite be in 24 hours?

$\nu_i = 0°$
$E_i = 0°$
$M_i = 0°$

$$n = \sqrt{\frac{\mu}{a^3}} = 6.31348 \times 10^{-4} \text{ rad/sec}$$

$$M_f - M_i = n(TOF)$$

$$M_f = (6.31348 \times 10^{-4})(86400 \text{ sec}) = \frac{54.5485 \text{ rad}}{2\pi \text{ rad/rev}} = (8.6817 \text{ rev} - 8 \text{ rev})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = 4.2830 \text{ rad}$$

$$E_f = M_f + e \sin E_f$$
$$= 4.2830 + .1 \sin 4.2830 = 4.1921$$
$$= 4.2830 + .1 \sin 4.1921 = 4.1962$$
$$= 4.2830 + .1 \sin 4.1962 = 4.1960$$
$$= 4.2830 + .1 \sin 4.1960 = 4.1960 \text{ rad} = E_f$$

$$\nu_f = \cos^{-1}\left(\frac{\cos E_f - e}{1 - e\cos E_f}\right) = \cos\left(\frac{\cos 4.1960 - .1}{1 - .1\cos 4.1960}\right) = 2.1722 \text{ rad}$$

Half Plane Check $\rightarrow$ $\nu_f = 4.1110$ rad $= \boxed{235.5417° = \nu_f}$

Problem #2

The Restricted Two-Body EOM made several simplifying assumptions. Several of the perturbations we talked about violate these assumptions. Talk about which assumptions atmospheric drag and the Earth's oblateness violate and how they affect an orbit's specific mechanical energy and COEs.

ATMOSPHERIC DRAG → violates assumption $F_{drag} = 0$
→ changes a + e and decreases $\mathcal{E}$

EARTH'S OBLATENESS → violated assumption that Earth is spherical and can be treated as a point mass
→ changes Ω + ω in predictable ways

Problem #3

What are the 4 assumptions we make for a Hohmann transfer?

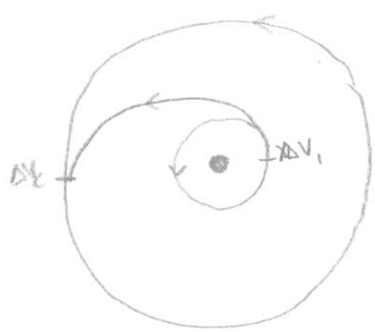
1. orbits are coplanar
2. orbits are coapsidal
3. ΔV's are instantaneous
4. ΔV's are tangential

Problem #4

NASA wants to place a communications satellite in a semi-synchronous circular orbit with an altitude of 20232 km and inclination of 0°. The satellite is currently in a circular orbit with an altitude of 300 km and inclination of 34.6°. What is the total ΔV needed, where should we perform the combined plane change, and how long will it take?

$R_2 = 6378.137 + 20232 = 26610.137 \text{ km}$ $\quad i_2 = 0°$

$R_1 = 6378.137 + 300 = 6678.137 \text{ km}$ $\quad i_1 = 34.6°$



$V_1 = \sqrt{\frac{\mu}{R_1}} = 7.7258 \text{ km/s}$

$V_2 = \sqrt{\frac{\mu}{R_2}} = 3.8703 \text{ km/s}$

$V_{t_1} = \sqrt{2\left(\frac{\mu}{R_1} + \varepsilon_t\right)} = 9.7686 \text{ km/s}$

$V_{t_2} = \sqrt{2\left(\frac{\mu}{R_2} + \varepsilon_t\right)} = 2.4516 \text{ km/s}$

$a_t = \frac{26610.137 + 6678.137}{2}$

$a_t = 16644.137 \text{ km}$

$\varepsilon_t = -\frac{\mu}{2a_t} = -11.9742 \text{ km}^2/\text{s}^2$

$\Delta V_1 = |V_1 - V_{t_1}| = 2.0428 \text{ km/s}$

$\Delta V_c = \sqrt{V_{t_2}^2 + V_2^2 - V_{t_2} V_2 2\cos\theta} = 2.3171 \text{ km/s}$

$\boxed{\Delta V_{TOT} = 4.3599 \text{ km/s}}$

Problem #5

A satellite is in a circular orbit with a radius of 7000 km, i = 90°, and Ω = 0°. It needs to maneuver so the Ω is 45°. Calculate the total delta V required to perform this maneuver.

$V = \sqrt{\frac{\mu}{R}} = \sqrt{\frac{398600.5}{7000}} = 7.5461 \text{ km/s}$ $\qquad \theta = 45°$

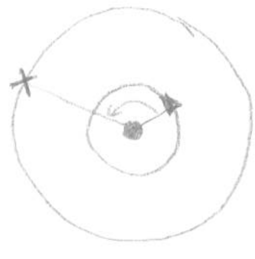
$\Delta V_s = 2V_i \sin\frac{\theta}{2}$

$= 2(7.5461)\sin\frac{45}{2}$

$\boxed{\Delta V_s = 5.7755 \text{ km/s}}$

Problem #6

The Space Shuttle is in a circular orbit with an altitude of 350 km and an inclination of 51.6°. It needs to maneuver to rendezvous with a disabled satellite which is in a circular orbit with an altitude of 600 km and an inclination of 51.6°. Currently the Shuttle has an argument of latitude of 30° and the disabled satellite has an argument of latitude of 150°. Calculate how long the Shuttle must wait before beginning the rendezvous and the total delta V required to perform this maneuver.



$$a_t = \frac{R_{shuttle} + R_{sat}}{2} = 6853.137\,km$$

$$\varepsilon_t = -\frac{\mu}{2a} = -29.08161\,km^2/s^2$$

$$TOF = \pi\sqrt{\frac{a_t^3}{\mu}} = 2823.6273\,sec$$

$$V_1 = \sqrt{\frac{\mu}{R_1}} = \sqrt{\frac{\mu}{6728}} = 7.6971\,km/s$$

$$V_2 = \sqrt{\frac{\mu}{R_2}} = \sqrt{\frac{\mu}{6978}} = 7.5579\,km/s$$

$$V_{t_1} = \sqrt{2\left(\frac{\mu}{R_1} + \varepsilon_t\right)} = 7.7670\,km/s$$

$$V_{t_2} = \sqrt{2\left(\frac{\mu}{R_2} + \varepsilon_t\right)} = 7.4886\,km/s$$

$$\Delta V_1 = |V_1 - V_{t_1}| = 0.0699\,km/s$$

$$\Delta V_2 = |V_2 - V_{t_2}| = 0.0693\,km/s$$

$$\boxed{\Delta V_{TOT} = 0.1392\,km/s}$$

$$\phi_i = 120^\circ = 2.0944\,rad$$

$$\omega_{shuttle} = \sqrt{\frac{\mu}{R^3}} = \sqrt{\frac{\mu}{(6728)^3}} = 0.001144\,rad/sec$$

$$\omega_{sat} = \sqrt{\frac{\mu}{R^3}} = \sqrt{\frac{\mu}{(6978)^3}} = 0.001083\,rad/s$$

$$\alpha_{Ld} = \omega_{tgt} \times TOF = 3.6573\,rad$$

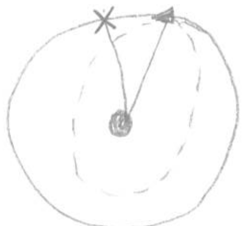
$$\phi_f = \pi - \alpha_{Ld} = 0.0843\,rad$$

$$W.T. = \frac{\phi_f - \phi_i\,(\pm 2k\pi)}{\omega_{tgt} - \omega_{int}}$$

$$W.T._1 = \frac{.0843 - 2.0944}{.001083 - .001144} = \boxed{32952.46\,sec}$$

Problem #7

The Space Shuttle is in a circular orbit with an altitude of 600 km and inclination of 51.6°. It must rendezvous with a disabled satellite which is in a circular orbit with an altitude of 600 km and inclination of 51.6°. The disabled satellite is currently 15° ahead of the Space Shuttle in its orbit. Calculate the size of the phasing orbit and the total ΔV required to perform this rendezvous.



$\phi_i = 15°$

$\phi_{travel} = 2\pi - \phi_i = 345° = 6.0214 \text{ rad}$

$\omega_{tgt} = \sqrt{\frac{\mu}{R^3}} = 0.001083 \frac{\text{rad}}{\text{sec}}$

$a_{phasing} = \sqrt[3]{\mu \left(\frac{\phi_{travel}}{2\pi\, \omega_{tgt}}\right)^2} = \boxed{6783.264 \text{ km}}$

$V_1 = \sqrt{\frac{\mu}{R}} = 7.5579 \text{ km/s}$

$V_p = \sqrt{2\left(\frac{\mu}{R} + \varepsilon_p\right)} = 7.4485 \text{ km/s}$

$\varepsilon_p = -\frac{\mu}{2a_{phasing}} = -29.3812 \text{ km}^2/\text{s}^2$

$\Delta V_1 = |V_1 - V_p| = 0.1094 \text{ km/s}$

$\boxed{\Delta V_{TOT} = 2\Delta V_1 = 0.2188 \text{ km/s}}$

Is this a valid phasing orbit? Why or why not?

$R_p = 2a_p - R_a$

Yes, perigee of the phasing orbit is greater than $R_\oplus$

What is the wait time required for this rendezvous?

There is no wait for a co-orbital rendezvous

Problem #8

MySat-1 is going to launch from Kennedy Space Center (N 28.5° W 80.5°) into a circular orbit with a radius of 8900 km, inclination of 28.5°, and Ω of 60°.

a.) How many launch opportunities are there in the next 24 hours?

1 opp $\quad i = L_o$

b.) What is the launch azimuth for each opportunity?

$\beta = 90°$

c.) If the LST is currently 0500 hrs. How long do you need to wait to launch into your first launch opportunity?

$LWST = \Omega + \delta = 60° + 90° = 150° = 10\text{ hrs}$

$W.T. = LWST - LST = 10 - 5$

$W.T. = 5\text{ hrs}$

Problem #9

FalconSat 2 was launched on a Falcon1 Launch Vehicle out of Omelek Island in the Kwajalein Atoll (N 9.05° W 167.7°). The desired orbit for FalconSat 2 is a circular orbit with an altitude of 360 km and an inclination of 52°. Calculate the ΔV_{design} required to put FalconSat 2 into the ascending node opportunity for this orbit. Assume ΔV_{Losses} = 1.5 km/s.

$V_{bo} = \sqrt{\frac{\mu}{R_{BO}}} = \sqrt{\frac{\mu}{6738.137}} = 7.6913\text{ km/s}$

$\sin \gamma = \frac{\cos \alpha}{\cos L_o}$

$\gamma = 38.57°$

$\beta_{AN} = \gamma = 38.57°$

$V_{Needed,S} = -V_{bo} \cos \phi \cos \beta = -6.0134\text{ km/s}$

$V_{Needed,E} = V_{bo} \cos\phi \sin \beta - .4651 \cos L_o = 4.3360\text{ km/s}$

$V_{Needed,Z} = V_{bo} \sin \phi + \sqrt{\frac{2\mu(R_{BO} - R_{LS})}{R_{BO} R_{LS}}} = 2.5842\text{ km/s}$

$\Delta V_{Design} = |\Delta V_{Needed}| + \Delta V_{Losses} = \boxed{9.3511\text{ km/s} = \Delta V_{design}}$

Problem #10

You have been tasked to assess the capability of a previously unseen three-stage rocket with specifications given in the table below.

	Structure Mass	Propellant Mass	Specific Impulse
Stage 1	8500	125000	287
Stage 2	3750	32500	293
Stage 3	980	12300	305

What is the maximum total ΔV the rocket is capable of delivering with a payload of 1000 kg (assume g_o=9.81 m/s^2)?

$$\Delta V_1 = (287)(9.81)\ln\left(\frac{184030}{59030}\right) = 3201.3386 \text{ m/s}$$

$$\Delta V_2 = (293)(9.81)\ln\left(\frac{50530}{18030}\right) = 2962.0838 \text{ m/s}$$

$$\Delta V_3 = (305)(9.81)\ln\left(\frac{14280}{1980}\right) = 5911.5820 \text{ m/s}$$

ΔV_{S1} 3.201 km/s

ΔV_{S2} 2.962 km/s

ΔV_{S3} 5.911 km/s

Total ΔV 12.074 km/s