gravitational parameter of earth (pg. 118) $\mu_{\oplus} = GM_{Earth} = Gm_{\oplus}$ 389 - pg. 394 Gabriel Staples 8-12-08

A310 EQUATION SHEET

$\mu_{\oplus} = 398600.5\ km^3/sec^2$

radius of Earth $R_{\oplus} = 6378.137$ km

2-Body Equation of Motion

$g_0 = \frac{\mu}{R^2}$

$$\ddot{\vec{R}} + \frac{\mu}{R^3}\vec{R} = \ddot{\vec{R}} + \frac{\mu}{R^2}\hat{R} = \vec{0}$$

$$R = alt + R_{\oplus}$$

$$R = \frac{a(1-e^2)}{1+e\cos\nu}$$

dist. to earth's ctr.

eccentricity

$$e = \frac{R_a - R_p}{R_a + R_p}$$

$$R_p = a(1-e) \qquad R_a = a(1+e)$$

apogee

$$a = \frac{R_a + R_{perigee}}{2}$$

142

$$Period = 2\pi\sqrt{\frac{a^3}{\mu}}$$

semi-major axis

Transfer Orbit TOF = $\frac{P}{2}$ (pg. 198)

pg. 181 (magnitude)

spec. mechanical E

$$\boxed{\varepsilon = KE + PE = \frac{V^2}{2} - \frac{\mu}{R} = -\frac{\mu}{2a}}$$

$$a = -\frac{\mu}{2\varepsilon}$$

pg. 141

vel.

$$V = \sqrt{2\left(\frac{\mu}{R} + \varepsilon\right)}$$

dist. spec. Mech. E

$$V_{Circular} = \sqrt{\frac{\mu}{R_{Circular}}}$$

Classical Orbital Elements

pg. 169

spec. ang. momentum vector

$$\vec{h} = \vec{R} \times \vec{V}$$

Unit vector through the North Pole

$$\vec{n} = \hat{k} \times \vec{h}$$

ascending node vector

$$\vec{e} = \frac{1}{\mu}\left[\left(V^2 - \frac{\mu}{R}\right)\vec{R} - \left(\vec{R} \bullet \vec{V}\right)\vec{V}\right] \qquad e = |\vec{e}|$$

$$\cos i = \frac{\hat{k} \bullet \vec{h}}{kh}$$

$$\frac{\vec{A} \cdot \vec{B}}{AB} = \cos\theta$$

$$\cos\Omega = \frac{\hat{i} \bullet \vec{n}}{in} \quad \text{(If } n_j < 0\text{, then } 180° < \Omega < 360°)$$

$$\cos\omega = \frac{\vec{n} \bullet \vec{e}}{ne} \quad \text{(If } e_k < 0\text{, then } 180° < \omega < 360°)$$

$$\cos\nu = \frac{\vec{e} \bullet \vec{R}}{eR} \quad \text{(If } \vec{R} \bullet \vec{V} < 0\text{, then } 180° < \nu < 360°)$$

Alternate Orbital Elements

For e=0, $\cos u = \frac{\vec{n} \bullet \vec{R}}{nR}$ (If $R_k < 0$, $180° < u < 360°$)

For Equatorial, $\cos\Pi = \frac{\hat{i} \bullet \vec{e}}{ie}$ (If $e_j < 0$, $180° < \Pi < 360°$)

For e=0 & **i=0**, $\cos l = \frac{\hat{i} \bullet \vec{R}}{iR}$ (If $R_j < 0$, $180° < l < 360°$)

Ground Tracks

Nodal Displacement

For direct orbits, $Per'd = \frac{\Delta N}{15^0/hr}$ $\quad a = \sqrt[3]{\mu\left(\frac{Per'd(sec)}{2\pi}\right)^2}$

$\Delta N = 360° -$ long. between successive ascending nodes

Do IN Radians!

Orbit Prediction

Eccentric Anomaly

$$\cos E = \frac{e + \cos\nu}{1 + e\cos\nu} \qquad \cos\nu = \frac{\cos E - e}{1 - e\cos E}$$

Mean Anomaly

$M = E - e\sin E$ (keep in radians) mean motion (avg. ang. velocity) pg. 263 $\boxed{n = \sqrt{\frac{\mu}{a^3}} = \frac{2\pi}{Period}}$

TOF

$M_f - M_i = n(t_{future} - t_{initial}) - 2k\pi$ ← Last term (only needed to get it < 2π) $-2\pi k$

$k = \cancel{number\ of\ times\ past\ perigee}$, $0 < M_{future} < 2\pi$

(# of orbits from the initial position)

$\vec{A} \times \vec{B} = AB\sin\theta$

$\vec{A} \cdot \vec{B} = AB\cos\theta$

Remote Sensing Payloads

$$f = \frac{c}{\lambda} \qquad \lambda_{max} = \frac{2898}{T}(\mu m) \qquad c = 3.0\times10^8 \frac{m}{sec}$$

λ for max output; object's temp (K)

$$Res = \frac{2.44\lambda h}{D} \qquad SW = 2R_g = \frac{2r_d h}{fl}$$

(m) Lense; λ of radiation (m) sensed; dist. from lense to viewed object; Instruments Aperture diam. (m)

ground image radius; radius of the detector; Focal length (dist. from lense to detector)

$$Res_{CCD} = \frac{D}{\#\ of\ pixels}$$

$$FOV = 2\tan^{-1}\left(\frac{r_d}{fl}\right) = 2\tan^{-1}\left(\frac{R_g}{h}\right)$$

(∠ your camera can see) (dist. your camera can see)

Perturbations

$$\dot{\Omega} \cong -2.06474x10^{14} a^{-7/2}(\cos(i))(1-e^2)^{-2} \quad deg/day$$

$$\dot{\omega} \cong 1.03237x10^{14} a^{-7/2}(4 - 5\sin^2(i))(1-e^2)^{-2} \quad deg/day$$

Launch Windows

pg. 295-302

$$\sin\gamma = \frac{\cos\alpha}{\cos L_o}$$

Launch Azimuth; Launch direction auxiliary ∠; Launch window location ∠

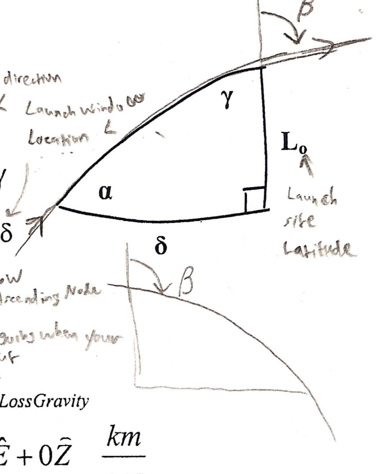
Inclination Auxiliary ∠ ($i = \alpha$ for prograde orbit)

$$\beta_{AN} = \gamma \qquad \beta_{DN} = 180° - \gamma$$

$$\cos\delta = \frac{\cos\gamma}{\sin\alpha} \qquad LWST_{AN} = \Omega + \delta$$

$LWST_{DN} = \Omega + 180 - \delta$

Launch Window Sidereal Time Ascending Node



Launch Velocity

How fast you're going when your rockets burn out

$$\vec{V}_{Needed} = \vec{V}_{burnout} - \vec{V}_{LaunchSite} + \vec{V}_{LossGravity}$$

$$\vec{V}_{Launch\,Site} = 0\hat{S} + 0.4651\cos L_0\ \hat{E} + 0\hat{Z} \quad \frac{km}{sec}$$

$R_{BO} = R_{earth} + Alt.$

$R_L = R_{earth}$

$$\Delta\vec{V}_{LossGravity} = 0\hat{S} + 0\,\hat{E} + \sqrt{\frac{2\mu(R_{BurnOut} - R_{Launch})}{R_{Launch}R_{BurnOut}}}\,\hat{Z} \quad \frac{km}{sec}$$

Flight path ∠ (0° for circ. orbits)

$$\begin{bmatrix} \Delta V_{Needed_{South}} \\ \Delta V_{Needed_{East}} \\ \Delta V_{Needed_{Zenith}} \end{bmatrix} = \begin{bmatrix} -V_{BurnOut}\cos\phi\cos\beta \\ V_{BurnOut}\cos\phi\sin\beta \\ V_{BurnOut}\sin\phi \end{bmatrix} - \begin{bmatrix} 0 \\ V_{LaunchSite} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ V_{LossGravity} \end{bmatrix} \begin{matrix} \hat{S} \\ \hat{E} \\ \hat{Z} \end{matrix}$$

$$\Delta V_{design} = \left|\Delta\vec{V}_{Needed}\right| + V_{Losses\,(other\ than\ gravity)}$$

Propulsion

$$g_o = 9.81\frac{m}{s^2}$$

$$F_{thrust} = \dot{m}V_{exit} + A_{exit}\left(P_{exit} - P_{atmosphere}\right)$$

vehicle's vel. change; spec. impulse

$$\Delta V = I_{sp}g_o \ln\left(\frac{m_{initial}}{m_{final}}\right) \left(\frac{m}{s}\right) \qquad I_{sp} = \frac{F}{\dot{m}g_o}$$

Ideal Rocket Eqn. gravitational constant

Hohmann Transfers

pg. 193-199

$$\Delta V = \left|V_{before\ burn} - V_{after\ burn}\right|$$

ΔV_1 or ΔV_c; MO; TO; PO

$$\Delta V_{Total} = \Delta V_1 + \Delta V_2 \qquad TOF_{Hohmann} = \pi\sqrt{\frac{a^3_{transfer}}{\mu}} = \frac{Period}{2}$$

$\Delta V_{Total} = \Delta V_1 + \Delta V_c$

Plane Changes

pg. 203-206

plane change ∠ $\boxed{\theta = |i_{final} - i_{initial}|}$

$$\Delta V_s = 2V_i\sin\left(\frac{\theta}{2}\right) \qquad \Delta V_c = \sqrt{V_i^2 + V_f^2 - 2V_iV_f\cos\theta}$$

vel. change for a simple plane change (only orbit's direction changes)

ΔV combined plane change (do it at apogee of transfer orbit, absorbing ΔV_1 of Hohmann transfer) (orbit's direction and magnitude change)

$\boxed{L_o \le i}$ (direct orbits)

Launch site's Latitude; orbit's inclination

$\boxed{\text{Wait Time} = LWST_{AN/DN} - LST_{Lo}}$

Gabriel Staples
9-16-08

(amt. by which the interceptor must lead the target (rad.))

Rendezvous (Coplanar) (i.e. timed Hohmann Transfer)

pg. 208-215

phase ∠ between the interceptor + the target as the transfer begins (rad.)

angular vel. of target or s/c

$$\omega = \sqrt{\frac{\mu}{R^3_{Circular\ Orbit}}} \ \frac{rad.}{sec} \qquad \phi_{final} = \pi - \alpha_{lead}$$

Lead →

$$\alpha_{lead} = \omega_{target} \times TOF \qquad 0 \le \alpha_{lead} \le 2\pi$$

TOF = ½ p'd. of the transfer orbit

$\phi_{initial}$ is measured from interceptor to target in the direction of satellite motion, at the start of the problem

$$WT = \frac{\phi_f - \phi_i \pm (2\pi)k}{\omega_{Target} - \omega_{Interceptor}}$$

Synodic Period $= \frac{2\pi}{|\omega_\oplus - \omega_{TGT\ planet}|}$ pg. 249 & 250

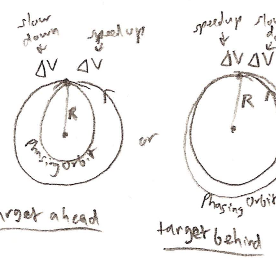
(Co-orbital) ∠ through which the target travels to reach the rendezvous location

If target ahead... $\phi_{travel} = 2\pi - \phi_{initial}$

If target behind... $\phi_{travel} = 4\pi - \phi_{initial}$

$$a_{phasing} = \sqrt[3]{\mu\left(\frac{\phi_{travel}}{2\pi\omega_{target}}\right)^2}$$

semimajor axis of the phasing orbit (km)



Control

dist. to target (alt.) · pointing accuracy (rad) · pointing stability (diam. of target) km

$$D = h\Psi \qquad \vec{H} = I\vec{\Omega} \qquad \vec{T} = \vec{R} \times \vec{F} = \dot{\vec{H}} = I\vec{\alpha}$$

$$\vec{a}_I = \vec{a}_N + \vec{a}_g \qquad \vec{a}_g = \frac{-\mu}{R^3}\vec{R} = \frac{-\mu}{R^2}\hat{R}$$

468
Electrical Power

Solar Input $= 1358 \frac{W}{m^2}$ at Earth

Earth's angular radius viewed from space (deg)

$$\rho = \sin^{-1}\left(\frac{R_\oplus}{R_\oplus + h}\right)$$

orbital altitude (km)

Max. Time of Eclipse

$$TE = \frac{2\rho}{360^0} \times \text{Period}$$

plane check! (E or W)

Power out Beg. of Life (w): $P_{BOL} = (SolarInput)\eta A_{Effective}$ — solar cells' efficiency (≈ .3); array's effective area

TS = Per'd − TE (time in sunlight)

End of Life (W), Degradation Rate: $P_{EOL} = P_{BOL}(1 - DG_{Array})^{Msn\_Life}$

E used by satellite during Eclipse → $E_{Eclipse} = (P_{P/L} + P_{Bus})TE$

E used by satellite during sunlit portion of p'd (W·s = J, convert to Watts): $E_{Sunlit} = (P_{P/L} + P_{Bus})TS$

$$P_{SA\_req} = \frac{E_{Eclipse} + E_{Sunlit}}{TS}$$

Solar array power req. during sunlit portion of p'd; TS: time in sunlight

$$Cap_{Batt\_Rqd} = \frac{E_{Eclipse}}{DoD}$$

Batt Cap Req.; Energy req. in Eclipse; Depth of Discharge

pg. 485
Environmental Control

Solar Input = 1358 W/m²

solar + albedo + earthshine

Albedo = 407 W/m² (sun reflected off Earth)

$q_{In} = (Heat\ Input)A\alpha$ (absorptivity)

Earthshine = 237 W/m²

$q_{internal} = P_{Bus} + P_{PL}$

transmissivity, reflectivity, absorptivity: $\tau + \alpha + \rho = 1$ = 100%

Stefan-Boltzmann's constant: $\sigma = 5.67x10^{-8} \frac{W}{m^2 K^4}$; Emissivity

Temp of Body (K):

$$T = \sqrt[4]{\frac{q_{out}}{\sigma \cdot A \cdot \sum_{i=1}^{N}(\varepsilon_i)}}$$

$q_{Out} = \sigma\varepsilon AT^4$ ("heat")^4

heat-power transfer per time (W); black-body Temp (K)

$q_{out} = q_{in} + q_{internal}$

Structures

pg. 505, 510

$$\sigma = \frac{F}{A} \qquad E = \frac{\sigma}{\varepsilon} \qquad \varepsilon = \frac{\Delta L}{L} \qquad f_{fund} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Stress (N/m²); modulus of elasticity, or Young's Modulus (N/m²); Strain (m/m) (change in length, or deformation); fundamental Freq. (Hz = cycles/sec); object's spring const. (N/m) (stiffness); mass

°C + 273.15 = K

625
Communications

Power of transmitter (W); Gain; Signal

Signal to Noise Ratio (W/W) (unitless)

$$\frac{S}{N} = \left(\frac{P_{Xmitr}G_{Xmitr}}{kB}\right)\left(\frac{\lambda}{4\pi R}\right)^2\left(\frac{G_{Rcvr}}{T}\right)$$

Rx Bandwidth (Hz); temp of Rx (K)

N = kTB (Noise)

$$R_{max} = \sqrt{(R_\oplus + h)^2 - R_\oplus^2} \qquad k = 1.381x10^{-23} \frac{J}{K}$$

Rmin = satellite directly overhead; Boltzmann's constant

$$Data\ Rate = \frac{S_{avg}}{E_b}\ (bits/sec)$$

Energy per bit (J/bit)

Re-entry

β=0.000139 m^{-1} (atmospheric scale height of Earth)   ρ$_o$=1.225 kg/m³ (atmospheric density at sea level)

Ballistic Coeff. pg. 330

$$BC = \frac{m}{C_D A} \qquad a_{drag} = \frac{\rho V^2}{2BC}$$

vehicle's decel.; flight-path ∠

336
$$a_{drag_{max}} = \frac{V^2_{entry}\beta\sin\gamma}{2e}$$

vehicle's max deceleration; e: base of the natural Logarithm: 2.7183

338
$$\dot{q} = 1.83\times10^{-4}V^3\sqrt{\frac{\rho}{r_{nose}}}$$

vehicle's heating rate (W/m²); vehicle's nose radius

$$h_{a_{dragmax}} = \frac{1}{\beta}\ln\left(\frac{\rho_o}{BC\beta\sin\gamma}\right)$$

altitude of vehicle's max decel.

338
$$h_{\dot{q}_{max}} = \frac{1}{\beta}\ln\left(\frac{\rho_o}{3BC\beta\sin\gamma}\right) \qquad V_{\dot{q}_{max}} = 0.846V_{entry}$$

alt. of vehicle's max heating rate (m); velocity (m/s) of max heating rate; vehicle's re-entry velocity

744
Ballistic Missiles

Λ = Range Angle   β = Azimuth Angle

Lat launch site; L = Latitude; l = longitude

$$Cos\Lambda = SinL_0 SinL_t + CosL_0 CosL_t Cos\Delta l \qquad \Delta l = l_t - l_o$$

target

plane check!

$$Cos\beta = \frac{SinL_t - SinL_0 Cos\Lambda}{CosL_0 Sin\Lambda} \qquad Range = \Lambda \frac{10{,}000\ km}{90^\circ}$$

$$Cos\Lambda = SinL_0 SinL_t + CosL_0 CosL_t Cos(\Delta l + \omega_{earth}TOF)$$

Trajectory Parameter $(0 \le Q \le 2+)$

$$Q_{Burnout} = \frac{V^2_{Burnout}R_{Burnout}}{\mu} \ ; \ Q_{Burnout_{min}} = \frac{2Sin\frac{\Lambda}{2}}{1 + Sin\frac{\Lambda}{2}}$$

$$Q_{Bo} = \frac{V_{Bo}^2}{V_{cir}^2}$$

$$\phi_{Burnout\_LOW} = \frac{1}{2}\left(Sin^{-1}\left[\left(\frac{2 - Q_{Burnout}}{Q_{Burnout}}\right)Sin\frac{\Lambda}{2}\right] - \frac{\Lambda}{2}\right)$$

in deg!

Flight path angle

$$\phi_{Burnout\_HIGH} = \frac{1}{2}\left(180^\circ - Sin^{-1}\left[\left(\frac{2 - Q_{Burnout}}{Q_{Burnout}}\right)Sin\frac{\Lambda}{2}\right] - \frac{\Lambda}{2}\right)$$

use this to calculate R_{max}!

Range ∠ →
$$\Lambda_{Max} = 2Sin^{-1}\left(\frac{Q_{Burnout}}{2 - Q_{Burnout}}\right)$$

$$(\phi_{Burnout})_{max\ Range} = 45^0 - \frac{\Lambda_{max}}{4}$$

TOF: use chart pg. 748 (for P'd$_{cir}$, use $R_{Bo} = a$)

227
Interplanetary 234

μSUN = 1.327 x 10^{11} km³/sec²

(like Hohmann Transfer)

$$\Delta V = |V_{Park} - V_{Hyperbolic}| \qquad V_\infty = |V_{Planet} - V_{TransferatPlanet}|$$

Boost/Retro; Park Orbit; Hyperbolic Orbit (@ Po); V_{Planet} (around the sun!); T@__

$$V_{hyperbolic} = \sqrt{2\left(\frac{\mu_{Planet}}{R} + \varepsilon_\infty\right)} \qquad \varepsilon_\infty = \frac{V_\infty^2}{2}$$

vel. of s/c at infinite distance from planet

$$TOF = \frac{1}{2}pd. = \pi\sqrt{\frac{a^3_{transfer}}{\mu_{sun}}}$$

pg. 248