gravitational parameter of earth (pg. 118) $\mu_{\oplus} = GM_{Earth} = Gm_{\oplus}$

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A310 EQUATION SHEET

$\mu_{\oplus} = 398600.5\ km^3/sec^2$

radius of Earth $R_{\oplus} = 6378.137$ km

2-Body Equation of Motion

$$\ddot{\vec{R}} + \frac{\mu}{R^3}\vec{R} = \ddot{\vec{R}} + \frac{\mu}{R^2}\hat{R} = \vec{0}$$

$g_g = \frac{\mu}{R^2}$

$R = alt + R_{\oplus}$

$$R = \frac{a(1-e^2)}{1+e\cos\nu}$$

dist. to earth's ctr.

eccentricity

$$e = \frac{R_a - R_p}{R_a + R_p}$$

$R_P = a(1-e)$ $\quad R_a = a(1+e)$

Semi-major axis apogee 142

$$a = \frac{R_a + R_{perigee}}{2}$$

Period $= 2\pi\sqrt{\frac{a^3}{\mu}}$ semi-major axis pg. 181 $\frac{P}{2}$ (pg. 198) Transfer Orbit TOF $= \frac{P}{2}$

spec. mechanical E (magnitude)

$$\boxed{\varepsilon = KE + PE = \frac{V^2}{2} - \frac{\mu}{R} = -\frac{\mu}{2a}}$$

Vall. 27

$$a = -\frac{\mu}{2\varepsilon}$$

pg. 141 Vall. 32 vel.

$$V = \sqrt{2\left(\frac{\mu}{R} + \varepsilon\right)}$$

dist. spec. Mech. E

$$V_{Circular} = \sqrt{\frac{\mu}{R_{Circular}}}$$

Classical Orbital Elements

pg. 169

spec. ang. momentum vector $\vec{h} = \vec{R} \times \vec{V}$

Unit vector through the North Pole

$\vec{n} = \hat{k} \times \vec{h}$ ascending node vector

$$\vec{e} = \frac{1}{\mu}\left[\left(V^2 - \frac{\mu}{R}\right)\vec{R} - \left(\vec{R} \bullet \vec{V}\right)\vec{V}\right] \qquad e = |\vec{e}|$$

$$\cos i = \frac{\hat{k} \bullet \vec{h}}{kh} \qquad \frac{\vec{A}\cdot\vec{B}}{AB} = \cos\theta$$

RAAN

$\cos\Omega = \frac{\hat{i} \bullet \vec{n}}{in}$ (If $n_j < 0$, then $180° < \Omega < 360°$) (the "j" component of the $\vec{n}$ vector)

$\cos\omega = \frac{\vec{n} \bullet \vec{e}}{ne}$ (If $e_k < 0$, then $180° < \omega < 360°$)

$\cos\nu = \frac{\vec{e} \bullet \vec{R}}{eR}$ (If $\vec{R} \bullet \vec{V} < 0$, then $180° < \nu < 360°$)

Alternate Orbital Elements

For e=0, $\cos u = \frac{\vec{n} \bullet \vec{R}}{nR}$ (If $R_k < 0$, $180° < u < 360°$)

For Equatorial, $\cos\Pi = \frac{\hat{i} \bullet \vec{e}}{ie}$ (If $e_j < 0$, $180° < \Pi < 360°$)

For e=0 & i=0, $\cos l = \frac{\hat{i} \bullet \vec{R}}{iR}$ (If $R_j < 0$, $180° < l < 360°$)

Ground Tracks

Nodal Displacement

For direct orbits, $Per'd = \frac{\Delta N}{15^0/hr}$ $\quad a = \sqrt[3]{\mu\left(\frac{Per'd(sec)}{2\pi}\right)^2}$

$\Delta N = 360° -$ long. between successive ascending nodes

Do IN Radians!

Orbit Prediction

Eccentric Anomaly $\cos E = \frac{e + \cos\nu}{1 + e\cos\nu}$ $\qquad \cos\nu = \frac{\cos E - e}{1 - e\cos E}$

Mean Anomaly $M = E - e\sin E$ (keep in radians) mean motion (avg. ang. velocity) pg. 263 $\boxed{n = \sqrt{\frac{\mu}{a^3}} = \frac{2\pi}{Period}}$

TOF

$M_f - M_i = n(t_{future} - t_{initial}) - 2k\pi$ $- 2\pi k$ last term (only needed to get it $< 2\pi$)

$k = \cancel{number\ of\ times\ past\ perigee}$, $0 < M_{future} < 2\pi$

(# of orbits from the initial position)

$\vec{A} \times \vec{B} = AB\sin\theta$

$\vec{A} \cdot \vec{B} = AB\cos\theta$

Half plane check: M, E, and ν must all be in the same half plane ($0°-180°$) or ($180°-360°$)! If anyone of the 3 equals $0°$ or $180°$, then they all must equal that value ($0°$ or $180°$)

$a = \frac{p}{(1-e^2)}$

Remote Sensing Payloads

$f = \frac{c}{\lambda}$ $\quad$ λ for max output $\lambda_{max} = \frac{2898}{T}(\mu m)$ object's temp (K) $\quad c = 3.0 \times 10^8 \frac{m}{sec}$

λ of radiation (m) sensed

Res $= \frac{2.44\lambda h}{D}$ (m) Lense dist. from lense to viewed object

Instruments Aperture diam. (m)

ground image radius, radius of the detector

$SW = 2R_g = \frac{2r_d h}{fl}$ Focal length (dist. from lense to detector)

$Res_{ccd} = \frac{D}{\#\ of\ pixels}$

$FOV = 2\tan^{-1}\left(\frac{r_d}{fl}\right) = 2\tan^{-1}\left(\frac{R_g}{h}\right)$ (dist. your camera can see)

($\angle$ your camera can see)

Perturbations

$\dot{\Omega} \cong -2.06474x10^{14}\,a^{-7/2}(\cos(i))(1-e^2)^{-2}$ deg/ day

$\dot{\omega} \cong 1.03237x10^{14}\,a^{-7/2}(4 - 5\sin^2(i))(1-e^2)^{-2}$ deg/ day

Launch azimuth β

Launch Windows

- pg. 295-302

Inclination Auxiliary $\angle$ ($i = \alpha$ for prograde orbit)

$\sin\gamma = \frac{\cos\alpha}{\cos L_o}$ Launch Azimuth

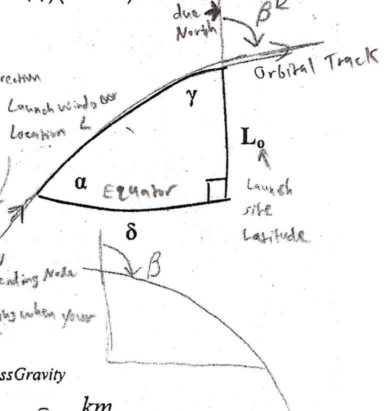
Launch direction auxiliary $\angle$ Launch window location $\angle$

$\beta_{AN} = \gamma$ $\qquad \beta_{DN} = 180° - \gamma$

$\cos\delta = \frac{\cos\gamma}{\sin\alpha}$ $\qquad LWST_{AN} = \Omega + \delta$

Launch Window Sidereal Time Ascending Node

$LWST_{DN} = \Omega + 180 - \delta$



Launch Velocity

How fast you're going when your rockets burn out

$\vec{V}_{Needed} = \vec{V}_{burnout} - \vec{V}_{LaunchSite} + \vec{V}_{LossGravity}$

$$\vec{V}_{Launch\ Site} = 0\hat{S} + 0.4651\cos L_0\ \hat{E} + 0\hat{Z}\ \frac{km}{sec}$$

$R_{BO} = R_{earth} + Alt.$

$R_L = R_{earth}$

$$\Delta\vec{V}_{LossGravity} = 0\hat{S} + 0\hat{E} + \sqrt{\frac{2\mu(R_{BurnOut} - R_{Launch})}{R_{Launch}R_{BurnOut}}}\ \hat{Z}\ \frac{km}{sec}$$

Flight path $\angle$ ($0°$ for circ. orbits)

$$\begin{bmatrix} \Delta V_{Needed_{South}} \\ \Delta V_{Needed_{East}} \\ \Delta V_{Needed_{Zenith}} \end{bmatrix} = \begin{bmatrix} -V_{BurnOut}\cos\phi\cos\beta \\ V_{BurnOut}\cos\phi\sin\beta \\ V_{BurnOut}\sin\phi \end{bmatrix} - \begin{bmatrix} 0 \\ V_{LaunchSite} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ V_{LossGravity} \end{bmatrix} \begin{matrix} \hat{S} \\ \hat{E} \\ \hat{Z} \end{matrix}$$

$$\Delta V_{design} = \left|\Delta\vec{V}_{Needed}\right| + V_{Losses\,(other\ than\ gravity)}$$

Propulsion

$g_o = 9.81\frac{m}{s^2}$

$F_{thrust} = \dot{m}V_{exit} + A_{exit}(P_{exit} - P_{atmosphere})$ $\quad \boxed{C = I_{sp}g_o}$

vehicle's vel. change, spec. impulse

$\Delta V = I_{sp}g_o\ln\left(\frac{m_{initial}}{m_{final}}\right)$ $\left(\frac{m}{s}\right)$ $\quad I_{sp} = \frac{F}{\dot{m}g_o}$ $\quad \boxed{F = c\dot{m}}$

Ideal Rocket Eqn. gravitational constant (coplanar)

Hohmann Transfers

pg. 193-199

$\Delta V = |V_{before\ burn} - V_{after\ burn}|$

MO, TO, ΔV_i, PO, ΔV_f or ΔV_e

Time of Flight

$\Delta V_{Total} = \Delta V_1 + \Delta V_2$ $\qquad TOF_{Hohmann} = \pi\sqrt{\frac{a^3_{transfer}}{\mu}} = \frac{Period}{2}$

$\Delta V_{Total} = \Delta V_i + \Delta V_f$

Plane Changes

pg. 203-206

plane change $\angle$ $\boxed{\theta = |i_{final} - i_{initial}|}$

$\Delta V_s = 2V_i\sin\left(\frac{\theta}{2}\right)$ $\qquad \Delta V_c = \sqrt{V_i^2 + V_f^2 - 2V_iV_f\cos\theta}$

vel. change for a simple plane change (only orbit's direction changes)

ΔV combined plane change (do it at apogee of transfer orbit, after doing ΔV_i of Hohmann transfer)

$\boxed{L_o \le i}$ (direct orbits)

Launch site's Latitude, orbit's inclination

$\boxed{Wait\ Time = LWST_{AN/DN} - LST_{L_o}}$

(orbit's direction and magnitude change)

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(amt. by which the interceptor must lead the target (rad.))

Vall 364

Rendezvous (Coplanar)

pg. 208-215 (i.e. timed Hohmann Transfer)

phase ∠ between the interceptor & the target as the transfer begins (rad.)

angular vel. of target s/c

$$\omega = \sqrt{\frac{\mu}{R^3_{Circular\ Orbit}}} \ \frac{rad.}{sec} \qquad \phi_{final} = \pi - \alpha_{lead}$$

Lead → $\alpha_{lead} = \omega_{target} \times TOF \qquad 0 \le \alpha_{lead} \le 2\pi$

= ½ p'd. of the transfer orbit

$\phi_{initial}$ is measured from interceptor to target in the direction of satellite motion, at the start of the problem

$$WT = \frac{\phi_f - \phi_i \pm (2\pi)k}{\omega_{Target} - \omega_{Interceptor}}$$

Synotic Period $= \frac{2\pi}{|\omega_{\oplus} - \omega_{TGT}|}$ pg. 249 & 250 (planet)

(Co-orbital)

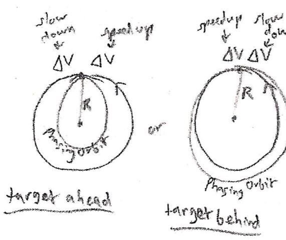
∠ through which the target travels to reach the rendezvous location

If target ahead... $\phi_{travel} = 2\pi - \phi_{initial}$

If target behind... $\phi_{travel} = 4\pi - \phi_{initial}$

$$a_{phasing} = \sqrt[3]{\mu\left(\frac{\phi_{travel}}{2\pi\omega_{target}}\right)^2}$$

semimajor axis of the phasing orbit (km)



Control

pointing stability (alt.) diff. to target pointing accuracy (rad) (km)

$$D = h\Psi \qquad \vec{H} = I\vec{\Omega} \qquad \vec{T} = \vec{R} \times \vec{F} = \dot{\vec{H}} = I\vec{\alpha}$$

$$\vec{a}_I = \vec{a}_N + \vec{a}_g \qquad \vec{a}_g = \frac{-\mu}{R^3}\vec{R} = \frac{-\mu}{R^2}\hat{R}$$

Electrical Power

468

Solar Input $= 1358 \frac{W}{m^2}$ at Earth

Earth's angular radius viewed from space (deg)

$$\rho = \sin^{-1}\left(\frac{R_{\oplus}}{R_{\oplus} + h}\right) \qquad TE = \frac{2\rho}{360^0} \times Period$$

orbital altitude (km); Max. Time of Eclipse; plane check! (E or W)

Power Out Beg. of Life P_{in} (W); solar cells' efficiency (≈ .3)

$$P_{BOL} = (SolarInput)\eta A_{Effective}$$

← array's effective area. $TS = Per'd - TE$ (time in sunlight)

End of Life (W); Degradation Rate

$$P_{EOL} = P_{BOL}(1 - DG_{Array})^{Msn\_Life}$$

E used by satellite during Eclipse

$$E_{Eclipse} = (P_{P/L} + P_{Bus})TE \qquad E_{Sunlit} = (P_{P/L} + P_{Bus})TS$$

E used by satellite during sunlit (W·s = J, convert to W·hrs) portion of p'd

$$P_{SA\_req} = \frac{E_{Eclipse} + E_{Sunlit}}{TS} \qquad Cap_{Batt\_Rqd} = \frac{E_{Eclipse}}{DoD}$$

Solar array power req. during sunlit portion of p'd.; time in sunlight; Batt Cap Req.; ← Energy req. in Eclipse; Depth of Discharge

Environmental Control

pg. 485

Solar Input = 1358 W/m²

Albedo = 407 W/m² (sun reflected off Earth)

Earthshine = 237 W/m²

solar + albedo + earthshine

$$q_{In} = (Heat\ Input)A\alpha$$

absorptivity; transmissivity; reflectivity

$$q_{internal} = P_{Bus} + P_R \qquad \tau + \alpha + \rho = 1 \ = 100\%$$

Stefan-Boltzmann's constant

$$\sigma = 5.67x10^{-8} \frac{W}{m^2 K^4}$$

Emissivity; Temp of Body (K)

$$T = \sqrt[4]{\frac{q_{out}}{\sigma \cdot A \cdot \sum_{i=1}^{N}(\varepsilon_i)}}$$

$$q_{Out} = \sigma\varepsilon A T^4 \quad (\text{"heat"})^4$$

heat-power transfer per time (W); black-body Temp (K)

$q_{out} = q_{in} + q_{internal}$

Structures

pg. 505, 510

$$\sigma = \frac{F}{A} \qquad E = \frac{\sigma}{\varepsilon} \qquad \varepsilon = \frac{\Delta L}{L} \qquad f_{fund} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Stress (N/m²); modulus of Elasticity, or Young's Modulus (N/m²); Strain (m/m) (change in length or deformation); fundamental Freq. (Hz = cycles/sec); objects spring const. (N/m) (stiffness); mass

°C + 273.15 = K

Communications

625

Power of transmitter (W); Gain; Signal

Signal to Noise Ratio (W/W) (unitless)

$$\frac{S}{N} = \left(\frac{P_{Xmitr}G_{Xmitr}}{kB}\right)\left(\frac{\lambda}{4\pi R}\right)^2\left(\frac{G_{Rcvr}}{T}\right)$$

Rx Bandwidth (Hz); temp of Rx (K)

$N = kTB$ (Noise)

$$R_{\max} = \sqrt{(R_{\oplus} + h)^2 - R_{\oplus}^2} \qquad k = 1.381x10^{-23} \frac{J}{K}$$

R_{min} = satellite directly overhead; Boltzmann's constant

$$Data\ Rate = \frac{S_{avg}}{E_b} \ (bits/sec)$$

Energy per bit (J/bit)

Re-entry

atmospheric density at sea level

β=0.000139 m⁻¹ ρ_o=1.225 kg/m³

atmospheric scale height of Earth

Ballistic Coeff. pg. 330

$$BC = \frac{m}{C_D A} \qquad a_{drag} = \frac{\rho V^2}{2BC}$$

flight-path ∠; vehicle's decel.

336

$$a_{drag_{\max}} = \frac{V^2_{entry}\beta\sin\gamma}{2e}$$

vehicle's max deceleration; ← base of the natural Logarithm ≈ 2.7183

338

$$\dot{q} = 1.83 \times 10^{-4} V^3 \sqrt{\frac{\rho}{r_{nose}}}$$

vehicle's heating rate (W/m²); ← vehicle's nose radius

$$h_{a_{drag\max}} = \frac{1}{\beta}\ln\left(\frac{\rho_o}{BC\beta\sin\gamma}\right)$$

altitude of vehicle's max decel.

338

$$h_{\dot{q}_{\max}} = \frac{1}{\beta}\ln\left(\frac{\rho_o}{3BC\beta\sin\gamma}\right) \qquad V_{\dot{q}_{\max}} = 0.846V_{entry}$$

alt. of vehicle's max heating rate (m); velocity (m/s) of max heating rate; vehicle's re-entry velocity

Ballistic Missiles

744

Λ = Range Angle β = Azimuth Angle

Lat launch site; L = Latitude; l = longitude; target

$$Cos\Lambda = SinL_0 SinL_t + CosL_0 CosL_t Cos\Delta l \qquad \Delta l = l_t - l_0$$

$$Cos\beta = \frac{SinL_t - SinL_0 Cos\Lambda}{CosL_0 Sin\Lambda} \qquad Range = \Lambda \frac{10{,}000\ km}{90^\circ}$$

$$Cos\Lambda = SinL_0 SinL_t + CosL_0 CosL_t Cos(\Delta l + \omega_{earth}TOF)$$

Trajectory Parameter $(0 \le Q \le 2)$

$$Q_{Burnout} = \frac{V^2_{Burnout}R_{Burnout}}{\mu} \ ; \ Q_{Burnout_{\min}} = \frac{2Sin\frac{\Lambda}{2}}{1 + Sin\frac{\Lambda}{2}}$$

$$Q_{BO} = \frac{V^2_{BO}}{V^2_{cir}}$$

$$\phi_{Burnout\_LOW} = \frac{1}{2}\left(Sin^{-1}\left[\left(\frac{2 - Q_{Burnout}}{Q_{Burnout}}\right)Sin\frac{\Lambda}{2}\right] - \frac{\Lambda}{2}\right)$$

in deg!

flight path angle

$$\phi_{Burnout\_HIGH} = \frac{1}{2}\left(180^\circ - Sin^{-1}\left[\left(\frac{2 - Q_{Burnout}}{Q_{Burnout}}\right)Sin\frac{\Lambda}{2}\right] - \frac{\Lambda}{2}\right)$$

use this to calculate R_{max}!

Range ∠ → $$\Lambda_{Max} = 2Sin^{-1}\left(\frac{Q_{Burnout}}{2 - Q_{Burnout}}\right)$$

$$(\phi_{Burnout})_{\max Range} = 45^0 - \frac{\Lambda_{\max}}{4}$$

TOF: use chart pg. 748 (for Pd ccr, use $R_{BO} = a$)

Interplanetary

227 - 234

μ_{SUN} = 1.327 x 10¹¹ km³/sec²

(i.e. Hohmann Transfer)

$$\Delta V = \left|V_{Park} - V_{Hyperbolic}\right| \qquad V_{\infty} = \left|V_{Planet} - V_{TransferatPlanet}\right|$$

Boost/Retro; Orbit; Orbit (@ Po); (around the sun); T@___; Period?

$$V_{hyperbolic} = \sqrt{2\left(\frac{\mu_{Planet}}{R} + \varepsilon_{\infty}\right)} \qquad \varepsilon_{\infty} = \frac{V_{\infty}^2}{2}$$

$$TOF = \frac{1}{2}pd. = \pi\sqrt{\frac{a^3_{transfer}}{\mu_{sun}}}$$

pg. 248

vel. of s/c at infinite distance from planet (hyperbolic excess vel.) in relationship to sun ???