gravitational parameter of earth (pg. 118) $\mu_\oplus = GM_{Earth} = Gm_\oplus$

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A310 EQUATION SHEET

$\mu_\oplus = 398600.5\ km^3/sec^2$

radius of Earth $R_\oplus = 6378.137$ km

2-Body Equation of Motion

$g_g = \frac{\mu}{R^2}$

$$\ddot{\vec{R}} + \frac{\mu}{R^3}\vec{R} = \ddot{\vec{R}} + \frac{\mu}{R^2}\hat{R} = \vec{0}$$

$$R = alt + R_\oplus$$

$$R = \frac{a(1-e^2)}{1+e\cos\nu}$$

dist. to earth's ctr.

eccentricity

$$e = \frac{R_a - R_p}{R_a + R_p}$$

$$R_P = a(1-e) \qquad R_a = a(1+e)$$

apogee, perigee

142

$$a = \frac{R_a + R_{p}}{2}$$

pg. 181

$$Period = 2\pi\sqrt{\frac{a^3}{\mu}}$$

semi-major axis

Transfer Orbit TOF $= \frac{P}{2}$ (pg. 198)

spe. mechanical E (magnitude)

$$\boxed{\varepsilon = KE + PE = \frac{V^2}{2} - \frac{\mu}{R} = -\frac{\mu}{2a}} \qquad a = -\frac{\mu}{2\varepsilon}$$

pg. 141

vel.

$$V = \sqrt{2\left(\frac{\mu}{R} + \varepsilon\right)}$$

dist. spec. Mech. E

$$V_{Circular} = \sqrt{\frac{\mu}{R_{Circular}}}$$

Classical Orbital Elements

spec. ang. momentum vector

pg. 169

$$\vec{h} = \vec{R} \times \vec{V}$$

Unit vector through the North Pole

$$\vec{n} = \hat{k} \times \vec{h}$$

ascending node vector

$$\vec{e} = \frac{1}{\mu}\left[\left(V^2 - \frac{\mu}{R}\right)\vec{R} - \left(\vec{R}\bullet\vec{V}\right)\vec{V}\right] \qquad e = |\vec{e}|$$

$$\cos i = \frac{\hat{k}\bullet\vec{h}}{kh}$$

$\frac{\vec{A}\cdot\vec{B}}{AB} = \cos\theta$

$$\cos\Omega = \frac{\hat{i}\bullet\vec{n}}{in}$$ (If $n_j < 0$, then $180° < \Omega < 360°$)

$$\cos\omega = \frac{\vec{n}\bullet\vec{e}}{ne}$$ (If $e_k < 0$, then $180° < \omega < 360°$)

$$\cos\nu = \frac{\vec{e}\bullet\vec{R}}{eR}$$ (If $\vec{R}\bullet\vec{V} < 0$, then $180° < \nu < 360°$)

Alternate Orbital Elements

For e=0, $\cos u = \frac{\vec{n}\bullet\vec{R}}{nR}$ (If $R_k < 0$, $180° < u < 360°$)

For Equatorial, $\cos\Pi = \frac{\hat{i}\bullet\vec{e}}{ie}$ (If $e_j < 0$, $180° < \Pi < 360°$)

For e=0 & i=0, $\cos l = \frac{\hat{i}\bullet\vec{R}}{iR}$ (If $R_j < 0$, $180° < l < 360°$)

Ground Tracks

Nodal Displacement

For direct orbits, $Per'd = \frac{\Delta N}{15^0/hr}$ $\qquad a = \sqrt[3]{\mu\left(\frac{Per'd(sec)}{2\pi}\right)^2}$

$\Delta N = 360° -$ long. between successive ascending nodes

Do IN Radians!

Orbit Prediction

Eccentric Anomaly

$$\cos E = \frac{e + \cos\nu}{1 + e\cos\nu} \qquad \cos\nu = \frac{\cos E - e}{1 - e\cos E}$$

Mean Anomaly, mean motion (avg. ang. velocity) pg. 263

$$M = E - e\sin E \text{ (keep in radians)} \qquad n = \sqrt{\frac{\mu}{a^3}} = \frac{2\pi}{Period}$$

TOF

$$M_f - M_i = n(t_{future} - t_{initial}) - 2k\pi$$

last term (only needed to get it $< 2\pi$)

$-2\pi k$

$k = \cancel{number\ of\ times\ past\ perigee}$, $0 < M_{future} < 2\pi$

(# of orbits from the initial position)

$\vec{A}\times\vec{B} = AB\sin\theta$

$\vec{A}\cdot\vec{B} = AB\cos\theta$

Remote Sensing Payloads

λ for max output, object's temp (K)

$$f = \frac{c}{\lambda} \qquad \lambda_{max} = \frac{2898}{T}(\mu m) \qquad c = 3.0\times10^8\ \frac{m}{sec}$$

λ of radiation (m) sensed; dist. from lense to viewed object; Lense (m)

$$Res = \frac{2.44\lambda h}{D}$$

ground image radius; radius of the detector; focal length (dist. from lense to detector)

$$SW = 2R_g = \frac{2r_d h}{fl}$$

$$Res_{CCD} = \frac{D}{\#\ of\ pixels}$$

Instruments Aperture diam. (m)

$$FOV = 2\tan^{-1}\left(\frac{r_d}{fl}\right) = 2\tan^{-1}\left(\frac{R_g}{h}\right)$$

($\angle$ your camera can see) (dist. your camera can see)

Perturbations

$$\dot{\Omega} \cong -2.06474x10^{14}\,a^{-7/2}(\cos(i))(1-e^2)^{-2} \quad deg/day$$

$$\dot{\omega} \cong 1.03237x10^{14}\,a^{-7/2}(4-5\sin^2(i))(1-e^2)^{-2} \quad deg/day$$

Launch Windows

- pg. 295-302

Inclination Auxiliary $\angle$ ($i = \alpha$ for prograde orbit)

Launch Azimuth; Launch direction auxiliary $\angle$; Launch window Location $\angle$

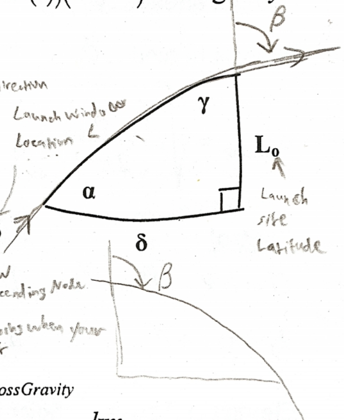
$$\sin\gamma = \frac{\cos\alpha}{\cos L_o}$$

$$\beta_{AN} = \gamma \qquad \beta_{DN} = 180° - \gamma$$

$$\cos\delta = \frac{\cos\gamma}{\sin\alpha} \qquad LWST_{AN} = \Omega + \delta$$

$$LWST_{DN} = \Omega + 180 - \delta$$

Launch Window Sidereal Time Ascending Node



Launch Velocity

How fast you're going when your rockets burn out

$$\vec{V}_{Needed} = \vec{V}_{burnout} - \vec{V}_{LaunchSite} + \vec{V}_{LossGravity}$$

$$\vec{V}_{Launch\ Site} = 0\hat{S} + 0.4651\cos L_0\ \hat{E} + 0\hat{Z} \quad \frac{km}{sec}$$

$R_{BO} = R_{earth} + Alt.$

$R_L = R_{earth}$

$$\Delta\vec{V}_{LossGravity} = 0\hat{S} + 0\,\hat{E} + \sqrt{\frac{2\mu(R_{BurnOut} - R_{Launch})}{R_{Launch}R_{BurnOut}}}\ \hat{Z} \quad \frac{km}{sec}$$

Flight path $\angle$ (0° for circ. orbits)

$$\begin{bmatrix}\Delta V_{Needed_{South}}\\ \Delta V_{Needed_{East}}\\ \Delta V_{Needed_{Zenith}}\end{bmatrix} = \begin{bmatrix}-V_{BurnOut}\cos\phi\cos\beta\\ V_{BurnOut}\cos\phi\sin\beta\\ V_{BurnOut}\sin\phi\end{bmatrix} - \begin{bmatrix}0\\ V_{LaunchSite}\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 0\\ V_{LossGravity}\end{bmatrix} \begin{array}{c}\hat{S}\\ \hat{E}\\ \hat{Z}\end{array}$$

$$\Delta V_{design} = \left|\Delta\vec{V}_{Needed}\right| + V_{Losses\,(other\ than\ gravity)}$$

Propulsion

$$g_o = 9.81\frac{m}{s^2}$$

$$F_{thrust} = \dot{m}V_{exit} + A_{exit}(P_{exit} - P_{atmosphere})$$

vehicle's vel. change; spec. impulse

$$\Delta V = I_{sp}g_o\ln\left(\frac{m_{initial}}{m_{final}}\right) \left(\frac{m}{s}\right) \qquad I_{sp} = \frac{F}{\dot{m}g_o}$$

Ideal Rocket Eqn. gravitational constant

Hohmann Transfers

pg. 193-199

ΔV_1 or ΔV_c, MO, TO, PO, ΔV_2

$$\Delta V = \left|V_{before\ burn} - V_{after\ burn}\right|$$

$$\Delta V_{Total} = \Delta V_1 + \Delta V_2 \qquad TOF_{Hohmann} = \pi\sqrt{\frac{a^3_{transfer}}{\mu}} = \frac{Period}{2}$$

$\Delta V_{Total} = \Delta V_1 + \Delta V_c$

Plane Changes

pg. 203-206

plane change $\angle$ $\boxed{\theta = |i_{final} - i_{initial}|}$

$$\Delta V_s = 2V_i\sin\left(\frac{\theta}{2}\right) \qquad \Delta V_c = \sqrt{V_i^2 + V_f^2 - 2V_iV_f\cos\theta}$$

vel. change for a simple plane change (only orbit's direction changes)

ΔV combined plane change (do it at apogee of transfer orbit, after doing ΔV_1 of Hohmann transfer)

$\boxed{L_o \leq i}$ (direct orbits)

Launch site's Latitude; orbit's inclination

$\boxed{Wait\ Time = LWST_{AN/DN} - LST_{L_o}}$

(orbit's direction and magnitude change) (amt. by which the interceptor must lead the target (rad.))

Gabriel Staples 9-11-08

Rendezvous (Coplanar)

pg. 208-215 (i.e. timed Hohmann Transfer)

angular vel. of target s/c

$$\omega = \sqrt{\frac{\mu}{R^3_{Circular\ Orbit}}} \quad \frac{rad.}{sec}$$

phase ∠ between the interceptor & the target as the transfer begins (rad.)

$$\phi_{final} = \pi - \alpha_{lead}$$

Lead ∠

$$\alpha_{lead} = \omega_{target} \times TOF \qquad 0 \le \alpha_{lead} \le 2\pi$$

TOF = ½ p'd. of the transfer orbit

$\phi_{initial}$ is measured from interceptor to target in the direction of satellite motion, at the start of the problem

$$WT = \frac{\phi_f - \phi_i \pm (2\pi)k}{\omega_{Target} - \omega_{Interceptor}}$$

Synotic Period $= \frac{2\pi}{|\omega_{\oplus} - \omega_{TGT\ planet}|}$ pg. 249+250

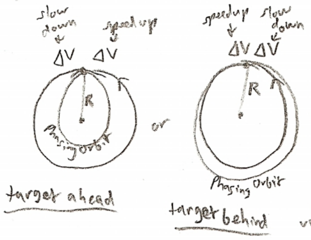
(Co-orbital) ∠ through which the target travels to reach the rendezvous location

If target ahead… $\phi_{travel} = 2\pi - \phi_{initial}$

If target behind… $\phi_{travel} = 4\pi - \phi_{initial}$

$$a_{phasing} = \sqrt[3]{\mu\left(\frac{\phi_{travel}}{2\pi\omega_{target}}\right)^2}$$

semimajor axis of the phasing orbit (km)



Control

dist. to target (alt.); pointing accuracy (rad); pointing stability (dimen. of target) km

$$D = h\Psi \qquad \vec{H} = I\vec{\Omega} \qquad \vec{T} = \vec{R}\times\vec{F} = \dot{\vec{H}} = I\vec{\alpha}$$

$$\vec{a}_I = \vec{a}_N + \vec{a}_g \qquad \vec{a}_g = \frac{-\mu}{R^3}\vec{R} = \frac{-\mu}{R^2}\hat{R}$$

Electrical Power

468 Solar Input $= 1358\frac{W}{m^2}$ at Earth

Earth's angular radius viewed from space (deg)

$$\rho = \sin^{-1}\left(\frac{R_\oplus}{R_\oplus + h}\right)$$

h = orbital altitude (km)

Max. Time of Eclipse

$$TE = \frac{2\rho}{360^0}\times \text{Period}$$

plane check! (E or W)

$TS = Per'd - TE$

Power Out Beg. of Life (W); P_{in}; solar cells' efficiency (≈.3); array's effective area

$$P_{BOL} = (SolarInput)\eta A_{Effective}$$

End of Life (W); Degradation Rate

$$P_{EOL} = P_{BOL}\left(1 - DG_{Array}\right)^{Msn\_Life}$$

time in sunlight

E used by satellite during sunlit portion of p'd (W·S = J, convert to Wahrs)

E used by satellite during Eclipse

$$E_{Eclipse} = (P_{P/L} + P_{Bus})TE \qquad E_{Sunlit} = (P_{P/L} + P_{Bus})TS$$

$$P_{SA\_req} = \frac{E_{Eclipse} + E_{Sunlit}}{TS}$$

Solar array power req. during sunlit portion of p'd.; time in sunlight

$$Cap_{Batt\_Rqd} = \frac{E_{Eclipse}}{DoD}$$

Energy req. in Eclipse; Batt Cap Req.; Depth of Discharge

Environmental Control

pg. 485

Solar Input = 1358 W/m²

Albedo = 407 W/m² (sun reflected off Earth)

Earthshine = 237 W/m²

solar + albedo + earthshine

$$q_{In} = (Heat\ Input)A\alpha$$

absorptivity; transmissivity; reflectivity

$$\tau + \alpha + \rho = 1 = 100\%$$

$q_{internal} = P_{Bus} + P_{PL}$

Stefan-Boltzmann's constant

$$\sigma = 5.67\text{x}10^{-8}\ \frac{W}{m^2\ K^4}$$

Emissivity

$$q_{Out} = \sigma\varepsilon AT^4 \quad (\text{"heat"})^4$$

heat-power transfer per time (W); black-body Temp (K)

Temp of Body (K); absorptivity

$$T = \sqrt[4]{\frac{q_{out}}{\sigma\cdot A\cdot\sum_{i=1}^{N}(\varepsilon_i)}}$$

$q_{out} = q_{in} + q_{internal}$

Structures

pg. 505, 510

$$\sigma = \frac{F}{A} \qquad E = \frac{\sigma}{\varepsilon} \qquad \varepsilon = \frac{\Delta L}{L} \qquad f_{fund} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Stress (N/m²); modulus of elasticity, or Young's Modulus (N/m²); Strain (m/m) (change in length, or deformation); fundamental freq. (Hz = cycles/sec); object's spring const. (N/m) (stiffness); mass

$°C + 273.15 = K$

Communications

625 Power of transmitter (W); Gain; Signal

Signal to Noise Ratio $\left(\frac{W}{W}\right)$ (unitless)

$$\frac{S}{N} = \left(\frac{P_{Xmitr}G_{Xmitr}}{kB}\right)\left(\frac{\lambda}{4\pi R}\right)^2\left(\frac{G_{Rcvr}}{T}\right)$$

Rx Bandwidth (Hz); temp of Rx (K)

$N = kTB$ Noise

$$R_{max} = \sqrt{(R_\oplus + h)^2 - R_\oplus^2} \qquad k = 1.381\text{x}10^{-23}\ \frac{J}{k}$$

R_{min} = satellite directly overhead; Boltzmann's constant

$$Data\ Rate = \frac{S_{avg}}{E_b}\ (bits/sec)$$

Energy per bit (J/bit)

Re-entry

β=0.000139 m⁻¹ atmospheric scale height of Earth; ρ₀=1.225 kg/m³ atmospheric density at sea level

Ballistic Coeff. pg. 330

$$BC = \frac{m}{C_D A} \qquad a_{drag} = \frac{\rho V^2}{2BC}$$

vehicle's decel.

flight-path ∠ 336

$$a_{drag_{max}} = \frac{V^2_{entry}\beta\sin\gamma}{2e}$$

e ← base of the natural Logarithm: 2.7183

vehicle's max deceleration

338

$$\dot{q} = 1.83\times10^{-4}V^3\sqrt{\frac{\rho}{r_{nose}}}$$

vehicle's heating rate (W/m²); vehicle's nose radius

$$h_{a_{drag\,max}} = \frac{1}{\beta}\ln\left(\frac{\rho_o}{BC\beta\sin\gamma}\right)$$

altitude of vehicle's max decel.

338

$$h_{\dot{q}_{max}} = \frac{1}{\beta}\ln\left(\frac{\rho_o}{3BC\beta\sin\gamma}\right) \qquad V_{\dot{q}_{max}} = 0.846V_{entry}$$

alt. of vehicle's max heating rate (m); velocity (m/s) at max heating rate; vehicle's re-entry velocity

Ballistic Missiles

744 Λ = Range Angle β = Azimuth Angle

Lat launch site; L = Latitude; l = longitude; target

$$Cos\Lambda = SinL_0SinL_t + CosL_0CosL_tCos\Delta l \qquad \Delta l = l_t - l_0$$

$$Cos\beta = \frac{SinL_t - SinL_0Cos\Lambda}{CosL_0Sin\Lambda} \qquad Range = \Lambda\frac{10{,}000\ km}{90°}$$

$$Cos\Lambda = SinL_0SinL_t + CosL_0CosL_tCos(\Delta l + \omega_{earth}TOF)$$

Trajectory Parameter $(0 \le Q \le 2+)$

$$Q_{Burnout} = \frac{V^2_{Burnout}R_{Burnout}}{\mu}\ ;\ Q_{Burnout_{min}} = \frac{2Sin\frac{\Lambda}{2}}{1 + Sin\frac{\Lambda}{2}}$$

$$Q_{BO} = \frac{V^2_{BO}}{V^2_{cir}}$$

$$\phi_{Burnout\_LOW} = \frac{1}{2}\left(Sin^{-1}\left[\left(\frac{2 - Q_{Burnout}}{Q_{Burnout}}\right)Sin\frac{\Lambda}{2}\right] - \frac{\Lambda}{2}\right)$$

in deg!

flight path angle

$$\phi_{Burnout\_HIGH} = \frac{1}{2}\left(180° - Sin^{-1}\left[\left(\frac{2 - Q_{Burnout}}{Q_{Burnout}}\right)Sin\frac{\Lambda}{2}\right] - \frac{\Lambda}{2}\right)$$

use this to calculate R_{max}!

Range ∠

$$\Lambda_{Max} = 2Sin^{-1}\left(\frac{Q_{Burnout}}{2 - Q_{Burnout}}\right)$$

$$(\phi_{Burnout})_{max\ Range} = 45^0 - \frac{\Lambda_{max}}{4}$$

TOF: use chart pg. 748 (for P'd cir, use $R_{BO} = a$)

Interplanetary

227-234 μSUN = 1.327 x 10¹¹ km³/sec²

(Hohmann Transfer)

$$\Delta V = \left|V_{Park\ Orbit} - V_{Hyperbolic\ Orbit}\right| \qquad V_\infty = \left|V_{Planet} - V_{TransferatPlanet}\right|$$

Boost/Retro; Orbit (@ P0); (around the sun!) T@__

$$V_{hyperbolic} = \sqrt{2\left(\frac{\mu_{Planet}}{R} + \varepsilon_\infty\right)} \qquad \varepsilon_\infty = \frac{V_\infty^2}{2}$$

vel. of s/c at infinite distance from planet

$$TOF = \tfrac{1}{2}\,pd. = \pi\sqrt{\frac{a^3_{transfer}}{\mu_{sun}}}$$

pg. 248